ANOMALOUS ABSORPTION OF AN ELECTROMAGNETIC WAVE BY A PLASMA

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The spectral energy density of ion-acoustic oscillations in a nonisothermal plasma situated in the field of an electromagnetic wave is determined. The high-frequency conductivity of a weakly-turbulent plasma with developed ion-sound noise is calculated for an electromagnetic-wave frequency close to the plasma frequency. It is shown that the electromagnetic-wave absorption coefficient connected with this conductivity is large compared with the usual absorption determined by electron-ion collisions.

EXPERIMENTS on the interaction of a plasma with electromagnetic waves in the high-frequency ($\omega_0 \approx 2 \times 10^{10}$ sec^{-1})[1-3] and the optical[4] ($\omega_0 \approx 2 \times 10^{15}$ sec^{-1}) ranges point to the existence of phenomena occurring outside the framework of the linear theory of absorption, reflection, and propagation of electromagnetic waves in a plasma. On the other hand, as early as in 1965, one of the authors predicted the phenomenon of rapid heating of a plasma by a high-frequency field, as a result of development of a parametric plasma instability.[5] The theory that determines the conditions for the occurrence of such an instability has by now been developed in sufficient detail.[6,7] At the same time, the theory of this turbulent state, which results from the parametric instability, is still far from complete. The need for such a theory is dictated by the need for constructing a detailed picture of the entire interaction between a strong electromagnetic field and a plasma. Definite progress towards the development of such a theory was made in [8,9], where a study was made of the influence of growing plasma oscillations on the distribution of the plasma particles, and it was concluded that the dissipation of the electromagnetic-field energy increases in anomalous fashion. These conclusions were confirmed by the results of a computer experiment,[10] in which the use of a model that is quite idealized but nevertheless reflects the important features of the real problem has made it possible to obtain numerical values for the plasma conductivity determined by the nonequilibrium field pulsations. The authors of [10] advanced some ideas concerning the possible physical causes of the stationary pulsation level, in analogy with the fact that, in accord with [8], plasma heating can suppress parametric instability. So far, however, no attempts have been made to study the role of the nonlinear interaction of growing perturbations in a parametrically unstable plasma. The results of such a study constitute the content of the present article.

1. STATIONARY SPECTRAL DENSITY OF ION-SOUND ENERGY

The theory of parametric resonance of a plasma situated in an external alternating electric field[5,6]

$$\mathbf{E}_0 \cos \omega_0 t \tag{1.1}$$

with constant and homogeneous amplitude $\mathbf{E}_0$ and with frequency ω_0, predicts a large number of effects on the basis of which one can attempt to explain the experimentally observed phenomena. In this paper we calculate the damping decrement $(\nu_{\rm eff} + \nu_{\rm ei})/2$, which depends on the amplitude $\mathbf{E}_0$, of a plane monochromatic wave of frequency ω_0 close to the electron Langmuir frequency $\omega_{\rm Le}$ in a plasma:

$$\omega_0 \gtrsim \omega_{Le}, \tag{1.2}$$

and having a negligibly small wave number compared with the wave numbers of the perturbations that build up in the plasma. Following [7], we confine ourselves here to the case of low electron oscillation velocities v_E in the field of the electromagnetic wave, compared with the thermal velocity v_{Te} (e is the electron charge and m its mass):

$$\mathbf{v}_E \equiv e\mathbf{E}_0 / m\omega_0, \quad v_E = |\mathbf{v}_E| \ll v_{Te}$$

and neglect completely the displacements of the ions in the field (1.1). Under these conditions, when the electric field E_0 exceeds a certain "threshold" value $E_{\rm thr}$ in a nonisothermal plasma with hot electrons (temperature T_e) and relatively cold ions (temperature T_i)

$$T_e \gg T_i \tag{1.3}$$

the instability gives rise to ion-acoustic oscillations with frequency

$$\omega = kv_s, \quad v_s \equiv \omega_{Li} r_{De}, \tag{1.4}$$

much lower than the ion Langmuir frequency $\omega_{\rm Li}$, and with a wave number k smaller than the reciprocal electron Debye radius $r_{\rm De}$,

$$(kr_{De})^2 \ll 1.$$

Such an instability can be interpreted as the decay of the external wave (1.1) into a low-frequency ($\omega \ll \omega_0$) long-wave ion-acoustic oscillation (1.4) and a Langmuir oscillation with a spectrum

$$\omega^2 = \omega_{Le}^2 + \omega_{Li}^2 + 3k^2 v_{Te}^2.$$

The equation connecting the frequencies of the oscillations that take part in the decay imposes a limitation (see (1.2)) on the frequency deviation

$$\Delta\omega_0 \equiv \omega_0 - (\omega_{Le}^2 + \omega_{Li}^2 + 3k^2 v_{Te}^2)^{1/2} > 0$$

and determines ($\Delta\omega_0 = kv_S$) the wavelength $2\pi/k_0$ of the plasma oscillations:

$$k_0 = \frac{1}{3r_{De}}\left\{\sqrt{\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6\frac{\omega_0 - \omega_p}{\omega_p}} - \frac{\omega_{Li}}{\omega_{Le}}\right\}, \quad \omega_p \equiv \sqrt{\omega_{Le}^2 + \omega_{Li}^2}. \quad (1.5)$$

The spectral energy density $W_S(\mathbf{k})$ of the excited ion-acoustic waves determines[8,9] the energy density per unit time, averaged over the period $(2\pi/\omega_0)$, released in the plasma by the field (1.1) ($\delta\sigma$ is the sought contribution made to the high-frequency conductivity resulting from the interaction with the developed plasma oscillations):

$$\frac{1}{2}\delta\sigma E_0^2 = \frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{(\mathbf{k}\mathbf{v}_E)^2}{(kr_{De})^2}\frac{W_s(\mathbf{k})}{\tilde{\gamma}(k)}. \quad (1.6)$$

