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TWO-FREQUENCY PRECESSION OF MUONIUM IN A MAGNETIC FIELD

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A new phenomenon—two-frequency precession of the spin of the μ^+ meson in muonium in a transverse magnetic field—is predicted and investigated experimentally. The two-frequency precession is used to determine the magnitude of the hyperfine splitting of the free muonium atom in ice, germanium, and quartz. The phenomenon of cessation of the precession of the muonium μ^+ meson in a strong magnetic field is theoretically considered. Finally, the effect of incoherent interactions of muonium with the medium on the precession of the spin of the μ^+ meson in the muonium atom is considered.

THIS paper is devoted to the study of the precession of the spin of the μ^+ meson in muonium in a transverse magnetic field. It is shown that in a weak magnetic field, the precession of the μ^+ meson is described by two frequencies and has the form of beats. This phenomenon has been called the two-frequency precession of muonium[2)]. The frequencies ω_0 of the hyperfine splitting of the muonium atom in different substances are found with the aid of the two-frequency precession method. The precession of μ^+ meson in the muonium atom is described in Sec. 1; in Sec. 2 the effect of incoherent interaction processes with the medium which lead to a relaxation of the muonium spin is considered; in Sec. 3 an experiment for the observation of the two-frequency precession and the determination of ω_0 is described; in Sec. 4 the results obtained are discussed; and in Sec. 5 brief conclusions are presented.

1. MUONIUM IN A MAGNETIC FIELD

A muonium atom is formed when a μ^+ meson is slowed down in matter to such an extent that its velocity becomes comparable with the velocities of the atomic electrons. The free muonium atom is usually observed by means of the Larmor precession in a transverse magnetic field H with frequency $\omega = eH/2m_ec$, where m_e is the mass of the electron. However, such a one-frequency dependence of the polarization P(t) of the μ^+ meson in muonium is an approximation, which is valid only for small observation times. In fact, the dependence P(t) is determined by several frequencies $\omega_{ik} = \omega_i - \omega_k$, where the ω_i's are the energy eigenvalues of the stationary states of muonium in a magnetic field ($\hbar = 1$). The following are the expressions for ω_i[2]:

$$\left.\begin{aligned} \omega_1 &= {}^1/_4\omega_0 + \omega_-, \qquad & m &= +1 \\ \omega_2 &= -{}^1/_4\omega_0 + Q, & m &= 0 \\ \omega_3 &= {}^1/_4\omega_0 - \omega_-, & m &= -1 \end{aligned}\right\} \quad I = 1, \tag{1}$$

$$\omega_4 = -{}^1/_4\omega_0 - Q, \quad m = 0; \quad I = 0,$$

$$Q = \sqrt{\frac{1}{4}\omega_0^2 + \omega_+^2}, \quad \omega_\pm = \omega\left(1 \pm \frac{\mu_\mu}{\mu_e}\right) = \omega\left(1 \pm \frac{m_e}{m_\mu}\right)$$

Here

$$\omega_0 = \frac{32\mu_\mu\mu_e}{3\hbar a^3} = 2\pi\Delta\nu, \quad \Delta\nu = 4463 \text{ MHz} \tag{2}$$

is the frequency of hyperfine splitting of the ground state of the muonium atom[3] [3)]; μ_e, m_e, and μ_μ, m_μ are the modulus of the magnetic moment and the mass of the electron and μ^+ meson respectively; and a is the Bohr radius of the electron in muonium. The four values of ω_i correspond to different values of the spin I = 0, 1 of muonium and its projection m on the direction of the magnetic field. A schematic dependence (1) of the frequencies ω_i on the magnetic field in the units $x = H/H_0$ is shown in Fig. 1. The quantity $H_0 = \omega_0/2\mu_e = 32\mu_\mu/6\hbar a^3 = 1594.5$ Oe is the magnetic field at the location of the electron due to the magnetic moment of the μ^+ meson and separates the domain of the Zeeman effect ($H \ll H_0$) from that of the Paschen-Back effect ($H > H_0$)[4)].

It follows from expressions (1) that the time dependence of any physical quantity pertaining to muonium is, generally speaking, determined by six frequencies ω_{ik}. However, the polarization P(t) of the μ^+ meson in muonium is determined by only four frequencies with $\Delta m = \pm 1$. The coefficients of the two other frequencies are identically equal to zero. The "operating" frequencies are

$$\begin{aligned} \omega_{12} &= {}^1/_2\omega_0 + \omega_- - Q, \\ \omega_{23} &= -{}^1/_2\omega_0 + \omega^- + Q, \\ \omega_{14} &= {}^1/_2\omega_0 + \omega_- + Q, \\ \omega_{34} &= {}^1/_2\omega_0 - \omega_- + Q. \end{aligned} \tag{3}$$

In weak fields $H \ll H_0$ the relation (3) may be written in the form

$$\begin{aligned} \omega_{12} &= \omega_- - \Omega \approx \omega - \Omega, \\ \omega_{23} &= \omega_- + \Omega \approx \omega + \Omega, \\ \omega_{14} &= \omega_0 + \omega_- + \Omega \approx \omega_0 + \omega + \Omega, \\ \omega_{34} &= \omega_0 - \omega_- + \Omega \approx \omega_0 - \omega + \Omega. \end{aligned} \tag{4}$$

Here

$$\Omega = \frac{\omega_+^2}{\omega_0} \approx \frac{\omega^2}{\omega_0} = \omega\frac{H}{2H_0} = \frac{1}{4}\omega_0\left(\frac{H}{H_0}\right)^2 \sim H^2. \tag{5}$$

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[2)] The two-frequency muonium precession was first described in the authors' paper [1].

