PROPAGATION OF STRONG ELECTROMAGNETIC WAVES IN A TWO-COMPONENT PLASMA

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The effect of an intense electromagnetic wave on the plasma concentration and on the heating of the carriers, which leads to a change in the dielectric constant, is investigated. The coordinate dependence of the wave amplitude is determined for different relations between the characteristic frequencies of the plasma.

INTRODUCTION

WHEN an electromagnetic wave of large amplitude[1)] propagates in a two-component (semiconductor or gas-discharge) plasma, an important role is played by the self-action effect (heating of the carriers)[1,2], as a result of which the propagation of the wave acquires a nonlinear character. In a plasma inhomogeneously heated by an electromagnetic field, another cause of the nonlinearity is the change of the carrier concentration, connected with the diffusion of the charges along the inhomogeneity[1].

We do not consider changes produced in the carrier density by processes such as impact ionization, the striction effect (the forcing-out of the plasma by the pressure of the inhomogeneous electric field of the wave), and the dependence of the recombination coefficient on the field.

If the quasineutrality condition

$$d\,|\operatorname{grad} E| \ll E \tag{1}$$

is satisfied, where E is the wave amplitude and d is the Debye radius, then we can assume that at any point of a single-component plasma the carrier density is equal to its equilibrium value. Therefore in a plasma with one type of carrier the nonlinearity due to the change of the concentration does not appear under the assumptions made above[3]. In a two-component plasma the inequality (1) leads to the requirement $N_+ \approx N_- = N$ (N_+ and N_- are the concentrations of the positively and negatively charged carriers) and, as was shown in[1], if the heating is not uniform the carriers become redistributed over the sample. This is connected with the fact that the free charges with the opposite signs can diffuse along the inhomogeneity without violating the quasineutrality of the plasma.

The time of establishment of a stationary distribution of the energy of the light carriers Δt_ϵ is much shorter than the time of establishment of the concentration Δt_N[1], and therefore if the total duration of the alternating-field pulse is shorter than Δt_ϵ (but longer than Δt_N), then the change of the concentration can be neglected and the temperature and the field are given by the expressions of[3,4].

In the present paper we study the propagation of a strong electromagnetic wave with a pulse duration exceeding Δt_N, when the change of the concentration is appreciable.

We consider the half-space $z > 0$ filled with a two-component plasma (the interface plane is $z = 0$), on which a plane monochromatic electromagnetic wave is incident normally to the interface. The wave in a vacuum is given by

$$E = E_0(e^{ikz} + Re^{-ikz}), \tag{2}$$

where E_0 is the amplitude of the incident wave, R is the reflection coefficient, $k = \omega/c$, ω is the frequency of the wave, and c is the speed of light in vacuum. In such a formulation, the problem is one-dimensional and all the quantities depend only on the single coordinate z.

In the construction of the theory we shall start from the kinetic equations for the distribution functions of the electrons $f^{(e)}$ and the ions $f^{(i)}$ (in semiconductors—the holes $f^{(n)}$). Expanding the distribution functions in spherical functions[5] and confining ourselves to the first two terms of these series, we represent $f^{(e)}$ and $f^{(i)}$ in the form

$$f^{(e)} = f_0^{(e)}(\varepsilon, z, t) + \mathbf{f}_1^{(e)}(\varepsilon, z, t)\,\mathbf{p}_e/p_e, \tag{3}$$
$$f^{(i)} = f_0^{(i)}(\varepsilon, z, t) + \mathbf{f}_1^{(i)}(\varepsilon, z, t)\,\mathbf{p}_i/p_i,$$

ϵ is the carrier energy, $\mathbf{p}_e$ and $\mathbf{p}_i$ are the momenta of the electrons and of the ions. The smallness of the function $|\mathbf{f}_1^{(e)}|$ compared with $f_0^{(e)}$ is ensured by the quasi-elasticity of the collisions of the electrons with the scattering centers (in a weakly-ionized plasma—by the small parameter m_e/M, where m_e is the mass of the electron, M is the mass of the molecule[6], and in semiconductors—$\hbar\omega_\eta/\epsilon$, where ω_η is the frequency of the phonons on which the energy relaxation takes place[7]).

The same reasoning holds for holes in semiconductors.