The damping decrement of the ion sound

$$\gamma(\mathbf{k}) = \gamma_s(k)\{1 - F(k)\cos^2\theta\}, \quad (1.7)$$

is determined by the damping of the ion-sound oscillations by the electrons

$$\gamma_s(k) = \sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{\omega_{Le}}kv_s \quad (1.8)$$

and by their buildup by electrons oscillating in the field (1.1)[7]

$$F(k) = \frac{1}{\sqrt{2\pi}}\left(\frac{v_E}{\omega_0 r_{De}}\right)^2 \frac{\omega_0\Delta\omega_0\tilde{\gamma}(k)kv_{Te}}{[(\Delta\omega_0)^2 - k^2v_s^2]^2 + 4k^2v_s^2\tilde{\gamma}^2(k)}. \quad (1.9)$$

The high-frequency damping decrement $\tilde{\gamma}(k)$ is determined by the Landau damping and by the collisions of the electrons with the ions (the effective collision frequency is $\nu_{ei} \ll kv_{Te}$)[7]

$$\tilde{\gamma}(k) = \sqrt{\frac{\pi}{8}}\frac{\omega_0}{(kr_{De})^3}\exp\left\{-\frac{1}{2}\left(\frac{\omega_0}{kv_{Te}}\right)^2\right\} + \frac{1}{2}\nu_{ei}. \quad (1.10)$$

A characteristic feature of the decrement (1.7) is its dependence on the direction of ion-sound propagation relative to the field (1.1) ($\theta \equiv \mathbf{kE}_0$). The sound is amplified if its damping decrement (1.7) is negative

$$1 - F(k)\cos^2\theta < 0.$$

We determine the spectral energy density $W_S(\mathbf{k})$ of long-wave ion sound (1.4) within the framework of the theory of nonlinear wave interaction in a plasma. The nonlinear mechanism limiting the level of the ion-acoustic noise is the induced scattering of ion sound by the ions. The corresponding equation describing the spectral energy density $W_S(\mathbf{k})$ has in the case of Coulomb scattering the form (κ is the Boltzmann constant, N_e is the number of electrons per unit volume, and the subscript i labels ionic quantities)

$$\kappa T_e = W_s(\mathbf{k})\left\{[1 - F(k)\cos^2\theta] + 2\frac{v_{Te}}{v_{Ti}}\right. \quad (1.11)^*$$

$$\left.\times\int d\mathbf{k}'\frac{W_s(\mathbf{k}')}{N_e\kappa T_e}\frac{k-k'}{kk'}\left(\frac{\mathbf{kk}'}{kk'}\right)^2\frac{[\mathbf{kk}']^2}{|\mathbf{k}-\mathbf{k}'|^3}\exp\left[-\frac{1}{2}\frac{r_{De}^2}{r_{Di}^2}\frac{(k-k')^2}{(\mathbf{k}-\mathbf{k}')^2}\right]\right\}.$$

The left side corresponds here to spontaneous emission of ion-acoustic oscillations by electrons, and the first term in the right-hand side gives the contribution of the linear processes of damping and buildup of the sound. An important property of the nonlinear term of (1.11) is the antisymmetry of its kernel relative to the replacement of the wave vectors $\mathbf{k} \rightleftarrows \mathbf{k}'$ of the interacting sound waves. In the nonisothermal plasma (1.3), this kernel can be greatly simplified by approximating the exponential with a δ function:[11]

$$\frac{k-k'}{|\mathbf{k}-\mathbf{k}'|^3}\exp\left\{-\frac{1}{2}\frac{r_{De}^2}{r_{Di}^2}\frac{(k-k')^2}{(\mathbf{k}-\mathbf{k}')^2}\right\} \cong \sqrt{2\pi}\frac{r_{Di}^3}{r_{De}^3}\frac{\partial}{\partial k'}\delta(k'-k). \quad (1.12)$$

Equation (1.11) can then be represented in the form

$$\kappa T_e = W_s(\mathbf{k})\left\{1 - F(k)\cos^2\theta + 2\sqrt{2\pi}\frac{r_{Di}^2}{r_{De}^2}\frac{\omega_{Le}}{\omega_{Li}}\right.$$
$$\left.\times\int d\mathbf{k}'\frac{W_s(\mathbf{k}')}{N_e\kappa T_e}(kk')\frac{\partial\delta(k'-k)}{\partial k'}\left(\frac{\mathbf{kk}'}{kk'}\right)^2\frac{[\mathbf{kk}']^2}{(kk')^2}\right\}. \quad (1.13)$$

Confining ourselves to the solution of (1.13) at the threshold for the buildup of the ion sound by the electromagnetic wave (1.1), and neglecting the small spontaneous term in the left-hand side of (1.13) we have

$$(\psi^2 + x^2 - 1) + \int_{x_1}^{x_2} dx'\frac{\partial\delta(x'-x)}{\partial x'}\int_0^{\psi_1(x')}\psi'\,d\psi'(\psi^2 + \psi'^2)\,y(x',\psi') = 0. \quad (1.14)$$

The sought function $y(x, \psi)$ of the dimensionless wave number ($x \gtrless 0$)

$$x \equiv \frac{k-k_0}{k_0}b_0\psi_0, \quad \psi_0^2 \equiv \frac{1}{F(k_0)-1} \gg 1, \quad b_0 \equiv \frac{k_0 v_{Te}}{\tilde{\gamma}(k_0)}\sqrt{\frac{\omega_{Li}^2}{\omega_{Le}^2} + 6\frac{\omega_0-\omega_p}{\omega_p}},$$

and of the angle $\psi = \theta\psi_0$ ($\theta \ll 1$) determines here the dependence of the spectral energy density $W_S(\mathbf{k})$ of the ion-sound oscillations on their wave vector $\mathbf{k}$ (with allowance for the cylindrical symmetry of the initial equation (1.1) about the electric intensity vector $\mathbf{E}_0$ of the wave (1.1)):

$$W_s(\mathbf{k}) = \frac{(2\pi)^{-3/2}}{2}\frac{r_{De}^2}{r_{Di}^2}\frac{\omega_{Li}}{\omega_{Le}}\frac{\psi_0}{b_0}\frac{N_e\kappa T_e}{k_0^3}y(x,\psi). \quad (1.15)$$

