LOW FREQUENCY ASYMPTOTIC BEHAVIOR OF THE PHONON GREEN FUNCTION AND KINETIC EQUATIONS FOR AN ELECTRON-PHONON SYSTEM IN A QUANTIZING MAGNETIC FIELD

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A sequence of the perturbation theory diagrams giving the main contribution to the hydrodynamic asymptotic behavior of the phonon Green function of an electron-phonon system in a quantizing magnetic field is summed. The summation reduces to the solution of a system of equations of the kinetic type by the Chapman-Enskog method. In the Green function there appear new poles relative to the unperturbed Green function; one pole corresponds to heat conduction, and, in the model in which Coulomb interaction and Umklapp processes are not taken into account, there are also poles corresponding to second sound. Anisotropy of the sound velocities and of the thermal conductivity and sound damping tensors is important. The transverse components of these tensors are calculated explicitly in quadratures. The stability condition of the low-frequency energy spectrum is discussed; this condition is violated at low temperatures if one of the Landau levels approaches the Fermi surface.

1. In a number of papers (e.g., [1,2]), the method of equilibrium Green functions is used to describe weakly non-equilibrium kinetic processes such as second sound in solids. In the present paper an analogous method, developed earlier in [3] for the electron-phonon system, is carried over to the case when this system is placed in a sufficiently strong (quantizing) magnetic field. The method reduces to distinguishing a sequence of the diagrams which give the main contribution to the phonon Green function $G_{ph}(E, \mathbf{k})$ in the hydrodynamic regime ($E\tau, kl \ll 1$, where τ is the relaxation time and l is the mean free path). The summation of the selected sequence reduces to the solution of a system of equations of the kinetic type.

For an electron-phonon system in a quantizing magnetic field, such an approach enables us to avoid introducing a temperature gradient; the introduction of a temperature gradient is necessary when intrinsically non-equilibrium methods are used and often leads to ambiguous results when the kinetic coefficients are calculated. In the Green function formalism the kinetic equations arise automatically in the process of summing the diagrams, without the use of any additional assumptions.

Usually, [4-6] the analog of the electron distribution function in the kinetic equations depends not only on physical variables such as the principal quantum number n and the longitudinal momentum $p_{\parallel}$ (and also on E and $\mathbf{k}$), but also on arguments such as the oscillator center y_0. Variables of the type y_0 cease to be constants of the motion on change in the gauge of the vector potential, and the energy spectrum does not depend on them. It is clear that these variables are unphysical, and it is desirable to obtain kinetic equations which do not contain them; such equations are derived in Sec. 2 of this paper.

The solution of the resulting system of equations by the Chapman-Enskog method is treated in Sec. 3. It is shown that the exact phonon Green function has poles relative to the unperturbed function; a pole appears which corresponds to heat conduction, and in the model in which the Coulomb interaction and Umklapp processes are not taken into account there are also poles corresponding to second sound. In the present paper we shall confine ourselves to treating a model in which only the electron-phonon interaction is taken into account. However, the method is also applicable when Coulomb interaction and Umklapp processes are taken into account, with an obvious transformation of a second sound branch into a plasma branch.

In a quantizing magnetic field the sound velocities and the tensors defining the sound damping and the heat propagation become essentially anisotropic. We note that it is found possible to calculate the transverse components of these tensors in quadratures.

To conclude, we discuss the condition which ensures the stability of the low-frequency energy spectrum. It is shown that this condition ceases to be fulfilled if, at a sufficiently low temperature, one of the Landau levels approaches the Fermi surface.

2. We write the Hamiltonian of the electron-phonon system in a constant magnetic field $\mathbf{H}$ ($\mathbf{H} = \text{curl } \mathbf{A}$, $\mathbf{A} = \frac{1}{2}\mathbf{H} \times \mathbf{x}$) in the form[1]

$$H' = H - \lambda N =$$

$$= \sum_s \int d^3 p\, a_s^{+}(\mathbf{p}) \left\{ \frac{1}{2m}\left(\mathbf{p} - \frac{ie}{2}[\mathbf{H}\nabla_{\mathbf{p}}]\right)^2 - \left(\frac{seH}{2m} + \lambda\right) \right\} a_s(\mathbf{p})$$

$$+ \int_{k<k_0} d^3 k\, \omega_0(\mathbf{k})\, b^{+}(\mathbf{k})\, b(\mathbf{k}) \tag{2.1*}$$

$$+ \frac{g}{(2\pi)^{3/2}} \int_{k<k_0} d^3 k\, d^3 p \left(\frac{\omega_0(\mathbf{k})}{2}\right)^{1/2} a_s^{+}(\mathbf{p}+\mathbf{k}) a_s(\mathbf{p})[b(\mathbf{k}) + b^{+}(-\mathbf{k})],$$

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[1] We use a system of units with $\hbar = k = 1$ ($\hbar$ and k are Planck's and Boltzmann's constants).

*[$\mathbf{H}\nabla_{\mathbf{p}}$] $\equiv \mathbf{H} \times \nabla_{\mathbf{p}}$.

where λ is the chemical potential, $s = \pm$ is the spin index, and $\omega_0(k) = c_0 k$. In the integrals over the phonon momenta in (2.1) there is a cut-off at the upper limit k_0.

