NORMAL COMPONENT DENSITY OF He³—He⁴ SOLUTIONS AT TEMPERATURES DOWN TO 0.4°K

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The normal component density was measured by the oscillating disk-stack method in He³—He⁴ solutions containing up to 0.54 He³ by weight at temperatures from the λ point down to 0.4°K. The Bose and Fermi branch parameters of the elementary excitation spectrum in the solutions are estimated on the basis of the data thus obtained.

ONE of the important characteristics of a superfluid-the density of the normal component ρ_n—has been studied in solutions of helium isotopes by different authors.[1-4] Solutions with a small concentration of He³ were chiefly studied in these researches, and the range of temperatures from T_0 to 1.3°K was covered; the results obtained in these researches are well described by the theory of Pomeranchuk[5], which is based on the assumption of the ideal nature of the gas of quasiparticles.

Only one of the works[6] was devoted to the determination of the normal component density in concentrated solutions. In it, a departure was observed from a linear dependence of ρ_n on the weight concentration x. This circumstance was connected with the dependence of the effective mass of the impurity excitations on the concentration, and with the possibility of change of the parameters of the phonon and roton spectra. Data necessary for a detailed consideration of this question could be obtained only in the measurement of ρ_n in the range of sufficiently low temperatures, when $\rho_{ni} \gg \rho_{n4}$ (ρ_{ni} and ρ_{n4} are the densities of the normal component due to the impurities and to He⁴, respectively.

In the present work, which is a continuation of[6], direct measurements are carried out of the density of the normal component over the entire range of existence of superfluidity in He³—He⁴ that is found at the saturated vapor pressure.

1. ARRANGEMENT AND EXPERIMENTAL PROCEDURE

The density of the normal component was measured by the oscillating disk-stack method;[7] however, in contrast with the aluminum disks usually employed, we used disks made of copper for the range of temperatures studied. This was done because the transition of aluminum to the superconducting state leads to the appearance of significant instabilities in the stack oscillations. The stark 1 consisted of 24 copper disks (Fig. 1) of diameter 30 mm and thickness 15 microns. The distance between the disks amounted to 0.22 mm and for all the temperatures and concentrations used was less than the penetration depth of the viscous wave. The stack of plate was suspended by an elastic thread, which was at room temperature. To obtain the initial
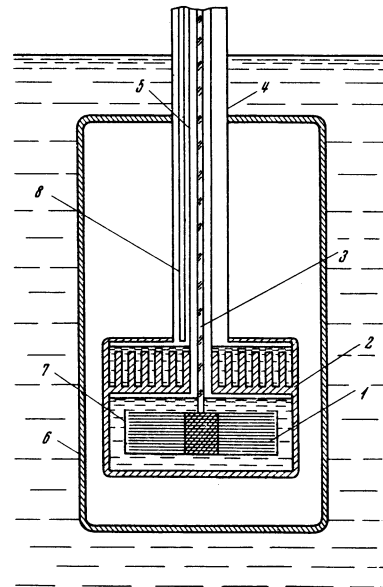

FIG. 1. Low-temperature part of the apparatus for measurement of the density of the normal component: 1—stack of disks, 2—copper block with reservoir for He³ (upper chamber) and He³-He⁴ solution (lower chamber), 3—straight glass rod, 4—tube for pumping off He³ vapor, 5—tube along which the condensation of the He³-He⁴ takes place, 6—vacuum jacket, 7—container to which the stack of disks is fastened, 8—tube connecting to the McLeod gauge.

condition of a departure of the system from the equilibrium position, we used a special repeating apparatus, which operated under the action of a magnetic field. The system was carefully adjusted so that various bendings and wobblings did not occur.

