STATIONARY THEORY OF STIMULATED MANDEL'SHTAM-BRILLOUIN SCATTERING IN MEDIA WITH SMALL LINEAR SOUND DAMPING

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The stationary solution of the problem of back-scattering of high-intensity light waves in a linear isotropic medium with a small linear sound damping is investigated by the method of slowly varying amplitude with account of the reaction of the sound on the light. An estimate is given of the range of applicability of the stationary approximation. Possible modifications of the solution are considered qualitatively by taking into account the acoustic nonlinearity and variability of the incident light intensity.

STIMULATED Mandel'shtam-Brillouin scattering (SMBS) is the process of the effective scattering of an incident electromagnetic field by the acoustic waves of the medium, which waves are amplified from the thermal noise level by parametric interaction with the incident (E_p) and reflected (E_1) fields. To solve the SMBS problem, we must find simultaneous solutions of the equations of hydrodynamics (elasticity theory) and Maxwell's equations. In the approximation of a homogeneous isotropic medium and slowly changing amplitudes of the interacting plane waves, the set of equations for the interacting amplitudes (in the case of back scattering) is written in the form

$$\frac{\partial p}{\partial z}+\frac{1}{S_0}\frac{\partial p}{\partial t}+\alpha_0 p+\sigma_1 E_1 E_p=0, \tag{1a}$$

$$\partial E_1/\partial z-\sigma_2 p E_p=0, \tag{1b}$$

$$\partial E_p/\partial z-\sigma_3 p E_1=0, \tag{1c}$$

$$\sigma_1=\frac{\varkappa}{16\pi}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_{\rho=\rho_0},\quad \sigma_2=8\pi k_1\Big/\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_{\rho=\rho_0}E_0^2,$$

$$\sigma_3=8\pi k_p\Big/\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_{\rho=\rho_0}E_0^2,$$

$$E_0^2=16\pi\varepsilon_0\rho_0 S_0^2/(\rho\,\partial\varepsilon/\partial\rho)^2_{\rho=\rho_0},$$

p is the amplitude of the pressure in the sound wave, α_0 the linear acoustic absorption coefficient, ϵ_0, ρ_0, and S_0 the dielectric permittivity, density and sound velocity in the unperturbed medium, and κ, k_p, k_1 the acoustic and optical wave vectors.

Several approaches to the solution of the set (1) have been developed for the solution of the SMBS. Kroll[1] solved the problem under the assumption E_p = const over the length of the interaction region (the given-field approximation). The solution obtained here gives the representation of the initial stage of the formulation of the process. In[2], the case was considered in which the damping $\alpha_0 \gg 1\ \text{cm}^{-1}$; here the process reaches the stationary regime very quickly (in a time $t'_c \sim 1/\alpha_0 S_0$[1]), and the spatial amplification is small. This case corresponds to the neglect of the first two terms in Eq. (1a), and, in practice, to scattering in liquids and solids at room temperature.

The approach developed in the present paper arose in connection with the appearance of SMBS at low temperatures $T \sim 4$–$20\,^\circ\text{K}$[3,4]. It is well known (see, for example,[5]) that the damping at such temperatures is $\alpha_0 \leq 1\ \text{cm}^{-1}$, and is considerably smaller in some cases, which makes it possible to neglect the linear damping of sound. It is assumed here that, because of the very great increase in reflected light, the system rapidly reaches the steady state. The approximations indicated cause Eq. (1a) to take the form

$$\partial p/\partial z=\sigma_1 E_1 E_p=0. \tag{1a$'$}$$

As is shown in[1], the amplitude of the scattered wave is proportional to the expression

$$E_1(z,\ t)\sim\exp\{2[(L-z)tE_{p0}^2k_p^2S_0/E_0^2]^{1/2}\}. \tag{2}$$

Here L is the length of the interaction region, t the time of influence of the light field, $E_{p0} = E_p(z = 0)$.