In a weakly-ionized plasma the ion mass m_i is of the order of M, and therefore the smallness of the function $|\mathbf{f}_1^{(i)}|$ as compared with $f_0^{(i)}$ can be insured only by weakness of the electric field. We shall henceforth assume

[1)]Pertinent numerical estimates are given below.

that the amplitude of the incident electromagnetic wave is bounded from above (the pertinent estimates are given below) and the symmetrical part of the ion distribution function is of the form

$$f_0^{(i)} = \frac{N}{(2\pi m_i T)^{3/2}} e^{-\varepsilon/T}.$$

Here T is the temperature of the ions and coincides in the given approximation with the temperature of the molecules. By weakly-ionized plasma is meant here a gas consisting of electrons, ions, and molecules, the concentration of the electrons and the ions being much smaller than the concentration of the molecules. This makes it possible to regard the distribution function of the molecules as Maxwellian and to neglect the electron-ion collisions compared with the collisions of the electrons or of the ions with the molecules.

The electron and ion subsystems in the plasma (in semiconductors, the electron and hole subsystems) are characterized by three collision frequencies $\nu_{e,n,i}$—the collision frequency connected with the momentum transfer, $\tilde{\nu}_{e,i,n}$—the collision frequency connected with energy transfer, with $\tilde{\nu}_{e,n} \ll \nu_{e,n}$ because of the quasi-elasticity[3], and $\nu_{ee,nn,ii}$—the collision frequency within the subsystem (the indices e, i, and n pertain respectively to the electron, ion, and hole subsystems; $\nu_{e,n,i}$ correspond to ν_e, ν_n, and ν_i, and $\nu_{ee,nn,ii}$ to ν_{ee}, ν_{nn}, and ν_{ii}). Depending on the relation between these frequencies, the following cases can arise.

1. $\nu_{ee} \gg \nu_e$, $\nu_{nn} \gg \nu_n$. Under this assumption, the carrier distribution function is a biased Maxwellian distribution[8], with ν and $\tilde{\nu}$ dependent on the temperatures of the carriers in the following manner:

$$\nu_{e,n} = \nu_{0e,n}(T)\,(\Theta_{e,n}/T)^{-q_{e,n}}, \quad \tilde{\nu}_{e,n} = \tilde{\nu}_{0e,n}(T)\,(\Theta_{e,n}/T)^{r_{e,n}-1}. \tag{4}$$

Here T is the temperature of the molecules (lattice), $\Theta_{e,n}$ is the temperature of the carriers, $m_{e,n}$ their mass, and $q_{e,n}$ and $r_{e,n}$ are parameters that depend on the concrete type of the mechanism. The values of r and q are given in[4].

Recognizing that (see[8])

$$\nu_{ee,\,nn} \sim \frac{40e^4 N}{m_{e,n}^{1/2} T^{3/2}} \left(\frac{T}{\Theta_{e,n}}\right)^{3/2}, \tag{5}$$

where e is the charge of the electron, to realize this case it is necessary to satisfy the inequality

$$N \gg \frac{\nu_{0e} m_e^{1/2} T^{3/2}}{40e^4} \left(\frac{\Theta_e}{T}\right)^{3/2-q_e}.$$

In semiconductors with $m_e \sim 10^{-28}$ g, $\nu_{0e} \sim 10^{12}$ sec^{-1}, $T \sim 10^{-14}$ erg, and $\Theta_e/T \sim 10$ we obtain a concentration $N \gg 10^{15}$ cm^{-3}. In a plasma with a molecule concentration $N_M \sim 10^{16}$ cm^{-3} and $T \sim 10^{-13}$ erg we get $N \gg 10^{12}$ cm^{-3}.

2. $\nu_e \gtrsim \nu_{ee} \gg \tilde{\nu}_e$, $\nu_n \gtrsim \nu_{nn} \gg \tilde{\nu}_n$. When these inequalities are satisfied, the symmetrical part of the distribution functions of the electrons and of the holes are Maxwellian[3], and for $\nu_{e,n}$ we have

$$\nu_{e,n}(\varepsilon) = \nu_{0e,n}(T)\,(\varepsilon/T)^{-q_{e,n}}, \tag{6}$$

while $\nu_{ee,nn}$ and $\tilde{\nu}_{e,n}$ are described by formulas (5) and (4). To realize this case at the same semiconductor and plasma parameters, we obtain the following values for the concentration: in semiconductors 10^{15} cm^{-3} $\gtrsim N \gg 10^{12}$ cm^{-3}, and in a plasma 10^{12} cm^{-3} $\gtrsim N \gg 10^9$ cm^{-3} (we take into account here the fact that $\nu_{0e,n} \sim 10^3\, \nu_{0e,n}$).