The chamber with the stack of disks and the He³ reservoir used to obtain the low temperatures consisted of a single copper block 2 on which 200 ribs of cross section 1.5 × 1.5 mm and height 15 mm were milled to assure good thermal contact of the liquid He³ with the apparatus. The low temperature part of the apparatus was located in a vacuum jacket and cooled by liquid He⁴ in the usual way (1.45°K). The temperature down to 0.4°K was obtained by pumping of He³ vapor through the tube 4 of diameter 14 mm by a carbon adsorption pump located in another dewar at a temperature of 4.2°K. The temperature in the apparatus was regulated by the rate of pumping of He³ vapor and

was kept constant to within $\pm 0.003°$K. Control of the stability of the temperature was provided by means of a carbon resistance thermometer, which was put in the bath with He³. The temperature was determined from the He³ vapor pressure (T_{62} temperature scale), which was measured by a McLeod gauge attached to the tube 8, which terminated at the He³ surface. The temperature was corrected for the thermomolecular pressure difference.[8]

The relative density of the normal component ρ_n/ρ_λ (ρ_λ is the density of the solution at the λ point) was determined from the formula

$$\frac{\rho_n}{\rho_\lambda} = \frac{\Theta^2 - \Theta_0^2}{\Theta_\lambda^2 - \Theta_0^2} \frac{\delta_\lambda \Psi(\delta_\lambda)}{\delta \Psi(\delta)} \tag{1}$$

where Θ, Θ_λ and Θ_0 are the oscillation periods for a specified temperature, the λ point and in a vacuum, respectively; $\delta = (\Theta \eta_n / \pi \rho_n)^{1/2}$ is the penetration depth of the viscous wave, η_0 is the viscosity of the normal component. The function

$$\Psi(\delta) = \frac{\text{sh}(a/\delta) + \sin(a/\delta)}{\text{ch}(a/\delta) + \cos(a/\delta)} + \frac{R_1^4}{2N(R^4 - r^4)}\left[\frac{\text{sh}(2h/\delta) - \sin(2h/\delta)}{\text{ch}(2h/\delta) - \cos(2h/\delta)}\right.$$
$$+ \frac{\text{sh}(2l/\delta) + \sin(2l/\delta)}{\text{ch}(2l/\delta) + \cos(2l/\delta)} + \frac{4L}{R_1} \cdot \frac{\text{sh}(2d/\delta) - \sin(2d/\delta)}{\text{ch}(2d/\delta) - \cos(2d/\delta)}$$
$$\left. + \frac{8L}{R_1} \frac{\text{sh}(q/\delta) + \sin(q/\delta)}{\text{ch}(q/\delta) + \cos(q/\delta)}\right], \tag{2}$$

where a is the distance between the disks, q that between the edges of the disks and the internal cylindrical surface of the copper container, d that between the wall of the chamber and the external lateral surface of the container, h that between the bottom of the chamber and the bottom of the stack and l the distance between the top disk and the surface of the liquid; R is the radius of the disks, R_1 the radius of the container, r the radius of the packing between the disks, L the height of the stack and N the number of disks.

The value of δ was determined from the damping decrement and the period of oscillation from the equation

$$\frac{\Psi(\delta)}{\chi(\delta)} = \frac{\Theta^2 - \Theta_0^2}{\Theta^2 + \Theta_0^2} \frac{2\pi}{\Theta(\gamma - \gamma_0)}, \tag{3}$$

where $\gamma_0 \Theta_0$ is the damping decrement in a vacuum. Equation (3) was solved graphically for δ. The function $\chi(\delta)$ is written down in the following fashion:

$$\chi(\delta) = \frac{\text{sh}(a/\delta) - \sin(a/\delta)}{\text{ch}(a/\delta) + \cos(a/\delta)} + \frac{R_1^4}{2N(R^4 - r^4)}\left[\frac{\text{sh}(2h/\delta) + \sin(2h/\delta)}{\text{ch}(2h/\delta) - \cos(2h/\delta)}\right.$$
$$+ \frac{\text{sh}(2l/\delta) - \sin(2l/\delta)}{\text{ch}(2l/\delta) + \cos(2l/\delta)} + \frac{4L}{R_1} \cdot \frac{\text{sh}(2d/\delta) + \sin(2d/\delta)}{\text{ch}(2d/\delta) - \cos(2d/\delta)}$$
$$\left. + \frac{8L}{R_1} \frac{\text{sh}(q/\delta) - \sin(q/\delta)}{\text{ch}(q/\delta) + \cos(q/\delta)} + \frac{6\delta}{R_1}\left(\frac{L}{R_1} + 0.8\right)\right]. \tag{4}$$

