CHARGE EXCHANGE OF A NEGATIVE ION IN A COLLISION WITH A HIGH-ENERGY PROTON

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Charge exchange of a negative ion with a proton is considered at proton energies $E \gtrsim 10$ keV. In the case of a negative hydrogen ion, the initial and final states of the electrons are described with the aid of symmetrized electronic functions. For all other negative ions, a two-parameter single-electron wave function is used to describe the bound state. The cross sections for the capture of an electron by a proton in states characterized by a principal quantum number n are calculated. The four-dimensional symmetry of the wave functions of the hydrogen atom is used in summing over the orbital and magnetic quantum numbers.

1. INTRODUCTION

We calculate in the present paper the cross section for the charge exchange of a negative ion colliding with a proton of energy $E \gtrsim 10$ keV. We consider here the capture of an electron by a proton in a state characterized by a principal quantum number n:

$$A^- + p \to A + \mathrm{H}(n). \tag{1}$$

The main role in the formation of the bound state of a negative ion is played by the exchange interaction of the electrons. Therefore, generally speaking, it is necessary to use many-electron functions to describe the negative ion that takes part in the collision. In the general case of collision of a negative ion, such a problem is quite complicated. This raises the question of the possibility of simplifying the problem by using the single-electron approximation. The possibility of using the single-electron approximation can be assessed by considering the charge exchange of a negative hydrogen ion with a proton using the symmetrized two-electron wave functions[1)]. Such a problem was formulated within the framework of the Born approximation, by Drukarev and Rokotyan[1]. In Sec. 2 of this article we consider the collision

$$\mathrm{H}^- + \mathrm{H}^+ \to \mathrm{H}(n') + \mathrm{H}(n) \tag{2}$$

within the framework of the impact-parameter method. It turns out that capture of a weakly-bound electron plays the principal role at collision velocities $v \lesssim 1$, in the region where the cross section reaches the maximum value. This means that at such velocities the one-electron approximation is justified. Section 3 of the article considers the case of collision of any negative ion with a proton (1) in the one-electron approximation; the weakly-coupled electrons is described by a two-parameter wave function.

Process (1) was considered in the case of low collision energies in[2]. The Landau-Zener formula was used for the transition probability at the point of quasi-intersection of the terms.

The cross section of the process (1) at energies $E \gtrsim 10$ keV will be calculated in the Brinkmann-Kramers (BK) approximation[3] using the impact-parameter method. In the indicated energy interval, the incoming proton and the atomic core of the negative ion can be regarded as classical particles, and their de Broglie wavelength is much smaller than the characteristic dimension or the effective radius of the atomic forces. Excluding the case of frontal collision, which makes a small contribution to the cross section, the trajectories of the colliding particles will be assumed linear.

The BK approximation was used earlier to calculate the charge-exchange cross sections in collisions between positive ions and atoms. This resulted in good agreement at high energies and in a qualitatively correct behavior of the cross section in the region of its maximum. Introduction of the term resulting from the non-orthogonality of the initial and final states has made it possible to improve the agreement with experiment[4,5]. In the present paper we use the initial variant of the BK approximation, since it gives the correct behavior of the cross section at high energies and describes qualitatively correctly the course of the cross section at medium energies. In addition, it makes it possible to carry out all the calculations in analytic form.

2. CHARGE EXCHANGE BETWEEN A NEGATIVE HYDROGEN ION AND A PROTON

The charge-exchange cross section calculated in the BK approximation by the impact-parameter method is

$$\sigma_{nn'}(v) = 2\pi \int_0^\infty \rho P_{nn'}(\rho, v)\, d\rho, \tag{3}$$

$P_{nn'}(\rho, v)$ is the probability, which depends on the impact parameter, that one electron of H^- is captured in a state with a principal quantum number n, and the other goes over into the state n′ of the H atom:

[1)] The correlation of the electrons is not taken into account in the wave function for the negative hydrogen ion, and therefore the variables can be separated.