[3)] An expression for ω_0 which takes into account relativistic and radiative corrections has been obtained in [4].

[4)] The field H_0 practically coincides with the field $H_c = \omega_0/2(\mu_e + \mu_\mu) = 1586.7$ Oe usually used in the treatment of the Zeeman and the Paschen–Back effects.

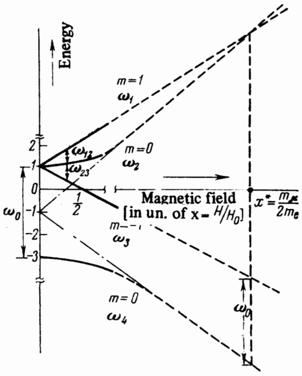

FIG. 1. Energy of the stationary states of muonium in a magnetic field. The arrows indicate the frequencies ω_{12} and ω_{23}, which determine the two-frequency precession in weak fields; $\hbar\omega_0$ is the energy of the hyperfine splitting of the ground state of muonium; $\Omega = (\omega_{23}-\omega_{12})/2$ is the beat frequency; $x^* = H^*/H_0 \approx m_\mu/2m_e$ is the field corresponding to the crossing point of the ω_1 and ω_2 terms.

In the case when the μ^+ meson is initially completely polarized, i.e., $P(t = 0) = 1$, we obtain for the time dependence of the polarization $P(t)$ of the μ^+ meson in the initial direction the expression (see Sec. 2):

$$P(t) = {}^1/_4[\cos\omega_{12}t + \cos\omega_{23}t + \cos\omega_{14}t + \cos\omega_{34}t] + \frac{\omega_+}{4Q}[\cos\omega_{12}t - \cos\omega_{23}t - \cos\omega_{14}t + \cos\omega_{34}t]. \tag{6}$$

Expression (6) for $P(t)$ might be simplified if the high frequencies ω_{14} and ω_{34} were not recorded in the experiment (as was the case in this experiment). For then the rapidly oscillating terms $\cos\omega_{14}t$ and $\cos\omega_{34}t$ would be averaged over and would not then contribute to the $P_{obs}(t)$:

$$P_{obs}(t) = \frac{1}{4}[\cos\omega_{12}t + \cos\omega_{23}t] + \frac{\omega_+}{4Q}[\cos\omega_{12}t - \cos\omega_{23}t] = \frac{1}{2}\cos\omega_-t\cos\Omega t + \frac{1}{2}\frac{\omega_+}{Q}\sin\omega_-t\sin\Omega t. \tag{7}$$

Here $\omega_- = (\omega_{12} + \omega_{23})/2 \approx \omega = eH/2m_ec$ is the muonium Larmor precession frequency in the field H; $\Omega = (\omega_{23} - \omega_{12})/2 \approx \omega^2/\omega_0$ is the modulation or "beat" frequency. In fields $H \ll H_0$ expression (7) for $P_{obs}(t)$ may be approximated by

$$P_{obs}(t) \approx \frac{1}{2}\cos\omega t\cos\Omega t + \frac{H}{2H_0}\sin\omega t\sin\Omega t \approx \frac{1}{2}\cos\omega t\cos\Omega t. \tag{8}$$

It may be seen from relation (8) that the time dependence of $P_{obs}(t)$ is determined by two frequencies and may be called a two-frequency precession (or beats) of the spin of the μ^+ meson in muonium.

The observation of the two-frequency precession enables us to find the frequencies ω and Ω and thus determine the frequency $\omega_0 = \omega^2/\Omega$ of hyperfine splitting of the muonium atom. The experimental determination of the frequencies ω_0 in different substances is described in Sec. 3.

In the region of the Paschen-Back effect, when $\omega_\pm \gg \omega_0$, we may observe still another beautiful phenomenon, namely, the cessation of the precession of the spin of the μ^+ meson in muonium. The cessation of the precession is related to the crossing of the ω_1 and ω_2 terms (see Fig. 1) in a field of intensity H* for which the frequency $\omega_{12} = 0$. The crossing of the ω_1 (m = 1) and the ω_2 (m = 0) terms may be explained by the fact that the resultant magnetic moment of muonium in the state 1 $(\mu_e - \mu_\mu)$ is smaller than in state 2 $(\mu_e + \mu_\mu)$. Consequently, the energy of the ω_2 term increases more rapidly with increase in the magnetic field than the energy of the ω_1 term. All the frequencies except ω_{12} will be very large in fields of intensity close to H* and the observed dependence $P_{obs}(t)$ given by (7) will be determined by the single frequency ω_{12}. At H = H*, when $\omega_{12} = 0$, the precession ceases and the μ^+ meson will, subsequently, maintain its polarization $P_{obs} = 1/2$ in the initial direction (see expression (7) for P(t) for $\omega_\pm \gg \omega_0$).

The magnitude H* of the field and the μ^+ meson precession frequency ω_{12} for fields close to H* are found from relation (3)

$$\omega_{12} = {}^1/_2\omega_0 + \omega_- - Q.$$

Here

$$\omega_\pm = \omega(1 \pm \zeta), \quad \zeta = \frac{m_e}{m_\mu}, \quad \omega = \frac{eH}{2m_ec}, \quad \omega_0 = \frac{eH_0}{m_ec}.$$

When $\omega_{12} = 0$, we obtain $\omega_0\omega_- = \omega_+^2 - \omega_-^2$, i.e.,

$$\omega^* = \frac{eH^*}{2m_ec} = \frac{1-\zeta}{4\zeta}\omega_0$$

or

$$H^* = \frac{1-\zeta}{2\zeta}H_0 = 1{,}64\cdot10^5\,\text{Oe}. \tag{9}$$

It may be seen from relation (9) that H* is equal to one-half the magnitude of the magnetic field produced at the location of the μ^+ meson by the electron in the muonium atom. Relation (9) gives the value of H* for H_0 = 1594.5 Oe, i.e., for the muonium atom in a vacuum. In a material medium, H_0 may differ from its vacuum value, and H* will correspondingly change. For H close to H*, the frequency ω_{12} is approximately equal to

$$\omega_{12} \approx 2\zeta\frac{e(H^* - H)}{2m_ec} = \frac{e\Delta H}{m_\mu c}, \tag{10}$$

where $\Delta H = H^* - H$. It follows from Eq. (10) that for $H \approx H^*$, the frequency ω_{12} is equal to the Larmor precession frequency of the free μ^+ meson in the field ΔH. An experimental determination of the field H* may also be used to determine the frequency ω_0 of the muonium atom in matter.

