CRYSTAL LATTICE DRAG BY CONDUCTION ELECTRONS AND THE ONSAGER RELATION BETWEEN ELECTROACOUSTIC COEFFICIENTS

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Submitted June 20, 1970

Zh. Eksp. Teor. Fiz. 59, 2116–2129 (December, 1970)

Expressions are obtained for the drag force exerted by the electrons on the lattice under conditions of nonstationary and inhomogeneous deformation. The relations are valid for arbitrary topology of the Fermi surface, including the case of open surfaces or hole groups. Elasticity theory equations in metals and semiconductors are discussed. The symmetry relations of the kinetic coefficients are considered.

INTRODUCTION

In a number of problems, for example, that of sound propagation in metals at low temperatures, it is important to know the form of the drag force exerted by the conduction electrons on the lattice in a deformed conductor. Because of the electron contribution, the elasticity theory equations become nonlocal at long mean free path lengths, and the dispersion law of the conduction electrons begins to play an important role, as does the presence of an external magnetic field, which affects the dynamics of the electron.[1,2]

It is not difficult to understand that the complete set of equations which describe the deformed conductor should consist of the equations of motion of the lattice with account of the drag force; the equations which describe the electron gas, for which we take the classical kinetic equations for the electron distribution function, and, finally, Maxwell's equations, which describe the macroscopic field due to the lattice deformation.

In the present paper we derive the equations of elasticity theory in metals for an arbitrary topology of the Fermi surface of the conduction electrons, including the presence of open surfaces and hole groups.[1)]

The analysis is phenomenological, in terms of a dispersion law of the conduction electrons that is assumed to be known. In such an approach, it is natural to use the variational principle to obtain the force or, what is equivalent, the conservation laws. Here, however, a specific difficulty appears and is connected in essence with the fact that the lattice loses its periodicity upon deformation and motion. In this case, in the laboratory system of coordinates (the L system) the dispersion law of the electron includes noninvariant components that are proportional to the quasi-momentum. Yet such a quantity as the total energy of the electrons is determined and the conservation laws (or the variational principle) formulated in this same set of coordinates.

[1)]We note that the derivation of the drag force given in [2] refers to the case of purely electronic groups. However, we shall show that the complete set of equations describing the deformed metal, I (4.7), I (4.4) and I (4.5) (the Roman numeral I refers to the formulas of [2]) keep their form for any dispersion law. At the same time, the drag-force expressions that contain explicitly the momentum or the quasimomentum transferred in collisions (I (4.1), I (4.6)) are sensitive to the form of the energy spectrum and will be obtained below for an arbitrary topology.

On the other hand, the Hamiltonian function of the conduction electron is formulated, in the phenomenological approach, in the co-moving local system (the C system), where the individual lattice element is at rest and the periodicity is preserved locally. The periodic (in the quasimomentum) Hamiltonian function $\epsilon = \epsilon_0 + \Lambda_{ik}u_{ik}$ used for the variation in[1] corresponds to the energy of the electrons in the C system (with neglect of its noninertial character, i.e., the Stewart-Tolman effect). But the energy in the C system is not the total energy of the electron and, correspondingly, the deformation force f^d found in[1] is only part of the drag force. The expression in[2] for the force has been obtained with the aid of the conservation laws in the L system. Because of the non-inertial character of the dispersion law in the L system, the conclusion reached in[2] requires refinement in the presence of hole groups and open surfaces. Nevertheless, the final result is valid for an arbitrary spectrum (see footnote 1). This result (with neglect of the Stewart-Tolman effect) was confirmed in[3] by means of a variational principle in which, however, the energy of the electrons in the C system was varied for fixed fields in the L system to obtain the correct result. It will be shown below that such a procedure arises from the usual formulation of the variational principle in the L system.

We also note that the results of the present research correspond with the expression for the drag force found in[4], with the exception of the deformation component, which corresponds in[4] to the case of free electrons (see below). This is connected with the fact that the effect of lattice deformation on the electron spectrum was not considered in[4].

1. THE EQUATIONS OF ELASTICITY THEORY IN METALS

In this section, we shall give a summary of the results.

The deviation of the distribution of conduction electrons in metals from the local equilibrium distribution (including the deviation due to deformation of the crystal) leads to the generation of appreciable forces exerted by the electrons on the lattice. These forces can be expressed in terms of functionals of the nonequilibrium part of the electron distribution function. One of the

functionals is the total current j, the other, the deformation force f^d:

$$\mathbf{j} = -e\langle \mathbf{v}\chi\rangle, \quad f_i^{\,d} = -\frac{\partial}{\partial x_k}\langle \Lambda_{ik}\chi\rangle. \tag{1.1}$$

The closed set of equations consists of the equations of elasticity (with account of the forces exerted by the electrons on the lattice) and the kinetic equation. This system has the form[2)]

$$\rho\ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - \frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{1}{c}[\mathbf{jH}]_i + f_i^{\,d}, \tag{1.2}*$$

$$\left(\frac{d}{dt_0} + \hat{\nu}\right)\chi = g \equiv -e\mathbf{v}\tilde{\mathbf{E}} - \Lambda_{ik}\dot{u}_{ik}, \tag{1.3}$$

$$\tilde{\mathbf{E}} = \mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}] - \frac{1}{e}\nabla\frac{\langle\lambda_{ik}\rangle}{\langle 1\rangle}u_{ik} - \frac{m\mathbf{u}}{e},$$

$$\Lambda_{ik}(\mathbf{p}) \equiv \lambda_{ik}(\mathbf{p}) - \frac{\langle\lambda_{ik}\rangle}{\langle 1\rangle}, \tag{1.4}$$

where the currents and fields that are involved are related by Maxwell's equations:[3)]

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \tag{1.5}$$

and Eqs. (1.1) play the role of the material equations.

The brackets in (1.1)–(1.4) denote averaging over the Fermi surface and $\chi\partial f_0/\partial\epsilon$ is the deviation of the distribution function from local equilibrium (see I (2.17)), e is the electron charge, $\hat{\nu}$ the (linearized) collision operator,[4)]

$$\frac{d}{dt_0} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial \tau} \equiv \frac{e}{c}[\mathbf{vH}]\frac{\partial}{\partial \mathbf{p}}, \quad \mathbf{v} = \frac{\partial \varepsilon_0}{\partial \mathbf{p}},$$

where τ is the period of rotation in the orbit in a magnetic field.