$$P_{nn'}(\rho, v) = |b_{n;\,n'}|^2 = \sum_{l,\,m} |b_{n,\,l,\,m;\,n'}|^2. \tag{4}$$

The sum extends over all the values of the quantum numbers l and m corresponding to given n. The amplitude of the charge-exchange probability $b_{n,l,m;n'}$ is calculated in the BK approximation by the formula

$$b_{n,\,l,\,m;\,n'} = \int_{-\infty}^{+\infty} dt \exp(-i\Delta t)\, V_{if}(t), \tag{5}$$

where $V_{if}(t)$ is the matrix element of the transition, equal to

$$V_{if}(t)\exp(-i\Delta t) = -\int dr_1^3\, dr_2^3 \chi_f^*(\mathbf{r}_1, \mathbf{r}_2, t)\left(\frac{1}{r_{1B}} + \frac{1}{r_{2B}}\right)\chi_i(\mathbf{r}_1, \mathbf{r}_2, t). \tag{6}$$

Here χ_i is the symmetrized wave function of the initial state, χ_f the symmetrized wave function of the final state, $\mathbf{r}_{iA}$, $\mathbf{r}_{iB}$, and $\mathbf{r}_i$ are the coordinates of the i-th electron reckoned respectively from the nucleus of the negative ion, from the incident proton, and from their geometric center.

Choosing the coordinate system such that the geometric center is at rest at the origin, we have

$$\chi_i = \Phi_i(\mathbf{r}_{1A}, \mathbf{r}_{2A})\exp\left\{\frac{i}{2}(\mathbf{v}\mathbf{r}_1 + \mathbf{v}\mathbf{r}_2) - i\left(E + \frac{v^2}{2}\right)t\right\}, \tag{7}$$

where

$$\Phi_i(\mathbf{r}_{1A}, \mathbf{r}_{2A}) = N\sqrt{\frac{\alpha^3\beta^3}{\pi^2}}\,\{\exp[-(\alpha r_{1A} + \beta r_{2A})] + \exp[-(\alpha r_{2A} + \beta r_{1A})]\}, \tag{8}$$

where Φ_i is the Chandrasekhar function for H^- with variational parameters α = 1039 and β = 0.283; v is the vector of the proton velocity relative to the negative ion; E is the total energy of H^-; N is a normalization factor, equal to $2^{-1/2}[1 + 64\alpha^3\beta^3/(\alpha+\beta)^6]^{-1/2}$, and

$$\chi_f = 2^{-1/2}\Big[\varphi_{n',l',m'}(\mathbf{r}_{1A})\,\varphi_{n,l,m}(\mathbf{r}_{2B}) \times \exp\left\{\frac{i}{2}(\mathbf{v}\mathbf{r}_1 - \mathbf{v}\mathbf{r}_2) - i\left(\varepsilon_n + \varepsilon_{n'} + \frac{v^2}{4}\right)t\right\} + (1 \leftrightarrow 2)\Big]. \tag{9}$$

Here $\varphi_{n,l,m}$ is the hydrogen function and ϵ_n the energy of the hydrogen state.

Substituting the expression for the wave functions (7) and (9) in (6), we obtain for the matrix element of the transition an expression consisting of four terms:

$$V_{if}(t) = -\sqrt{2}N\{I(\alpha, 1, n')V(\beta, 0, n) + I(\alpha, 0, n')V(\beta, 1, n) + I(\beta, 1, n')V(\alpha, 0, n) + I(\beta, 0, n')V(\alpha, 1, n)\}, \tag{10}$$

where

$$I(\alpha, k, n) = \sqrt{\frac{\alpha^3}{\pi}}\int d^3r \exp(-\alpha r_A)\frac{1}{r_B^k}\varphi_{n,l,m}(\mathbf{r}_A), \tag{11}$$

