

THE QUESTION OF THE IDENTITY OF ELEMENTARY PARTICLES

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The problem of a continuous transition between the properties of a system of identical and nonidentical particles is considered. It is shown that in the presence of superpositions with respect to any internal quantum number, the continuous parameter of such a transition is the degree of nonorthogonality of these superpositions. A logically consistent scheme, which does not contradict any experimental facts, is described, within the framework of which the identity of the particles is approximate.

1. INTRODUCTION

THE concepts generally accepted in quantum mechanics about identical particles are in excellent agreement with experiment. At the same time they possess two interrelated distinctive features, which sharply distinguish them from the circle of other physical concepts. The object of the present work consists in an analysis of these distinctive features.

The first of these concerns the behavior of a system of particles—for simplicity we shall only speak of two particles—and may be formulated thus: upon a transition from two arbitrarily similar particles to rigorously identical particles, the behavior of the system changes discontinuously, i.e., the continuous convergence of the particles' properties is apparently not accompanied by a continuous change of the properties of the system.

If we restrict our attention to states of elementary particles which correspond to different values of certain discrete quantum numbers, such a situation is not paradoxical, since we do not know of any physically realizable method for continuously bringing the properties of such states together.

The situation is different in the case when the intrinsic states of the particles are described by superpositions of certain basis states and the continuous convergence being discussed is quite feasible: it simply reduces to the convergence of the coefficients of the corresponding superpositions. Naturally the question arises of the behavior of a system of two particles described by such superpositions, i.e., the question of the presence or absence of a discontinuity associated with coincidence of the states of the particles forming the system. The natural answer, which is given in Secs. 2 and 3, is the following: in connection with a continuous convergence of the coefficients of superposition, the properties of the system being analyzed continuously change into the properties of a system of identical particles.

In the specific case of degeneracy with respect to the different components of the spin, many of the relations given in Sec. 2 were known earlier. However, in the present article more general significance is attached to them, and they are considered, as far as we know, from a new point of view.

The second of the distinctive features mentioned above consists in the very initial ideas about the existence of identical particles since nowhere, except in quantum mechanics, do we encounter absolutely identical objects. In this connection it is of interest to ascertain whether it is possible to replace the strict identity of elementary particles by an approximate identity without disturbing the agreement of theory with all presently known experimental data.

One can extract certain leading indications from an analysis of the properties of neutral K mesons. For example, let us consider the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0 + n\pi$. Usually it is assumed that in all reaction events of this type exactly identical particles $K^0 = (K_1 + K_2)/\sqrt{2}$ are produced, independently of the energy of the initial π meson, the angles of flight of the secondary particles, etc. This is, of course, true if only the strong interactions are taken into consideration, where in the present case these play the decisively important role. However, strictly speaking it is also necessary to take the strangeness-nonconserving weak interactions into account, whose contribution depends on all of the kinematical factors listed above. The final result is that very similar but nevertheless not completely identical superpositions of K_1 and K_2 mesons are produced in different events, i.e., the identity of different specimens of the K mesons turns out not to be absolute.

In accordance with what has been said, in Sec. 4 it is proposed that the elementary particles of a definite type (for example, electrons) which we observe correspond to a superposition of two or of several subsidiary particles having very nearly the same masses, and the possibility of constructing an internally consistent scheme of such a type, which is in agreement with experiment, is indicated. It seems to us that the ideas developed here may at least claim a certain logical value. In any case, from them it follows that the traditional opinion about the absolute identity of elementary particles is not the only possibility from a logical point of view.

2. ELASTIC SCATTERING OF PARTICLES HAVING NONORTHOGONAL INTRINSIC STATES

For simplicity we assume that the potentials describing the interactions between two particles of type

A, two particles of type B, and between particles A and B, are identical, and we also assume that the amplitudes of the transitions $AA \rightarrow BB$, $AA \rightarrow AB$, and $BB \rightarrow AB$ vanish.