Below it is found convenient to use a variant of the temperature diagram technique[7] in which the expressions for the bare phonon function and vertex are written with δ-functions corresponding to the momentum conservation law. The expressions for the lines and vertices have the following form:

$$\underset{p \,\leftarrow\, i\omega_n,\, s}{\overline{\qquad}}\,q \qquad G_e^{(0)}(i\omega_n, s, \mathbf{p}, \mathbf{q}), \tag{2.2}$$

$$\underset{\kappa}{\text{--}}\underset{i\omega_n}{\text{--}}\underset{\kappa'}{\text{--}} \quad \delta(\kappa-\kappa')\,G_{ph}^{(0)}(i\omega_n,\kappa) \equiv \delta(\kappa-\kappa')\,\omega_0^2(k)[(i\omega_n)^2 - \omega_0^2(k)]^{-1}$$

$$\underset{p}{\overset{\uparrow\, k}{\longrightarrow}}\,q \qquad g\,(2\pi)^{-3/2}\,\delta(\mathbf{p}-\mathbf{q}-\kappa).$$

We take the Green function $G_e^{(0)}$ of an electron in a magnetic field in a form convenient for calculating sums over the frequencies $\omega_n = (2n+1)\pi T$:

$$G_e^{(0)}(i\omega_n, s; \mathbf{p}, \mathbf{q}) = \sum_{n=0}^{\infty} P_n(\mathbf{p}, \mathbf{q})\,[i\omega_n - \varepsilon(p_\parallel, n, s)]^{-1}. \tag{2.3}$$

Here, $\epsilon(p_\parallel, n, s)$ denotes the energy of the Landau level with number n:

$$\varepsilon(p_\parallel, n, s) = \frac{p_\parallel^2}{2m} + \frac{eH}{m}\left(n + \frac{1}{2}(1-s)\right) - \lambda, \tag{2.4}$$

and the function $P_n(\mathbf{p}, \mathbf{q})$ is defined by the formula

$$P_n(\mathbf{p}, \mathbf{q}) = \frac{2}{\pi eH}\,\delta(p_\parallel - q_\parallel)\exp\left(\frac{-2i(\mathbf{H}[\mathbf{pq}])}{eH^2}\right)\varphi_n\left(\frac{2(\mathbf{p}_\perp - \mathbf{q}_\perp)^2}{eH}\right), \tag{2.5}$$

where $p_\parallel$ is the component of the momentum $\mathbf{p}$ along the magnetic field, $p_\perp$ is the transverse component, and φ_n is the Laguerre function:

$$\varphi_n(x) = e^{-x/2}L_n(x) \equiv \frac{1}{n!}\,e^{x/2}\frac{d^n}{dx^n}(x^n e^{-x}). \tag{2.6}$$

We note that $P_n(\mathbf{p}, \mathbf{q})$ is the kernel of the projection operator on states with the n-th Landau level, and, therefore, the equality

$$\int P_n(\mathbf{p}, \mathbf{q})P_m(\mathbf{q}, \mathbf{r})\,d^3q = \delta_{mn}P_n(\mathbf{p}, \mathbf{r}), \tag{2.7}$$

is valid and will be used below.

The exact electron and phonon Green functions G_e and G_{ph} are depicted by thick continuous and dashed lines respectively. They can be expressed in terms of the self-energy parts Σ and P:

$$G_e = (G_e^{(0)-1} - \Sigma)^{-1}, \qquad G_{ph} = (C_{ph}^{(0)-1} - P)^{-1}. \tag{2.8}$$

The following exact diagrammatic equalities (in (2.9) all the lines are thick)

$$\tag{2.9}$$

show that the function P (the polarization operator) is expressed in terms of the vertex part D_1 and the exact electron Green function, while the vertex parts D_1 and D_2 satisfy a system of linear equations. The thick electron and phonon lines in the diagrams (2.9) correspond to exact Green functions. Irreducible diagrams, which cannot be cut along a vertical by cutting two lines, contribute to the blocks $K_1 - K_4$.

The derivation of the kinetic equations from the diagrammatic equalities (2.9) is analogous to that performed in the previous article[3] for a system without a magnetic field. Here we shall dwell chiefly on the singularities appearing in a quantizing magnetic field. As also in [3], for the functions Σ, P and K_i we confine ourselves to the simplest perturbation theory diagrams:[2)]

$$\Sigma \approx \Sigma_2 = \quad , \qquad P \approx P_2 = \quad$$

$$K_1 \approx \quad , \qquad K_2 \approx \quad , \qquad K_3 \approx \quad , \qquad K_4 \approx 0. \tag{2.10}$$

The system 12.9) reduces to a system of equations for the limiting values of the analytic functions which arise from the vertex parts D_1 and D_2 on making the analytic continuation $i\omega_1 \to z$; the system for the limiting values $D_1^{(i)}$ and $D_2^{(i)}$ on the inner edges of the cuts (Im $z = +0$, $\omega - 0$) is found to be non-trivial. The values $D_1^{(e)}$ and $D_2^{(e)}$ on the external edges (Im $z = -0$, $\omega + 0$) in the first approximation are equal to

$$D_1^{(e)} = g(2\pi)^{-3/2}\delta(\mathbf{p}-\mathbf{q}-\mathbf{k}), \qquad D_2^{(e)} = 0. \tag{2.11}$$