and

$$V(\alpha, k, n) = \sqrt{\frac{\alpha^3}{\pi}}\int d^3r \exp(-\alpha r_A + i\mathbf{v}\mathbf{r})\frac{1}{r_B^k}\varphi_{n,l,m}(\mathbf{r}_A), \tag{12}$$

$$\Delta = E - \varepsilon_n - \varepsilon_{n'}. \tag{13}$$

The first and terms in (10) lie outside the framework of the BK approximation, since they result from the non-orthogonality of the functions of the initial and final states. Neglecting these terms, we obtain for the probability amplitude

$$b_{n,l,m;n'} = -\sqrt{2}N\Big\{I(\alpha, 0, n')\int_{-\infty}^{+\infty} dt \exp(-i\Delta t)\, V(\beta, 1, n) + I(\beta, 0, n')\int_{-\infty}^{+\infty} dt \exp(-i\Delta t)\, V(\alpha, 1, n)\Big\}. \tag{14}$$

Thus, the probability of capture of one of the H^- electrons in a state with n and the transition of the remaining electron into a state with n′ is expressed in the form of a sum of two terms:

$$P_{nn'} = \sum_{l,m}|b_{n,l,m;n'}|^2 = 2|N|^2\Big\{|I(\alpha, 0, n')|^2 \times \sum_{l,m}\Big|\int_{-\infty}^{+\infty} dt \exp(-i\Delta t)\, V(\beta, 1, n)\Big|^2 + |I(\beta, 0, n')|^2\sum_{l,m}\Big|\int_{-\infty}^{+\infty} dt \exp(-i\Delta t)\, V(\alpha, 1, n)\Big|^2\Big\}, \tag{15}$$

the first of which describes the capture of a weakly-bound electron (β = 0.283), and the second the capture of an internal electron. Substituting in (11) the wave function of the homogeneous state, we obtain

$$I(\alpha, 0, n') = \frac{8\sqrt{\alpha^3}(\alpha - 1)}{n'^{3/2}(\alpha - n'^{-2})^2}\left(\frac{\alpha - 1/n'}{\alpha + 1/n'}\right)^{n'}.$$

We have, for example, for n′ = 1

$$|I(\alpha, 0, 1)|^2 \approx 1, \quad |I(\beta, 0, 1)|^2 \approx 0.33.$$

For n′ ≠ 1, all $|I(\alpha, 0, 1)|^2$ can be regarded as equal to zero. From this it follows that in charge exchange in which after collision there is a slow excited hydrogen atom, the principal role is played by capture of an internal electron.

In order to estimate which process plays the principal role in a charge exchange after which the hydrogen remains in the ground state (n′ = 1), let us calculate the charge-exchange cross section. We substitute (15) in (3) and obtain

$$\sigma_{nn'}(v) = 4\pi|N|^2\{|I(\alpha, 0, n')|^2 Q_n^\beta + |I(\beta, 0, n')|^2 Q_n^\alpha\},$$

$$Q_n^\alpha = \int_0^\infty \rho\, d\rho \sum_{l,m}\Big|\int_{-\infty}^{+\infty} dt \exp(-i\Delta t)\, V(\alpha, 1, n)\Big|^2. \tag{16}$$

The matrix element of the transition $V(\alpha, 1, n')$ can be represented in the form

$$V(\alpha, 1, n') = \left(\frac{\alpha}{2}\right)^{5/2}\frac{(2\pi)^{3/2}}{2n^2}\int d^3p\, \frac{(1 + n^2p^2)\,u^*_{n,l,m}(\mathbf{p})\exp(i\mathbf{p}\mathbf{R} - i\mathbf{R}\mathbf{v}/2)}{(|\mathbf{p} - \mathbf{v}|^2 + \alpha^2)^2} \tag{17}$$

by using the Fourier transformation

$$\exp(-\alpha r) = \frac{\alpha}{\pi^2}\int\frac{\exp(-i\mathbf{p}\mathbf{r})}{(p^2 + \alpha^2)^2}d^3p, \tag{18}$$