Now let two packets with momenta p and $-p$ collide and scatter, where the first packet is described by the superposition

$$|C\rangle^{(p)} = \alpha|A\rangle^{(p)} + \beta|B\rangle^{(p)}, \quad (1)$$

and the second packet is described by the superposition

$$|D\rangle^{(-p)} = \gamma|A\rangle^{(-p)} + \delta|B\rangle^{(-p)}. \quad (2)$$

From the normalization conditions it follows that

$$|\alpha|^2 + |\beta|^2 = 1, \quad |\gamma|^2 + |\delta|^2 = 1. \quad (3)$$

Thus, before scattering the state of the two particles has the form of the direct product of the vectors

$|C\rangle^{(p)}$ and $|D\rangle^{(-p)}$

$$|\Phi_0\rangle = (\alpha|A\rangle^{(p)} + \beta|B\rangle^{(p)}) \times (\gamma|A\rangle^{(-p)} + \delta|B\rangle^{(-p)}). \quad (4)$$

Let $f(\theta)$ be the amplitude for the scattering of non-identical particles, corresponding to the given interaction potential, and let θ be the angle between the vectors p and p' , where p and p' denote the momenta of one of the considered particles before and after scattering. Let us call this amplitude the amplitude of the direct process. It is well known (see, for example,^[1]) that the amplitude for the exchange process, in which either the initial or the final particles change places, satisfies the relation

$$f_{ex}(\theta) = \pm f(\pi - \theta). \quad (5)$$

Here the plus sign corresponds to particles with integer spin, and the minus sign corresponds to particles with integer spin, and the minus sign corresponds to particles with half-integer spin.

With (5) taken into consideration, the amplitude for the transition from the state $|\Phi_0\rangle$ to the state $|A\rangle^{(p')} \times |A\rangle^{(-p')}$ is given by

$$F_1 = \alpha\gamma(f(\theta) \pm f(\pi - \theta)). \quad (6)$$

For the amplitude of a transition from the state $|\Phi_0\rangle$ to the state $|B\rangle^{(p')} \times |B\rangle^{(-p')}$ we obtain the expression

$$F_2 = \beta\delta(f(\theta) \pm f(\pi - \theta)). \quad (7)$$

As to the amplitude for the transition from the state $|\Phi_0\rangle$ to the state $|A\rangle^{(p')} \times |B\rangle^{(-p')}$, it is equal to the sum of the amplitudes for transitions from the states $|A\rangle^{(p)} \times |B\rangle^{(-p)}$ and $|B\rangle^{(p)} \times |A\rangle^{(-p)}$ with the appropriate coefficients. These transitions differ by an exchange of the initial particles. Therefore, with (5) taken into consideration, we have

$$F_3 = \alpha\delta f(\theta) \pm \beta\gamma f(\pi - \theta). \quad (8)$$

Similarly, the amplitude for the transition from the state $|\Phi_0\rangle$ to the state $|B\rangle^{(p')} \times |A\rangle^{(-p')}$ is given by

$$F_4 = \beta\gamma f(\theta) \pm \alpha\delta f(\pi - \theta). \quad (9)$$

Now let us find the differential cross section for the scattering of superpositions of the states $|C\rangle$ and $|D\rangle$, summed over the four final states (the over-all

cross section). With the normalization condition (3) taken into account, after simple transformations we obtain

$$\frac{d\sigma(\theta)}{d\Omega} = \sum_{i=1}^4 |F_i|^2 = |f(\theta)|^2 + |f(\pi - \theta)|^2 \pm 2|\langle D|C\rangle|^2 \operatorname{Re} f(\theta)f^*(\pi - \theta), \quad (10)$$

where

$$\langle D|C\rangle = \alpha\gamma^* + \beta\delta^* \quad (11)$$

is a measure of the nonorthogonality of the states $|C\rangle$ and $|D\rangle$.

It is important to emphasize that in the assumed hypothesis about the nature of the interaction (the interaction does not depend on the type of particles, and the amplitudes of transitions involving a change of the internal quantum numbers are equal to zero) relation (10) is valid for any arbitrary number of basis states A, B, C, \dots (in this connection, see^[2]). From formula (10) it follows that if $\langle C|D\rangle = 0$, the states $|C\rangle$ and $|D\rangle$ behave like different particles. However, if the superpositions $|C\rangle$ and $|D\rangle = 0$ are identical ($\alpha = \gamma, \beta = \delta$), then $\langle C|D\rangle = 1$, and we obtain the well-known formula describing the scattering of identical particles. Thus, the quantity $\langle C|D\rangle$ is a continuous parameter characterizing the degree of difference of the superpositions.¹⁾