On integration over the energy variable, characteristic "energy denominators" arise:

$$Z_e = E - \varepsilon_1(p_\parallel + k_\parallel, n, s) + \varepsilon_1(p_\parallel, n, s) + 2i\Delta_1(p_\parallel, n, s),$$
$$Z_\Phi = E - \varepsilon_2(\mathbf{k}_1 + \mathbf{k}) + \varepsilon_2(\mathbf{k}_1) + 2i\Delta_2(\mathbf{k}_1), \tag{2.12}$$

where $\epsilon_1 \equiv \epsilon(p_\parallel, n, s)$ (cf. (2.4)) and $\epsilon_2(\mathbf{k}_1) = c_0 k_1$ are the energy spectra of the electrons and phonons, and Δ_1 and Δ_2 are their imaginary parts, defined by the diagrams (2.10). The function Δ_2 can be expressed in terms of the imaginary part of the polarization operator by the formula

$$-\frac{\omega_0(\mathbf{k}_1)}{2}\,\text{Im}\,P_2(E = \varepsilon_2(\mathbf{k}_1) + i0, \mathbf{k}_1, \mathbf{k}_2) = \Delta_2(\mathbf{k}_1)\delta(\mathbf{k}_1 - \mathbf{k}_2). \tag{2.13}$$

The function Δ_1 can be obtained from the expression

$$\Sigma_2(i\omega, s; \mathbf{p}, \mathbf{q}) =$$

$$= -\frac{2g^2 T}{\pi eH(2\pi)^3}\,\delta(p_\parallel - q_\parallel)\exp\left(\frac{-2i(\mathbf{H}[\mathbf{pq}])}{eH^2}\right)\sum_{m=0}^{\infty}\varphi_m\left(\frac{2(\mathbf{p}_\perp - \mathbf{q}_\perp)^2}{eH}\right)$$

$$\times \int d^3k \sum_{\omega_1}\omega_0^2(k)\,[(i\omega_1)^2 - \omega_0^2(k)]^{-1}[i\omega - i\omega_1 - \varepsilon_1(p_\parallel - k_\parallel, m, s)]^{-1}.$$

$$\times \exp\left(\frac{-2i(\mathbf{H}[\mathbf{k}, \mathbf{p}-\mathbf{q}])}{eH^2}\right). \tag{2.14}$$

In this formula the integral over the momentum $\mathbf{k}$ is a function of $(\mathbf{p}_\perp - \mathbf{q}_\perp)^2$, the product of which with $\varphi_m(2(\mathbf{p}_\perp - \mathbf{q}_\perp)^2/eH)$ can be expanded in the Laguerre functions φ_n. As a result, the expression (2.14) for Σ_2 can be rewritten in the form

$$\Sigma_2(i\omega, s; \mathbf{p}, \mathbf{q}) = \sum_{n=0}^{\infty} P_n(\mathbf{p}, \mathbf{q})A_n(i\omega, s, p_\parallel), \tag{2.15}$$

where

$$A_n(i\omega, s, p_\parallel)$$

$$= -\frac{Tg^2}{(2\pi)^3}\sum_{\omega_1}\sum_{n=0}^{\infty}\int \frac{2d^2(\mathbf{p}_\perp - \mathbf{q}_\perp)}{\pi eH}\,\varphi_n\left(\frac{2(\mathbf{p}_\perp - \mathbf{q}_\perp)^2}{eH}\right)\varphi_m\left(\frac{2(\mathbf{p}_\perp - \mathbf{q}_\perp)^2}{eH}\right)$$

[2)] For $E\tau \lesssim 1$ the value of P differs substantially from P_2. However, in Eqs. (2.9) the approximation $P \approx P_2$ is legitimate since the phase volume of the region in which it is not true is small.

$$\times \int d^3 k \omega_0{}^2(\mathbf{k}) \left[(i\omega_1)^2 - \omega_0{}^2(\mathbf{k}) \right]^{-1} \left[i\omega_n - i\omega_1 - \varepsilon_1(p_{\parallel} - k_{\parallel}, m, s) \right]^{-1}$$

$$\times \exp\left(\frac{-2i(\mathbf{H}, [\mathbf{k}, \mathbf{p} - \mathbf{q}])}{eH^2} \right). \qquad (2.16)$$

Δ_1 is expressed by the imaginary part of the function $A_n(i\omega, s, p_{\parallel})$ after the replacement $i\omega \to \epsilon_1(p_{\parallel}, n, s) + i0$.

We are interested in the analytic continuation $i\omega \to E \lesssim \tau^{-1}$ for $k \lesssim l^{-1}$, where τ is the relaxation time and l is the mean free path. We can put $E = 0$, $\mathbf{k} = 0$ in the expressions for the diagrams $K_1 - K_4$ and in the bare vertex in the first of Eqs. (2.9), as is usually done in deriving the kinetic equations without a magnetic field; this can also be justified when the magnetic field is taken into account. As a result, the dependence on the variables E and $\mathbf{k}$ remains only in the energy denominators Z_e and Z_{ph} (2.12).