The deformation potential[5] $\lambda_{ik}(\mathbf{p})$ takes into account the change in the energy spectrum of the electron in the deformed lattice, which has the form I (3.11) in the C system in the linear approximation

$$\varepsilon'(\mathbf{r}', \mathbf{p}', t) = \varepsilon_0(\mathbf{p}') + \lambda_{ik}(\mathbf{p}')u_{ik} - m\dot{\mathbf{u}}\mathbf{v}'. \tag{1.6}$$

The last component in (16) is connected with the noninertial character of the accompanying system of coordinates and takes into account the D'Alembertian force $m\ddot{\mathbf{u}}$ acting in this system, where m is the mass of the free electron. It is responsible for the Stewart-Tolman effect, a detailed microscopic theory of which is developed in[8,9].

Transformation to the C system (see I (2.9)–(2.11)), in which the dispersion law of the conduction electrons is formulated, is an essential point of the phenomenological approach used.[6,7] In such an approach, we not only exclude motion of the deformed lattice, but we "straighten it out" (locally) with the required accuracy, which follows from the properties of the transformation. The band spectrum can be introduced only in the C system, while the Brillouin zone possesses a center of symmetry and the dispersion law is expressed in terms of the functions $\epsilon_0(\mathbf{p}')$ and $\lambda_{ik}(\mathbf{p}')$, $\mathbf{v}'$ being periodic in $\mathbf{p}'$ and having the period of the unperturbed reciprocal lattice. Conversely, the Hamiltonian $\epsilon(\mathbf{r}, \mathbf{p}, t)$ of the electron in the L system is equal to (see I (2.4), (3.13))

$$\varepsilon = \varepsilon_0(\mathbf{p}) + (\lambda_{ik} + p_i v_k)\frac{\partial u_i}{\partial x_k} + (p_i - mv_i)\dot{u}_i, \tag{1.7}$$

and is not identical with the energy of the electron, and neither of these quantities is a periodic function of the quasimomentum p, as ought to be the case in the deformed lattice. The corresponding quantum approach was developed recently by V. Gurevich, Lang, and Pavlov,[8] who, in particular, obtained a microscopic expression for $\lambda_{ik}(\mathbf{p})$.

The deformation force f^d (1.1) plays an important role in the theory under consideration. A similar component first appeared in the work of Silin.[1] Like the current j, it has the character of an electron flow and, as we shall see below, plays an important part in the Onsager relations. Because of the condition of electrical neutrality it is convenient to write in (1.1) the renormalized tensor Λ_{ik} in place of λ_{ik} (see (1.4)). As in the components in a force proportional to the current, only part of the force enters into f^d, the part which is due to the deviation from local equilibrium. The equilibrium part is included in the renormalization of the stress tensor σ_{ik}.

The research of Pekar and Tsekvava[4] was recently devoted to the calculation of the force in Eq. (1.2). The microscopic component in the force $\nabla_k\langle mv_i v_k\chi\rangle$ obtained by them is proportional to the limiting value of the deformation force (1.1) for the case of free electrons. In order to show this, following[8], we rewrite the dispersion law of the electrons in the laboratory set of coordinates (1.7) in the form

$$\varepsilon(\mathbf{r}, \mathbf{p}, t) = \varepsilon_0(\mathbf{p}) + (\lambda_{ik} + mv_i v_k)u_{ik} + (p_i - mv_i)\left(\dot{u}_i + v_k\frac{\partial u_i}{\partial x_k}\right).$$

It is then seen at once that ϵ, $\epsilon_0 \to p^2/2m$, $\mathbf{p} \to m\mathbf{v}$, $\lambda_{ik} \to -mv_i v_k$, corresponds to the limiting transition to free electrons, not coupled to the lattice.[5)] Here the deformation force f^d (1.1) tends to $\nabla_k\langle mv_i v_k\chi\rangle$, i.e., exactly to that component which was found in[4], but for which, for an arbitrary spectrum, there is generally no basis.

We also write out the momentum flux density tensor Π_{ik} computed in[2], but written there in somewhat inconvenient form (we include the components that renormalize σ_{ik} (see I (3.16)):

$$\Pi_{ik} = T_{ik} - \sigma_{ik} + \langle \Lambda_{ik}\chi\rangle. \tag{1.8}$$

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2) The given set of equations is contained in [2] (see I (4.7), (4.4), (4.5)), and we shall show below that it is applicable for an arbitrary spectrum. Here we shall be interested in the form of the equations of motion of an elastic medium, since the form of the kinetic equation in the deformed metal has been well studied (see [5-8]). In a number of cases, we refer in the text to the formulas of [2], where a more detailed discussion is given.

3) In the notation of Maxwell's equations (1.5), the condition of electrical neutrality $\langle\chi\rangle = 0$ is automatically satisfied. In the dynamical case, this is equivalent to the condition div j = 0. For semiconductors, where there is no electrical neutrality, the equations are given in Sec. 6.

4) It is assumed that scatterers (for example, impurities) are completely entrained in the motion of the lattice. A detailed analysis of the electron drag in electron-phonon interaction in the presence of an external sound field was carried out by Holstein. [9] His results do not contradict this assumption.

*$[\mathbf{jH}] \equiv \mathbf{j} \times \mathbf{H}$.

5) The author is grateful to V. L. Gurevich for this remark.

It is seen that Π_{ik} is symmetric if we assume that the Maxwell tension tensor T_{ik} and the renormalized stress tensor σ_{ik} are symmetric, in correspondence with the momentum conservation law. The energy flux is given by expression (2.7) below.

The boundary conditions for the system (1.2)–(1.5) include conditions for the electron distribution function (for example, the reflection conditions), which ought to be formulated on the deformed surface of the metal, in the local set of coordinates in which the given surface element is at rest. In the linear approximation considered, it is convenient to transpose these conditions to the undeformed boundary of the metal. Therefore, the boundary conditions, which are formulated for the function χ, and which describe the departure from local equilibrium, take on the same force as in the undeformed metal. For an arbitrary dispersion law, see, for example,[10].