$$\frac{1}{r}\varphi_{n,l,m}(\mathbf{r}) = \frac{1}{2n^2(2\pi)^{3/2}}\int(1 + n^2p^2)\,u_{n,l,m}(\mathbf{p})\exp(-i\mathbf{p}\mathbf{r})\,d^3p \tag{19}$$

and taking into account the geometry of the system

$$\mathbf{r}_A = \mathbf{r} + {}^1\!/_2\mathbf{R}, \quad \mathbf{r}_B = \mathbf{r} - {}^1\!/_2\mathbf{R}.$$

In the approximation where the flight is linear, $\mathbf{R} = \boldsymbol{\rho} + \mathbf{v}t$, $\boldsymbol{\rho}\mathbf{v} = 0$, we obtain

$$\int_{-\infty}^{+\infty}\exp(-i\Delta t)\, V(\alpha, 1, n')\,dt = \frac{(2\alpha)^{5/2}}{2n^2}\int d^3p\,\frac{\delta(\mathbf{v}\mathbf{p} - {}^1\!/_2v^2 - \Delta)\,(1 + n^2p^2)\,u^*_{n,l,m}(p)\exp(i\boldsymbol{\rho}\mathbf{p})}{(|\mathbf{p} - \mathbf{v}|^2 + \alpha^2)^2}. \tag{20}$$

Ratio of the quantities $|I(\alpha, 0, 1)|^2 Q_n^\beta / |I(\beta, 0, 1)|^2 Q_n^\alpha$ at different n and v

v	n				
	1	2	3	4	5
0,3	$4{,}4\cdot10^{-3}$	$5{,}6\cdot10^{-1}$	5,2	18,5	24,6
0,5	$3{,}5\cdot10^{-2}$	$8{,}1\cdot10^{-1}$	2,8	3,5	7,2
0,8	$3{,}6\cdot10^{-2}$	$4{,}1\cdot10^{-1}$	$8{,}1\cdot10^{-1}$	1,1	1,2
1,0	$9{,}4\cdot10^{-2}$	$4{,}0\cdot10^{-1}$	$3{,}8\cdot10^{-1}$	$1{,}7\cdot10^{-1}$	$1{,}9\cdot10^{-1}$
2,0	$3{,}1\cdot10^{-2}$	$1{,}7\cdot10^{-1}$	$6{,}3\cdot10^{-2}$	$6{,}6\cdot10^{-2}$	$6{,}6\cdot10^{-2}$

The summation of the squares of the moduli of the last expression can be carried out by using the four-dimensional symmetry of the wave functions of hydrogen (see [8]). We ultimately obtain for the cross section (see the Appendix)

$$Q_n^\alpha = \frac{2^8\alpha^5\pi}{v^2 n^3}\left[\frac{1}{3\varepsilon_\alpha^6(\varepsilon_n^2-\varepsilon_\alpha^2)^2} - \frac{1}{\varepsilon_\alpha^4(\varepsilon_n^2-\varepsilon_\alpha^2)^3} + \frac{3}{\varepsilon_\alpha^2(\varepsilon_n^2-\varepsilon_\alpha^2)^4} + \frac{1}{\varepsilon_n^2(\varepsilon_n^2-\varepsilon_\alpha^2)^4} - \frac{4}{(\varepsilon_n^2-\varepsilon_\alpha^2)^5}\ln\frac{\varepsilon_n^2}{\varepsilon_\alpha^2}\right], \quad (21)$$

where

$$Q_n^\beta = Q_n^\alpha(\alpha\to\beta),\ \varepsilon_\alpha^2 = \left(\frac{\Delta}{v}+\frac{v}{2}\right)^2 + \alpha^2 - 2\Delta,\ \varepsilon_n^2 = \left(\frac{\Delta}{v}+\frac{v}{2}\right)^2 + \frac{1}{n^2}.$$