Equations (1)—(4) were obtained from the solutions of the Navier-Stokes equation for the appropriate boundary conditions. The method of solution of this problem given in[2,7] was used, but, in addition, corrections were introduced for the so-called corner effect;[9] furthermore, use was made of the more accurate asymptotic form of the Bessel functions and the drag of the liquid in the space between the edges of the disks and the internal surface of the container was taken into account.

Measurement of the period and damping of the oscillations was carried out in the following way. When the oscillatory system passed through the zero position, a ray of light, reflected from a mirror attached to the upper part of the glass rod, was incident on the photocell STsV-51. At this instant, the set of counters attached to it activates the electronic frequency meter ChZ-3, which operates so as to measure time intervals. The pulse which is generated in the return passage of the system through the zero position (i.e., the corresponding half-period) is delayed by the counter unit and the next pulse shuts off the frequency meter. Thus the complete period of oscillation of the system is measured. The damping was determined by means of the same apparatus, but now two photocells are used, located at a distance y from one another, while the time t_n of passage of this distance by the ray was measured. The quantities y and t_n are connected by the following equation:

$$y = A_0 e^{-\gamma(n\Theta + t_n)} \sin \omega t_n, \tag{5}$$

where $\gamma\Theta$ is the logarithmic damping decrement, $\omega = 2\pi/\Theta$, A_0 is the initial amplitude (for n = 0), n the number of oscillations. The quantity $\gamma\Theta$ was determined by the slope of the straight line in the graph of $\ln \sin \omega t_n$ against n.

The experiment was performed as follows. The He³—He⁴ solution (15 n.l.) and pure He³ (15 n.l.) were condensed in the corresponding chamber at a temperature of $1.45°$K. Then the adsorbi i_S pump was connected, cooling the He³ vapors, and the required temperature was established. After 40—50 min at constant temperature, measurements were made of the period and damping of the oscillations of the stack of disks. At each temperature, the period was measured no fewer than 20 times. For all the solutions, measurements were made both decreasing and increasing temperature; no hysteresis was observed.

When the temperature fell below the temperature of stratification of the solution, the stack of disks was in the superfluid phase, the concentration of He⁴ in which could be determined from the known temperature with the use of the phase diagram of the solutions.[10] Measurements under these conditions was made until the layer boundary was above the stack of disks.

The period and damping of the oscillations of the stack of disks were measured in the temperature range from T_λ to $0.41°$K in He³—He⁴ solutions with weight concentrations of He³ equal to 0.085, 0.156, 0.256, 0.380, and 0.534. Control experiments were carried out in pure He⁴ before beginning the measurements in solutions and after these measurements had been completed. The period of the oscillations in a vacuum amounted to 14.745 ± 0.005 sec.

The data obtained on the period and damping of the oscillations of the stack of disks in solutions were used for the calculation of ρ_n/ρ_λ by means of Eqs. (1)—(4).

Estimate of errors. At temperatures above $1.4°$K, the accuracy of the determination of the temperature was within $\pm 0.002°$K, and at lower temperatures the error increased; at the minimum temperature it amounted to $0.005°$K. The accuracy with which the concentration of the solution was known lay within $\pm 0.1\%$ He³, and the change in concentration due to the vapor phase did not exceed 0.05% He³. The period of the oscillations could be measured to within ± 0.001

sec; however, because of the dependence of the elastic properties of the thread on the temperature, which could change by several degrees during the course of the experiment, and because of the shaking of the apparatus, which we have not been able to eliminate completely at the present time, the actual accuracy of the determination of the period was within ±0.005 sec.