Equation (1.14) is an elementary consequence of (1.3) for the propagation of ion sound almost parallel to the field $\mathbf{E}_0$ of the wave (1.1) ($\theta \ll 1$) and in the fields $\mathbf{E}_0$ close to the threshold E_{thr}:[7]

$$E_{thr}^2 = 8(2\pi)^{3/2}N_e\kappa T_e\frac{\omega_{Li}}{\omega_{Le}}\frac{\tilde{\gamma}(k_0)}{\omega_0}, \quad (1.16)$$

determined by the vanishing of the damping decrement (1.7) at the maximum value of the function (1.9)

$$F(k) \approx \frac{F(k_0)}{1 + x^2[F(k_0)-1]}; \quad \max_k F(k) \approx F(k_0) = \frac{E_0^2}{E_{thr}^2} \gg 1.$$

The case of antiparallel propagation of the sound relative to $\mathbf{E}_0$ reduces to (1.14) because of the symmetry of the spectral energy density (see (1.11))

$$W_s(k, \theta) = W_s(k, \pi - \theta).$$

According to (1.14), the nonequilibrium spectral density is defined in the wave-number interval

$$x_1 \leqslant x \leqslant x_2 \quad (1.17)$$

and in the angle interval

$$0 \leqslant \psi \leqslant \psi_1(x). \quad (1.18)$$

The end points of the interval (1.17) and the wave-number function $\psi_1(x)$ must also be found. The meaning of the function $\psi_1(x)$ is that it delineates on the (x, ψ) plane the region in which the level of the ion-sound noise is much higher than the spontaneous noise

$$y(x, \psi_1(x)) = 0. \quad (1.19)$$

The existence of such a line $\psi = \psi_1(x)$ follows naturally

*$(\mathbf{kk}) \equiv \mathbf{k}\cdot\mathbf{k}$; $[\mathbf{kk}'] \equiv \mathbf{k}\times\mathbf{k}'$.

from general physical considerations concerning the attenuation of the sound with increasing angle θ between the field $\mathbf{E}_0$ and the propagation direction $\mathbf{k}$ of the oscillations, and is confirmed by the solution of (1.14) given below; the validity of this solution can be easily verified directly:[1)]

$$y(x,\psi)=\frac{15}{2}\frac{x-C_1}{\psi_1^5(x)}\psi[\psi_1^2(x)-\psi^2],\quad \psi_1^2(x)=\frac{7}{3}\frac{C_2+x-{}^1/_3x^3}{C_1-x}.$$

The integration constants C_1 and C_2 which arise here are determined by the boundary values of the spectral energy density at the end points of the wave-number interval (1.17). We assume that at the end points of this interval the noise level decreases to the spontaneous value (i.e., to zero, in accordance with the approximation assumed on going from (1.13) to (1.14))

$$y(x_1,\psi)=0,\quad \psi_1(x_2)=0. \tag{1.20}$$

The end points x_1 and x_2 determining these constants

$$C_1=x_1,\quad C_2={}^1/_3x_2^3-x_2,$$

are determined, in turn, from the positiveness condition $\psi_1^2(x)>0$

$$x_1=-2,\quad x_2=1.$$

As a result we obtain a final formula for the nonequilibrium spectral energy density (1.15) of ion-sound oscillations built up by the electromagnetic wave (1.1) when its electric field intensity E_0 slightly exceeds the threshold value (1.16) ($E_0 \gtrsim E_{thr}$):

$$W_s(\mathbf{k})=\frac{15}{4}(2\pi)^{-3/2}\frac{\omega_{Li}}{\omega_{Le}}\frac{r_{De}^2}{r_{Di}^2}\frac{\tilde\gamma(k_0)}{k_0v_{Te}}\left(\frac{E_0^2}{E_{thr}^2}-1\right)^{-1/2}\left(\frac{\omega_{Li}^2}{\omega_{Le}^2}+6\frac{\omega_0-\omega_p}{\omega_p}\right)^{-1/2}$$
$$\times\frac{N_e\varkappa T_e}{k_0^3}\frac{x+2}{\psi_1^5(x)}\psi[\psi_1^2(x)-\psi^2],\quad \psi_1(x)\equiv\frac{\sqrt7}{3}(1-x). \tag{1.21}$$

2. HIGH-FREQUENCY CONDUCTIVITY OF TURBULENT PLASMA

In accordance with (1.21), the spectral energy density of the sound reaches a maximum (with respect to the angle ψ) on the straight line $\psi=\psi_1(x)/\sqrt3$ of the (x,ψ) plane:

$$\operatorname*{Max}_{\psi} y(x,\psi)=y\left(x,\frac{\psi_1}{\sqrt3}\right)=\frac{15\sqrt3}{7}\frac{2+x}{(1-x)^2}.$$

The maximum with respect to the wave numbers is reached on the line ($y'_x=0$)

$$\psi=\psi_1(x)\sqrt{\frac{7+2x}{11+4x}}.$$

For the total energy density W (over the entire spectrum) of the ion-sound oscillations we have

$$W\equiv\int\frac{d\mathbf{k}}{(2\pi)^3}W_s(\mathbf{k})=\frac{9}{2(2\pi)^{7/2}}\frac{r_{De}^2}{r_{Di}^2}\frac{\omega_{Li}}{\omega_{Le}}\frac{N_e\varkappa T_e}{b_0^2\psi_0^2}. \tag{2.1}$$

In particular, it can be verified with the aid of (2.1) that the condition used in the derivation of the initial equation (1.11), namely that the total energy density W is small compared with the energy density of the thermal motion of the plasma particles, is satisfied:

$$W/N_e\varkappa T_e\ll 1. \tag{2.2}$$

This, however, imposes a limitation on the degree of nonisothermy of the plasma:

$$\frac{r_{De}^2}{r_{Di}^2}\ll\frac{2(2\pi)^{7/2}}{9}\frac{\omega_{Le}}{\omega_{Li}}\left(\frac{\omega_{Li}^2}{\omega_{Le}^2}+6\frac{\omega_0-\omega_p}{\omega_p}\right)\left(\frac{E_0^2}{E_{thr}^2}-1\right)^{-1}\frac{(k_0v_{Te})^2}{\tilde\gamma^2(k_0)}, \tag{2.3}$$