The considerations discussed above pertain, in particular, to the scattering of identical particles with non-zero spin in the case when the interaction potential does not depend on the spin. In this connection, states having a definite projection of the spin on the distinguished axis play the role of the particles A, B, \dots . In connection with this we note that for superpositions of arbitrary nature, one can introduce the concept of a generalized "spin" $s = (m - 1)/2$, where m is the number of basis states. The case when the interaction does not depend on the generalized "spin" is considered above. In the general case the amplitudes of the direct and exchange processes are represented by matrices in the "spin" space of the

¹⁾Evidently the degree of nonorthogonality of the intrinsic states $\langle C|D\rangle$ is a universal parameter characterizing the continuous transition from nonidentical particles to identical ones (also see [3]). In particular, this also pertains to processes involving the production of particles. In this connection, the example cited in article [4] is very instructive. One might understand the footnote on page 867 of this article as an assertion that upon coincidence of the states of the K_S and K_L mesons ($\langle K_S|K_L\rangle \rightarrow 1$) the probability for the production of a $K_S K_L$ pair in a p-wave nevertheless remains unequal to zero, in contrast to the case of identical bosons. At first glance this appears to be correct since the intrinsic wave function of a $K^0 \bar{K}^0$ pair with odd orbital angular momentum may be written in the form

$$|\Psi\rangle = \frac{|K_S^{(p)}\rangle \times |K_L^{(-p)}\rangle - |K_L^{(p)}\rangle \times |K_S^{(-p)}\rangle}{\sqrt{2(1 - |\langle K_S|K_L\rangle|^2)}}.$$

However, for nonorthogonal superpositions of K_S and K_L , it does not follow at all from this notation that the state $|\Psi\rangle$ corresponds to a $K_S K_L$ pair. It is easy to see that the probability for the detection of the state $|K_S\rangle$ by one of the detectors, and the state $|K_L\rangle$ by the other is given by

$$P = |\langle K_S^{(p)} \times K_L^{(-p)} | \Psi \rangle|^2 = 1/2(1 - |\langle K_S|K_L\rangle|^2).$$

If $\langle K_S|K_L\rangle \rightarrow 1$, the quantity $P \rightarrow 0$ as must occur for identical particles.

two packets, and the specific formulas become more complicated. It is essential, however, that in the case of the collision of identical superpositions, all of the transition amplitudes (corresponding both to elastic as well as to inelastic processes), independently of the "spin" structure of the interaction, are symmetric (for bosons) and antisymmetric (for fermions) with respect to interchange of the momentum of the packets before scattering (for more details, see^[2]). We note that in the case of identical superpositions the cross sections of the processes in general depend on just exactly how the superpositions participate in them. The case of "spin" independence considered above, constitutes an exception.

3. SCATTERING OF NONSTATIONARY SUPERPOSITIONS

Now let us consider collisions of nonstationary superpositions. Let us assume that we have two generators, in each of which both particles A and particles B can be produced, where the masses of particles A and B are similar but not identical. If the conditions of production are such that in principle it is impossible to determine from the state of the generator which of the particles—A or B—will be produced, then a packet of particles emitted in any arbitrary direction is represented by a nonstationary superposition of particles A and B of the type $\alpha_0|A\rangle + \beta_0|B\rangle$. The generation of such superpositions is possible if the duration of the process $\Delta t \ll \hbar/\Delta m c^2$, where Δm is the mass difference between particles A and B.^[2]

The coefficients α_0 and β_0 are related to the amplitudes for the production of particles A and B by the relations

$$\alpha_0 = \frac{f_A}{\sqrt{|f_A|^2 + |f_B|^2}} \quad \beta_0 = \frac{f_B}{\sqrt{|f_A|^2 + |f_B|^2}} \quad (12)$$

Now let the first generator produce the state $|C_0\rangle(p) = \alpha_0|A\rangle(p) + \beta_0|B\rangle(p)$, and the second generator—the state $|D_0\rangle(-p) = \gamma_0|A\rangle(-p) + \delta_0|B\rangle(-p)$ (p and $-p$ are the average momenta of the packets). In the region of scattering occurring at a distance R_1 from the first generator and at a distance R_2 from the second generator, the corresponding superpositions will have the form (1) and (2), where

$$\begin{aligned} \alpha &= \alpha_0 e^{im_A \tau_1}, & \beta &= \beta_0 e^{im_B \tau_1}, \\ \gamma &= \gamma_0 e^{im_A \tau_2}, & \delta &= \delta_0 e^{im_B \tau_2}. \end{aligned} \quad (13)$$