Comparing the quantities $|I(\alpha, 0, 1)|^2 Q_n^\beta$ and $|I(\beta, 0, 1)|^2 Q_n^\alpha$, we can see that at high collision velocities $v \gtrsim 1$ capture of the internal electron predominates, in agreement with the results of [1] (see the table). At lower collision velocities, capture of the external weakly-bound electron in a state with principal quantum number $n \geq 2$ predominates. This fact enables us to conclude that in the case of charge exchange of negative ions with protons one can use the single-electron approximation for an external weakly-bound electron at velocities $v \lesssim 1$.

3. CHARGE EXCHANGE OF NEGATIVE ION WITH PROTON

We calculate the cross section $\sigma_n(v)$ of the process (1) in the single-electron approximation, assuming that the active electron is an external weakly-bound electron. In the BK approximation using the impact-parameter method, $\sigma_n(v)$ is equal to

$$\sigma_n(v) = 2\pi\int_0^\infty \rho P_n(\rho, v)\, d\rho, \quad (22)$$

where $P_n(\rho, v)$ is the probability, which depends on the impact parameter, of the charge exchange of the external electron of the negative ion with capture into the state of the hydrogen atom with principal quantum number n:

$$P_n(\rho, v) = |b_n|^2 = \sum_{l,m} |b_{n,l,m}|^2. \quad (23)$$

The sum extends over all values of the quantum numbers l and m corresponding to the given n. The amplitude of the charge-exchange probability in the BK approximation, $b_{n,l,m}$, is calculated from the formula

$$b_{n,l,m} = \int_{-\infty}^{+\infty} dt \int d^3r\, \varphi_{n,l,m}(\mathbf{r}_B)\, V(\mathbf{r}_B)\, \varphi_\gamma(\mathbf{r}_A) \exp(i\mathbf{v}\mathbf{r} + i\Delta t); \quad (24)$$

here $\varphi_{n,l,m}$ is the electronic wave function of the hydrogen; φ_γ is the wave function of the external electron of the negative ion; $V(r) = -1/r$ is the potential of the interaction between the proton and the external electron of the negative ion; v is the relative velocity of the colliding negative ion and the proton; $\Delta = E_n - E_\gamma$ is the difference between the binding energies of the electron in the final and initial states.

We choose as the wave function of the external electron of the negative ion

$$\varphi_\gamma = C(\beta, \gamma)\frac{1}{r}[\exp(-\gamma r) - \exp(-\beta r)],\quad \gamma < \beta, \quad (25)$$

where $C(\beta, \gamma)$ is a normalization factor. Such a function has the correct asymptotic behavior at large r, if γ is chosen from the condition $\gamma = \sqrt{-2E_\gamma}$, and behaves sufficiently well at small r, if β is chosen from a comparison with the more exact variational functions. In order of magnitude, β^{-1} is equal to the operating radius of the atomic forces (see [6]).

To calculate the cross section (22), we proceed in the same manner as in Sec. 2. Changing over to Fourier transforms for the wave function (25) and using (19), we obtain for the amplitude of the transition the expression

$$b_{n,l,m} = -\frac{(\gamma+\beta)\sqrt{\gamma\beta(\gamma+\beta)}}{n^2} \times \int d^3q \frac{(1+n^2q^2)\, u_{n,l,m}(\mathbf{q})\, \delta(vq - v^2/2 - \Delta)\exp(-i\rho\mathbf{q})}{(|\mathbf{q}-\mathbf{v}|^2+\gamma^2)(|\mathbf{q}-\mathbf{v}|^2+\beta^2)}. \quad (26)$$

