SHOCK WAVES PROPAGATING ALONG A MAGNETIC FIELD IN A COLLISIONLESS PLASMA

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We investigate perturbations excited by a pulsed magnetic "piston" and traveling along the magnetic field. It is observed that under certain conditions shock waves of the "switch-on wave" type are formed. The internal structure of the wave front is investigated.

INTRODUCTION

THE possible occurrence of shock waves in a plasma without collisions is closely connected with the existence of collective motions of the plasma particles. The existence of collisionless shock wave propagating across the magnetic field and at an angle to it in a plasma has been experimentally demonstrated[1]. The question of the existence of shock waves propagating along a magnetic field is at the present time under discussion. Its solution is of importance both from the point of view of astrophysical applications and from the point of view of a detailed study of collective turbulent processes in a plasma. Hydromagnetic longitudinal waves without allowance for dissipation effects have been theoretically investigated by Pataraya[2] and by Saffman[3]. It was shown that there exist stationary nonlinear waves traveling along the field. The characteristic frequencies of such waves turn out to be of the order of ω_{He}. Under real conditions, when the initial perturbation is not very rapid, and under the influence of dissipative effects, it is apparently possible to expect the frequencies of the stationary motions (if they exist) to be determined by the lower "dispersion frequency," which equals ω_{Hi}.

Perturbations propagating along the magnetic field were investigated experimentally by Watson-Munro, Gross, and James[4] and by Nesterikhin, Ponomarenko, and Yablochnikov[5]. Under the conditions of these experiments, however, they either obtained no stationary profile of the perturbation, or the role of pair collisions was appreciable.

In the present paper we present results of experiments on longitudinal waves in a collisionless plasma. It has been observed that under certain conditions there arises the so-called "switch-on wave" with an oscillatory structure, and the characteristic frequencies of the motions turn out to be close to ω_{Hi}.

DESCRIPTION OF SETUP

The experiments were performed with the UN-6 setup (Fig. 1). The setup consists of an evacuated glass volume of diameter D = 40 cm and length L = 400 cm, placed in a quasistationary magnetic field ($H_0 \sim$ 100–1000 Oe, t = 3 msec) with mirror configuration. The measurements were carried out in a hydrogen plasma.

The preliminary ionization of the gas (at a pressure $p_0 = 10^{-4}$ 10^{-3} mm Hg) was effected by a linear discharge between two grid electrodes (C = 24 μF, U = 10 kV, I_{max} = 2 kA). After 50 μsec, a cylindrical loop produced a rapid discharge of the θ-pinch type (C = 4 × 1.25 μF, U = 120 kV, t = 1.5 μsec). Thus, there was produced in the volume a plasma with density $n_0 = 10^{12}$–10^{14} cm^{-3}, a temperature T = 1–10 eV, and a degree of ionization $\alpha \sim$ 70–80%. The effective radius of the plasma column was approximately half the radius of the tube, i.e., 10 cm.

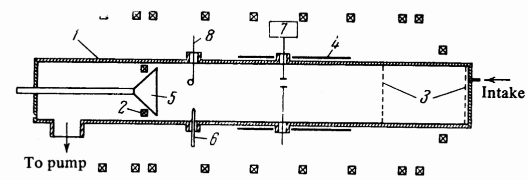

FIG. 1. Experimental setup: 1—vacuum volume, 2—coils of quasistationary field, 3—grid electrodes, 4—pre-ionization loops of the θ—pinch type, 5—conical shock loop, 6—electric probe, 7—microwave interferometer, 8—magnetic probe.

To excite the waves, a conical loop of the θ-pinch type was used, located on one end of the plasma volume. The pulsed magnetic field produced by the loop imparted to the plasma a rapid jolt in the direction of the force lines of the initial magnetic field. Simultaneously, this heated the plasma in the region of the shock loop.

The resultant perturbations were registered at several points distributed over the propagation direction. The magnetic field was registered with magnetic probes placed in insulating tubes. The plasma density was measured with the aid of a 4-mm radio interferometer. The transmitting and receiving antennas were placed directly in the plasma. The initial parameters of the unperturbed plasma (n_0, T_0, α) were measured with a triple Langmuir probe, analogous to that described by Sin Li-Chen and Sekiguchi[6], which was connected through an isolating transformer.