Here $\tau_1 = R_1/v\gamma$ and $\tau_2 = R_2/v\gamma$ are the proper times of flight of the packets $|C\rangle(p)$ and $|D\rangle(-p)$ from the generators to the point where the packets meet, v is the group velocity of the packets, and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor.^[6]

Substituting expressions (1) and (2) into (10) and taking (13) into consideration, we obtain the following formula for the over-all differential scattering cross section:

$$\begin{aligned} \frac{d\sigma(\theta)}{d\Omega} &= |f(\theta)|^2 + |f(\pi - \theta)|^2 \pm 2 \operatorname{Re} f(\theta) f^*(\pi - \theta) [|C_0|D_0\rangle|^2 \\ &- 4 \operatorname{Im} \alpha_0 \gamma_0^* \delta_0 \beta_0^* \exp\{i(m_A - m_B)(\tau_1 - \tau_2)\} \sin \frac{(m_A - m_B)(\tau_1 - \tau_2)}{2}]. \end{aligned} \quad (14)$$

The cross section for the scattering of two nonstationary superpositions $|C\rangle$ and $|D\rangle$ contains interference terms which depend on the difference between the proper times τ_1 and τ_2 corresponding to the different distances of the first and second generators from the point where the packets meet. This conclusion not only pertains to the differential cross section, but also to the total scattering cross section (see^[2]).³⁾

Now let us consider the case when identical superpositions arise in both generators, i.e., $\alpha_0 = \gamma_0$, $\beta_0 = \delta_0$. Then from formula (14) it follows that the over-all scattering cross section is

$$\frac{d\sigma(\theta)}{d\Omega} = |f(\theta) \pm f(\pi - \theta)|^2 \mp 8 |\alpha_0 \beta_0|^2 \sin^2 \frac{(m_A - m_B)(\tau_1 - \tau_2)}{2} \quad (15)$$

For $\Delta m |\tau_1 - \tau_2| \ll 1$ one can neglect the last term in (15), and we obtain the formula describing the scattering of identical particles.

Thus, if the difference in the masses of particles A and B is very small, then even for a large value of the difference between the proper times the nonstationary states, each of which has the form $|C_0\rangle = \alpha_0|A\rangle + \beta_0|B\rangle$ at the moment of generation, behave like identical particles. This conclusion remains valid even in the case of an arbitrary dependence of the interaction on the "spin".⁴⁾

Let us now assume that owing to some kind of dynamical reasons, one and the same (or almost one and the same) superposition $|C_0\rangle$ of the particles A and B having very similar masses is always produced in the generation processes. From the foregoing analysis it follows that for times $\tau \ll 1/\Delta m$ experimental investigation of the interactions of nonstationary states with each other and with other particles (and also the study of the static properties of an aggregate of the states $|C_0\rangle$) does not enable us to answer the question as to whether $|C_0\rangle$ is an elementary particle in the generally accepted sense of this word, or whether two types of particles exist (the superposition $|C_0\rangle$ and a superposition $|E_0\rangle$ which is orthogonal to it) which can undergo transitions into one another for $\tau > 1/\Delta m$.

Let us illustrate this situation by the example of neutral K mesons. It is well known that two types of neutral K mesons exist— K^0 and \bar{K}^0 . At sufficiently low energies only K^0 mesons are produced as a result of collisions of particles possessing zero strangeness. If we were to confine the investigation to only these processes, where the mass difference between the K_2

³⁾We note that if the detection of particles after scattering is carried out with the aid of detection filters, registering not the stationary states A and B themselves but certain superpositions of them, then the probability of detection not only depends on the quantity $\tau_1 - \tau_2$, but also on the difference between the proper times τ_1' and τ_2' corresponding to the distances of the first and second detectors from the scattering region (see^[6,7]).

⁴⁾In the general case the cross section for collisions of nonstationary superpositions not only depends on the difference, but also depends on the sum $(\tau_1 + \tau_2)$ of the proper times.