We see the function D_1 in the form

$$D_1(i\omega_1, i\omega, s, \mathbf{p}, \mathbf{q}, \mathbf{k}) = \frac{g}{(2\pi)^{3/2}} \sum_{n=0}^{\infty} P_n(\mathbf{p}, \mathbf{q}) F_n(i\omega_1, i\omega, s, p_{\parallel}, \mathbf{k}). \quad (2.17)$$

In place of D_1 and D_2 we introduce, as the unknowns, the new functions h and f, defining them by the formulas

$$h(n, s, p_{\parallel}, E, \mathbf{k}) = Z_e^{-1} F_n(i\omega_1 \to \varepsilon_1(p_{\parallel}, n, s), \ i\omega \to E, s, p_{\parallel}, \mathbf{k}), \qquad (2.18)$$

$$f(\mathbf{k}_1, E, \mathbf{k})\delta(\mathbf{k}_1 - \mathbf{k}_2) = \frac{g\varepsilon_2(\mathbf{k}_1)}{(2\pi)^{3/2} 2 Z_\Phi} D_2(i\omega_1 \to \varepsilon_2(\mathbf{k}_1), i\omega \to E, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}).$$

Then the system (2.9) takes the following form:

$$(E - 2\varepsilon_1'(k_1)_{\parallel} k_{\parallel})h + iI_1(h, f) = 1,$$

$$(E - 2\varepsilon_2'(\mathbf{k}_1 \mathbf{k}))f + iI_2(h, f) = 0. \qquad (2.19)$$

The first of the Eqs. (2.19) is obtained if, having represented the δ-function $\delta(\mathbf{p} - \mathbf{q})$ in the form

$$\delta(\mathbf{p} - \mathbf{q}) = \sum_{n=0}^{\infty} P_n(\mathbf{p}, \mathbf{q}), \qquad (2.20)$$

we then equate the coefficients of $P_n(\mathbf{p}, \mathbf{q})$ on both sides of the first Eq. (2.9); in doing this we must make use of property (2.7).

In the system (2.19), for $k_1 \gg k$ we have made the replacements

$$\varepsilon_1((k_1)_{\parallel} + k_{\parallel}, n, s) - \varepsilon_1((k_1)_{\parallel}, n, s) \to \partial\varepsilon_1 / \partial(k_1)_{\parallel} 2(k_1)_{\parallel} k_{\parallel}$$
$$\equiv 2\varepsilon_1'(k_1)_{\parallel} k_{\parallel},$$
$$\varepsilon_2(\mathbf{k}_1 + \mathbf{k}) - \varepsilon_2(\mathbf{k}_1) \to \partial\varepsilon_2 / \partial\mathbf{k}_1 2(\mathbf{k}_1\mathbf{k}) \equiv 2\varepsilon_2'(\mathbf{k}_1\mathbf{k}).$$

I_1 and I_2 in (2.19) denote the expressions

$$I_1 = \frac{g^2}{(2\pi)^2}(1 + \exp\{-\beta\varepsilon_1(p_{\parallel}, n, s)\}) \sum_{m=0}^{\infty} \int d^3k_1 \, dq_{\parallel} W_{mn}(\mathbf{k}_1) \cdot \qquad (2.21)$$

$$\left(\frac{\varepsilon_2(\mathbf{k}_1)}{2} \right) n_1(q_{\parallel}, m, s) n_2(\mathbf{k}_1) \left[\exp\{\beta\varepsilon_1(q_{\parallel}, m, s)\} \delta(\varepsilon_1(p_{\parallel}, n, s) - \varepsilon_1(q_{\parallel}, m, s) \right.$$

$$+ \varepsilon_2(\mathbf{k}_1)) \delta(p_{\parallel} - q_{\parallel} + (k_1)_{\parallel})(h(p_{\parallel}, n, s) - h(q_{\parallel}, m, s) + f(\mathbf{k}_1))$$
$$+ \exp\{\beta\varepsilon_1(p_{\parallel}, n, s)\} \delta(\varepsilon_1(p_{\parallel}, n, s) - \varepsilon_1(q_{\parallel}, m, s)$$
$$\left. - \varepsilon_2(\mathbf{k}_1)) \delta(p_{\parallel} - q_{\parallel} - (k_1)_{\parallel})(h(p_{\parallel}, n, s) - h(q_{\parallel}, m, s) - f(\mathbf{k}_1)) \right],$$

$$I_2 = \frac{g^2 eH}{2\pi}(1 - \exp\{-\beta\varepsilon_2(\mathbf{k}_1)\}) \sum_{n, m, s} \int dp_{\parallel} dq_{\parallel} W_{mn}(\mathbf{k}_1) \left(\frac{\varepsilon_2(\mathbf{k}_1)}{2} \right)$$

$$\times n_1(p_{\parallel}, n, s) n_1(q_{\parallel}, m, s) \exp\{\beta\varepsilon_1(p_{\parallel}, n, s)\} \, \delta(\varepsilon_2(\mathbf{k}_1) - \varepsilon_1(p_{\parallel}, n, s)$$
$$+ \varepsilon_1(q_{\parallel}, m, s)) \delta((k_1)_{\parallel} - p_{\parallel} + q_{\parallel})(f(\mathbf{k}_1) - h(p_{\parallel}, n, s) + h(q_{\parallel}, m, s)),$$

where

$$n_1(p_{\parallel}, n, s) = (\exp\{\beta\varepsilon_1(p_{\parallel}, n, s)\} + 1)^{-1},$$
$$n_2(\mathbf{k}_1) = (\exp\{\beta\varepsilon_2(\mathbf{k}_1)\} - 1)^{-1}. \qquad (2.22)$$