The logarithmic damping decrement was determined to within 2%, which produced an error in the determination of the correction to Eq. (1) (i.e., to the quantity $1 - \delta_\lambda \Psi(\delta_k)/\delta\Psi(\delta)$) of the order of 10%. The correction increased with decrease in temperature and concentration; thus, for pure He^4 it amounted to about 30% at the minimum temperature, and for solutions with concentrations of 0.085, 0.156, 0.256, 0.380 and 0.534, it was 19, 11, 9, 3 and 1.5%, respectively. As the λ point was approached, the corrections decreased to zero.

The error in the determination of ρ_n/ρ_λ consisted of the errors in the measurement of the period of the oscillations and in the determination of the correction.

The absolute error in ρ_n/ρ_λ associated with the period did not in practice depend on the temperature and increased somewhat with increase in the concentration, inasmuch as the contribution of the liquid to the moment of inertia of the system decreased for decrease in density of the solution. This error was equal to 0.10 for pure He^4 and was 0.011, 0.011, 0.012, 0.013 and 0.014 for 0.085, 0.156, 0.256, 0.380 and 0.534, respectively. These quantities constituted the absolute error in ρ_n/ρ_λ close to T_λ; however, as the temperature decreased, the error connected with the correction increased, so that the absolute error at the minimum temperatures for pure He^4 and for solutions with concentrations of 0.085, 0.156, 0.256, 0.380 and

0.534 amounted to 0.011, 0.015, 0.015, 0.017, 0.015, and 0.016, respectively.

The relative error increased with decrease in ρ_n/ρ_λ and at the lowest temperatures, for pure He^4 and solutions with the concentrations 0.085, 0.156, 0.256, 0.380, and 0.534, amounted to 40, 9, 5, 4, 2 and 2%, respectively.

2. RESULTS AND DISCUSSION

The dependence of the relative density of the normal component ρ_n/ρ_λ on the temperature T for solutions with a different content of He^3 is shown in Fig. 2, from
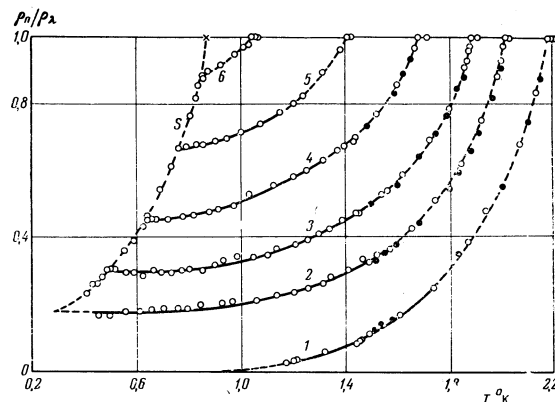

FIG. 2. The dependence of the density of the normal component on the temperature in He^3–He^4 solutions: 1—pure He^4, 2—0.085 He^3, 3—0.156 He^3, 4—0.256 He^3, 5—0.380 He^3, 6—0.534 He^3, S the stratification line of the solutions in the coordinates ρ_n/ρ_γ, T. The continuous lines correspond to values computed from Eq. (9) with the values of the parameters given in Table 2. The dashed lines are drawn through the experimental points; O—data of the present research, ●—[6], X—[11].

Table I. Dependence of the relative density of the normal component on the temperature