RESULTS OF EXPERIMENTS

The observations have shown that two different perturbations traveling along the magnetic field and following one another there are produced after the "jolt" of the magnetic piston. Such a "splitting" of the perturbation into two is clearly seen on the oscillograms of the longitudinal component of the magnetic field. Typical oscillograms of the longitudinal field (ΔH_z) together with the oscillograms of the azimuthal

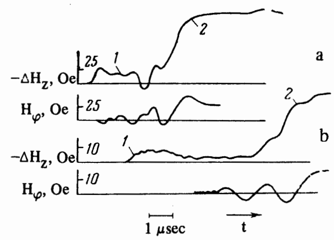

FIG. 2. Oscillograms of the longitudinal (ΔH_z) and azimuthal (H_φ) components of the magnetic field at distances z = 40 cm (a) and z = 120 cm (b) from the piston. The distance from the axis of the system is r = 5 cm, $n_0 = 3 \times 10^{13}$ cm^{-3}, H_0 = 300 Oe.

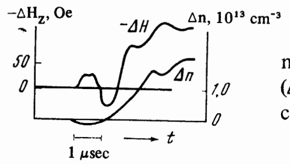

FIG. 3. Oscillogram of the longitudinal field (ΔH_z) and of the plasma density (Δn). z = 40 cm, r = 5 cm, $n_0 = 2 \times 10^{13}$ cm^{-3}, $H_0 \sim 420$ Oe.

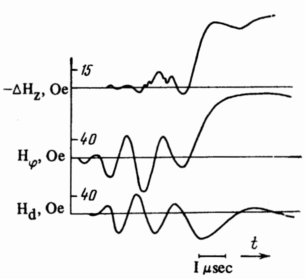

FIG. 4. Oscillograms of the longitudinal, azimuthal, and radial components of the magnetic field in the wave z = 80 cm, r = 5 cm, $H_0 \sim 300$ Oe, $n_0 = 2 \times 10^{13}$ cm^{-3}.

component are shown in Fig. 2. The oscillograms pertain to points located at a distance 40 cm (a) and 120 cm (b) from the piston and 5 cm away from the tube axis. The leading front of the perturbations, denoted 1 and 2 respectively, move with different velocities and the distance between them increases.

The characteristic velocity of the "fast" perturbations under the experimental conditions is $u \sim 10^8$ cm/sec, and that of the "slow" ones is 10^7 cm/sec. The amplitude of the magnetic field of these perturbations is respectively 0.1--0.2 H_0 and 0.3–0.5 H_0 at a distance 40 cm from the piston. At a distance 120 cm the amplitude increases by a factor 3–5.

The following features of the signals should be noted.

The signs of the registered signals correspond to a decreasing field in the perturbed region of the plasma. The field outside the plasma (at the same distance from the shock turn) increases at the same time, i.e., the magnetic field is being "pushed out" from the perturbed region. This is obviously connected with the fact that in this region the kinetic pressure of the plasma increases.

The plasma density behaves in this case in the following manner: in the region of the fast perturbation it decreases somewhat, and in the region of the slow perturbation the density increases simultaneously with the diamagnetic decrease of the field (see Fig. 3). In addition, in this region there appears an azimuthal component of the magnetic field (Figs. 2 and 4).

Thus, it can be stated that the fast perturbation is connected with the transport of heat only, without transport of matter as a whole. (The decrease of the density is due here obviously to the transverse diamagnetic expansion of the plasma.) These "fast" signals have apparently the same nature as the perturbations investigated in[7-9]. We shall henceforth consider only "slow" signals. The slow perturbation is connected with the transport of matter, and it represents a discontinuity in the pressure, density, and azimuthal component of the magnetic field; this discontinuity travels along the magnetic field.

The most essential feature of the process is the fact that when the perturbation propagates the width of this discontinuity remains unchanged over a large distance (80 cm at a discontinuity width $\Delta \sim 10$ cm). It follows therefore that the appearance of the diamagnetic signal cannot be connected, for example, with the free penetration of the heated plasmoid into the unperturbed plasma, for in this case a transformation of the profile of the perturbation would take place. (The thermal velocity of the particles, obtained from the diamagnetic signal, is comparable with the propagation velocity.)

Thus, the slow perturbation is a stationary motion of the shock-wave type. It has been established that the wave velocity depends little on the amplitude (the velocity measurements were carried out simultaneously at different distances from the piston) and in a sufficiently wide range of parameters ($n_0 \sim 10^{13}$–10^{14} cm^{-3}, $H_0 \sim 100$–1000 Oe) it turns out to be close to the Alfven velocity V_A.