²⁾In this connection it is assumed that the production of the superpositions is not forbidden by superselection rules, i.e., the particles have identical electric charges, baryon charges, and so forth. [⁵] Neutral K mesons may serve as an example; they correspond to nonstationary superpositions of K_1 and K_2 .

and K_1 mesons (playing the role of the A and B particles) would be a vanishingly small quantity, then we would not have any indications of the existence of the second neutral K meson (\bar{K}^0). The difficulty of observing \bar{K}^0 would increase in connection with the so-called coherent regeneration, for at a sufficiently small value of Δm the latter effect leads to the result that inside matter—in contrast to the situation in vacuum—the stationary states become not K_1 and K_2 but K^0 and \bar{K}^0 . However, in fact processes are known in which \bar{K}^0 mesons are produced, and furthermore the difference in the masses of K_1 and K_2 is large enough to observe the transition $K^0 \rightarrow \bar{K}^0$ both in vacuum as well as in a dense medium. Below we shall attempt to develop a scheme, in the framework of which nonstationary states of another type $|E_0\rangle$, analogous to \bar{K}^0 mesons, exist together with the states $|C_0\rangle$, and moreover it is difficult to observe these states experimentally both in production processes as well as in secondary interactions, even during the course of a long observation time.

4. DOUBLING OF THE UNIVERSE. STERILE PARTICLES

Now we may precisely formulate the hypothesis presented at the end of Sec. 1.

1. Let us assume for definiteness that two types of electrons exist, $|C_0\rangle$ and $|E_0\rangle$, where $\langle C_0|E_0\rangle = 0$.⁵⁾ However, in our real universe it is assumed that only the electrons $|C_0\rangle$ participate in the different processes. As to the electrons $|E_0\rangle$, to a high degree of accuracy they are sterile with regard to all of the interactions known to us. Thus, in the representation of the states $|C_0\rangle$ and $|E_0\rangle$ the potential describing the interaction of electrons with the particles of our universe (and the corresponding scattering amplitudes) can be written in matrix form

$$V = V_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (16)$$

If an electron is created or annihilated (the question may concern β decay, electron-positron annihilation $e^+e^- \rightarrow \gamma\gamma$, and so forth), the amplitude of the corresponding process has the structure

$$W = W_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad W' = W'_0 (10),$$

i.e., the amplitude for the creation or annihilation of an electron $|E_0\rangle$ associated with the interaction with the particles of our universe vanishes (to within terms of the order of the "superweak interaction," which is discussed below).

2. Let us further assume that $|C_0\rangle$ and $|E_0\rangle$ correspond to nonstationary orthogonal superpositions of certain particles A and B having definite masses:

$$|C_0\rangle = \alpha|A\rangle + \beta|B\rangle, \quad |E_0\rangle = \beta^*|A\rangle - \alpha^*|B\rangle. \quad (17)$$

Let the masses of particles A and B be equal, respectively, to m_A and m_B , where the mass difference $\Delta m = |m_A - m_B|$ is very small. Then in the repre-

sentation of the states $|C_0\rangle$ and $|E_0\rangle$ the mass operator has the following form:

$$\hat{M} = \begin{pmatrix} m + \frac{1}{2}\Delta m(|\alpha|^2 - |\beta|^2) & \alpha\beta\Delta m \\ \alpha^*\beta^*\Delta m & m - \frac{1}{2}\Delta m(|\alpha|^2 - |\beta|^2) \end{pmatrix}. \quad (18)$$

We assume that the mass difference between particles A and B is due to a hypothetical "superweak interaction" which mixes the states $|C_0\rangle$ and $|E_0\rangle$. It is precisely the presence of the indicated "superweak interaction" which leads to the result that the states $|C_0\rangle$ and $|E_0\rangle$, defined above, turn out to be nonstationary. In the absence of the "superweak interaction" the masses of the electrons $|C_0\rangle$ and $|E_0\rangle$ would coincide and would be equal to m .

3. The question arises, how can the electrons $|C_0\rangle$ and $|E_0\rangle$ have different masses in the absence of the "superweak interaction" if we postulate that $|E_0\rangle$ does not interact with any of the particles known to us? In order to resolve this paradox within the framework of the field-theoretic concept of mass, it is necessary to assume that the doubling of the electrons implies a doubling of all of the particles with which the electrons interact. We arrive at the concept of two universes, where the particles of each are "sterile" with respect to the particles of the other. However, the particles which are sterile with respect to our universe, like the particles of our universe, interact among themselves. In this connection the mirror universe is quite identical to ours, i.e., double degeneracy occurs.⁶⁾

4. The "superweak interaction" between the electrons, which we postulated above, intermixes the states $|C_0\rangle$ and $|E_0\rangle$, removing the degeneracy in the mass. For the remaining particles, within the framework of the scheme under consideration such a "superweak interaction" may have another value, larger or smaller.