The function $W_{mn}(\mathbf{k})$ in (2.21) is the integral

$$W_{mn}(\mathbf{k}) = \frac{2}{\pi eH} \int d^2 p_{\perp} \varphi_n \left(\frac{2p_{\perp}{}^2}{eH} \right) \varphi_m \left(\frac{2p_{\perp}{}^2}{eH} \right) \exp\left\{ \frac{2i}{eH^2}(\mathbf{H}[\mathbf{p}_{\perp}\mathbf{k}]) \right\}$$

$$= \int dx \, \varphi_n(x) \varphi_m(x) J_0(2\sqrt{xy}), \qquad (2.23)$$

where

$$y = k_{\perp}{}^2 / 2eH. \qquad (2.24)$$

Evaluation of the integral (2.23) (cf. [8], formula 7.422.2) gives

$$W_{mn} = (-1)^{m+n} e^{-y} L_n^{m-n}(y) L_m^{n-m}(y) \qquad (2.25)$$
$$\equiv (-1)^{m+n} e^{-y} \left(\frac{e^y}{n!} \frac{d^n}{dy^n} y^m e^{-y} \right) \left(\frac{e^y}{m!} \frac{d^m}{dy^m} y^n e^{-y} \right) = \frac{e^y y^{m-n}}{m! \, n!} \left(\frac{d^m}{dy^m} y^n e^{-y} \right)^2.$$

Formula (2.25) shows that W_{mn} is non-negative, as for physical reasons it should be.

Equations (2.19) form a system of linearized inhomogeneous kinetic equations in which the functions h and f play the role of corrections to the equilibrium electron and phonon distribution functions respectively, while I_1 and I_2 play the role of collision integrals. The analog of the electron distribution function h, as noted already in Sec. 1, does not contain unphysical variables of the type y_0 (the oscillator center).

The polarization operator $P(E, \mathbf{k})$ (the coefficient of the δ-function in the expression $P(E, \mathbf{k}, \mathbf{k}') = \delta(\mathbf{k} - \mathbf{k}') \times P(E, \mathbf{k})$) for $E\tau \lesssim 1$ is expressed in terms of the function h by the formula

$$P(E, \mathbf{k}) = g^2 \beta \langle Eh - 1 \rangle_1, \qquad (2.26)$$

where $\langle \ldots \rangle_1$ is the first of the abbreviations:

$$\langle \varphi(k_{\parallel}, n, s) \rangle_1 = \frac{eH}{(2\pi)^2} \int dk_{\parallel} \sum_{n=0}^{\infty} \sum_{s=\pm} \varphi(k_{\parallel}, n, s) \frac{\exp\{\beta\varepsilon_1(k_{\parallel}, n, s)\}}{(\exp\{\beta\varepsilon_1(k_{\parallel}, n, s)\} + 1)^2},$$

$$\langle \psi(\mathbf{k}) \rangle_2 = (2\pi)^{-3} \int d^3 k \, \psi(\mathbf{k}) \frac{e^{\beta\varepsilon_2(\mathbf{k})}}{(e^{\beta\varepsilon_2(\mathbf{k})} - 1)^2}. \qquad (2.27)$$

The derivation of formula (2.26) is analogous to that given in [3] for a system without a magnetic field.

3. Solving the system (2.19) in the hydrodynamic regime ($E\tau$, $kl \ll 1$) by the Chapman-Enskog method enables us to treat the propagation and damping of the sound vibrations, and also the phenomenon of heat conduction in a quantizing magnetic field.

We seek the first (acoustic) approximation for the functions h and f in the form of linear combinations of functions which cause the collision integral to vanish

$$h((k_1)_{\parallel}, n, s) = a + b(k_1)_{\parallel} + c\varepsilon_1((k_1)_{\parallel}, n, s),$$
$$f(\mathbf{k}_1) = b(k_1)_{\parallel} + c\varepsilon_2(\mathbf{k}_1); \qquad (3.1)$$

For I_1 and I_2 the following orthogonality conditions hold:

$$\langle I_1 \rangle_1 = 0, \quad \langle I_1(k_1)_{\parallel} \rangle_1 + \langle I_2(k_1)_{\parallel} \rangle_2 = 0, \quad \langle I_1\varepsilon_1 \rangle_1 + \langle I_2\varepsilon_2 \rangle_2 = 0, \quad (3.2)$$

which, for the coefficients a, b and c, give the expressions

$$a = \Delta_1 / \Delta, \quad b = k_{\parallel} \Delta_2 / \Delta, \quad c = \Delta_3 / \Delta. \qquad (3.3)$$

We obtain for the determinants Δ, Δ_1, Δ_2 and Δ_3 the formulas

$$\Delta = E^3 \langle k_{\parallel}{}^2 \rangle_{12}(\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1{}^2)$$
$$- 4Ek_{\parallel}{}^2(\langle \varepsilon^2 \rangle_{12} \langle \varepsilon' k_{\parallel}{}^2 \rangle_1{}^2 + \langle 1 \rangle_1 \langle \varepsilon\varepsilon' k_{\parallel}{}^2 \rangle_1{}^2 - 2\langle \varepsilon \rangle_1 \langle \varepsilon' k_{\parallel}{}^2 \rangle_1 \langle \varepsilon\varepsilon' k_{\parallel}{}^2 \rangle_{12})$$
$$\equiv \langle k_{\parallel}{}^2 \rangle_{12}(\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1{}^2) E(E^2 - u^2 k_{\parallel}{}^2),$$