2. DERIVATION OF THE ELASTICITY EQUATIONS FOR AN ARBITRARY DISPERSION LAW

We use the conservation laws. The electron momentum density is equal to $-mj_{el}/e$ (see I (2.8)). By virtue of the conservation of momentum of the system consisting of the conduction electrons, the elastic medium (the lattice) and the electromagnetic field, we have for an arbitrary volume V (I (3.7)–(3.8)):

$$\frac{d\mathcal{P}_i}{dt} \equiv \int_V dV\left\{\rho_p \ddot{u}_i + \frac{m}{e}\frac{\partial j_i^{el}}{\partial t} - \frac{1}{c}[\mathbf{jH}]_i - \frac{\partial T_{ik}}{\partial x_k}\right\} = \int_S dS_k \psi_{ik}, \quad (2.1)$$

where S is the surface surrounding the volume V, T_{ik} the Maxwell stress tensor, ψ_{ik} an arbitrary tensor that is symmetric because of momentum conservation. In view of the arbitrariness of the volume, the expression for the force

$$\rho\ddot{u}_i = -\frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{1}{c}[\mathbf{jH}]_i + \frac{\partial}{\partial x_k}(T_{ik} + \psi_{ik}) \quad (2.2)$$

then follows. In order to determine ψ_{ik}, the law of energy conservation can be used for a closed system. Our system is placed in a thermostat, its energy is not conserved, but the time derivative of the free energy $\mathcal{F}$ of a nonequilibrium system ought to be a sign-definite (negative) quantity. (In the state of equilibrium, $\mathcal{F}$ has a minimum and the time average of its derivative $\overline{d\mathcal{F}/dt}$ determines the dissipated power.) This condition suffices for the determination of ψ_{ik}. Actually, separating in $\overline{d\mathcal{F}/dt}$ the dissipative component due to the collisions, and also the surface term that describes the energy flux, we should require the vanishing of the remaining volume components, which also gives us the equality necessary for the determination of ψ_{ik} (see footnote 7).

Making use of the known relations for the energy $\mathcal{E}_{el}$ and the entropy S_{el} of a nonequilibrium electron gas, we find $\mathcal{F}_{el} \equiv \mathcal{E}_{el} - TS_{el}$. Although the condition $\overline{d\mathcal{F}/dt} < 0$ itself is formulated in the L system, the finding of $d\mathcal{F}_{el}/dt$ is considerably simplified if the calculation is performed in the accompanying set of coordinates, where the dispersion law contains no noninvariant components.[6)]

[6)] We specially emphasize the fact that we are dealing with a transition to the local system only for convenience in calculations; the "conservation law" (or the variational principle, see below) is formulated in the laboratory system of coordinates. The author's attention to the advantage of the transition to the local system was called by I. M. Lifshitz in a discussion of [2].

According to the known transition formulas, the energy of the electrons $\mathcal{E}_{el}$ (in the L system) is written in terms of the spectrum and the distribution of electrons in the C system in the following way:

$$\mathcal{E}_{el} = \int dV'\left\{\langle\langle \varepsilon' f'\rangle\rangle + \dot{u}\left\langle\!\!\left\langle m\frac{\partial \varepsilon'}{\partial \mathbf{p}} f'\right\rangle\!\!\right\rangle + \frac{nm\dot{u}^2}{2}\right\}. \quad (2.3)$$

The double brackets denote integration over the Brillouin zone, the prime on $\mathbf{p}$ is omitted,

$$\varepsilon' = \varepsilon_0' + \delta\varepsilon', \quad \delta\varepsilon' = \lambda_{ik}u_{ik} - m\frac{\partial \varepsilon'}{\partial p_i}\dot{u}_i, \quad \delta\tilde{\varepsilon}' = \Lambda_{ik}u_{ik} - m\frac{\partial \varepsilon'}{\partial p_i}\dot{u}_i,$$

$$\frac{df'}{dt} + \hat{\nu}f' = 0 \quad \hat{\nu}f_0(\varepsilon_0' + \delta\tilde{\varepsilon}') = 0, \quad f' = f_0(\varepsilon_0' + \delta\tilde{\varepsilon}') + \frac{\partial f_0}{\partial \varepsilon}\chi.$$

Here d/dt and $\hat{\nu}$ are the total derivative and the collision operator in the local system. We use the same designation $\hat{\nu}$ both for the exact and the linearized operators, since this cannot lead to a misunderstanding. The last term in (2.3) can naturally be omitted, since it is small in comparison with the component $\rho_p\dot{u}^2/2$ in F_{el}. The equation for χ is given above (see (1.3)), $\tilde{E}_i = E_i' - \frac{1}{e}\nabla_i\frac{\langle\lambda_{lk}\rangle}{\langle 1\rangle}u_{lk} + (m/e)\ddot{u}_i$, where

$$\mathbf{E}' = \mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}] \quad (2.4)$$

is the electric field in the local set of coordinates, expressed in terms of the field $\mathbf{E}$ in the laboratory system.

We now find the quantity $\partial\mathcal{E}_{el}/\partial t$. According to I (3.3), we have

$$\left\langle\!\!\left\langle \frac{\partial}{\partial t}(\varepsilon' f')\right\rangle\!\!\right\rangle = \left\langle\!\!\left\langle \frac{d\varepsilon'}{dt} f'\right\rangle\!\!\right\rangle + \left\langle\!\!\left\langle \varepsilon'\frac{df'}{dt}\right\rangle\!\!\right\rangle - \frac{\partial}{\partial x_k}\left\langle\!\!\left\langle \frac{\partial \varepsilon'}{\partial p_k}\varepsilon' f'\right\rangle\!\!\right\rangle,$$

$$\left\langle\!\!\left\langle \frac{d\varepsilon'}{dt} f'\right\rangle\!\!\right\rangle = \dot{u}_{ik}\langle\langle\lambda_{ik}f'\rangle\rangle - \ddot{u}_i\left\langle\!\!\left\langle m\frac{\partial \varepsilon'}{\partial p_i}f'\right\rangle\!\!\right\rangle + \mathbf{jE}', \quad \left\langle\!\!\left\langle \varepsilon'\frac{df'}{dl}\right\rangle\!\!\right\rangle = \langle\delta\varepsilon\hat{\nu}\chi\rangle.$$

Here the equations and expressions for f′ and ϵ' are used. We get

$$\frac{\partial\mathcal{E}_{el}}{\partial t} = \int dV\left\{\dot{u}_i\left(\frac{m}{e}\frac{\partial j_i}{\partial t} - \frac{\partial}{\partial x_k}\langle\langle\lambda_{ik}f'\rangle\rangle\right) + \mathbf{jE}' + \langle\delta\varepsilon\hat{\nu}\chi\rangle - \frac{\partial}{\partial x_k}\left(\left\langle\!\!\left\langle \frac{\partial\varepsilon'}{\partial p_k}\varepsilon' f'\right\rangle\!\!\right\rangle - \dot{u}_i\langle\langle\lambda_{ik}f'\rangle\rangle\right)\right\}. \quad (2.5)$$