The high level of nonequilibrium noise of ion-sound oscillations, compared with the spontaneous noise

$$W_s(\mathbf{k})\gg\varkappa T_e\left(\frac{E_0^2}{E_{thr}^2}-1\right)^{-1}, \tag{2.4}$$

also follows from the solution (1.21), thereby confirming the assumption made above that the left-hand side of (1.13) can be neglected on going to (1.14). It must be emphasized at the same time that it is precisely because of the inequality (2.4) that the electric field intensity E_0 of the electromagnetic wave (1.1) cannot be too close to the threshold value E_{thr} (in this estimate we put $y(x,\psi)=0(1)$):

$$\left(\frac{E_0^2}{E_{thr}^2}-1\right)\gg32\pi^3\frac{r_{Di}^4}{r_{De}^4}\frac{(k_0v_{Te})^2}{\tilde\gamma^2(k_0)}\left(1+6\frac{\omega_{Le}^2}{\omega_{Li}^2}\frac{\omega_0-\omega_p}{\omega_p}\right)\frac{k_0^6}{N_e^2}. \tag{2.5}$$

To obtain an explicit expression for the contribution $\delta\sigma$ to the high-frequency conductivity due to the interaction with the plasma oscillations that develop as a result of the instability, we substitute the spectral energy density (1.21) in the right-hand side of (1.6), and simplify the latter in the near-threshold region of ion-sound buildup

$$\delta\sigma\equiv\frac{e^2N_e}{m\omega_0^2}\nu_{eff}. \tag{2.6}$$

It turns out here that the effective collision frequency ν_{eff} is determined by the total energy density W (over the the spectrum) of the ion-sound oscillations (2.1):

$$\nu_{eff}=\frac{1}{2}\frac{\omega_{Le}^2}{\tilde\gamma(k_0)}\frac{W}{N_e\varkappa T_e}. \tag{2.7}$$

Thus, the effective frequency ν_{eff}, which determines the contribution of the interaction with the oscillations to the high-frequency plasma conductivity (2.6), and consequently also to the damping decrement of the electromagnetic wave (1.1), takes the form

$$\nu_{eff}=\frac{81}{4(2\pi)^{7/2}}\tilde\gamma(k_0)\left(\frac{E_0^2}{E_{thr}^2}-1\right)\frac{r_{De}^2}{r_{Di}^2}\frac{\omega_{Li}}{\omega_{Le}}$$
$$\times\left(\frac{\omega_{Li}^2}{\omega_{Le}^2}+6\frac{\omega_0-\omega_p}{\omega_p}\right)^{-1}\left(\left[\frac{\omega_{Li}^2}{\omega_{Le}^2}+6\frac{\omega_0-\omega_p}{\omega_p}\right]^{1/2}-\frac{\omega_{Li}}{\omega_{Le}}\right)^{-2}. \tag{2.8}$$

We recall that here $\tilde\gamma(k_0)$ is the high-frequency damping decrement (1.10) at the decay value (1.5) of the ion-sound wave number, and the threshold intensity E_{thr} of the electric field of the wave (1.1) is given by (1.16).

It should also be noted that with decreasing deviation Δ of the frequency ω_0 of the wave (1.1) from the plasma frequency $\omega_p=\sqrt{\omega_{Le}^2+\omega_{Li}^2}$,

$$\Delta=6\frac{\omega_{Le}^2}{\omega_{Li}^2}\frac{\omega_0-\omega_p}{\omega_p},$$

the last factor in the right-hand side of (2.8) increases, together with the effective collision frequency ν_{eff}. Actually, however, the deviation Δ is limited even within the framework of the theory that is linear with respect to the plasma perturbation[7]

[1)]The conditions (1.20) ensure the vanishing of the result of integration by parts (with respect to dx′) in the second term of (1.14).

$$\Delta_1 < \Delta < \Delta_2, \tag{2.9}$$

With the end points Δ_1 and Δ_2 of the interval (2.9) determined by the condition

$$k_0 r_{De} > \tilde{\gamma}(k_0) / \omega_{Li}, \tag{2.10}$$

which must be satisfied by the wave number (1.5) of the ion-sound oscillations produced as a result of the decay of the wave (1.1). Namely, the lower limit is determined by the equation

$$\Delta_1 = \left(\frac{3}{2}\frac{\nu_{ei}}{\omega_{Li}}\frac{\omega_{Le}}{\omega_{Li}} + 1\right)^2 - 1$$

and results from (2.10) if one can neglect in the high-frequency damping decrement (1.10) the contribution of the Cerenkov damping by the electrons ($\tilde{\gamma}(k_0) \approx \nu_{ei}/2$). The upper limit Δ_2 of the deviations (2.9)

$$\Delta_2 \approx \frac{9}{2}\frac{\omega_{Le}^2}{\omega_{Li}^2}\left(\ln\frac{\omega_{Le}^2}{\omega_{Li}^2}\right)^{-1},$$

is determined from (2.10) in the opposite limit, when the high-frequency damping decrement (1.10) is determined only by the second term ($\tilde{\gamma}(k_0) \gg \nu_{ei}/2$). Thus, the effective collision frequency (2.8) is defined in the deviation interval (2.9) and is given by

$$\nu_{\text{eff}} = \frac{81}{8(2\pi)^{7/2}}\nu_{ei}\frac{r_{De}^2}{r_{Di}^2}\left(\frac{E_0^2}{E_{\text{thr}}^2} - 1\right)\left(\frac{\omega_{Le}}{\omega_{Li}}\right)^3 \frac{1}{(1+\Delta)(\sqrt{1+\Delta}-1)^2}$$
$$\times\left\{1 + 27\sqrt{\frac{\pi}{2}}\frac{\omega_{Li}}{\nu_{ei}}\left(\frac{\omega_{Le}}{\omega_{Li}}\right)^4(\sqrt{1+\Delta}-1)^{-3}\exp\left[-\frac{9}{2}\frac{\omega_{Le}^2}{\omega_{Li}^2}(\sqrt{1+\Delta}-1)^{-2}\right]\right\}. \tag{2.11}$$