$$\Delta_1 = E^2 \langle k_\parallel^2 \rangle_{12} (\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2)$$
$$- 4k_\parallel^2 (\langle 1 \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12}^2 - \langle \varepsilon \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12} \langle \varepsilon' k_\parallel^2 \rangle_1), \quad (3.4)$$
$$\Delta_2 = 2E \langle \varepsilon' k_\parallel^2 \rangle_1 (\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2),$$
$$\Delta_3 = 4k_\parallel^2 \langle \varepsilon' k_\parallel^2 \rangle_1 (\langle 1 \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12} - \langle \varepsilon \rangle_1 \langle \varepsilon' k_\parallel^2 \rangle_1),$$

where, e.g., $\langle \varepsilon \rangle_1 = \langle \varepsilon_1 (k_\parallel, n, s) \rangle_1$, $\langle \varepsilon^2 \rangle_{12} = \langle \varepsilon_1^2 \rangle_1 + \langle \varepsilon_2^2 \rangle_2$. Knowing h, we find from (2.26) the first approximation for P(E, k):

$$P(E, k) = \frac{4g^2\beta \langle \varepsilon' k_\parallel^2 \rangle_1^2}{\langle k_\parallel^2 \rangle_{12}} \frac{k_\parallel^2}{E^2 - u^2 k_\parallel^2} \equiv \frac{\delta^2 k_\parallel^2}{E^2 - u^2 k_\parallel^2}. \quad (3.5)$$

By equating the denominator of the function G_{ph} to zero we obtain the equation

$$(E^2 - c_0^2 k^2)(E^2 - u^2 k_\parallel^2) - c_0^2 \delta^2 k^2 k_\parallel^2 = 0, \quad (3.6)$$

which gives two anisotropic sound branches in the energy spectrum. The appearance of second sound is a consequence of the choice of model, which does not take Coulomb interaction and Umklapp processes into account. If the indicated interactions are small, so that the relaxation time τ_1 corresponding to them is appreciably greater than that τ_0 for the electron-phonon interaction, second sound can exist in reality in the interval $\tau_1^{-1} \ll E \ll \tau_0^{-1}$.

To study the heat conduction and sound damping, the second (viscosity) approximation of the Chapman-Enskog method is necessary. To calculate this, we find the corrections δh and δf to the functions (3.1) of the first approximation, and calculate the coefficients a, b and c with greater accuracy.

The corrections δh and δf have the form

$$\delta h = -i[b k_\parallel (\eta_1)_\parallel + a k_\parallel (k_1)_\parallel \delta_1 + c k_\parallel (k_1)_\parallel (\tau_1)_\parallel],$$
$$\delta f = -i[b k_\parallel (\eta_2)_\parallel + a k_\parallel (k_1)_\parallel \delta_2 + c k_\parallel (k_1)_\parallel (\tau_2)_\parallel$$
$$+ b(k_1)_\parallel (k_{1\perp} k_\perp) \eta_{2\perp} + c(k_{1\perp} k_\perp) \tau_{2\perp}]. \quad (3.7)$$

The five pairs of functions $((\eta_1)_\parallel, (\eta_2)_\parallel)$, (δ_1, δ_2), (τ_1, τ_2), $(0, \eta_{2\perp} k_{1\perp}(k_1)_\parallel)$ and $(0, \tau_{2\perp} k_{1\perp})$ occurring in (3.7) are obtained by inversion of an operator, which is defined by the collision integrals I_1 and I_2 and will be denoted by M, from the formulas

$$((\eta_1)_\parallel, (\eta_2)_\parallel) = 2M^{-1}(F_1, F_2),$$
$$((k_1)_\parallel \delta_1, (k_1)_\parallel \delta_2) = 2M^{-1}\left((k_1)_\parallel \left(\varepsilon_1' - \frac{\langle \varepsilon' k_\parallel^2 \rangle_1}{\langle k_\parallel^2 \rangle_{12}}\right), -(k_1)_\parallel \frac{\langle \varepsilon' k_\parallel^2 \rangle_1}{\langle k_\parallel^2 \rangle_{12}}\right),$$
$$((k_1)_\parallel \tau_1, (k_1)_\parallel \tau_2) = 2M^{-1}\left((k_1)_\parallel \left(\varepsilon_1 \varepsilon_1' - \frac{\langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12}}{\langle k_\parallel^2 \rangle_{12}}\right),\right. \quad (3.8)$$
$$\left.(k_1)_\parallel \left(\varepsilon_2 \varepsilon_2' - \frac{\langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12}}{\langle k_\parallel^2 \rangle_{12}}\right)\right),$$
$$(0, (k_1)_\parallel k_{1\perp} \eta_{2\perp}) = 2M^{-1}(0, \varepsilon_2'(k_1)_\parallel k_{1\perp}),$$
$$(0, k_{1\perp} \tau_{2\perp}) = 2M^{-1}(0, \varepsilon_2 \varepsilon_2' k_{1\perp}),$$

where

$$F_1 = \varepsilon_1'(k_1)_\parallel^2 + \varepsilon_1 \frac{\langle \varepsilon \rangle_1 \langle \varepsilon' k_\parallel^2 \rangle_1 - \langle 1 \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12}}{\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2}$$
$$+ \frac{\langle \varepsilon \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12} - \langle \varepsilon^2 \rangle_{12} \langle \varepsilon' k_\parallel^2 \rangle_1}{\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2}, \quad (3.9)$$
$$F_2 = \varepsilon_2'(k_1)_\parallel^2 + \varepsilon_2 \frac{\langle \varepsilon \rangle_1 \langle \varepsilon' k_\parallel^2 \rangle_1 - \langle 1 \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12}}{\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2},$$