2. TWO-FREQUENCY PRECESSION OF MUONIUM IN MATTER

As has been said in Sec. 1, muonium is formed in matter. The muonium atom thus formed interacts with the matter. These interactions may be coherent or incoherent. The coherent interactions distort the muonium wave function and manifest themselves in a change in the hyperfine splitting frequency ω_0 of the muonium atom. The incoherent interactions with the medium lead to the depolarization of the muonium electron. Chemical reactions between the muonium atoms and the molecules of the medium also constitute an incoherent effect. The depolarization of the muonium electron leads to the transitions

$$(I = 1) \rightleftarrows (I = 0) \tag{11}$$

in the muonium atom. The transitions (11) lead to the depolarization of the μ^+-meson in the time $t \approx 1/\nu$, where ν is the flipping frequency of the spin of the electron in muonium. When the muonium atom reacts chemically to form a diamagnetic μ^+ molecule, the coupling between the spins of the μ^+ meson and the electron is severed and, with respect to the external magnetic field, the μ^+ meson begins to behave as a free particle. The time interval within which muonium forms a chemical compound is usually very small ($\lesssim 10^{-10}$ sec) and this makes a direct observation of muonium in most substances impossible.

Thus, the experimental observation of muonium precession is possible only in those substances in which its lifetime with respect to the processes of depolarization and capture in molecule formation is long enough ($\tau \gtrsim 0.1$ μsec). Furthermore, the incoherent transitions (11) lead to some change in the observed dependence $P_{obs}(t)$ (7). This is expressed as a change in the beat frequency $\Omega(\nu)$. In the experiments described below, these changes in the frequency Ω are small and may be neglected. We calculate below the dependence $\Omega(\nu)$ and the general form of the polarization $P(t)$ of the μ^+ meson in the muonium atom, taking into account the incoherent reactions with the medium.

The spin density matrix of muonium in a medium in the presence of an external magnetic field is described by the Wangsness-Bloch equations. Following[6], we shall analyze this system of equations in dimensionless variables. For the elements of the density matrix

$$\rho = \sum_{\varkappa, k=0}^{3} \rho_{\varkappa k} u_{\varkappa} u_{k}, \qquad (12)$$

where $u_0 = \chi/\sqrt{2}$ and $\mathbf{u} = \boldsymbol{\sigma}/\sqrt{2}$ are orthogonal spin operators; χ is the 2×2 unit matrix, σ is the Pauli operator, the system of equations has the form[6]

$$\begin{aligned} \frac{d\rho_{\varkappa 0}}{d\tilde{t}} &= -e_{\varkappa\lambda l}\rho_{\lambda l}(\tilde{t}) - 2\zeta x_{\mu} e_{\varkappa\mu\lambda}\rho_{\lambda 0}(\tilde{t}), \\ \frac{d\rho_{0k}}{d\tilde{t}} &= e_{k\lambda l}\rho_{\lambda l}(\tilde{t}) + 2x_m e_{kml}\rho_{0l}(\tilde{t}) - \gamma\rho_{0k}(\tilde{t}), \\ \frac{d\rho_{\varkappa k}}{d\tilde{t}} &= e_{\varkappa k\lambda}\rho_{\lambda 0}(\tilde{t}) - e_{\varkappa kl}\rho_{0l}(\tilde{t}) - 2\zeta x_{\mu} e_{\varkappa\mu\lambda}\rho_{\lambda k}(\tilde{t}) \\ &\quad + 2x_m e_{kml}\rho_{\varkappa l}(\tilde{t}) - \gamma\rho_{\varkappa k}(\tilde{t}). \end{aligned} \qquad (13)$$

Here e_{klm} is the antisymmetric unit tensor;

$$\tilde{t} = \frac{\omega_0}{2} t, \ \zeta = \frac{m_e}{m_\mu}, x_i = \frac{H_i}{H_0}, \gamma = \frac{4\nu}{\omega_0};$$

and ν is the frequency of the transitions (11). We must remember here that H_0 and ω_0 may, in a medium, differ from their vacuum values. The first (Greek) index of an element of the density matrix corresponds to the μ^+ meson while the second (Latin) index corresponds to the electron. In formula (13) and in all other formulas that follow, letter designations for the indices are used for their "vector" values 1, 2, 3 while the "scalar" meaning 0 is explicitly written out. As usual, if a letter index is repeated in an expression, summation over the "vector" values is implied.