Correspondingly, it follows from the expressions

$$TS_{sa} = -kT\int dV\langle\langle(1-f)\ln(1-f) + f\ln f\rangle\rangle,$$

$$\mathcal{F}_{elast} = \int dV\left(\frac{1}{2}\rho_p\dot{u}^2 + \frac{\sigma_{lm}u_{lm}}{2}\right), \quad \sigma_{ik} = \lambda_{iklm}u_{lm}$$

and from Maxwell's equations that

$$\frac{\partial}{\partial t}(TS_{sa}) = \int dV\left\{\langle(\delta\varepsilon' + \chi)\hat{\nu}\chi\rangle + \frac{\partial}{\partial x_k}\left\langle\frac{v_k}{2}(\delta\varepsilon' + \chi)^2\right\rangle\right\},$$

$$\frac{\partial\mathcal{F}_{elast}}{\partial t} = \int dV\left\{\dot{u}_i\left(\rho_p\ddot{u}_i - \frac{\partial\sigma_{ik}}{\partial x_k}\right) + \frac{\partial}{\partial x_m}(\dot{u}_i\sigma_{im})\right\}.$$

$$\frac{\partial\mathcal{H}_{em}}{\partial t} = -\int dV\left\{\mathbf{jE} + \operatorname{div}\frac{c}{4\pi}[\mathbf{EH}]\right\}.$$

Finally, using the relation (2.4), we get the equation

$$\frac{\partial\mathcal{F}}{\partial t} = \int dV\left\{\dot{u}_i\left(\rho\ddot{u}_i - \frac{\partial\sigma_{ik}}{\partial x_k} - \frac{\partial}{\partial x_k}\langle\langle\lambda_{ik}f\rangle\rangle + \frac{m}{e}\frac{\partial j_i}{\partial t} - \frac{1}{c}[\mathbf{jH}]_i\right) - \operatorname{div}\mathbf{q}\right\} - \int dV\langle\chi\hat{\nu}\chi\rangle, \quad (2.6)$$

where the flux density of the free energy, $\mathbf{q}$, is equal to

$$q_k = \left\langle\!\!\left\langle \frac{\partial\varepsilon}{\partial p_k}\varepsilon f\right\rangle\!\!\right\rangle + \frac{c}{4\pi}[\mathbf{EH}]_k - \dot{u}_i(\sigma_{ik} - \langle\Lambda_{ik}\chi\rangle) - \left\langle\frac{v_k}{2}(\delta\tilde{\varepsilon}' + \chi)^2\right\rangle.$$

The last term in (2.6) has a definite sign by virtue of the properties of the collision integral. The corresponding proof is given in[11]; the nonstationarity does not play a role, since time enters simply as a parameter. The first component in (2.6) should be identically zero[7] in order that $d\mathcal{F}/dt$ be negative for arbitrary $\mathbf{u}(\mathbf{r}, t)$, which leads to

$$\frac{\partial \mathcal{F}}{\partial t} = -\int dS_k q_k - \int dV \langle \chi \hat{\nu} \chi \rangle,$$

$$\rho \ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - \frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{1}{c}[\mathbf{jH}]_i + \frac{\partial}{\partial x_k}\langle\langle \lambda_{ik} f' \rangle\rangle + [\mathbf{u}\dot{\Phi}]_i. \quad (2.6')$$

The vector Φ cannot be determined from energy considerations, but because of (2.2) it must be set equal to zero. Similarly, (2.2) uniquely determines the form of the components in (2.6) which are proportional to $\dot{\mathbf{u}}$ and which have the form of a time derivative, in which connection there is no necessity of distinguishing between the conditions $d\mathcal{F}/dt < 0$ and $\overline{d\mathcal{F}/dt} < 0$.

Finally, by including the equilibrium part in the renormalization of σ_{ik}, we obtain Eq. (1.2) and the expression (1.8) for the momentum flux density. We note that the contribution of the equilibrium electrons to σ_{ik} is not limited to this part and, in principle, a detailed calculation could be made only by the successive microscopic methods developed successfully in recent times.[12]

The force in the elasticity equations can be calculated in the same way as the variational derivative of the internal energy with respect to the displacement for constant entropy and constant potential of the electromagnetic field. Thus the electron contribution to the force is equal to

$$-\left(\frac{\delta}{\delta \mathbf{u}}\mathscr{E}_{el}\right)_{S,A}, \quad (2.8)$$

where all the quantities $\mathscr{E}_{el}$, S, and A refer to the L system. Fixing of the entropy means fixing of the distribution function, inasmuch as $S \equiv S\{f\}$. We express $\mathscr{E}_{el}$, in accord with (2.3), in terms of a quantity referred to the X system. By taking it into account that $dV d\tau_p f = dV' d\tau_{p'} f'$, we get the result that the force can be computed as the following variational derivative:

$$-\left(\frac{\delta}{\delta \mathbf{u}}\int dV' d\tau_{p'} f' \left[\varepsilon_0\left(\mathbf{P}' - \frac{e}{c}\mathbf{A}'\right) + \Lambda_{ik}\left(\mathbf{P}' - \frac{e}{c}\mathbf{A}'\right)u_{ik}\right]\right)_{f',A} \quad (2.9)$$

This expression looks as if, neglecting of the Stewart-Tolman effect, we varied the energy of the electron in the C system at a fixed vector potential in the L system and a fixed distribution function in the C system (cf.[3]).

3. ELECTRON DRAG OF THE LATTICE AND COLLISION MOMENTUM TRANSFER

The additional force acting on the lattice in metals can be represented in the form of a force exerted by the electrons (the drag force $\mathbf{f}^{dr}$) and of the Lorentz force acting on the ions:

$$\rho_p \ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - en\left(\mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]\right)_i + f_i^{dr}. \quad (3.1)$$

[7] It is seen from Eqs. (1.3) and (1.4), which determine χ as a functional of $\mathbf{u}(\mathbf{r}, t)$, and from (2.6) that in the opposite case one can always choose $\mathbf{u}$ so that $\partial \mathcal{F}/\partial t$ changes sign.

Here $\rho_p = (\rho - nm)$ is the lattice density; of course, the difference between ρ_p and ρ is purely symbolic. In the microscopic approach to $\mathbf{f}^{dr}$, it is natural for the component associated with collisions to be separated. In this connection, the writing down of Eq. (1.2) is of interest. In this expression, the electric fields acting on the lattice, and the part of the momentum transferred by the electrons in collisions with the lattice (the quasimomentum), appear explicitly.