We find the second approximation for a, b and c from the orthogonality conditions (3.2). The function P(E, k) in the second approximation is given by formula (2.26) in terms of the second approximation for the function h and is found to be equal to

$$(3.10)$$
$$P(E, k) = \frac{\delta^2 k_\parallel^2 (E - i\alpha E^2 + i\varkappa_{2\parallel} k_\parallel^2 + i\varkappa_{2\perp} k_\perp^2)}{(E + i\varkappa_{1\parallel} k_\parallel^2 + i\varkappa_{1\perp} k_\perp^2)[(E + i\eta_\parallel k_\parallel^2 + i\eta_\perp k_\perp^2)^2 - u^2 k_\parallel^2]}.$$

In (3.10), δ^2 and u^2 were defined above, and the kinetic coefficients α, $\eta_\parallel$, $\eta_\perp$, $\varkappa_{1\parallel}$, $\varkappa_{2\parallel}$, $\varkappa_{1\perp}$ and $\varkappa_{2\perp}$ are defined by the formulas

$$\alpha = \frac{\langle k_\parallel^2 \rangle_{12} \langle \varepsilon' k_\parallel^2 \delta \rangle_1}{2 \langle \varepsilon' k_\parallel^2 \rangle_1^2},$$
$$\eta_\parallel = \frac{\langle \varepsilon' k_\parallel^2 \eta_\parallel \rangle_{12}}{\langle k_\parallel^2 \rangle_{12}}$$
$$+ 4 \frac{\langle \varepsilon' k_\parallel^2 \delta \rangle_1 p^2 + \langle \varepsilon \varepsilon' k_\parallel^2 \tau_\parallel \rangle_{12} q^2 - (\langle \varepsilon \varepsilon' k_\parallel^2 \delta \rangle_{12} + \langle \varepsilon' k_\parallel^2 \tau_\parallel \rangle_1) pq}{u^2 \langle k_\parallel^2 \rangle_{12} (\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2)^2},$$
$$\eta_\perp = \frac{\langle \varepsilon' k_\parallel^2 k_\perp^2 \eta_\perp \rangle_2}{2 \langle k_\parallel^2 \rangle_{12}} + 2 \frac{\langle \varepsilon \varepsilon' k_\perp^2 \tau_\perp \rangle_2 q^2}{u^2 \langle k_\parallel^2 \rangle_{12} (\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2)}, \quad (3.11)$$
$$\varkappa_{2\parallel} = 2 \langle 1 \rangle_1 [\langle \varepsilon' k_\parallel^2 \delta \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12}^2 + \langle \varepsilon \varepsilon' k_\parallel^2 \tau_\parallel \rangle_{12} \langle \varepsilon' k_\parallel^2 \rangle_1^2$$
$$- (\langle \varepsilon \varepsilon' k_\parallel^2 \delta \rangle_{12} + \langle \varepsilon' k_\parallel^2 \tau_\parallel \rangle_1) \langle \varepsilon' k_\parallel^2 \rangle_{12} \langle \varepsilon' k_\parallel^2 \rangle_1] \cdot$$
$$\cdot [\langle \varepsilon' k_\parallel^2 \rangle_1^2 (\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2)]^{-1},$$
$$\varkappa_{2\perp} = \frac{\langle 1 \rangle_1 \langle \varepsilon \varepsilon' k_\perp^2 \tau_\perp \rangle_2}{\langle 1 \rangle_1 \langle \varepsilon^2 \rangle_{12} - \langle \varepsilon \rangle_1^2},$$
$$\frac{\varkappa_{2\parallel}}{\varkappa_{1\parallel}} = \frac{\varkappa_{2\perp}}{\varkappa_{1\perp}} = \frac{\langle 1 \rangle_1 \langle k_\parallel^2 \rangle_{12} u^2}{4 \langle \varepsilon' k_\parallel^2 \rangle_1^2},$$

where

$$p = \langle \varepsilon^2 \rangle_{12} \langle \varepsilon' k_\parallel^2 \rangle_1 - \langle \varepsilon \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12},$$
$$q = \langle 1 \rangle_1 \langle \varepsilon \varepsilon' k_\parallel^2 \rangle_{12} - \langle \varepsilon \rangle_1 \langle \varepsilon' k_\parallel^2 \rangle_1. \quad (3.12)$$

Formula (3.10) leads to the appearance of one more (thermal) pole in the phonon Green function

$$E = -i \frac{1 - g^2\beta \langle 1 \rangle_1}{1 - \delta^2 u^{-2}} [\varkappa_{1\parallel} k_\parallel^2 + \varkappa_{1\perp} k_\perp^2]. \quad (3.13)$$