DISCUSSION OF RESULTS

It is known that in magnetohydrodynamics there are two possible types of shock waves propagating along the magnetic field. These are ordinary hydrodynamic waves, in which the field does not change on passing through the front, and the so-called "switch-on waves," in which the transverse component of the magnetic field differs from zero behind the front.[10-12]. The velocity of such waves lies in the interval from V_A to $2V_A$. The presence in the observed wave of an azimuthal field and the proximity of its velocity to V_A offer evidence that this wave belongs to the second type.

As is well known, "switch-on" shock waves exist only in a definite intensity range. Thus, the Mach number M cannot exceed 2, and the transverse component of the field behind the front should be $H_\perp/H_0 \leq 1.5$. The experimentally measured maximal values of these parameters turn out to be smaller than follows from the theory of the plane one-dimensional wave. This apparently can be attributed to the following geometrical effect: behind the front of the wave, the heated plasma tends to expand across the magnetic force lines, and in the state behind the front the plasma column has a larger cross section; consequently, the plasma density behind the front will be lower than in the planar case.

We can attempt to take qualitatively into account the influence of this effect on the wave parameters. To this end we write down the ordinary relations for the continuity of the mass, momentum, and energy fluxes with allowance for the variation of the plasma-column cross section on going through the front (for waves without a transverse magnetic field, such calculations were made in[13]):

$$\rho_1 v_{1x} S_1 = \rho_2 v_{2x} S_2,$$

$$S_1\left(-\frac{H_{1x}^2}{8\pi} + \rho_1 v_{1x}^2 + p_1\right) = S_2\left(-\frac{H_{2x}^2}{8\pi} + \rho_2 v_{2x}^2 + p_2 + \frac{H_{2\varphi}^2}{8\pi}\right),$$

$$S_1(\tfrac{1}{2}\rho_1 v_{1x}^3 + \tfrac{5}{2}p_1 v_{1x}) = S_2(\rho_2 v_{2x}(v_{2\varphi}^2 + v_{2x}^2) + \tfrac{5}{2}p_2 v_{2x}),$$
$$\rho_2 v_{2x} v_{2\varphi} - H_{2x}H_{2\varphi}/4\pi = 0, \qquad \text{(A)}$$

where S, H_x, ρ, p, and v_x are respectively the cross section of the plasma column, the longitudinal magnetic field, the density, pressure, and longitudinal plasma velocity, and H_φ and v_φ are certain values of the azimuthal components of the field and of the velocity, averaged over the cross section. The indices 1 and 2 denote quantities pertaining to the regions ahead and behind the wave front.

It is necessary to add to these equations the balance equation, the equation for the frozen-in magnetic field, and the condition that the velocity and the magnetic field be parallel behind the wave:

$$p_2 - p_1 = \frac{H_{1x}^2 - H_{2x}^2}{8\pi}, \quad S_1H_1 = S_2H_2, \quad \frac{v_{2x}}{v_{2\varphi}} = \frac{H_{2x}}{H_{2\varphi}}. \qquad \text{(B)}$$

(Account is taken here of the fact that the diameter of the plasma cylinder is much smaller than the diameter of the external coils producing the magnetic field, and therefore no account is taken of the change of the field outside the plasma.)

The quantities ρ_1, p_1, $H_{1x} = H_0$, S_1, and H_{2x} can be regarded as specified. Then, from the value of the diamagnetic signal $\Delta H = H_{1x} - H_{2x}$, measured directly by the probe, it is possible to determine, by using these relations, all the wave parameters.

Figure 5 shows the experimental plots of M, H_φ/H_0, and $\eta = \rho_2/\rho_1$ against $\Delta H/H_0$. The solid lines show the results of calculation in accordance with relations (A) and (B) with $\beta = 0$. We see that the results of the experiment agree well with the "switch-on wave" model with allowance for the geometrical correction.