We shall henceforth assume that initially the polarization of the μ^+ meson is directed along the axis 1 while the magnetic field is along the axis 2. Then, the polarization may be written in the form of a complex quantity

$$\mathbf{P}(t) = P_1(t) + iP_3(t),$$

while the initial conditions for the system (13) take the form[6]

$$\begin{aligned} &\rho_{10}(0) = 1, \quad \rho_{20}(0) = 0, \quad \rho_{30}(0) = 0, \\ &\rho_{0k}(0) = 0, \quad \rho_{\varkappa k}(0) = 0. \end{aligned} \qquad (14)$$

The system (13) then reduces to a system of four equations for the complex quantities[6]

$$\begin{aligned} &\rho_\mu = \rho_{10} + i\rho_{30}, \quad \rho_e = \rho_{01} + i\rho_{03}, \\ &\rho_\mu^t = \rho_{21} + i\rho_{23}, \quad \rho_e^t = \rho_{12} + i\rho_{32}. \end{aligned} \qquad (15)$$

The system has the form

$$\begin{aligned} \frac{d\rho_\mu}{d\tilde{t}} &= -i\rho_e^t + i\rho_\mu^t + 2i\zeta x\rho_\mu, \\ \frac{d\rho_e}{d\tilde{t}} &= i\rho_e^t - i\rho_\mu - (\gamma + 2ix)\rho_e, \\ \frac{d\rho_e^t}{d\tilde{t}} &= i\rho_e - i\rho_\mu - (\gamma - 2i\zeta x)\rho_e^t, \\ \frac{d\rho_\mu^t}{d\tilde{t}} &= i\rho_\mu - i\rho_e - (\gamma + 2ix)\rho_\mu^t. \end{aligned} \qquad (16)$$

Since we are interested in the polarization of the μ^+ meson, we may restrict ourselves to obtaining the general solution only for

$$\mathbf{P}(\tilde{t}) = \rho_\mu(\tilde{t}) = \sum_{k=1}^{4} A_k e^{\lambda_k \tilde{t}}, \qquad (17)$$

where λ_k are the roots of the characteristic equation of the system (16). Following[6], we determine the coefficients A_k with the aid of the relation

$$A_k = -M(\lambda_k)/D'(\lambda_k), \qquad (18)$$

where

$$M(\lambda) = -i[\alpha\beta^2 - (\alpha + \beta)], \quad \alpha = (\lambda + \gamma)i + 2\zeta x,$$

$$\beta = (\lambda + \gamma)i - 2x, \ D(\lambda) = \prod_{k=1}^{4} (\lambda - \lambda_k).$$

Before analyzing the system (16) in the general case, we show how to obtain the formulas for the polarization when the incoherent interactions between the muonium atoms and the medium may be neglected. To do that, we clearly have to put in the system (16) $\gamma = 0$. It is then easy to obtain the characteristic equation of the system (16):

$$(\lambda - 2i\zeta x)^2(\lambda + 2ix)^2 + 4(\lambda + ix_-)^2 = 0. \qquad (19)$$

Using (17)—(19), we obtain

$$\mathbf{P}(\tilde{t}) = \rho_\mu(\tilde{t}) = \frac{1}{2} e^{ix_- \tilde{t}} \cos\tilde{t} \left[\cos\sqrt{1 + x_+^2}\,\tilde{t} + \frac{ix_+}{\sqrt{1 + x_+^2}} \sin\sqrt{1 + x_+^2}\,\tilde{t} \right]. \qquad (20)$$

Here $x_\pm = x(1 \pm \zeta) = 2\omega_\pm/\omega_0$.

It is easy to verify that the expression for Re $\mathbf{P}(t)$, which we obtain from (20), coincides with the dependence (6) for $\mathbf{P}(t)$.

The incoherent interaction with the medium leads to the damping of the polarization in time. In order that the phenomenon of beats may be observed, it is necessary for the damping to be small—at least, small enough for the polarization to be different from zero in the time interval $\tilde{t} \sim 1/\Omega$. The solution of the system (16) in the case when $x \ll 1$ and $\gamma \ll 1$ has been obtained in[6]. We shall not give the exact formulas for the large roots $\lambda_{2,3} \approx \pm 2i$, since the frequencies ω_{ik} corresponding to these roots are averaged out in the experiments. For small roots, which are responsible for the beats, we have

$$\lambda_{1,4} = {}^1/_4[-3\gamma - 4ix_- \pm i\Delta^{1/2}] \qquad (21)$$

with the corresponding coefficients

$$A_{1,4} = \frac{1}{4}\left[1 \pm \frac{\gamma \pm 2ix_+^3}{i\Delta^{1/2}}\right], \qquad (22)$$

where

$$\Delta = (-\gamma^2 + 4x_+^4) - 4i\gamma x_+^3 + O(\gamma^2 x^2,\ x^6,\ \gamma^3 x). \qquad (23)$$

It follows from (21) and (22) that the observable part of the polarization equals

$$\mathbf{P}_{\text{obs}}\ (\tilde{t}) = \frac{1}{2} e^{-ix_-\tilde{t} - 3\gamma\tilde{t}/4}\left[\cos\frac{1}{4}\Delta^{1/2}\tilde{t} + \frac{\gamma + 2ix_+^3}{\Delta^{1/2}}\sin\frac{1}{4}\Delta^{1/2}\tilde{t}\right]. \quad (24)$$

As can be seen from (24), beats may be observed only in the case, when Re $\Delta > 0$ or $4x^4 > \gamma^2$. The phenomenon is observed at its sharpest when Re $\Delta \gg$ Im Δ. In accordance with this, let us consider further the case when $4x^4 \gg \gamma^2$ or Im $\Delta \approx 0$. Then we have from (24)

$$\operatorname{Re}\mathbf{P}_{\text{obs}}\ (\tilde{t}) = \frac{1}{2} e^{-3\gamma\tilde{t}/4}\left[\left(\cos\tilde{\Omega}_\gamma\tilde{t} + \frac{\gamma}{2x_+^2(1-\gamma^2/4x_+^2)^{1/2}}\sin\tilde{\Omega}_\gamma\tilde{t}\right)\right.$$
$$\left.\times\cos x_-\tilde{t} + \frac{x_+}{(1-\gamma^2/4x_+^4)^{1/2}}\sin\tilde{\Omega}_\gamma\tilde{t}\sin x_-\tilde{t}\right], \quad (25)$$

where

$$\tilde{\Omega}_\gamma = \frac{x_+^2}{2}\left(1 - \frac{\gamma^2}{4x_+^4}\right)^{1/2}. \qquad (26)$$