We substitute further (26) in (23) and (23) in (22). In calculating the cross section $\sigma_n(v)$ in (22), we employ the method used to calculate Q_n^α (see the Appendix), and obtain ultimately

$$\sigma_n(\gamma, \beta) = \frac{64\gamma\beta(\gamma+\beta)^3\pi}{n^3 v^2}\int_0^\infty dp \frac{p}{(p^2+\varepsilon_n^2)^4 (p^2+\varepsilon_\beta^2)^2} \quad (27)$$

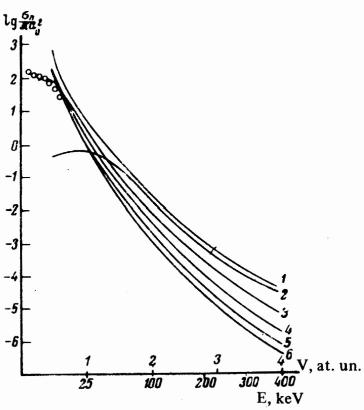
or

$$\sigma_n(\gamma, \beta) = \frac{32\gamma\beta(\gamma+\beta)^3\pi}{n^3 v^2(\beta^2-\gamma^2)^5}\left[\frac{1}{3}\left(\frac{\beta^2-\gamma^2}{\varepsilon_n^2}\right)^3 - \left(\frac{\beta^2-\gamma^2}{\varepsilon_n^2}\right)^2 + 3\left(\frac{\beta^2-\gamma^2}{\varepsilon_n^2}\right) + \frac{\beta^2-\gamma^2}{\varepsilon_n^2+\beta^2-\gamma^2} - 4\ln\left(\frac{\beta^2-\gamma^2}{\varepsilon_n^2}\right)\right], \quad (28)$$

where

$$\varepsilon_\beta^2 = \left(\frac{\Delta}{v}+\frac{v}{2}\right)^2 + \beta^2 - 2\Delta,\quad \varepsilon_n^2 = \left(\frac{\Delta}{v}+\frac{v}{2}\right)^2 + \frac{1}{n^2}. \quad (29)$$

The figure shows the dependence of the charge-exchange cross section of H^- with capture of an elec-



Cross section σ_n of the process $H^- + p \to H(1) + H(n)$. O—experimental data of [7], 1—summary cross section $\sigma = \sum_{n=1}^{5} \sigma_n$, 2—n = 1, 3—n = 2, 4—n = 3, 5—n = 4, 6—n = 5.

tron in the state H(n), n = 1, ... 5, and the total cross section $\sigma = \sum_{n=1}^{5} \sigma_n$ as functions of the energy of the incoming proton.

The parameters γ and β of the function (25) are equal respectively to 0.24 and 0.77. The available experimental data (see[7]) pertain to the energy range $E < 10$ keV, where the BK approximation no longer holds. These data are marked by circles in the figure. We see that in this energy range the main contribution to the cross section is made by processes with capture in the state with $n \geq 2$, and when $E > 100$ keV the principal role is played by capture in the state with n = 1.

APPENDIX

Let us calculate the single-particle cross section Q_n^α defined in (16). Substituting (20), we obtain

$$Q_n^\alpha = \frac{\alpha^5 2^3}{n^4} \int_0^\infty \rho\, d\rho \int d^3p\, d^3q\, \frac{\delta(\mathbf{vp} - {}^1/_2 v^2 - \Delta)\, \delta(\mathbf{vq} - {}^1/_2 v^2 - \Delta)}{(|\mathbf{p} - \mathbf{v}|^2 + \alpha^2)^2 (|\mathbf{q} - \mathbf{v}|^2 + \alpha^2)^2}$$

$$\times (1 + n^2p^2)(1 + n^2q^2) \exp[i\rho(\mathbf{p} - \mathbf{q})] \sum_{l,m} u^*_{n,l,m}(\mathbf{p})\, u_{n,l,m}(\mathbf{q}). \quad (30)$$