Let us examine the internal structure of the "switch-on wave." We consider here a one-dimensional plane problem. We use the model of two-fluid hydrodynamics, analogous to the procedure used by Karpman[14]. Since the mean free path is large under the experimental conditions, it is necessary to take into account the influence of dispersion effects on the structure of the wave front. The frequencies of the motions will be assumed to be $\omega \ll \omega_{He}$, in accord with the experimental value of the characteristic frequency (see Fig. 4) $\omega \sim 5 \times 10^{-6}\ \text{sec}^{-1} \ll \omega_{He}$. The plasma is assumed to be cold ($\beta \ll 1$), which also agrees with the experimental conditions. In addition, in the first approximation we consider only weak waves, when the kinetic pressure of the plasma can be neglected. From among the dissipative factors, we take into account only the finite conductivity. We shall assume here that the collision frequency is $\nu_{ei} \ll \omega_{He}$. Then the system of equations of the two-fluid hydrodynamics coincides with that given in[14]. This system can be reduced to a system of equations of magnetohydrodynamics "with ion dispersion"[5]:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{4\pi mn}[\mathbf{H}\,\mathrm{rot}\,\mathbf{H}], \qquad (1)^*$$

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}[\mathbf{v}\mathbf{H}] + \frac{c^2}{4\pi\sigma}\Delta\mathbf{H} + \frac{c}{4\pi e}\mathrm{rot}\left(\frac{1}{n}[\mathbf{H}\,\mathrm{rot}\,\mathbf{H}]\right), \qquad (2)$$

$$\frac{\partial n}{\partial t} + \mathrm{div}(n\mathbf{v}) = 0, \quad \mathrm{div}\,\mathbf{H} = 0. \qquad (3)$$

*$[\mathbf{H}\,\mathrm{rot}\,\mathbf{H}] \equiv \mathbf{H} \times \mathrm{curl}\,\mathbf{H}$.

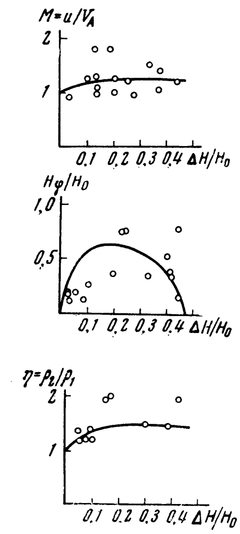

FIG. 5. Experimental dependences of the Mach number M, the azimuthal field behind the front H_φ/H_0, and of the density discontinuity $\eta = \rho_2/\rho_1$ on the intensity of the wave $\Delta H/H_0$. Solid lines—calculation.

where v is the mass velocity and m is the ion mass.

We change over to a coordinate system moving with the wave. We choose the directions of the axes such that the x axis is directed along the magnetic field and the y axis is directed along the transverse component of the magnetic field behind the front of the wave. Then the boundary conditions at $x \to -\infty$ are written in the form: $\nu_x = u_1$, $\nu_{1y} = \nu_{1z} = 0$, $H_{1y} = H_{1z} = 0$ (the index 1 labels quantities at $x \to -\infty$).

Equations (3) yield

$$nv_x = j = \text{const}, \quad H_x = H_0 = \text{const}. \qquad (4)$$

Taking into account the boundary conditions, we can obtain from Eqs. (1) and (2)

$$v_x = u_1 - \frac{H_y^2 + H_z^2}{8\pi mj}, \quad v_y = \frac{H_0H_y}{8\pi mj}, \quad v_z = \frac{H_0H_z}{8\pi mj}, \qquad (5)$$

$$v_yH_0 - H_yv_x + \frac{c^2}{4\pi\sigma}H_y' - \frac{mc}{e}v_xv_z' = 0, \qquad (6)$$

$$v_zH_0 - H_zv_x + \frac{c^2}{4\pi\sigma}H_z' - \frac{mc}{e}v_xv_y' = 0. \qquad (7)$$

Substituting (5) in (6) and (7) we obtain

$$\left(\frac{H_y^2 + H_z^2}{8\pi mj} - u_0\right)H_y + \frac{c^2}{4\pi\sigma}H_y' + \frac{cH_0v_x}{4\pi ej}H_z' = 0, \qquad (8)$$

$$\left(\frac{H_y^2 + H_z^2}{8\pi mj} - u_0\right)H_y + \frac{c^2}{4\pi\sigma}H_z' - \frac{cH_0v_x}{4\pi ej}H_y' = 0, \qquad (9)$$

where we have introduced the notation $u_0 = u_1 - H_0^2/4\pi mj = H_{2y}^2/8\pi mj$ (the index 2 labels the quantities behind the wave front).