We introduce the characteristic damping time $\tau_1 = 8/3\gamma\omega_0$. It is to be emphasized that this damping time τ_1 is determined by the incoherent transitions (11); the damping of the polarization P(t) given by (6), which is related to the capture of the muonium atom for the formation of a molecule, does not lead to any change in the beat frequency Ω. Therefore, it is convenient to write (25) in the form

$$\operatorname{Re}\mathbf{P}_{\text{obs}}\ (t) = \frac{1}{2} e^{-t/\tau_1}\left[\left(\cos\Omega_\gamma t + \frac{\sin\Omega_\gamma t}{3\tau_1\Omega[1-(3\tau_1\Omega)^{-2}]}\right)\cos\omega_- t\right.$$
$$\left. + \frac{2\omega_+}{\omega_0[1-(3\tau_1\Omega)^{-2}]}\sin\Omega_\gamma t\sin\omega_- t\right], \quad (27)$$

where

$$\Omega_\gamma = \Omega[1 - (3\tau_1\Omega)^{-2}]^{1/2}. \qquad (28)$$

It follows from the relations (27) and (28) that the precession damping caused by the incoherent interactions of the muonium with matter, leads to a more complex dependence of $\Omega(H)$ than $\Omega \sim H^2$ (see (5)), which obtains in the absence of damping. Thus, the beat effect enables us to verify experimentally the applicability of the Wangsness-Bloch equations (system (13)) to the treatment of the behavior of muonium in a given medium. In those media where the beat effect can be observed, this method is the most direct and the most effective.

It is easy to deduce the effect of damping on the slow μ^+-meson precession in very strong fields $H \to H^*$ described in Sec. 1. Here

$$H^* = \frac{m_\mu}{2m_e}\left(1 - \frac{m_e}{m_\mu}\right)H_0 = 1.64\cdot 10^5\,\frac{\omega_0\,(\text{in medium})}{\omega_0\,(\text{in vacuum})}\,[\text{Oe}]. \quad (29)$$

The characteristic equation of the system (16) has the form

$$\left(\frac{1}{\delta+\gamma} + \frac{1}{\delta+\gamma+2ix_+}\right)\left(\frac{1}{\delta} + \frac{1}{\delta+\gamma+2ix_+}\right) = -1, \quad (30)$$

where $\delta = \lambda - 2i\zeta x$. In the region close to the intersection of the terms, $2x_+ \sim 1/\zeta \gg 1$ and, since we are interested in roots of the order of magnitude not greater than unity, we may neglect in (30) terms which contain x_+. We then have

$$\lambda_{1,2} = 2i\zeta x + \frac{i}{2x_+} - \frac{\gamma}{2} \pm i\sqrt{1-\gamma^2/4} \qquad (31)$$

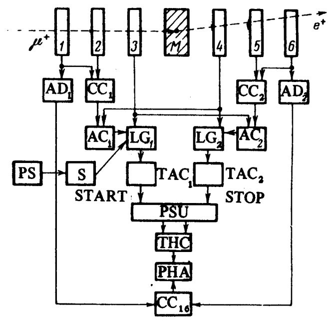

FIG. 2

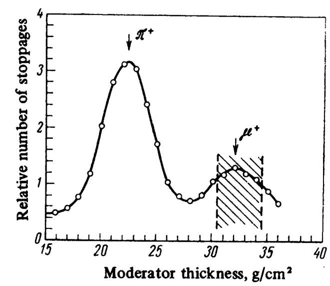

FIG. 3

FIG. 2. Diagram of the experimental setup for observing the two-frequency muonium precession in a transverse magnetic field. T is the target in which muonium is produced; CC—coincidence circuit; AC—anti-coincidence circuit; LG—linear gate; PS—pulse shaper; TAC—time adjustment circuit; S—photosensor on the accelerator buncher; PSU—pair-selector unit; THC—time-height converter; PHA—pulse-height analyzer.

FIG. 3. Range curve for the particles of a beam of polarized μ^+ mesons. The shaded area indicates the position of the target in the μ^+ meson beam.

with the corresponding coefficients

$$A_{1,2} = \frac{1}{2}\left[1 \pm \frac{\gamma/2}{i\sqrt{1-\gamma^2/4}}\right]. \qquad (32)$$

From which we see that the polarization attenuates with the characteristic damping time $\tau_2 = 4/\gamma\omega_0$ and the point of cessation of the precession $x^* = H^*/H_0$ is determined by the condition

$$\frac{1}{x^*(1+\zeta)} + 2\zeta x^* = \sqrt{1-\gamma^2/4}. \qquad (33)$$

Since the time dependence of the polarization can be observed within a time of the order of τ_2, it is clear that, with out present technology, only the cases when $\gamma \ll 1$ can be of real interest to us. As can be from (33), the change in x^* will then be small.