T, °K	ρ_n/ρ_λ	T, °K	ρ_n/ρ_λ	T, °K	ρ_n/ρ_λ	T, °K	ρ_n/ρ_λ	T, °K	ρ_n/ρ_λ
He⁴		1.490	0.326	1.559	0.541	1.680	1.000	1.313	0.893
		1.465	0.335	1.536	0.530	1.667	0.969	1.236	0.822
2.203	1.000	1.409	0.301	1.452	0.471	1.596	0.858	1.203	0.802
2.188	1.000	1.343	0.282	1.442	0.472	1.519	0.771	1.151	0.773
2.180	1.000	1.311	0.261	1.389	0.454	1.440	0.700	1.076	0.738
2.139	0.838	1.256	0.248	1.338	0.426	1.432	0.687	1.004	0.714
2.074	0.681	1.202	0.235	1.294	0.411	1.396	0.674	0.946	0.694
1.940	0.481	1.136	0.227	1.253	0.392	1.370	0.662	0.903	0.685
1.875	0.387	1.059	0.212	1.208	0.381	1.315	0.632	0.859	0.678
1.740	0.248	0.966	0.209	1.137	0.366	1.253	0.598	0.831	0.675
1.610	0.168	0.930	0.204	1.101	0.346	1.205	0.581	0.792	0.671
1.492	0.113	0.845	0.198	1.049	0.341	1.126	0.556	0.762	0.665
1.458	0.095	0.796	0.186	0.985	0.341	1.033	0.529	0.732*	0.609*
1.454	0.091	0.758	0.188	0.932	0.330	0.972	0.493	0.688*	0.542*
1.441	0.082	0.708	0.187	0.900	0.319	0.913	0.482	0.641*	0.462*
1.322	0.058	0.660	0.184	0.851	0.301	0.879	0.476		
1.210	0.034	0.614	0.177	0.803	0.305	0.815	0.464	53.4% He³	
1.200	0.032	0.561	0.178	0.772	0.299	0.775	0.460		
1.170	0.025	0.501	0.166	0.732	0.292	0.721	0.452	1.066	1.000
		0.456	0.166	0.694	0.293	0.674	0.452	1.057	1.000
				0.664	0.300	0.656	0.453	1.053	1.000
8.5% He³				0.624	0.284	0.639	0.451	1.042	1.000
		15.6% He³		0.589	0.294	0.625*	0.431*	1.028	0.977
2.035	1.000			0.561	0.292	0.589*	0.388*	1.012	0.968
2.031	1.000	1.907	1.000	0.519	0.304	0.553*	0.358*	0.979	0.948
2.017	1.000	1.888	1.000	0.498	0.304	0.489*	0.302*	0.923	0.915
2.007	0.978	1.884	1.000	0.471*	0.280*	0.433*	0.259*	0.873	0.895
1.999	0.933	1.878	0.974	0.450*	0.260*			0.855	0.875
1.983	0.885	1.875	0.963	0.410*	0.231*			0.852	0.880
1.922	0.754	1.868	0.930			38.0% He³		0.837*	0.855*
1.845	0.624	1.857	0.911	25.6% He³				0.827*	0.815*
1.746	0.512	1.797	0.789			1.419	1.000	0.806*	0.765*
1.655	0.429	1.722	0.696	1.712	1.000	1.407	1.000		
1.572	0.366	1.620	0.592			1.382	0.962		
1.520	0.348								

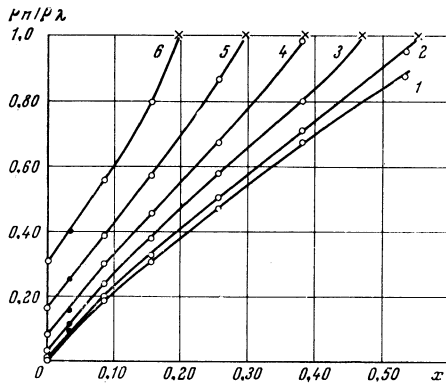*Data were obtained in the region of stratification of the solutions.

FIG. 3. Concentration dependence of the density of the normal component for various temperatures: $1-0.85$, $2-1.0$, $3-1.2$, $4-1.4$, $5-1.6$, $7-1.8°K$, O—present research, ●—[1], X—[10].

which it follows that upon a lowering of the temperature, ρ_n/ρ_λ decreases monotonically in solutions of all concentrations. For T_λ and T_S the curves experience kinks. The curve S represents the line of stratification of the solutions in the coordinates ρ_n/ρ_λ, T and terminates with a point corresponding to the critical concentration 0.603 (weight parts of He³), for which ρ_n/ρ_λ 1.000.[11] Furthermore, points have been plotted in the graph that were obtained earlier[6]; these are in excellent agreement with the results of the current work. The experimental values of ρ_n/ρ_λ are given in Table 1.