Let us consider the specific case when the diagonal matrix elements of the "superweak interaction" are equal to zero, i.e., when complete intermixing of the states $|C_0\rangle$ and $|E_0\rangle$ occurs. In this connection, in formula (18) one should set⁷⁾ $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$. Then the mass matrix will have the form:

$$\hat{M} = \begin{pmatrix} m & \frac{\Delta m}{2} e^{i\varphi} \\ \frac{\Delta m}{2} e^{-i\varphi} & m \end{pmatrix}. \quad (19)$$

Since the states $|C_0\rangle$ and $|E_0\rangle$ are defined to within a phase, without any loss of generality we may set $\varphi = 0$. In this connection, definite values of the mass correspond to the states

$$|A\rangle = \frac{|C_0\rangle + |E_0\rangle}{\sqrt{2}}, \quad |B\rangle = \frac{|C_0\rangle - |E_0\rangle}{\sqrt{2}}. \quad (20)$$

⁵⁾ A doubling of the particles in another connection has been repeatedly discussed in the literature (see, for example, [8,9]). It should be noted that in all such schemes the actual existence of the mirror universe is not at all suggested. The mirror particles, can, of course, be produced in collisions of "our" particles, but with a smaller probability than the weak "superweak interaction" under consideration.

⁷⁾ If the nondiagonal matrix elements of the "superweak interaction" are much smaller than the difference between the diagonal elements, the $\beta \ll \alpha$, and the stationary states $|A\rangle$ and $|B\rangle$ are almost the same as the states $|C_0\rangle$ and $|E_0\rangle$.

⁵⁾ We are discussing, of course, electrons with identical projections of the spins along a specified axis.

The state $|C_0\rangle$ is nonstationary, and in the course of the proper time τ changes according to the law:

$$|C\rangle_\tau = [|A\rangle e^{-im_A\tau} + |B\rangle e^{-im_B\tau}] / \sqrt{2}. \quad (21)$$

Thus, during a time interval $\tau \sim 1/\Delta m$ an ordinary electron changes into a "sterile" electron.

5. In the representation of the stationary states A and B, the potential for the interaction of the electrons with the particles of our universe has the following matrix structure:

$$V = \frac{1}{2} V_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (22)$$

and the amplitudes for the creation and annihilation of electrons have the structure

$$W = \frac{W_0}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad W' = \frac{W'_0}{\sqrt{2}} (1 \ 1). \quad (23)$$

From here it immediately follows that the charge operator of the electron has the form

$$\hat{e} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (24)$$

i.e., the intermixing interaction does not conserve charge.⁸⁾ The eigenstates of the charge operator are $|C_0\rangle$ (charge 1) and $|E_0\rangle$ (charge zero). A "sterile" electron possesses a "charge" $e' = 1$, which describes its interaction with "sterile" photons. In the representation of the particles A and B

$$\hat{e}' = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (25)$$

In this connection it is easy to see that for all superpositions of A and B

$$\hat{e} + \hat{e}' = 1, \quad \hat{e}'^2 + \hat{e}^2 = 1. \quad (26)$$

The first of these relations indicates that the "total" charge is conserved, and the second corresponds to the equality of the field masses of both types of electrons in the absence of the "superweak interaction."

The average charge of an electron, existing in vacuum, must change in accordance with (21) according to the following law:

$$e(\tau) = \langle C_\tau | \hat{e} | C_\tau \rangle = \cos^2 \Delta m \tau. \quad (27)$$

The amplitudes for the scattering of electrons by protons or by nuclei should change during the course of time according to the same law. However, during time intervals $\tau \ll 1/\Delta m$ in principle we cannot observe these variations.

6. Earlier we talked about the interaction of the electrons with other particles. In the representation of the states $|C_0\rangle$ and $|E_0\rangle$, the amplitude for the scattering of two electrons one on the other has the following structure:

$$\hat{F}(\theta) = \frac{1}{2} [f(\theta) - f(\pi - \theta)] \left[I + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{(1)} \times \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{(2)} \right], \quad (28)$$

where I is the unit operator in the "spin space" of the two particles 1 and 2.