Formulas (2.8) and (2.11) for the effective collision frequency due to the plasma oscillations were obtained under the assumption that the high-frequency dissipation is determined by the decrement $\tilde{\gamma}(k)$. This assumption is valid only if $\nu_{\text{eff}} \ll \tilde{\gamma}$. At sufficiently high plasma-oscillation intensity, when the interaction with the oscillations is important for high-frequency dissipation, it is necessary to replace $\tilde{\gamma}$ by $\tilde{\gamma} + \nu_{\text{eff}}/2$. Such a replacement in (2.8) leads to the following expression:

$$\nu_{\text{eff}} = 2\tilde{\gamma}(k_0)\left(\frac{E_0^2}{E_{\text{thr}}^2} - 1\right)\left\{1 + \frac{8(2\pi)^{7/2}}{81}\frac{r_{Di}^2}{r_{De}^2}\times\left(\frac{\omega_{Li}}{\omega_{Le}}\right)^3(1+\Delta)(\sqrt{1+\Delta}-1)^2\right\}^{-1}, \tag{2.12}$$

in which the threshold value of the field is given by (1.16). We see therefore that in the near-threshold region considered by us ν_{eff} is always smaller than the high-frequency decrement $\tilde{\gamma}(k_0)$. Therefore the interaction with the plasma oscillations, in accordance with (2.12), can lead to an effective collision frequency ν_{eff} larger than the usual electron-ion frequency ν_{ei}, only in the frequency-deviation interval

$$\Delta_c < \Delta < \Delta_2, \tag{2.13}$$

in which the contribution of the ordinary collisions to $\tilde{\gamma}$ is negligibly small, i.e.,

$$2\tilde{\gamma}(k_0) = 27\sqrt{\frac{\pi}{2}}\frac{\omega_{Le}^4}{\omega_{Li}^3}(\sqrt{1+\Delta}-1)^{-3}\exp\left\{-\frac{9}{2}\frac{\omega_{Le}^2}{\omega_{Li}^2}(\sqrt{1+\Delta}-1)^{-2}\right\}.$$

The lower limit Δ_c of the frequency-deviation interval (2.13) is given by

$$1 = 27\sqrt{\frac{\pi}{2}}\frac{\omega_{Le}}{\nu_{ei}}\left(\frac{\omega_{Le}}{\omega_{Li}}\right)(\sqrt{1+\Delta_c}-1)^{-3}\exp\left\{-\frac{9}{2}\frac{\omega_{Le}^2}{\omega_{Li}^2}(\sqrt{1+\Delta_c}-1)^{-2}\right\}$$

and its order of magnitude is

$$\Delta_c \approx \frac{9}{2}\left(\frac{\omega_{Le}}{\omega_{Li}}\right)^2\left(\ln\frac{\omega_{Le}^2}{\nu_{ei}^2}\right)^{-1}.$$

In the interval (2.13), the effective collision frequency (2.12) increases monotonically with increasing frequency deviation, reaching a maximum value at the upper limit $\Delta = \Delta_2$:

$$\operatorname*{Max}_{\Delta}\nu_{\text{eff}} = \frac{1}{\sqrt{2}(2\pi)^{7/2}}\omega_{Li}\frac{\omega_{Li}}{\omega_{Le}}\left(\ln\frac{\omega_{Le}^2}{\omega_{Li}^2}\right)^{3/2}\frac{r_{De}^2}{r_{Di}^2}\left(\frac{E_0^2}{E_{\text{thr}}^2} - 1\right). \tag{2.14}$$

Such a growth of the absorption is accompanied by a growth of the threshold field (1.16) with increasing Δ

$$E_{\text{thr}}^2 = 54(2\pi)^2 N_e \varkappa T_e\left(\frac{\omega_{Le}}{\omega_{Li}}\right)^2(\sqrt{1+\Delta}-1)^{-3}\times\exp\left\{-\frac{9}{2}\frac{\omega_{Le}^2}{\omega_{Li}^2}(\sqrt{1+\Delta}-1)^{-2}\right\},$$

which assumes at the upper limit Δ_2 of (2.13) the value

$$E_{\text{thr}}^2 = \frac{8(2\pi)^{3/2}}{\sqrt{2}}\left(\frac{\omega_{Li}}{\omega_{Le}}\right)^2\left(\ln\frac{\omega_{Le}^2}{\omega_{Li}^2}\right)^{1/2}N_e\varkappa T_e. \tag{2.15}$$

The effective collision frequency (2.14) is much higher here than the electron-ion collision frequency[12] ($N_{De} = \frac{4}{3}\pi N_e r_{De}^3$ is the number of electrons in a sphere having the electronic Debye radius and e_i is the ion charge):

$$\nu_{ei} = \frac{2}{9(2\pi)^{1/2}}\frac{e_i}{|e|}\omega_{Le}\frac{\ln N_{De}}{N_{De}},$$

and in a plasma with sufficiently high electron temperature we have

$$\frac{1}{\nu_{ei}}\operatorname*{Max}_{\Delta}\nu_{\text{eff}} = \frac{3}{\sqrt{2}(2\pi)^2}\frac{T_e}{T_i}\frac{\omega^2}{\omega_{Le}^2}\left(\ln\frac{\omega_{Le}^2}{\omega_{Li}^2}\right)^{3/2}\frac{N_e r_{De}^3}{\ln N_{De}}\left(\frac{E_0^2}{E_{\text{thr}}^2} - 1\right) \gg 1. \tag{2.16}$$

In a hydrogen plasma $(\omega_{Le}/\omega_{Li}) \approx 43$, this condition (2.16) takes the form (the temperature is in eV, N_e is the number of electrons per cm^3)

$$2.5\cdot 10^5(\ln N_{De})^{-1}\frac{T_e}{T_i}T_e^{3/2}N_e^{-1/2}\left(\frac{E_0^2}{E_{\text{thr}}^2} - 1\right) \gg 1. \tag{2.17}$$

In a hot plasma defined by (2.17), the absorption of a transverse electromagnetic wave with frequency $\omega_0 \approx \omega_p$ defined by the effective collision frequency (2.14) is anomalously large compared with the ordinary absorption due to ν_{ei}.