We have for the observable polarization

$$\operatorname{Re}\mathbf{P}_{\text{obs}}\,(t) = e^{-\gamma\tilde{t}/2}\left[\frac{1}{2}\cos 2\zeta(x^* - x)\tilde{t} + \frac{\gamma}{2\sqrt{1-\gamma^2/4}}\sin 2\zeta(x - x^*)\tilde{t}\right]. \quad (34)$$

It can be seen from (33) and (34) that in fields, which slightly differ from H*, the damping causes practically nothing more than a shift in the position of the "nodes" by the amount

$$\Delta\tilde{t} \approx \gamma/2(x^* - x)\zeta.$$

3. EXPERIMENTAL STUDY OF THE TWO-FREQUENCY PRECESSION

a) Apparatus. A diagram of the experimental setup for observing the two-frequency precession of the μ^+ meson in muonium and a block diagram of the recording equipment are shown in Fig. 2. A beam of longitudinally polarized μ^+ mesons from the JINR synchrocyclotron was slowed down and then stopped in the target T prepared from the material under investigation. The target was situated in a magnetic field H perpendicular to the spin of the μ^+ meson. The beam contained a considerable admixture of π^+ mesons. However, practically all of them were stopped in the moderator and did not, therefore, reach the target. Figure 3 shows the range curve for the particles. It can be seen from the figure that the ranges of the π^+ and μ^+ mesons of the beam considerably differ from each other and, hence, the admixture of π^+ meson stoppage in the target may be made small. A typical thickness of the target T was 4 g/cm^2.

The polarization $P_{obs}(t)$ (8) of the μ^+ mesons was determined from the asymmetry in the measured angular distribution of the positrons from the $\mu^+ \to e^+$-decay. This was done in the following way. The instant t_μ when a μ^+ meson was stopped in the target T was fixed by a system of signals from the scintillation counters $123\bar{4}$ (coincidence of signals from counters 1, 2, 3 and anticoincidence from counter 4) while the instant t_e of emission of a $\mu \to e$ decay positron was recorded by a system of signals $456\bar{3}$. The intervals of time $t = t_e(456\bar{3}) - t_\mu(123\bar{4})$ for each case of $\mu^+ \to e^+$ decay were analyzed with the aid of a time-height converter in an AI-4096 pulse-height analyzer. The time spectrum N(t) of the $\mu^+ \to e^+$ decay positrons, which was thus obtained, is related to the variation in time of the polarization $P_{obs}(t)$ (8) of the μ^+ mesons by the expression

$$N(t) = N_0 e^{-t/\tau_0}[1 - c e^{-t/\tau} P_{\text{набл}}(t)] = N_0 e^{-t/\tau_0}\left[1 - \frac{c}{2} e^{-t/\tau} \cos \Omega t \cos \omega t\right]. \qquad (35)$$

Here, $\tau_0 = 2.2 \times 10^{-6}$ sec is the lifetime of the μ^+-meson, τ the lifetime of muonium, and c the experimental coefficient of the $\mu^+ \to e^+$ decay positron angular distribution asymmetry. c is determined by the polarization of the μ^+ mesons in the beam, the probability of production in the target material of muonium with a long lifetime, the acceptance spatial angle of the positron telescope of the counters, and the time resolution of the apparatus.

We briefly describe below the operation of the electronic equipment, which was used to determine the spectrum N(t) (see Fig. 2). The "start" pulse $123\bar{4}$, which fixed the instant t_μ when a μ^+ meson was stopped in the target, was formed in the following way. The signal $12\bar{4}$, generated by the coincidence CC_1 and anticoincidence AC_1 circuits, opens the linear gate LG_1 which then admits a pulse from counter 3. This pulse triggers the converter with the time adjustment circuit TAC_1. The TAC_1 circuit eliminates the time spread in the start pulses which is connected with the non-monochromatic nature of the amplitude spectrum of the signals from counter 3. Standard pulses from the TAC_1 were used as the "start" pulses. The gate LG_1 can be triggered by pulses from $12\bar{4}$ for only the duration $\Delta t = 2 \times 10^{-3}$ sec of extraction of particles from the accelerator. This reduces the background of spurious start pulses. The corresponding signal arrives at gate LG_1 from the photosensor S of the accelerator buncher through the pulse shaper PS. The "stop" pulse, which determines the moment t_e of emission of a $\mu \to e$ decay positron, is generated in a similar manner. The "start" and "stop" pulses are admitted by the pair-selector unit PSU which transmits them to the time-height converter THC only if the start pulse is followed by the stop pulse within the time interval under investigation. The PSU unit was introduced into the network in order to reduce the loading of the pulse-height analyzer with large pulses delivered by the time-height converter when a "start" pulse is not accompanied by a "stop" pulse. The total (physical) time resolution of all the recording and electronic equipment was measured under working conditions in a beam of μ^+ mesons and was found to be equal to $\Delta = \pm 0.8$ nsec. The differential nonlinearity of the time analyzer used here does not exceed 2% and is a stable characteristic of the this THC unit. During the processing of the experimental time spectra, the data were corrected for the 2% nonlinearity of the analyzer. This had been measured with a high degree of accuracy.

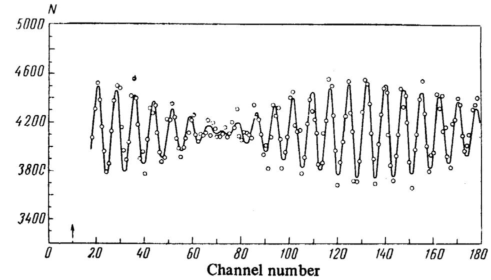

FIG. 4. Two-frequency precession (beats) of the muonium μ^+-meson spin in fused quartz. The continuous curve represents the theoretical dependence N(t) (35) parameters for which were selected by means of the method of least squares. The theoretical and experimental values for N(t) were "corrected" for the exponential decay of μ^+ mesons e^{-T/τ_0} (τ_0 = 2200 nsec). The number of counts N in the channel is plotted along the ordinate. The channel width of the time analyzer was 1 nsec; the magnetic field H = 95 Oe. The arrow indicates the channel corresponding to t = 0.