In the representation of the stationary states A and B the same scattering amplitude has the structure

$$\hat{F}(\theta) = \frac{1}{2} [f(\theta) - f(\pi - \theta)] \left[I + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{(1)} \times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{(2)} \right]. \quad (29)$$

Within the framework of the present scheme, our instruments only detect the state $|C_0\rangle$; the sterile particles do not interact with them. In view of this the measured scattering cross section, as one can easily see, will be proportional to the quantity

$$\frac{d\sigma(\theta)}{d\Omega} = \langle C_0 C_0 | \hat{F}(\theta) | C_\tau C_\tau \rangle = |f(\theta) - f(\pi - \theta)|^2 \cos^2 \Delta m \tau_1 \cos^2 \Delta m \tau_2, \quad (30)$$

where τ_1 and τ_2 are the proper times which have elapsed since the moments of production of the two packets.

Thus, under the assumptions we have adopted, for all values of τ_1 and τ_2 the electrons are scattered one by the other like identical particles, but their interaction with each other and with the other particles changes with the passage of time. If $\Delta m \leq 1/\tau_1, 1/\tau_2$, these changes are experimentally unobservable. In this case the scheme under consideration leads to the same results as the usual theory, even though it is based on an assumption about the existence of two types of electrons.

7. We note that formula (19) determines the elements of the mass matrix in vacuum. However, since the interaction of the electrons with a medium or with an external electromagnetic field has the structure (16), it is clear that if the quantity Δm is much smaller than the energy of interaction of the state $|C_0\rangle$ with an external field or with the medium, then the stationary states having definite effective masses are not particles A and B but the states $|C_0\rangle$ and $|E_0\rangle$ (compare with the comment about coherent regeneration in the last paragraph of Sec. 3). Since under real conditions there is always some kind of field and some kind of medium, then for sufficiently small values of Δm even the change considered above in the electron's properties vanishes, with the passage of time, i.e., the described scheme actually does not differ at all from the conventional one. In particular, all of the electrons in atoms are found in the state $|C_0\rangle$; the admixture of $|E_0\rangle$ electrons amounts to a quantity of the order of Δm in atomic units.

8. If the general theory of relativity is valid, the "sterile" particles must interact with the particles of our universe by means of the gravitational field.^[9] In contrast to (16), for all particles the structure of the gravitational interaction must have the form

$$V = V_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad \text{This means that within the framework}$$

of the scheme which has been developed, processes involving the participation of gravitons should go differently than according to the usual theory. In particular, the total cross section for the production of electron-positron pairs by gravitons must exceed by two times the value which the usual theory gives. We note, however, that the cross section for the production of "our" pairs coincides with the usual result.

5. CONCLUSION

The case of two basis states for the electrons has been considered above. One could carry out similar

⁸⁾We emphasize in this connection that the extent of the charge nonconservation is slight, and it is likely that the small photon mass [10,11] associated with the presence of this charge nonconservation does not contradict the current experimental data.

arguments for three or for a larger number of basis states. Accordingly we would arrive at the concept of a "tripling" of the universe, its "quadrupling," and so forth. In connection with such an approach, of the N forms of the basic particles only one is realized in our universe; all remaining particles turn out to be "sterile."

Here it is appropriate to especially stress the following: to no extent do we assert that the developed scheme is actually realized, or that any kind of experimental facts are in its favor; on the contrary the usual ideas about the absolute identity of all electrons, of all protons, etc. immediately appear in many respects to be more natural. It is, however, worth calling attention to the fact that these ideas, in spite of the traditional assertion, cannot be regarded as uniquely proven.⁹⁾

In conclusion let us estimate an upper limit for the value of Δm which would follow from presently known experimental facts under the assumption that the described scheme corresponds to reality. It appears to us that the best way is to start from the following direct estimate. One can regard it as established that particles, under the conditions of a sufficiently good vacuum, traverse a path of the order of one kilometer without any noticeable change of their properties. Hence the period of the spatial pulsations is a large quantity, which leads to the estimate

$$(\Delta mc^2) \ll 10^{-10} \text{ eV.}$$

If it is assumed that photons have a mass $\sim \Delta m$, then on the basis of the arguments contained in^[10,11] a more stringent indirect estimate is possible:

$$(\Delta mc^2) \ll 10^{-15} \text{ eV.}$$

⁹⁾ It is of interest to note that E. Fermi held a similar point of view.
^[12] We thank Academician B. M. Pontecorvo, who called our attention to article [12].

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