We first make use of the equation of quasimomentum transfer, which we obtain by multiplying the kinetic equation (1.3) by $\mathbf{p}$ and integrating it over the Fermi surface. We have

$$\frac{\partial}{\partial t}\langle p_i \chi \rangle + \frac{\partial}{\partial x_k}\langle p_i v_k \chi \rangle + \langle p_i \hat{\nu} \chi \rangle = -e\bar{E}_k \langle p_i v_k \rangle - \frac{1}{c}[\mathbf{jH}]_i + \frac{e}{c}\varepsilon_{klm} H_m \oint_\Sigma p_i v_l \frac{\partial f_0}{\partial \varepsilon}\chi \, d\Sigma_k. \quad (3.2)$$

The last term is an integral over the surface of the Brillouin zone. Because of the presence of the factor $\mathbf{p}$, the integrand is not periodic and, if the Fermi surface intersects the boundary of the Brillouin zone, the integral generally differs from zero. For closed surfaces, one can so choose the elementary cell that its boundaries will be intersected by the Fermi surfaces. Then $\langle p_i v_k \rangle = (n_- - n_+)\delta_{ik}$, where n_- is the concentration of electrons in electron bands and n_+ is the concentration of holes, while the transport equation takes the form

$$\frac{\partial}{\partial t}\langle p_i \chi \rangle + \frac{\partial}{\partial x_k}\langle p_i v_k \chi \rangle + \langle p_i \hat{\nu} \chi \rangle = -e\bar{E}_i(n_- - n_+) - \frac{1}{c}[\mathbf{jH}]_i.$$

Correspondingly, the elasticity equation (1.2) can be rewritten in the following way:

$$\rho_p \ddot{u}_i = -e\bar{E}_i(n_- - n_+) + \frac{\partial \sigma_{ik}}{\partial x_k} - \frac{\partial}{\partial x_k}\langle (\lambda_{ik} + p_i v_k)\chi \rangle - \langle p_i \hat{\nu} \chi \rangle - \frac{\partial}{\partial t}\langle (p_i - m v_i)\chi \rangle. \quad (3.3')$$

For $n_+ = 0$, i.e., for purely electron groups, this expression goes over into I (4.1). For open surfaces, the quantity $\langle p_i v_k \rangle$ is no longer equal to $\delta_{ik}(n_- - n_+)$ (cf.[13]). In this case, the last component in (3.2), where the integration is over the strip constituting the intersection of the open Fermi surface with the boundaries of the Brillouin zone, is different from zero. The elasticity equation for an arbitrary topology thus has the form

$$\rho_p \ddot{u}_i = -e\bar{E}_k \langle p_i v_k \rangle + \frac{\partial \sigma_{ik}}{\partial x_k} - \frac{\partial}{\partial x_k}\langle (\lambda_{ik} + p_i v_k)\chi \rangle - \langle p_i \hat{\nu} \chi \rangle - \frac{\partial}{\partial t}\langle (p_i - m v_i)\chi \rangle + \frac{e}{c}\varepsilon_{klm} H_m \oint d\Sigma_k \chi p_i v_l \frac{\partial f_0}{\partial \varepsilon}. \quad (3.3)$$

In place of Eq. (3.2), we can use the equation for electron momentum transfer

$$\frac{\partial}{\partial t}\langle m v_i \chi \rangle + \frac{\partial}{\partial x_k}\langle m v_i v_k \chi \rangle + \left\langle m v_i \frac{\partial \chi}{\partial \tau} \right\rangle + \langle m v_i \hat{\nu} \chi \rangle = -e\langle m v_i v_k \rangle \bar{E}_k, \quad (3.4)$$

Replacing

$$-\frac{m}{e}\frac{\partial j_i}{\partial t} \equiv \frac{\partial}{\partial t}\langle m v_i \chi \rangle$$

in the initial equation (1.2) with the aid of (3.4), we get

$$\rho_0 \dot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - e\bar{E}_k \langle m v_i v_k \rangle - \frac{\partial}{\partial x_k} \langle (\lambda_{ik} + m v_i v_k)\chi \rangle - \langle m v_i \hat{\nu} \chi \rangle + \varphi_i(\mathbf{H}), \tag{3.5}$$

where

$$\varphi_i(\mathbf{H}) \equiv \frac{1}{c}[\mathbf{jH}]_i - m\left\langle v_i \frac{\partial \chi}{\partial \tau} \right\rangle.$$

Integrating the last term by parts, we find that φ is equal to

$$\varphi_i(\mathbf{H}) = -\frac{e}{c}\varepsilon_{klj} H_j \left\langle \left(\delta_{ik} - m \frac{\partial^2 \varepsilon}{\partial p_i \partial p_k} \right) v_l \chi \right\rangle. \tag{3.6}$$

The appearance of the factor

$$\delta_{ik} - m\partial^2 \varepsilon / \partial p_i \partial p_k$$

in the induction terms described by φ simultaneously with the grouping of the deformation potential in the combination

$$\lambda_{ik} + m v_i v_k,$$

which vanishes on going to the free electrons, is in agreement with the results of[8]. If one can introduce the effective mass, then $\partial^2 \epsilon / \partial p_i \partial p_k = \delta_{ik}/m^*$ and φ has the form (cf.[4])

$$\varphi(\mathbf{H}) = \left(1 - \frac{m}{m^*}\right) \frac{1}{c}[\mathbf{jH}]. \tag{3.7}$$

Comparing (3.1) with (3.3)–(3.7), we can obtain an explicit expression for the drag force.

4. THE ONSAGER RELATIONS

The dissipated energy, which is equal to the rate of decrease of the free energy of the system (see (2.6′))

$$-\frac{\partial \mathcal{F}}{\partial t} = \int \langle \chi \hat{\nu} \chi \rangle \, dV \tag{4.1}$$

(the fluxes through the surface are omitted here), is conveniently transformed by using the kinetic equation (1.3), with which we define $\hat{\nu}\chi$:

$$-\frac{\overline{\partial \mathcal{F}}}{\partial t} = \int \overline{\langle \chi g \rangle} \, dV. \tag{4.2}$$

The bar indicates time averaging. We have used the fact that in the identity

$$\langle \chi \hat{\nu} \chi \rangle \equiv \langle \chi g \rangle - \frac{\partial}{\partial t}\left\langle \frac{\chi^2}{2} \right\rangle - \nabla \langle \chi \mathbf{v} \chi \rangle - \left\langle \frac{\partial}{\partial \tau} \frac{\chi^2}{2} \right\rangle$$

the second term drops out in the time averaging, and the third in integration over the volume, the latter being identically equal to zero.

Substituting g from (1.3) in (4.2), we get

$$-\frac{\overline{\partial \mathcal{F}}}{\partial t} = \int \overline{(\mathbf{j}\tilde{\mathbf{E}} - \mathbf{f}^{\mathrm{d}}\dot{\mathbf{u}})} \, dV. \tag{4.3}$$

In the second component, integration is now carried out by parts and the surface integral is omitted.