We introduce the quantity $h = h_y + iH_z$ and, multiplying (9) by i, we add it to (8). We obtain the following equation for h:

$$\frac{|h|^2 - H_{2y}^2}{8\pi mj}h - \left(\frac{icH_0}{4\pi en} - \frac{c^2}{4\pi\sigma}\right)h' = 0. \qquad (10)$$

We now write the complex-conjugate equation

$$\frac{|h^2| - H_{2y}^2}{8\pi mj}\bar{h} - \left(-\frac{icH_0}{4\pi en} - \frac{c^2}{4\pi\sigma}\right)\bar{h}' = 0. \qquad (11)$$

From (10) and (11) we obtain an equation for $|h|^2 = H_y^2 + H_z^2$:

$$|\,|h|^2| = h'\bar{h} + \bar{h}'h =$$
$$= (H_{2y}^2 - |h|^2)\frac{c^2}{2\pi\sigma}|h|^2\frac{1}{8\pi mj}\left[\left(\frac{cH_0}{4\pi en}\right) + \left(\frac{c^2}{4\pi\sigma}\right)\right]^{-2}$$

$$= (H_{2y}^2 - |h|^2)\frac{\nu}{\omega_{He}}\left(\frac{\Omega_0}{c}\left(1+\frac{\nu^2}{\omega_{He}^2}\right)\right)\ |h|^2/2H_0\sqrt{H_0^2+{}^1/_2H_{2y}^2},$$

$\Omega_0^2 = 4\pi e^2 n/m$ (we have taken into account here the fact that $\nu \ll \omega_{He}$). This equation has a solution

$$|h|^2 = \frac{H_{2y}^2}{2}\left[\operatorname{th}\left(\frac{x}{\Delta}\right)+1\right],$$

$$\Delta = \frac{2c}{\Omega_0}\frac{\omega_{He}}{\nu}\frac{H_0\sqrt{H_0^2+{}^1/_2H_{2y}^2}}{H_{2y}^2},$$

where Δ is the effective width of the transition region. We now consider the structure of the wave in the front part, where $|h|^2 \ll H_{2y}^2$. Equation (11) has the following solution:

$$h \sim \exp i\left(\frac{x}{\delta}\left(1+i\frac{\nu}{\omega_{He}}\right)\right),$$

$$\delta = \frac{c}{\Omega_0}\frac{2H_0\sqrt{H_0+{}^1/_2H_{2y}^2}}{H_{2y}^2}.$$

Thus, in the leading part of the front there exist oscillations of the magnetic field, and the vector of the direction of the field in the oscillations is circularly polarized. The size of the oscillations is $\sim c/\Omega_0$, their frequency is $\sim\omega_{Hi}$, the damping coefficient is $\kappa = \nu/\omega_{He}$, and the effective total width of the transition region is $\Delta \sim (c/\Omega_0)(\nu/\omega_{He})^{-1}$. The oscillograms shown in Figs. 3 and 4 agree well with such a picture of the wave front.

Thus, the investigated perturbation traveling along the magnetic field comprises a "switch-on wave." The wave has an oscillatory structure determined by the ion dispersion near the frequency ω_{Hi}.

From the measured damping coefficient of the oscillations in the forward part of the front we can determine the collision frequency. The value $\kappa = \nu/\omega_{He}$ under the experimental conditions, in those cases when a stationary front is observed, lies in the range $^1/_2$ – $^1/_5$. Such a high collision frequency cannot be due to pair collisions, since the frequency of the pair collisions under typical experimental conditions ($n_0 \sim 10^{13}\ cm^{-3}$, $T \sim 1$ eV, $H_0 \sim 500$ Oe) is $\nu_{ei} \sim 10^8\ sec^{-1}$, and $\omega_{He} \sim 10^{10}\ sec^{-1}$.

The finite conductivity is connected in this case apparently with the collective mechanisms. The question of the concrete reason for the low conductivity calls for additional research. So far we can note only that the damping coefficient of the oscillations, i.e., the quantity ν/ω_{He}, decreases sharply with increasing magnetic field and with decreasing plasma density, and at sufficiently small n_0 ($n_0 \lesssim 10^{13}\ cm^{-3}$ and $H_0 \gtrsim 500$ Oe) a stationary wave profile is no longer observed.

It is interesting that the azimuthal field behind the wave front occurs "spontaneously," since the piston does not produce an azimuthal field. This probably may be connected with instability, against Alfven perturbations, of ordinary hydrodynamic waves traveling along the field, with velocities[12]

$$V_A < u < V_A\left(1+\frac{2}{\gamma-1}(1+\gamma\beta_1)\right).$$

It would be of interest to excite in the plasma a wave with velocity much larger than the Alfven velocity, when the occurrence of the switch-on wave is impossible, and to establish whether there exists some mechanism, other than the regular magnetic field, that leads to the establishment of the wave front.

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