Summarizing, let us emphasize certain characteristic features of the basic formulas (2.11), (2.12), and (2.14) obtained in this section. Namely, the dependence of the effective collision frequency ν_{eff} on the field $\mathbf{E}_0$, the absorption of which by the plasma is determined by this frequency, has a threshold character, since the anomalous absorption indicated above sets in only when the field E_0 exceeds a certain threshold value E_{thr}. Such a threshold value of the field is much lower than the field in which the electromagnetic-wave energy density becomes comparable with the plasma pressure (see (1.16), (2.15)). The dependence of ν_{eff} on the frequency ω_0 of the absorbed wave is resonant: an increase of the relative frequency deviation $(\omega_0 - \omega_p)/\omega_p$ within certain limits increases the absorption sharply (see (2.11) and (2.12)) within the interval (2.13). Finally, the dependence of the effective collision frequency on the plasma temperature and density, given by (2.11), (2.12), and (2.14), differs strongly from the usual one for ν_{ei}.

CONCLUSION

The main results of this paper are formulas (1.21), (2.11), and (2.12) for the stationary spectral energy density of the plasma ion-sound oscillations and the effective collision frequency that determines the high-frequency conductivity of the weakly-turbulent plasma. The method of solving the integral equation (1.11) for the spectral energy density in a wide range does not depend on the concrete mechanism whereby the unstable oscillations are excited. The ion-sound noise excited in the plasma by an electron current[11] is determined by this method in the Appendix.

The obtained effective collision frequency $\nu_{\rm eff}$ determines the damping decrement $(\nu_{\rm eff} + \nu_{\rm ei})/2$ (in reciprocal seconds) of the transverse electromagnetic wave, its wave vector $\tilde{k}$, and the absorption coefficient μ (in reciprocal centimeters) (see formulas (6.22) on p. 43 in the book [12]):

$$\tilde{k}=\frac{\omega_0}{c}\left\{\frac{\omega_0-\omega_p}{\omega_p}+\left[\left(\frac{\omega_0-\omega_p}{\omega_p}\right)^2+\left(\frac{\nu_{\rm eff}+\nu_{ei}}{2\omega_0}\right)^2\right]^{1/2}\right\}^{1/2},$$

$$\mu=\frac{\nu_{\rm eff}+\nu_{ei}}{2c}\left\{\frac{\omega_0-\omega_p}{\omega_p}+\left[\left(\frac{\omega_0-\omega_p}{\omega_p}\right)^2+\left(\frac{\nu_{\rm eff}+\nu_{ei}}{2\omega_0}\right)^2\right]^{1/2}\right\}^{-1/2}.$$

In the case of large frequency deviations from the interval (2.13)

$$\tilde{k}=\sqrt{2}\frac{\omega_0}{c}\left(\frac{\omega_0-\omega_p}{\omega_p}\right)^{1/2},\quad \mu=\frac{1}{2\sqrt{2}}\frac{\nu_{\rm eff}+\nu_{ei}}{c}\left(\frac{\omega_0-\omega_p}{\omega_p}\right)^{-1/2},$$

the absorption coefficient μ increases with increasing frequency deviation together with the effective collision frequency (2.12), and reaches a maximum value at the upper limit $\Delta = \Delta_2$ of the deviations $(\omega_0 - \omega_{\rm p} = {}^3\!/_4\omega_{\rm p} \times \ln^{-1} \omega_{\rm Le}^2/\omega_{\rm Li}^2)$:

$$\tilde{k}=\left(\frac{2}{3}\ln\frac{\omega_{Le}^2}{\omega_{Li}^2}\right)^{-1/2}\frac{\omega_0}{c},\quad \mu=\left(\frac{1}{6}\ln\frac{\omega_{Le}^2}{\omega_{Li}^2}\right)^{1/2}\frac{\nu_{\rm eff}+\nu_{ei}}{c}.$$

Here $\nu_{\rm eff}$ is given by (2.14) and greatly exceeds $\nu_{\rm ei}$ under the conditions (2.16). In a hydrogen plasma $(\omega_{\rm Le}/\omega_{\rm Li} = 43)$ with an electron density $N_{\rm e} \approx 10^{21}$ cm^{-3}, the light of a neodymium laser $(\omega_0 = 1.78 \times 10^{15}$ sec$^{-1})$ is absorbed approximately ten times more strongly than as a result of electron-ion collisions $\nu_{\rm ei}$, if the electron temperature is of the order of 10 keV ($\kappa T_{\rm e} \approx 16$ keV), and the electric field intensity of the light wave E_0 exceeds by 20% the threshold value $E_{\rm thr} \approx 2 \times 10^8$ V/cm (in this estimate, the nonisothermy of the plasma is given by $T_{\rm e} \approx 12T_{\rm i}$). In such a plasma ($r_{\rm De} \approx 3 \times 10^{-6}$ cm) the wave number of the developed ion-sound oscillations $k_0 \approx r_{\rm De}^{-1}[2 \ln(\omega_{\rm Le}^2/\omega_{\rm Li}^2)]^{-1/2}$ exceeds the wave number $\tilde{k}$ of the light wave $(\sqrt{3}\,\omega_0 r_{\rm De}/c \ll 1)$ in accordance with the initial assumptions (see formula (1.1)).