It is seen from this expression that the scattering takes place essentially at the forward boundary z = 0. It is logical to assume (this is confirmed by the calculation carried out below) that, for high intensity of reflected light (of the order of the incident intensity) all the scattering will take place in the region $z_0 \sim E_0/k \sim E_0/k_pE_{p0}$. It follows from this assumption that first, specifying the boundary will be unimportant for $L \gg z_0$, and we can set $L = \infty$; second, for $z \gg z_0$, the amplitude of the incident light will approach the threshold value for nonlinear SMBS (the threshold value of $E_p(z = 0)$ is determined by the linear sound damping) and, inasmuch as it is assumed in the given case that $E_{p0} \gg E_p(z = 0)$ is threshold, then one can set $E_p(z = \infty) = 0$. Taking into account what was pointed out above, we solve the set (1) with the equation (1a) in the form (1a′) for the following boundary condition:[1)]

$$E_p(z=0)=E_{p0},\ E_p(z=\infty)=0,\ E_1(z=\infty)=0,\ p(z=0)=p_1.$$

Then the solution of the system has the form[2)]

[1)]The solution of the set (1) with Eq. (1a′) for the case of finite values of L was given in [6].

[2)]Such a value of p_∞ can be obtained from the Manley-Rowe relations.

$$p(z)=p_\infty\frac{1-A^2\exp\{-2p_\infty z\sqrt{\sigma_2\sigma_3}\}}{1+A^2\exp\{-2p_\infty z\sqrt{\sigma_2\sigma_3}\}}$$

$$E_1(z)=2p_\infty A\sqrt{\frac{\sigma_2}{\sigma_1}}\frac{\exp\{-p_\infty z\sqrt{\sigma_2\sigma_3}\}}{1+A^2\exp\{-2p_\infty z\sqrt{\sigma_2\sigma_3}\}} \qquad (3)$$

$$E_p(z)=[\omega_p/(\omega_p-\Omega)]^{1/2}E_1(z),$$

$$p_\infty^2=p_1^2+\frac{\varkappa}{4\pi k_p}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_{\rho=\rho_0}E_{p0}^2E_0^2\approx E_{p0}^2E_0^2|_{p_1\ll p_\infty};$$

$$A=\left\{p_\infty\sqrt{\frac{\sigma_2}{\sigma_1}}+\left[p_\infty^2\frac{\sigma_2}{\sigma_1}-E_1^2(z=0)\right]^{1/2}\right\}\approx 1+\frac{p_1}{E_{p0}E_0}\Big|_{p_1\ll p_\infty}$$

ω_p is the frequency of the incident light, Ω the frequency of sound in the medium.

The value of p_1 is connected with the nonstationary process (thermal fluctuations) and its use for the solution of the stationary problem is not completely correct. It is seen from the solution (3) that for $p_1 \ll p_\infty$ the result is practically independent of p_1. We therefore set $p_1 = 0$, assuming that E_{p0} is sufficiently large.

We estimate the "damping depth" of the amplitudes of the light waves. From the solutions (3) we get $z_{damp} = z_0 \sim E_0/k_i E_{i0}$. The qualitative picture of the behavior of the amplitudes is shown in the drawing.

We estimate the region of applicability of the stationary approximation to the given problem. A strictly stationary regime can be achieved in a time $t'_c \approx 1/\alpha_0 S_0$ [1], as has been mentioned (or in the time $t''_c \sim L/S_0$ in the case $L < 1/\alpha_0$). In the case of large E_{p0}, the regime, which is close to stationary (quasistationary), can be achieved earlier—the amplitude of the sound manages to increase to such a large value that practically all the incident light is reflected in a narrow range at the forward boundary. One can estimate this moment t_c by setting $E_1 \cong E_{p0}$ in the distribution (2) or, what amounts to the same thing, $p = p_\infty$ for $z \sim z_0$. Then

$$t_c=E_0^2\ln^2(p_\infty/p_1)/4E_{p0}^2k_p^2S_0(L-z_0).$$

At the instant t_c, the amplitude of the reflected wave becomes of the order of the amplitude of the incident wave and for $t > t_c$ the conditions of applicability of the given field approximation are violated.