Figure 3 shows the concentration dependence of the relative density of the normal component. The data of[1] are given in this figure for a solution of weight content of He³ x = 0.033 as well as the points corresponding to the λ concentrations for a given temperature[10] (i.e., concentrations for which the corresponding temperature is the temperature of the λ transition). Attention is called to the fact that the departure from a linear law for the concentrated solutions has different signs, depending on the temperature.

On the basis of the data on the density of the normal component, some estimates are given of the parameters of the spectrum of elementary excitations in He³—He⁴ solutions. As is known, the first attempt to obtain information on such parameters was made in the use of data on ρ_n, which were obtained on the basis of measurements of the speed of fourth sound in solutions.[12] Similar to[12], we shall assume that the system of interacting bosons (He⁴) and fermions (He³) can be described within the framework of the model of a gas of quasiparticles. In this case, the density of the normal component ρ_n can be represented in the form

$$\rho_n = \rho_{ni} + \rho_{nr} + \rho_{nph}, \qquad (6)$$

where ρ_{ni}, ρ_{nr} and ρ_{nph} are the impurity, proton and d phonon parts, respectively, of the density of the normal component. Moreover, we shall assume that ρ_{ni} does not depend on the temperature, and that ρ_{nr} in the solutions is described by the Landau formula[13]

$$\rho_{nr} = bT^{-1/2}e^{-\Delta/T}, \qquad (7)$$

where

$$b = {}^{2}/_{3}(m_r{}^{*})^{1/2}p_0{}^{4}/(2\pi)^{3/2}\hbar^3 k^{1/2}, \qquad (8)$$

while m_r^*, p_0 and Δ are the parameters of the roton spectrum, which can depend on the concentration; k is Boltzmann's constant.

By assuming further that the corresponding Landau formula is also valid for ρ_{nph} in He³—He⁴ solutions,[14] we can easily compute this quantity, knowing the speed of first sound in solutions. Such calculations show that for all the temperatures and concentrations used by us, ρ_{nph} is negligibly small. Thus,

$$\frac{\rho_n}{\rho_\lambda} = \frac{\rho_{ni}}{\rho_\lambda} + \frac{b}{\rho_\lambda\sqrt{T}}e^{-\Delta/T}. \qquad (9)$$

It should be noted that the number of rotons increases strongly with increase in temperature; therefore, the use of the quasiparticle model becomes incorrect as we approach the λ point. For pure He⁴, for example, for temperatures $T > 0.08T_\lambda$, departures are observed for the values of ρ_n computed from the energy spectrum, which were found from experiments on neutron scattering in HeII,[15] from the values measured by means of the stack of disks and computed from the speed of second sound. Inasmuch as there is no strict criterion for estimating the temperature range of applicability of the relation (9) in solutions, we shall assume its validity for $T < 0.8T_\lambda$, just as in[12].

In connection with this, the treatment of the experimental values of ρ_n/ρ_λ was carried out by the method of least squares with the use of (9). Inasmuch as the absolute error in the determination of ρ_n/ρ_λ changes little with change in temperature, the measurements were assumed to be of equal accuracy and the values of the parameters ρ_{ni}, b and Δ were determined from the condition of a minimum value of the quantity

$$\Phi = \sum\left(\frac{\rho_n}{\rho_\lambda} - \frac{\rho_{ni}}{\rho_\lambda} - \frac{b}{\rho_\lambda\sqrt{T}}e^{-\Delta/T}\right)_i^2 \qquad (10)$$

where the summation is carried out over all the experimental points (for $T < 0.8T_\lambda$). This was performed in the following way. The parameter Δ was specified in the range from 3 to $10°K$ with a step of $0.1°$, while the remaining parameters were found for a given Δ in the usual manner, inasmuch as the problem became linear. The calculations were carried out on the electronic computer M-20.