We now consider the problem of the response of the system to periodic excitation with frequency ω. Inasmuch as the system is located in a thermostat, it follows, as is well known,[14] that the statistical average of the rate of change of the energy of the system is equal to the rate of change of its free energy. Thus, we arrive at the expression (4.3), with the aid of which we can introduce the generalized forces and coordinates in order to apply the general symmetry relations of kinetic coefficients:[14]

$$-\frac{\partial \mathcal{F}}{\partial t} = \sum_a \int x_a(\mathbf{r}) f_a(\mathbf{r}) \, d\mathbf{r}, \quad x_a(\mathbf{r}) = \sum_b \int \alpha_{ab}(\mathbf{r}, \mathbf{r}') f_b(\mathbf{r}') \, d\mathbf{r}'.$$

It follows from (3.3) that if we choose the quantities $\tilde{\mathbf{E}}(\mathbf{r})$ and $\mathbf{u}(\mathbf{r})$ as the generalized forces then the role of generalized coordinates will be played by the fluxes $\mathbf{j}(\mathbf{r})/i\omega$ and $-\mathbf{f}^{\mathrm{d}}(\mathbf{r})$:

$$f_a(\mathbf{r}) \to \tilde{\mathbf{E}}(\mathbf{r}), \quad \mathbf{u}(\mathbf{r}); \qquad x_a(\mathbf{r}) \to \mathbf{j}(\mathbf{r})/i\omega, \quad -\mathbf{f}^{\mathrm{d}}(\mathbf{r}). \tag{4.4}$$

The role of index is played by the combination of the discrete index a and the continuous index r. We write down the symmetry relations of the kinetic coefficients for a homogeneous medium, where

$$\alpha_{ab}(\mathbf{r} - \mathbf{r}', \mathbf{H}) = \alpha_{ba}(\mathbf{r}' - \mathbf{r}, -\mathbf{H}),$$

or, in spatial Fourier components,

$$\alpha_{ab}(\mathbf{k}, \mathbf{H}) = \alpha_{ba}(-\mathbf{k}, -\mathbf{H}).$$

Returning to the material equations that follow from (1.1) and (1.3):

$$j_{il} = \sigma_{il}\bar{E}_l + \partial_{il} u_l, \quad f_i^{\mathrm{a}} = c_{il}\bar{E}_l + b_{il} u_l, \tag{4.5}$$

we see that the material tensors $\hat{\sigma}$,[8] $\hat{\partial}$, $\hat{c}$, and $\hat{b}$ should satisfy the symmetry relations which, as follows from (3.4), (4.5), have the form

$$\sigma_{il}(\mathbf{k}, \mathbf{H}) = \sigma_{li}(-\mathbf{k}, -\mathbf{H}), \quad b_{il}(\mathbf{k}, \mathbf{H}) = b_{li}(-\mathbf{k}, -\mathbf{H}),$$
$$\partial_{il}(\mathbf{k}, \mathbf{H}) = -i\omega c_{li}(-\mathbf{k}, -\mathbf{H}). \tag{4.6}$$

In particular, the relations (4.6) allow us to construct a "field" part of the deformation force, determined by c_{ik}, from the well known expression for the flux (its "deformation" part, described by the tensor c_{ik}).

Introducing the Green operator of the kinetic equation (1.3) and its symmetric and antisymmetric (in $\mathbf{p}$) parts $R^{s,a}(\mathbf{p}) = (R(\mathbf{p}) \pm R(-\mathbf{p}))/2$, we represent the material tensors in the form I (5.1)–(5.3):

$$\sigma_{il} = e^2 \langle v_i R^s v_l \rangle, \quad \partial_{il} = e\omega k_j \langle v_i R^a \Lambda_{lj} \rangle,$$

$$c_{il} = iek_j \langle \Lambda_{ij} R^a v_l \rangle, \quad b_{il} = i\omega k_j k_i \langle \Lambda_{ij} R^s \Lambda_{il} \rangle. \tag{4.7}$$

From the form of the kinetic equation (1.3) it follows, if the collision integral is even in $\mathbf{p}$, that $R(-\mathbf{k}, \mathbf{p}) = R(\mathbf{k}, -\mathbf{p})$. Actually, $\mathbf{k}$ enters in R only in the form of the product $\mathbf{k} \cdot \mathbf{v}$, while the orbiting time τ is even in $\mathbf{p}$. Thus, for $\hat{\nu}(\mathbf{p}) = \hat{\nu}(-\mathbf{p})$

$$R^a(-\mathbf{k}, \mathbf{p}) = -R^a(\mathbf{k}, \mathbf{p}), \quad R^s(-\mathbf{k}, \mathbf{p}) = R^s(\mathbf{k}, \mathbf{p}). \tag{4.8}$$

It then follows, in accord with (4.7) that in this case $\hat{\sigma}$, $\hat{\partial}$, $\hat{c}$ and $\hat{b}$ are even functions of $\mathbf{k}$, and, correspondingly, we get the symmetry relations in the form

$$\sigma_{il}(\mathbf{k}, \mathbf{H}) = \sigma_{li}(\mathbf{k}, -\mathbf{H}), \quad b_{il}(\mathbf{k}, \mathbf{H}) = b_{li}(\mathbf{k}, -\mathbf{H}),$$
$$\partial_{il}(\mathbf{k}, \mathbf{H}) = -i\omega c_{li}(\mathbf{k}, -\mathbf{H}). \tag{4.9}$$

In particular, (4.9) is true for the presence of a center of inversion. In the simplest case, $\mathbf{H} = 0$, for strong spatial dispersion

$$R^s = \pi\delta(\mathbf{kv}), \quad R^a = -i\frac{\mathrm{P}}{\mathbf{kv}};$$

(for an expression for $(\mathbf{k} \cdot \mathbf{v})R$ see I (6.2)). In this case, the symmetry relations are easily verified directly, since R is a multiplication operator.

[8] The designation of the conductivity tensor and the stress tensor by the same symbol does not lead to any misunderstandings, since they never appear simultaneously.

If we introduce the extraneous fluxes f^{extr} and f^{d} in the equations for fields and deformations (1.2), (1.5) and (4.5), then, according to fluctuation-dissipation theorem (see, for example,[13]), the Green's functions of this system determine the Fourier components of the correlation functions of the fields and deformations and, correspondingly, the currents and forces in deformed metals. We shall not dwell here on this question, but only note that the contribution of the electrons generally leads to significant spatial dispersion.