Up to now we have neglected acoustic nonlinearities. Together with this, it is known[3)] that in an inviscid medium ($\alpha_0 = 0$) a wave of any amplitude sooner or later goes over into a shock wave. As follows from what was pointed out above, the considered case approximates the case of an inviscid medium and apparently nonlinear effects should be strongly evident.

In acoustics, the Mach number $M = p/\rho_0 S_0^2 \sim E_{p0}/E_0$ for $p = p_\infty$ is used as a characteristic of the nonlinear processes, indicating the degree of prominence of nonlinear effects in the given medium. Use is also made of the Reynolds number $Re = \Theta\kappa M/2\alpha_0$, which expresses the features of nonlinear distortions of the shape of the wave profile.[4)] Of the nonlinear effects that can appear, we shall be interested most of all in the nonlinear sound absorption (absorption of the energy of the first harmonic due to transition into higher harmonics with their consequently greater absorption) in its generation and propagation, which can lead to a change in the solutions (3).

Because of the impossibility at the present time of obtaining an exact solution of the system with account of the acoustic nonlinearity, we shall carry out a qualitative study of possible changes in the solutions (3).

Nonlinear absorption begins from the instant of time μ when the wave transforms from a sinusoid into a shock wave with formation of a discontinuity (weak, since M does not exceed unity). Therefore, if $t_c \lesssim \mu = \alpha_0 Re S_0$, then the solution (3) remains valid. The inequality

$$E_{p0}\gtrsim E_0\Theta\ln^2(p_\infty/p_1)/k_pL=\hat{E}.$$

corresponds to this condition. If now $E_{p0} \ll \hat{E}$, then the shock wave that is formed does not allow the sound to increase to the value $p \sim p_\infty$, intensely absorbing the sound energy at the front (here the given field approximation remains valid). Let us make some estimates. Thus, for $k_p \sim 10^5\ cm^{-1}$, $S_0 \sim 10^5$ cm/sec, $L \sim 1$ cm, $E_{p0} \sim 10^7$ W/cm, $\alpha_0 \sim 1\ cm^{-1} \to M \sim 0.1$–$0.01$, $Re \sim 10^3$–10^4, $t_c \sim 10^{-8}$–10^{-9} sec, $\hat{E} \sim \Theta(10^6$–$10^7)$ W/cm.

Inasmuch as sound intensities can reach large values in strong fields, then the sound can be one of the reasons (at least initially) for the breakdown of solids in laser fields. Thus, for many solids, the static elastic limit is $P_{el} - 10^4\ k\Gamma/cm^2$, since $p_\infty \sim 10^5\ k\Gamma/cm^2$ for $E_{i0} \sim 10^7$ W/cm. A similar reason for destruction was given in[8].

It should be remarked here that, in spite of such high intensities of the reflected light $|E_1|^2$, multiple scattering practically does not occur, since, as follows from[1] and the previous results, there is always a small parameter $\gamma \sim z_0/L \ll 1$ in the problem. Experimentally (to be sure, for $T \sim 300°$K) repeated scattering has not been observed.[9]

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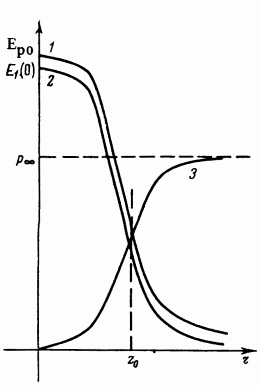

Dependence of the amplitude of sound and light waves on the distance to the leading edge z = 0; 1—amplitude of the incident light, 2—amplitude of the reflected light, 3—amplitude of the sound pressure.

[3)]The discussions of nonlinear acoustics here and below are taken from [7].

[4)]Θ is the parameter of nonlinearity of the medium. If we use the material equation of the form $p = p^*(\rho/\rho_0)^\Gamma$, then $\Theta = (\Gamma + 1)/2$. For an ideal case, $\Gamma = c_p/c_V$ and for liquids and solids, $\Gamma \sim 3$-14. [7]

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