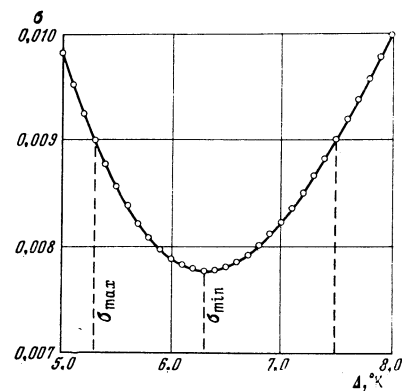For each solution, the curve for the dependence of

FIG. 4. Mean square error in the density of the normal component σ as a function of the parameter Δ for a solution with weight concentration of He³ equal to 0.085.

Table II. Parameters of the energy spectrum of He³—He⁴ solutions

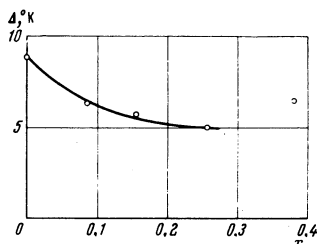| x	Δ, °K	p_0/h, A⁻¹	v_c, m/sec	ρ_{ni}/ρ_λ	m_s*/m_s
0.000	8.8±0.2	2.0±0.1	58±2	—	—
0.085	6.3±1.0	1.5±0.5	55±10	0.180±0.005	2.12±0.08
0.156	5.7±0.9	1.4±0.4	53±9	0.294±0.005	1.88±0.05
0.256	5.0±0.8	1.4±0.4	47±7	0.444±0.005	1.73±0.03
0.380	6.5±1.0	1.9±0.6	45±11	0.658±0.005	1.73±0.02

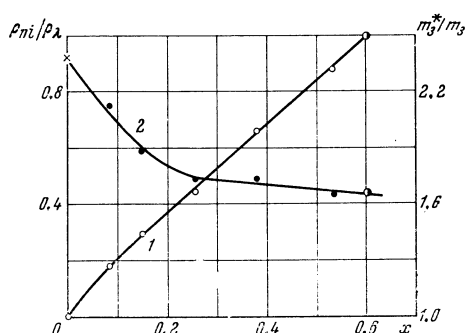FIG. 5. Dependence of the roton gap on the He³ concentration.

FIG. 6. Dependence of the impurity part of the density of the normal component (curve 1) and the effective mass of the impurity excitations (curve 2) on the concentration: O and ●—from the data of the present research, ◑—from [11], X—from [17].

the mean square error σ on Δ was constructed; here

$$\sigma = \sqrt{\Phi / (n - m)} \qquad (11)$$

(n is the number of experimental points, m the number of parameters determined). The desired value of Δ corresponded to a minimum value of σ. Figure 4 shows a typical curve of $\sigma(\Delta)$, constructed for a concentration of 0.085. An estimate of the error in the determination of Δ can be made with the use of the quantity

$$\sigma_{max} = \sigma_{min}(1 + [2(n - m)]^{-1/2}), \qquad (12)$$

which makes it possible to determine the error in Δ from the dependence of σ on Δ. The errors in the other parameters are determined by the error in Δ.

The dependence of Δ on the weight concentration x of the solution is shown in Fig. 5, from which it follows that Δ increases monotonically with increase in x. The strong departure from the monotonic curve for x = 0.380 is possibly connected with the inapplicability of the quasiparticle model for the description of the properties of sufficiently concentrated solutions. These values of Δ agree within the limits of error with similar values given in[12,16]. However, it is appropriate to remark that the values of ρ_n/ρ_λ, calculated from the data on the speed of fourth sound,[12] differ

from those found in the present research; the divergences become important for high concentrations of He³. One of the reasons for such divergences could be the necessity of the use in the calculation of ρ_n of quantities, other than the speed of fourth sound, that were found in other experiments. However, this fact does not play any significant role, inasmuch as the calculations of Δ are carried out for solutions that are not very concentrated.