5. CALCULATION IN THE L SYSTEM

In the transition to the local C system, the formal features of the deformed crystal are lost to a considerable degree. It is therefore of interest to derive the equations in the laboratory system of coordinates. If we are not interested in the renormalization of σ_{ik}, then for closed Fermi surfaces the calculations in the L system are not much more complicated than in the accompanying system.

The derivation given in[2] was done in the L system and pertained to the case of a purely electron spectrum, when $f_0 = 0$ on the surface of the Brillouin zone. In the general case of an arbitrary spectrum, integrals over the surface of the elementary cell of the reciprocal lattice appear (they are absent from[2]) in the transformation. We shall trace the contributions of these additional terms below.

We note first that integration over $\mathbf{p}$ in the L system is carried out not only over the Brillouin zone in $\mathbf{p}$ space but also over the region V_p', which corresponds in $\mathbf{p}'$ space (i.e., in the C system) to the Brillouin zone. If we denote integration over this region by $\langle\langle \ldots \rangle\rangle'$ and integration over the Brillouin zone in $\mathbf{p}$ space by $\langle\langle \ldots \rangle\rangle$, then, using the relation I (2.11) $\mathbf{p} = \mathbf{p}' - \nabla(\mathbf{u}\cdot\mathbf{p}')$, we get, in the linear approximation,

$$\int\limits_{V_{p'}'} \varphi\, d\tau_p \equiv \langle\langle\varphi\rangle\rangle' = \langle\langle\varphi\rangle\rangle + \oint\limits_{\Sigma} d\Sigma_k \Delta p_k \varphi, \qquad \Delta p_k \equiv -p_l \frac{\partial u_l}{\partial x_k}. \quad (5.1)$$

In the case of the intersection of the boundaries of the Brillouin zone with the closed Fermi surfaces, it is convenient to choose the displaced cell with no intersections as the elementary cell in $\mathbf{p}'$ space. In this case V_p', Σ, and the brackets in (5.1) denote these displaced cells and their surfaces.

We shall illustrate the appearance of surface components by the example of the derivation of (1.1) for the current density $\mathbf{j} = \mathbf{j}_{el} - e\mathbf{n}\dot{\mathbf{u}}$. In linear approximation,

$$\mathbf{j}_{el} = e\int\limits_{V_{p'}'} \frac{\partial\varepsilon}{\partial\mathbf{p}} f\, d\tau_p = e\int\limits_{V_{p'}'} \mathbf{v} f_0\, d\tau_p + e\left\langle\!\!\left\langle \frac{\partial\delta\varepsilon}{\partial\mathbf{p}} f_0 \right\rangle\!\!\right\rangle + e\left\langle\!\!\left\langle \mathbf{v}(\delta\tilde{\epsilon}+\chi)\frac{\partial f_0}{\partial\varepsilon}\right\rangle\!\!\right\rangle. \quad (5.2)$$

Here f satisfies the equation

$$\frac{df}{dt} + \hat{\mathbf{v}} f = 0, \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \frac{\partial\varepsilon}{\partial\mathbf{p}}\frac{\partial}{\partial\mathbf{r}} + \left(\mathbf{F} - \frac{\partial\varepsilon}{\partial\mathbf{r}}\right)\frac{\partial}{\partial\mathbf{p}},$$

$$\mathbf{F} = e\mathbf{E} + \frac{e}{c}\left[\frac{\partial\varepsilon}{\partial\mathbf{p}}\mathbf{H}\right]$$

$\epsilon \equiv \epsilon_0 + \delta\epsilon$ was given above (1.7), $\delta\tilde{\epsilon} \equiv \delta\epsilon - \mathbf{p}\cdot\dot{\mathbf{u}} - \delta\mu$ (see I (2.15)—(2.17) and (3.11)).

We compute the first term in (5.2) by means of (5.1) and integrate the second by parts, which gives

$$j_i^{el} = e\oint \Delta p_k v_i f_0\, d\Sigma_k + e\oint \delta\varepsilon f_0\, d\Sigma_i - e\langle v_i(\delta\tilde{\varepsilon} - \delta\varepsilon + \chi)\rangle. \quad (5.3)$$

By comparing this expression with I (2.19), we see that it contains two additional components which are integrals over the surface of the cell and differs from zero only for hole groups. Let us examine these terms. The expression for $\delta\epsilon$, according to (1.7), contains two periodic components proportional to $\mathbf{p}$, which also make a non-zero contribution to the second integral in (5.3):

$$\oint \delta\varepsilon f_0\, d\Sigma_i = \frac{\partial u_l}{\partial x_k}\oint p_i v_k f_0\, d\Sigma_i + \dot{u}_k \oint p_k f_0\, d\Sigma_i.$$

Transforming to volume integrals, it is easy to prove the symmetry of the tensor $\oint p_i v_k f_0 d\Sigma_i$ with respect to the index ik, in view of which the terms with $\partial u_l/\partial x_k$ cancel each other in the expression for f_{el} in (5.3). (In addition, these terms in the undisplaced cell are identically equal to zero because of spherical symmetry.). Similarly,

$$\oint p_i f_0 d\Sigma_k = n\delta_{ik} - \langle p_i v_k\rangle. \quad (5.3')$$

Finally, we get $\mathbf{j}^{el} = -e\langle\mathbf{v}\chi\rangle - e\mathbf{n}\dot{\mathbf{u}}$ (in correspondence with the result I (2.19)) and thus the total current $\mathbf{j}$ is expressed by Eq. (1.1) for arbitrary topology of the Fermi surfaces. The additional components arising in the derivation of the expression for the force have the same origin as those in the simple example considered.

The conservation laws contain time derivatives of the integrals over the cells in p space, which are conveniently treated by using the kinetic equation. The corresponding identity has the form (see I (3.3))

$$\left\langle\!\!\left\langle \frac{\partial\varphi}{\partial t}\right\rangle\!\!\right\rangle = \left\langle\!\!\left\langle \frac{d\varphi}{dt}\right\rangle\!\!\right\rangle - \left\langle\!\!\left\langle \frac{\partial}{\partial x_k}\frac{\partial\varepsilon}{\partial p_k}\varphi\right\rangle\!\!\right\rangle - \oint\left(F_i - \frac{\partial\varepsilon}{\partial x_i}\right)\varphi\, d\Sigma_i. \quad (5.4)$$

The last component appears as a result of integration by parts in the transformation to field derivatives and generally does not disappear in the L system in the presence of hole groups.