We note that the considerable excess above $E_{\rm thr}$ for light pulses of nanosecond duration is already difficult to accomplish. To the contrary, in experiments[1,2] on the interaction of a plasma ($N_{\rm e} \approx 10^{11}$ cm^{-3}, $\kappa T_{\rm e} = 4$ eV) with an electromagnetic wave in the 10-cm band, the threshold field $E_{\rm thr} \approx 32$ V/cm (in accordance with formula (2.15)) is exceeded by approximately 10–100 times.

The formula (2.12) obtained above for the effective collision frequency is valid, strictly speaking, only in the near-threshold region, when $E_0^2 - E_{\rm thr}^2 \ll E_{\rm thr}^2$ (such an approximation was essentially used to solve the integral equation (1.11) for the spectral energy density $W_{\rm s}(\mathbf{k})$ of the ion-sound oscillations). Extrapolating (for estimating purposes) formula (2.14) for the maximum effective collision frequency to the case of fields E_0 far from threshold ($E_0^2 \gg E_{\rm thr}^2$, $\kappa T_{\rm e}$ is in eV and $N_{\rm e}$ is in cm^{-3}),

$$\frac{\nu_{\rm eff}}{\nu_{ei}}\sim 10^4\frac{T_e}{T_i}T_e^{3/2}N_e^{-1/2}\frac{E_0^2}{E_{\rm thr}^2},$$

we find that the absorption of a transverse electromagnetic wave of frequency $\omega_0 \approx 1.78 \times 10^{-10}$ sec^{-1} in a hydrogen plasma with an electron density $N_{\rm e} \approx 10^{11}$ cm^{-3} and with electron and ion temperatures $\kappa T_{\rm e} \approx 4$ eV and $\kappa T_{\rm i} \approx 0.2$ eV is 2–3 times more intense than absorption due to ordinary electron-ion collisions $\nu_{\rm ei}$, if $E_0 \approx 10$–$30\ E_{\rm thr}$. This estimate agrees with measurements of the absorption coefficient.[1,2] In this sense, it is vital to extend the theory developed above to include the case of fields E_0 much above threshold. On the other hand, a direct comparison of the experimental data with the characteristic dependences of the effective frequency (2.11) and (2.12) and of the absorption coefficient on the density $N_{\rm e}$, on the plasma particle temperatures $T_{\rm e}$ and $T_{\rm i}$, on the frequency deviation $(\omega_0 - \omega_{\rm p})/\omega_{\rm p}$, and on the field intensity E_0 near the threshold is presently made difficult by the lack of detailed experimental data on this subject.

From this point of view it is of interest to use the spectral energy density (1.21) obtained above for ion-sound oscillations to calculate the distribution function of the fast ions accelerated by the sound (within the framework of the quasilinear theory), since there are published experimental data[13] on the angular and energy distributions of the fast ions in a plasma produced when a solid target is heated with light from a laser (for nanosecond pulses). A comparison of the calculated and measured fast-ion distributions will make it possible not only to determine their origin, but also to "verify" to some degree the obtained solution (1.21). We plan to carry out such a comparison in the future. A direct application of formula (2.14) for $\nu_{\rm eff}$ can be the use of $\nu_{\rm eff}$ in the problem of the hydrodynamics of spreading of a hot ($\kappa T_{\rm e} > 10$ keV) laser plasma for the determination of the density profile of a plasma close to critical, $N_{\rm e} \approx 10^{21}$ cm^{-3}.

APPENDIX

SPECTRAL ENERGY DENSITY OF ION-SOUND OSCILLATIONS EXCITED BY A CONSTANT ELECTRON CURRENT

The anomalous high-frequency conductivity of the weakly-turbulent plasma, which is proportional to $\nu_{\rm eff}$, was determined in this paper in terms of the spectral energy density of ion-acoustic oscillations, which is the solution of the corresponding integral equation. It is possible to calculate in perfect analogy the anomalous resistance of the plasma to a direct current of electrons,[14] which is also proportional to the effective collision frequency. The exposition presented below is devoted to a solution of the integral equation for the spectral energy density of ion sound excited by a stream of electrons moving in a nonisothermal ($T_{\rm e} \gg T_{\rm i}$) plasma with constant and homogeneous velocity $\mathbf{u}$ exceeding the speed of sound $v_{\rm s}$. Such a problem of determining $W_{\rm s}(\mathbf{k})$ was considered in [11]. The nonlinear mechanism limiting the noise level will be assumed as before (and as in [11]) to be the induced scattering of the ion sound by the ions (see [14]). We have

$$\varkappa T_e = W_s(\mathbf{k}) \left\{ 1 - \frac{u}{v_s}\cos\theta + 2\frac{v_{Te}}{v_{Ti}} \int dk' \frac{W_s(\mathbf{k}')}{N_e \varkappa T_e} \right. \tag{A.1}$$
$$\left. \times \frac{k-k'}{kk'} \left(\frac{\mathbf{k}\mathbf{k}'}{kk'} \right)^2 \frac{[\mathbf{k}\mathbf{k}']^2}{|\mathbf{k}-\mathbf{k}'|^3} \exp\left[-\frac{1}{2}\frac{r_{De}^2}{r_{Di}^2}\frac{(k-k')^2}{(\mathbf{k}-\mathbf{k}')^2} \right] \right\}, \quad \theta \equiv \widehat{\mathbf{k}\mathbf{u}}.$$

This equation differs from the one used above (1.11) only in the form of the linear term in the right-hand side. Using the approximation (1.12), first employed in [11], and neglecting the small spontaneous term in the left-hand side of (A.1) at the threshold of the build-up of the ion sound ($\psi_0^2 \equiv (u/v_S - 1)^{-1} \gg 1$), we obtain an equation analogous to (1.14):

$$\left(\frac{1}{2}\psi^2 - 1 \right) + x \int_{x_1}^{x_2} x'^3 dx' \frac{\partial \delta(x'-x)}{\partial x'} \int_0^{\psi_1(x')} \psi' d\psi' (\psi^2 + \psi'^2) y(x', \psi') = 0. \tag{A.2}$$

Here the sought function $y(x, \psi)$ of the dimensionless wave number $x \equiv kr_{De}$ and of the angle $\psi = \theta\psi_0$ ($\theta \ll 1$) determines the dependence of $W_S(\mathbf{k})$ on the wave vector $\mathbf{k}$ (see (1.15)):

$$W_s(\mathbf{k}) = \frac{(2\pi)^{-3/2}}{2} \frac{r_{De}^2}{r_{Di}^2} \frac{\omega_{Li}}{\omega_{Le}} (N_e r_{De}^3) \psi_0^2 \varkappa T_e y(x, \psi).$$

The spectral density is determined in this case in the wave-number and angle intervals (1.17) and (1.18), and satisfies the condition (1.19).