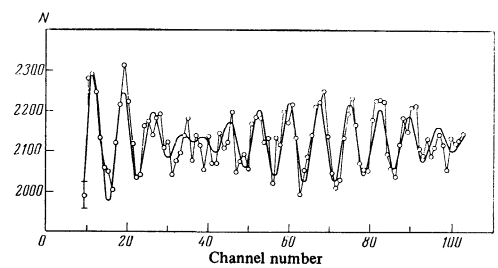

FIG. 5. Two-frequency precession of the muonium μ^+-meson spin in germanium. The number of counts N in the channel is plotted along the ordinate. The channel width of the time analyzer was 1 nsec; the magnetic field H = 98 Oe.

b) Results. Experimentally, the two-frequency precession (beats) of the μ^+ meson in muonium was observed in substances in which free muonium atoms have a sufficiently long lifetime. At present, only the noble gases, quartz, germanium, and ice are known to be such substances[7]. Figures 4 and 5 show the experimental dependence N(t) (35) for fused quartz and germanium. The experimental values for the frequencies ω and Ω, and also the values for τ, N_0, and c in (35), were found from a least-squares-fit comparison of the theoretical dependence (35) and the experimental spectrum $N_{exp}(t)$. The values thus obtained for ω, Ω, and τ for all the investigated substances are given in Table 1. We give in the same table the Pearson correspondence parameters χ^2 and their mean values $\overline{\chi^2}$, which are equal to the number of experimental points minus the number of selected parameters. Beats were observed in quartz at different values of the magnetic

Table I. Parameters of the two-frequency μ^+-meson precession in ice, germanium and fused quartz

H, Oe	τ, nsec	ω, 10^6 sec^{-1} *	Ω, 10^6 sec^{-1}	$\Delta\nu = \frac{1}{2\pi}\frac{\omega^2}{\Omega}$, MHz	χ^2	$\overline{\chi^2}$
Ice, t = 77°K						
98	160	858	24.5±1.5	4791±300	285	289
Germanium, t = 77°K						
98	100	868	48.1±1,1	2494±60	323	290
96	100	848	42.7±1.3	2682±80	330	307
Fused quartz, t = 300°K						
47	1500	417	6,1±0,9	4534±680	79	57
68	1500	597	11.6±0.7	4879±300	62	67
78	1500	686	16,8±0.8	4469±200	204	186
89	1500	783	21.3±0.7	4575±150	115	69
95	1500	837	25.7±0.4	4335±70	317	305
118	1500	1043	41.6±2.0	4160±200	257	185

*The errors in ω do not exceed 0.3% and have practically no effect on the accuracy of determination of Δν.

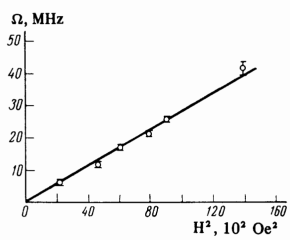

FIG. 6. Experimental dependence of the beat frequency Ω in quartz on the square of the magnetic field intensity H. The straight line represents the theoretical dependence (5).

field intensity H, making it possible to verify experimentally the dependence $\Omega \sim H^2$, which follows from the relation (5). Figure 6 shows the obtained dependence $\Omega(H^2)$. It can be seen from the figure that the experimental values $\Omega_{exp}(H^2)$ are in good agreement with the prediction of the theory. It follows from the relation $\Omega \sim H^2$ (see also Table I) that it is convenient to observe the two-frequency precession in relatively strong fields H, when Ω is sufficiently high. In weak fields H, when $\omega \gg \Omega$, it is necessary to record a very large number of Larmor precession periods in order to observe a single beat period. For such an experiment, the field H should be, to a high degree of accuracy, uniform in the target. Consequently, the field H was varied within the limits H = 50–120 Oe when the dependence $\Omega(H^2)$ was being measured in this experiment.

4. DISCUSSION

The frequencies ω and Ω given in Table I for different substances enable us to determine the magnitudes $\Delta\nu = \omega_0/2\pi$ of the hyperfine splitting of the ground state of the muonium atom in these substances. The values of Δν thus obtained for ice, germanium, and quartz are shown in Table I. It follows from the table that the frequencies in ice and fused quartz coincide, within the error limits, with the vacuum value $\Delta\nu_{vac}$ = 4463 MHz. For ice $\Delta\nu_{H_2O}$ = 4791 ± 300 MHz; for quartz the frequencies $\Delta\nu_{SiO_2}$, for all values of the field H, are close to the mean value $\overline{\Delta\nu}_{SiO_2}$ = 4404 ± 70 MHz. The coincidence of $\Delta\nu_{H_2O}$ and $\Delta\nu_{SiO_2}$ with the vacuum frequency $\Delta\nu_{vac}$ means that the "dimensions" of muonium in the ground state in ice and quartz are the same as in vacuum. The frequencies $\Delta\nu_{Ge}$ (the mean of the values for two field intensities $\Delta\nu_{Ge}$ = 2580 ± 50 MHz) in germanium is considerably lower than the vacuum magnitude $\Delta\nu_{vac}$. This means that the Bohr radius of the muonium atom (see (2))

$$a = \left(\frac{32\mu_e\mu_\mu}{3\hbar\omega_0}\right)^{1/3} \sim \left(\frac{1}{\omega_0}\right)^{1/3} \sim \left(\frac{1}{\Delta\nu}\right)^{1/3} \quad (36)$$

in germanium is larger than in vacuum:

$$a_{Ge} / a_{vac} = (\Delta\nu_{vac} / \Delta\nu_{Ge})^{1/3} = 1{,}2. \quad (37)$$