The sound branches in the spectrum acquire imaginary corrections $\sim k^2$. The corresponding formulas have the form

$$E_1(k) = E_1^{(0)}(k) - \frac{i}{E_1^2 - E_2^2} \left\{ (E_1^2 - c_0^2 k^2)(\eta_\parallel k_\parallel^2 + \eta_\perp k_\perp^2) \right. \quad (3.14)$$
$$\left. + \frac{1}{2} c_0^2 \delta^2 k^2 k_\parallel^2 \left[\alpha - (\varkappa_{2\parallel} - \varkappa_{1\parallel}) \frac{k_\parallel^2}{E_1^2} - (\varkappa_{2\perp} - \varkappa_{1\perp}) \frac{k_\perp^2}{E_1^2} \right] \right\},$$
$$E_2(k) = E_2^{(0)}(k) - \frac{i}{E_2^2 - E_1^2} \left\{ (E_2^2 - c_0^2 k^2)'(\eta_\parallel k_\parallel^2 + \eta_\perp k_\perp^2) \right.$$
$$\left. + \frac{1}{2} c_0^2 \delta^2 k^2 k_\parallel^2 \left[\alpha - (\varkappa_{2\parallel} - \varkappa_{1\parallel}) \frac{k_\parallel^2}{E_2^2} - (\varkappa_{2\perp} - \varkappa_{1\perp}) \frac{k_\perp^2}{E_2^2} \right] \right\},$$

where $E_1^{(0)}(k)$ and $E_2^{(0)}(k)$ are determined by Eq. (3.6) for the spectra branch in the acoustic approximation.

The "longitudinal" kinetic coefficients α, $\varkappa_{1\parallel}$, $\varkappa_{2\parallel}$ and $\eta_\parallel$, occurring in the formulas (3.11) and formally obtained by inversion of the operator M in accordance with (3.8), must in practice be calculated by approximate methods. It is possible to calculate the "transverse" coefficients $\varkappa_{1\perp}$, $\varkappa_{2\perp}$ and $\eta_\perp$ in explicit form in quadratures, since inversion of the operator M in the corresponding formulas (3.8) reduces to division by the function $2\Delta_2(k_1)$:

$$(k_1)_\parallel k_{1\perp} \eta_{2\perp} = (k_1)_\parallel k_{1\perp} \varepsilon_2' \Delta_2^{-1}, \quad k_{1\perp} \tau_{2\perp} = k_{1\perp} \varepsilon_2 \varepsilon_2' \Delta_2^{-1}. \quad (3.15)$$

As a result, explicit formulas

$$\langle \varepsilon \varepsilon' k_\perp^2 \tau_{2\perp} \rangle_2 = \langle \varepsilon_2^2 (\varepsilon_2')^2 k_\perp^2 \Delta_2^{-1} \rangle_2,$$
$$\langle \varepsilon' k_\parallel^2 k_\perp^2 \eta_{2\perp} \rangle_2 = \langle k_\parallel^2 k_\perp^2 (\varepsilon_2')^2 \Delta_2^{-1} \rangle_2. \quad (3.16)$$

are obtained for the averages $\langle \varepsilon \varepsilon' k_\perp^2 \tau_{2\perp} \rangle_2$ and $\langle \varepsilon' k_\parallel^2 k_\perp^2 \eta_{2\perp} \rangle_2$; the transverse kinetic coefficients can be expressed in terms of these averages.

Formulas, analogous in structure, for the transverse kinetic coefficients have been derived by other methods, e.g., in [4-6].

To conclude we consider the question of the stability of the low-frequency energy spectrum defined by the

poles of the phonon Green function. One can show that if the inequality

$$g^2\beta\langle 1\rangle_i < 1, \qquad (3.17)$$

holds, the spectrum is stable, since all its branches have a negative imaginary part. The condition (3.17) for a system without a magnetic field reduces at low temperatures, as was shown in [3], to the constraint on the coupling constant obtained by Migdal.[9]

In a quantizing magnetic field, the inequality (3.17) can be violated even when Migdal's constraint on the coupling constant is satisfied. This violation occurs when, at a sufficiently low temperature, one of the Landau levels approaches the Fermi surface. When the Fermi and Landau levels coincide, we have a lower bound for $g^2\beta\langle 1\rangle_i$

$$g^2\beta\langle 1\rangle_i > \frac{2g^2\beta eH}{(2\pi)^2}\int_{-\infty}^{\infty} dp\,\frac{e^{\beta p^2/2m}}{(e^{\beta p^2/2m}+1)^2} = g^2(2m\beta)^{1/2}eHa, \quad (3.18)$$

where $a = 2(2\pi)^{-5/2}(1-2^{-3/2})\zeta(3/2)$ is a numerical constant. It is clear from (3.18) that the stability condition (3.17) is violated at sufficiently low temperatures.

To study the system when the condition (3.17) is violated requires a modification of perturbation theory. It is possible that it is necessary to introduce anomalous Green functions to take into account the formation of pairs of the Cooper type. The formation of particle-hole pairs in a two-band model was studied by Abrikosov.[10,11] In the case considered here, however, such a symmetry-breaking mechanism is clearly impossible. We remark again that, in the model we have treated, coincidence of the Fermi and Landau levels, and not only a large value of the magnetic field, is important. The experimental observation of the instability, and possibly of the phase transition associated with it, requires low temperatures and strong magnetic fields.

For example, for $g^2m^2v_F\pi^{-2} \sim 1/2$, $\epsilon_F \sim 1$ eV, a possible phase transition occurs at $HT^{-1/2} \sim 10^7$ Gauss. deg$^{-1/2}$, i.e., at temperature $T = 1°K$, a magnetic field $H \sim 10^7$ Gauss is required.

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