Integrating in the second term of the left-hand side of (A.2) with respect to the wave numbers

$$\left\{ x x'^3 \delta(x'-x) \int_0^{\psi_1(x')} \psi' d\psi' (\psi^2 + \psi'^2) y(x', \psi') \right\}_{x'=x_1}^{x'=x_2} = 0, \tag{A.3}$$

we arrive at a pair of integral equations for $y(x, \psi)$ (C_1 and C_2 are integration constants):

$$\frac{\ln x}{2} - x^3 \int_0^{\psi_1(x)} \psi\, d\psi\, y(x,\psi) = C_1; \quad \ln x + x^3 \int_0^{\psi_1(x)} \psi^3 d\psi\, y(x,\psi) = C_2.$$

The solution of these equations can readily be found (it can be sought, for example, in the form $y(x, \psi) = y_1(x) P_1(\psi/\psi_1) + y_3(x) P_3(\psi/\psi_1)$, where P_1 and P_3 are the first and third Legendre polynomials) and determines $y(x, \psi)$ apart from the two unknown constants C_1 and C_2:

$$y(x,\psi) = \frac{15}{4} \frac{\ln x - 2C_1}{x^3 \psi_1^5(x)} \psi [\psi_1^2(x) - \psi^2], \quad \psi_1^2(x) = \frac{14}{3} \frac{C_2 - \ln x}{\ln x - 2C_1}. \tag{A.4}$$

The boundary conditions in (1.20) determine the constants in (A.4)

$$2C_1 = \ln x_1, \quad C_2 = \ln x_2$$

in terms of the end points of the wave-number interval (1.17). Equation (A.3) is then satisfied identically, and the sought spectral energy density of the ion-sound oscillations is (see (1.21))

$$W_s(\mathbf{k}) = \frac{15}{8} (2\pi)^{-3/2} \frac{r_{De}^2}{r_{Di}^2} \frac{\omega_{Li}}{\omega_{Le}} (N_e r_{De}^3) \varkappa T_e \frac{\ln(x/x_1)}{x^3 \psi_1^3(x)}$$
$$\times \psi [\psi_1^2(x) - \psi^2] \left(\frac{u}{v_s} - 1 \right)^{-1} \quad \psi_1(x) = \left\{ \frac{14}{3} \frac{\ln(x_2/x)}{\ln(x/x_1)} \right\}^{1/2};$$
$$x \equiv kr_{De}, \quad \psi \equiv \theta \left(\frac{u}{v_s} - 1 \right)^{-1/2}. \tag{A.5}$$

Thus, the non-equilibrium spectral energy density of the acoustic noise differs from zero (i.e., from the spontaneous noise) in the region of the (x, ψ) plane bounded by the curve $\psi = \psi_1(x)$ and by the straight lines $x = x_1$ and $\psi = 0$. In addition to having a different dependence on the wave number, the solution (A.5) differs from the spectral energy density (1.21) of sound excited by an electromagnetic wave (1.1) also in that the end points x_1 and x_2 of the wave-number interval (1.17) are not determined by the solution (A.5) itself, for to determine x_1 and x_2 it is necessary to advance additional physical considerations (see [11]).

The spectral density (A.5) of the ion sound excited by the electron current differs significantly from that obtained in [11] mainly in its angular dependence.[2)] Averaging the spectral density (A.5) over the angles

$$\int W_s(\mathbf{k}) \sin\theta\, d\theta \approx \frac{1}{\psi_0^2} \int_0^{\psi_1(x)} \psi W_s(\mathbf{k})\, d\psi = \frac{(2\pi)^{-3/2}}{4} \frac{r_{De}^2}{r_{Di}^2} \frac{\omega_{Li}}{\omega_{Le}} N_e r_{De}^3 \varkappa T_e \frac{\ln(x/x_1)}{x^3},$$

we obtain an expression that differs from that given in [11] by a numerical factor and in the logarithmic dependence ($\ln(x/x_1)$ in place of $\ln(x_2/x)$ in [11] with $x_2 \approx 1$). The difference between the logarithms does not, however, affect the total noise (over the spectrum) with density (A.5) (see (2.1)):

$$W \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} W_s(\mathbf{k}) = \frac{1}{8(2\pi)^{7/2}} \frac{r_{De}^2}{r_{Di}^2} \frac{\omega_{Li}}{\omega_{Le}} N_e \varkappa T_e \left(\ln\frac{x_2}{x_1} \right)^2. \tag{A.6}$$

Namely, the total noise (A.6) coincides (apart from a numerical factor) with the total noise that can be calculated with the aid of the angle-averaged spectral density.[11]

Thus, the method employed by us in the main body of the article to find the spectral energy density of ion-sound oscillations excited by the electromagnetic field (1.1), in addition to being more consistent, is perfectly suitable for determining the density of the acoustic noise excited by a constant electron current.

2) The author of [11] (see also the review [15]) solved an angle-averaged equation for the spectral density $W_S(\mathbf{k})$ in a "narrow cone of directions of $\mathbf{k}$ with aperture angle θ_0." This aperture angle θ_0, the order of magnitude of which was determined in [11], is equivalent to our function $\psi_1(x)$, which serves as the demarcation line between the regions of spontaneous and turbulent noise on the (k, θ) plane of the wave vector $\mathbf{k}$.

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