The relations (36) and (37) are, of course, valid only on the assumption that the muonium atom in germanium is hydrogen-like. In fact, the interaction with the medium distorts the wave function of the electron in muonium. It is precisely this distortion that is responsible for the deviation of Δν in matter from the quantity $\Delta\nu_{vac}$. Consequently, the frequency Δν is, strictly speaking, determined by only the value $|\psi(0)|^2$ of the electron density at the origin[3]:

$$|\psi(0)|^2 = \frac{3}{16}\frac{\hbar}{\mu_e\mu_\mu}\Delta\nu.$$

The quantities Δν given in Table I were found from the relation $\Delta\nu = \omega^2/2\pi\Omega$ without taking into account the effect of the medium on the experimentally measured beat frequency Ω. We showed in Sec. 2 that the incoherent interactions of the muonium atoms with the medium, which are related to the depolarization of the electron in muonium, lead to the damping of the precession and to a change ΔΩ in the beat frequency (see (28)):

$$\Delta\Omega / \Omega \approx 1 / 18(\tau_1\Omega)^2. \quad (38)$$

It follows from (38) that in the present experiment the possible magnitudes of ΔΩ/Ω are negligibly small. Even if we assume that the observed damping time τ is wholly determined by the depolarization of the electron in muonium ($\tau = \tau_1$), we have

$$(\Delta\Omega / \Omega)_{H_2O} = 4\cdot 10^{-3}, \quad (\Delta\Omega / \Omega)_{Ge} = 3\cdot 10^{-3},$$

In quartz, for which τ = 1.5 μsec, the quantity $(\Delta\Omega/\Omega)_{SiO_2}$ is even smaller.

The values of Δν found for the muonium atom in a material may be compared with the value $\Delta\nu_H$ for the hyperfine splitting of the hydrogen atom present in the same substance as an impurity. The quantities $\Delta\nu_H$ for fused quartz and ice were measured by means of the electron paramagnetic resonance (EPR) method in[8,9]. The value obtained for $\Delta\nu_H$ are given in Table II, where, for comparison, the values for Δν for the muonium atom in the same substances, are also given.

The value for $\Delta\nu_H$ in ice cited in Table II was obtained at a temperature of 4°K. The search for a free hydrogen atom in ice at 77°K using the EPR method did not yield positive results.

It follows from Table II that the hyperfine splitting of the hydrogen and muonium atoms in a medium coincide. This coincidence shows that the interactions of these atoms with matter are similar and may be studied by means of the EPR ($\Delta\nu_H$) as well as the two-frequency muonium precession (Δν) method. One

Table II. Hyperfine splitting frequencies for free hydrogen and muonium atoms ($\Delta\nu_H$ and $\Delta\nu$, respectively) in ice and quartz expressed in units of $\Delta\nu_{vac}$

	$\Delta\nu/\Delta\nu_{vac}$	$\Delta\nu_H/\Delta\nu_{H, vac}$
Ice	1.07±0.07	1.00 [8]
Fused quartz	0.987±0.016	0.985 [9]

advantage of the two-frequency precession method must be noted, namely, that this method enables us to detect equally effectively free muonium atoms in crystals like quartz and ice, where there are no free electrons, and in semiconductors (germanium).

5. CONCLUSIONS

1. A new phenomenon—a two-frequency precession of the spin of the μ^+ meson in the muonium atom in a transverse magnetic H—has been discovered:

$$P(t) = {}^1/_2 \cos\omega t \cdot \cos\Omega t, \quad \Omega = \omega^2/\omega_0,$$

where P(t) is the polarization of the μ^+ meson. The measurement of the frequencies Ω and ω enables us to determine the frequency ω_0 of hyperfine splitting of the muonium atom in a given material.

2. The two-frequency muonium precession was observed in ice, germanium, and fused quartz-substances in which muonium has a sufficiently long lifetime. The frequencies ω_0 of hyperfine splitting of the muonium atom in ice and quartz, it turns out, coincide with the vacuum value for this quantity ω_{0vac}. The frequency $\omega_{0Ge} = 0.56\ \omega_{0vac}$ in germanium is considerably smaller than ω_{0vac}. The small magnitude of ω_{0Ge} indicates that the wave function of the muonium electron in germanium is appreciably distorted, i.e., muonium in a germanium crystal is "swollen":

$$|\psi_{vac}(0)|^2/|\psi_{Ge}(0)|^2 = 1.8,$$

where $\psi(0)$ is the amplitude of the wave function of the electron at the origin.

3. The theoretical prediction that the best frequency Ω is proportional to the square of the magnetic field intensity H ($\Omega = \omega^2/\omega_0 \sim H^2$) was experimentally confirmed for fields $H < 120$ Oe during the observation in quartz.

4. The two-frequency muonium precession phenomenon in matter was theoretically investigated. The incoherent interaction processes of muonium in the medium were here taken into account.

It is shown that the effect of these incoherent interactions on the determination of the quantities Ω and ω_0 was negligible in our experiment and, hence, the observed deviation of the frequency ω_{0Ge} in germanium from its vacuum value is wholly related to the distortion in the medium of the wave function of the electron in the muonium atom.

5. The values obtained for the hyperfine splitting frequencies $\Delta\nu = \omega_0/2\pi$ for muonium in ice and quartz are compared with the corresponding values $\Delta\nu_H$ for the free hydrogen atom in these substances which were obtained by means of the EPR method.

6. The cessation of the μ^+-meson precession in a strong magnetic field

$$H^* = \frac{m_\mu}{2m_e}\left(1 - \frac{m_e}{m_\mu}\right)H_0,$$

due to the crossing of the Zeeman terms of the ground state of muonium, is predicted. In vacuum H* = 164000 Oe.

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