Using the values of the parameter b and the quantity $m_r^* = 0.16m_4$[15] (m_4 is the mass of the He⁴ atom), we can determine p_0 from Eq. (8). The resultant value of p_0 for He⁴ agrees within the limits of error (5%) with the known value of this quantity.[15] In the case of He³—He⁴ solutions, an assumption is made on the independence of m_r^* of the concentration which, however, should not strongly affect the determination of the quantity p_0, since a change in m_r^* by a factor of, say, three makes an uncertainty in p_0 of less than 15%. Estimates of the total error in p_0 lead to a quantity of the order of 30%.

Using the values found for Δ and p_0, we can estimate the critical speed of motion of the superfluid component $v_c = \Delta/p_0$[14] in He³—He⁴ solutions, a velocity that is associated with roton formation. The values of the quantities Δ, p_0 and v_c are given in Table 2. The values of the impurity part of the density of the normal component are also given in this same table and in Fig. 6. Inasmuch as ρ_{ni} practically coincides with the minimal value of ρ_n, the determination of the value of ρ_{ni} does not depend on the method of treatment of the data on $\rho_n(T)$.

The properties of weak solutions are well described on the basis of the ideal gas model of impurity excitations.[5] In this case, the relation

$$\rho_{ni} = m_{std}^*\dot{N}_{3_k} \qquad (13)$$

is valid for the impurity part of the normal component density. Here m_{3id}^* is the effective mass, N_3 the number of He³ atoms per unit volume. As has already been pointed out in[6], (13) is satisfied for any distribution law for the quasiparticles.

However, as the concentration of the solution increases, the interaction between the impurity excitations also increases, which can lead to the renormalization of the effective mass. Such a situation arises, for example, in strongly degenerate He³—He⁴ with account of the Fermi-liquid interaction.[18,19] The value of ρ_{ni}/N_3 here is not the same as the effective mass that enters into the dispersion relation $\epsilon(p)$.

Taking this into account, and having in mind also the fact that different effective masses can correspond to different processes, it is convenient in the case of the process of macroscopic motion of a gas of impurity

excitations to use the following determination of the effective mass:

$$m_3^* = \rho_{ni}/N_3 = m_3\rho_{ni}/\rho_\lambda x, \qquad (14)$$

where m_3 is the mass of the He³ atom. The values of m_3^*/m_3 computed from (14) for solutions of various concentrations are shown in Fig. 6 and in Table 2.

We point out a simple empirical formula which describes the dependence of m_3^* on x for $x \le 0.256$, within the limits of error:

$$m_3^* = m_{30}^*/(1 + ax); \qquad (15)$$

$m_{30}^* = 2.4m_3$, and $a = 1.6$. The value of the effective mass for $x = 0$ agrees well with the value obtained from data on the specific heat in solutions of low concentrations.[17] [1)].

Thus, within the limits of accuracy of the present research, the density of the normal component of superfluid He³—He⁴ solutions can be described by Eq. (9) with the values of the parameters given in Table 2, for the temperature range $T \le 0.8T_\lambda$ to $0.41°K$ in solutions with weight content of He³ up to 0.256.

However, it should be noted that the method described for the treatment of the experimental data on ρ_n in solutions is not unique, but, because of the absence of a theory of concentrated solutions, it is not possible to give a more rigorous consideration of this problem at the present time.

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1) As is known, [18] $m_3^{*\prime} = m_{30}^{*\prime}(1 + F_1/3)$, where $m_{30}^{*\prime}$ is the value of the effective mass in the limit of low concentration, F_1 is a parameter characterizing the Fermi-liquid interaction. This relation holds for the effective mass $m_3^{*\prime}$ which enters into the dispersion relation $\epsilon(p)$ in the case of degenerate solutions. Using known values of F_1 for weak solutions, [18] we can put the indicated relation in the form $m_3^{*\prime} = m_{30}^{*\prime}(1 + a'x)$. It is curious to note that $m_{30}^{*\prime} \approx m_{30}^*$ and $a' \approx a$ where m_{30}^* and a quantities entering into Eq. (15).

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