We shall consider the case of closed Fermi surfaces and assume that the cell is so chosen that its boundaries do not intersect the Fermi surfaces. Then additional (surface) terms can contain only the equilibrium distribution function, because the nonequilibrium part $(\delta\tilde{\epsilon} + \chi)\partial f_0/\partial\epsilon$ can make a contribution only in the intersection of the cell boundaries with the Fermi surface. The integrals over the surface of the elementary cell can differ from zero only for functions that are not periodic in $\mathbf{p}$. Terms with f_0 not containing the electric fields, and only some deformations, make a contribution only in the renormalization of σ_{ik} and we shall not write them down. According to I (3.4), the free energy of the electron gas in the L system is equal to

$$\mathcal{F}_{el} = \int\left\langle\!\!\left\langle \left(\varepsilon - \dot{u}\left(p - m\frac{\partial\varepsilon}{\partial p}\right)\right) f + kT((1-f)\ln(1-f) + f\ln f)\right\rangle\!\!\right\rangle' dV. \quad (5.5)$$

In the absence of intersections of the boundaries of the elementary cell with the Fermi surface, we have (omitting the fluxes)

$$T\frac{dS}{dt} = \int\langle(\delta\tilde{\varepsilon}+\chi)\hat{\mathbf{v}}\chi\rangle\, dV, \quad \left\langle\!\!\left\langle \frac{d}{dt}(ef)\right\rangle\!\!\right\rangle = \left\langle\!\!\left\langle \frac{\partial\varepsilon}{\partial t} f\right\rangle\!\!\right\rangle + \mathbf{E}\mathbf{j}^{el} + \langle\delta\varepsilon\hat{\mathbf{v}}\chi\rangle,$$

$$\oint\left(F_k - \frac{\partial\varepsilon}{\partial x_k}\right)\varepsilon f\, d\Sigma_k = eE_i\dot{u}_k\oint p_k f_0\, d\Sigma_i.$$

Using (5.1) and (5.4), we find that $d\mathcal{F}/dt$ is expressed by the formulas I (3.10) with the additional component $-\int dV\, eE_i u_k \oint p_k f_0 d\Sigma_i$, account of which leads to the replacing of the electron density n by the difference $n_- - n_+$. As a result, we get the expression (3.3′) for the force in elasticity equations.

6. EQUATIONS OF ELASTICITY THEORY IN SEMICONDUCTORS

All the discussions above are also applicable to semiconductors. It is only necessary to give up the condition of electrical neutrality. With this aim, we write down the condition of normalization of the distribution function in more detail than before.

Let Q be the ionic charge density in the L system. In the absence of deformation, $Q = -en_0$, where $n_0 = \langle\langle f_0 \rangle\rangle$ is the equilibrium electron density in the undeformed lattice. In deformation, the ionic charge $-en_0 dV'$ falls off in the volume dV and by virtue of conservation of ionic charge, $QdV = -en_0 dV'$, whence

$$Q = -en_0(1 - \operatorname{div} \mathbf{u}). \tag{6.1}$$

The electron, locally equilibrium, distribution function $f_0(\epsilon_0 + \delta\epsilon)$ is normalized to the true density n of electrons in the L system. The condition of electrical neutrality $Q = -en$ and the normalization condition $\langle\chi\rangle = 0$ then follow. The condition $Q = -en$ in metals serves as the equation for the determination of the chemical potential. From $-e\langle\langle f \rangle\rangle = Q$, with account of (6.1), it follows that $\delta\mu = \langle\lambda_{ik}\rangle u_{ik}/\langle 1\rangle$, which leads to the replacing of λ_{ik} by Λ_{ik} (see (1.3)). The longitudinal electric field is determined here from the equation $\operatorname{div} \mathbf{j} = 0$, which, for the found values of $\delta\mu$ and $\omega \neq 0$, corresponds to the equation $\langle\chi\rangle = 0$. The equation $\langle\chi\rangle = 0$ is the zeroth approximation of the equation $\operatorname{div} \tilde{\epsilon}_p \mathbf{E} = -4\pi e\langle\chi\rangle$ in the small parameter, which is the ratio of the frequencies ω, ν, ω_c, $\mathbf{k}\cdot\mathbf{v}$ to the plasma frequency ω_p.[10]

We now consider the case of a semiconductor where the frequencies ω_p are not large and the condition of electrical neutrality is not satisfied. Then the equation for χ takes the form

$$\frac{d\chi}{dt} + \hat{\nu}\chi = -e\tilde{\mathbf{E}}\mathbf{v} - \lambda_{ik}\dot{u}_{ik} + \delta\dot{\mu}, \tag{6.2}$$

where

$$\tilde{\mathbf{E}} = \mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}] - \frac{1}{e}\nabla\delta\mu - \frac{m}{e}\ddot{\mathbf{u}}. \tag{6.3}$$

We integrate (6.2) over the Fermi surface and take into account the normalization condition $\langle\chi\rangle = 0$. We get

$$\delta\dot{\mu}\langle 1\rangle = \langle\lambda_{ik}\rangle\dot{u}_{ik} - \frac{1}{e}\operatorname{div}\mathbf{j}, \tag{6.4}$$

where the total current density $\mathbf{j} = -e\langle\mathbf{v}\chi\rangle$. (As is easily seen, the expression for the current is not changed in the linear approximation.) Using the continuity equation

$$\frac{\partial}{\partial t}(Q + en) + \operatorname{div}\mathbf{j} = 0$$

and the equation

$$\operatorname{div}\hat{\epsilon}_{\text{lat}}\mathbf{E} = 4\pi(Q + en),$$

where $\hat{\epsilon}_{\text{lat}}$ is the tensor of the lattice dielectric permittivity, we get

$$\delta\mu = \frac{\langle\lambda_{ik}\rangle}{\langle 1\rangle}u_{ik} - \frac{1}{4\pi\langle 1\rangle}\operatorname{div}\hat{\epsilon}_{\text{lat}}\mathbf{E}. \tag{6.5}$$

Finally, the total set of equations for the deformed semiconductor consists of the elasticity equation (1.2)[9]; the expression for the fluxes j and f^d (1.1) in terms of the distribution function χ, which is described by Eq. (6.2); the expressions for the effective field $\hat{\mathbf{E}}$ and the chemical potential $\delta\mu$, determined from (6.3) and (6.5), and finally, Maxwell's equations

$$\operatorname{rot}\operatorname{rot}\mathbf{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{\epsilon}_{\text{lat}}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}. \tag{6.6}$$

In conclusion, the author expresses his sincere thanks to V. L. Gurevich, I. G. Lang and S. T. Pavlov for the communication of a number of their results before their publication, R. N. Gurzhi, I. M. Lifshitz, L. P. Pitaevskiĭ and V. M. Tsukernik for stimulated discussion.

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Translated by R. T. Beyer
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