The sum under the integral sign is equal to (see[8])

$$\frac{8}{n^4\pi^2(p^2 + n^{-2})^2(q^2 + n^{-2})^2} \frac{\sin n\omega}{\sin\omega},$$

where $\cos\omega$ is the cosine of the angle between the points having the coordinates $(\alpha_p, \vartheta_p, \varphi_p)$ and $(\alpha_q, \vartheta_q, \varphi_q)$ on the four-dimensional sphere. The connection between the four-dimensional sphere and three-dimensional space is established with the aid of the formula

$$\cos\alpha_p = (1 - n^2p^2) / (1 + n^2p^2).$$

Directing the polar axis in the integral (11) along v and integrating with respect to the angles ϑ_p and ϑ_q, we obtain after the substitution

$$p \to \left|\sqrt{p^2 + \left(\frac{\Delta}{v} + \frac{v}{2}\right)^2}\right|, \quad q \to \left|\sqrt{q^2 + \left(\frac{\Delta}{v} + \frac{v}{2}\right)^2}\right|$$

the following expression for Q_n^α;

$$Q_n^\alpha = \frac{2(2\alpha)^5}{\pi^2 v^2 n^4} \int_0^\infty p\, dp \int_0^\infty q\, dq \int_0^\infty \rho\, d\rho \int_0^{2\pi} d\varphi_p \int_0^{2\pi} d\varphi_q$$

$$\times \frac{\sin n\omega}{\sin\omega} \frac{\exp[i\rho(p\cos\varphi_p - q\cos\varphi_q)]}{(p^2 + \varepsilon_\alpha^2)^2 (q^2 + \varepsilon_\alpha^2)^2 (p^2 + \varepsilon_n^2)(q^2 + \varepsilon_n^2)} \quad (31)$$

where

$$\cos\omega = 1 - \frac{2}{n^2} \frac{p^2 + q^2 - 2pq\cos(\varphi_p + \varphi_q)}{(p^2 + \varepsilon_n^2)(q^2 + \varepsilon_n^2)}. \quad (32)$$

In order to integrate with respect to the angles φ_p and φ_q, we expand $\sin n\omega / \sin\omega$ in powers of $\cos\varphi_p$ and $\cos\varphi_q$, using formula (32). We then have

$$\int_0^{2\pi} d\varphi_p \int_0^{2\pi} d\varphi_q \frac{\sin n\omega}{\sin\omega} \exp[i\rho(p\cos\varphi_p - q\cos\varphi_q)]$$

$$4\pi^2 \sum_{i=0}^{E\left(\frac{n-1}{2}\right)} \sum_{j=0}^{n-1-2i} \sum_{k=0}^{j} (-1)^{i+j} 2^{n-1-2i+j} \binom{n-1-i}{i} \binom{n-2i-1}{j} \binom{j}{k}$$

$$\cdot \frac{(p^2 + q^2)^{j-k}(-pq)^k}{n^{2j}(p^2 + \varepsilon_n^2)^j (q^2 + \varepsilon_n^2)^j}$$

$$\times \left\{ \sum_{l=0}^{E\left(\frac{k-1}{2}\right)} 2\binom{k}{l} J_{k-2l}(\rho p) J_{k-2l}(\rho q) + \delta_{l,2m} \binom{l}{\frac{l}{2}} J_0(\rho p) J_0(\rho q) \right\}. \quad (33)$$

We substitute (33) in (31) and integrate over the impact parameter ρ. Since only products of Bessel functions of equal order depend on ρ in the integrand expression, we can use the orthogonality condition

$$\int_0^\infty \rho J_m(\rho p) J_m(\rho q)\, d\rho = \frac{\delta(p - q)}{\sqrt{pq}}.$$

After integration with respect to q, all the sums can be taken, and for Q_n^α we obtain the expression

$$Q_n^\alpha = \frac{2^8\alpha^5}{v^2 n^3} \int \frac{p\, dp}{(p^2 + \varepsilon_\alpha^2)^4 (p^2 + \varepsilon_n^2)^2}. \quad (34)$$

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