3. $\nu_{ee} \ll \nu_e$, $\tilde{\nu}_{nn} \ll \tilde{\nu}_n$. In this case the distribution functions have the form given in[4]; $\nu_{ee,nn}$ is described as before by formula (5), $\nu_{e,n}$ by formula (6), and

$$\tilde{\nu}_{e,n} = \tilde{\nu}_{0e,n}(T)\,(\varepsilon/T)^{r_{e,n}-1}. \tag{7}$$

Estimates yield here: in semiconductors $N \ll 10^{11}$ cm^{-3}, and in a plasma $N \ll 10^9$ cm^{-3}. We note that $\nu_{0e,n}(T)$ and $\tilde{\nu}_{0e,n}(T)$ in formula (4) differ from $\nu_{0e,n}(T)$ and $\tilde{\nu}_{0e,n}(T)$ in formulas (6) and (7) respectively by a numerical factor on the order of unity.

For concreteness we shall henceforth refer to case 1 as strong electron-electron (hole-hole) interaction, to the case 2 as moderate, and to the case 3 as absence of electron-electron (hole-hole) interaction. If the frequency of the incident electromagnetic wave is $\omega \gg \tilde{\nu}_{e,n}$, then, as shown in[9], the symmetrical part of the distribution function in the zeroth approximation in ν/ω does not depend on the time, i.e., a wave of the same frequency will propagate in the medium.

1. STRONG ELECTRON-ELECTRON (HOLE-HOLE) INTERACTION

The propagation of strong electromagnetic waves in a gas-discharge plasma, with satisfaction of the inequalities corresponding to the case 1, was investigated in[1], where the following expression was obtained for the concentration:

$$N = N_0 \cdot 2T / (\Theta_e + T). \tag{8}$$

We take into account here the fact that the temperature of the carriers at infinity is equal to T, since heat is transferred at infinity from the electron subsystem to the molecules, and there is no heating of the carriers because the field attenuates and is equal to zero at infinity. Following the procedure of[1], we easily obtain expressions for the concentration in a semiconductor, where both the electrons and holes are effectively heated:

$$N = N_0 \cdot 2T / (\Theta_e + \Theta_n). \tag{8a}$$

We shall henceforth consider the normal skin effect, i.e., we assume that the depth of attenuation of the field L is much larger than the carrier mean free paths $\tilde{l}_{e,n}$ connected with the energy transfer

$$L \gg \tilde{l}_{e,n}. \tag{9}$$

The expressions for E(z) and Θ(z) in the plasma, with allowance for the inequality (9), were found in[1]. In semiconductors these relations can be easily obtained by following[2] and taking into account the fact that formulas (1.11) and (1.12) of[2] must be replaced by expressions (5) and (8a) of the present article.

We henceforth consider for simplicity two cases of orientation of the magnetic field **H**: $H = H_z$ and $H = H_y$. Since the electromagnetic wave propagates always along the z axis, we shall refer to the first case as longitudinal propagation and to the second as transverse.

If the wave attenuates weakly

$$\frac{\nu_{e,n}}{|\omega_{H_{e,n}}-\omega|}\ll 1, \quad \frac{\nu_{e,n}}{\omega}\ll 1,$$

and also

$$\frac{N}{N}(a_e+a_n)\ll 1$$

($\omega H_{e,n} = \pm |e| H/m_{e,n}c$, $a_{e,n}$ are given in[2]) then, taking into account the remarks made above, we obtain for the field $F = ue^{is}$ and for the dimensionless temperature $\vartheta_{e,n} = \Theta_{e,n}/T$:

in the region where $\vartheta_{e,n} \gg 1$

$$u=|F_0|\left[1-\frac{2(1+q)}{r+q}\xi(0)z\right]^{(r+q)/2(1+q)},$$

$$s=kz-\frac{r+q}{q\sqrt{2}}(a_e+a_n)\left[\left(\frac{b_e}{\tilde{\nu}_{0e}}\right)^{1/(r+q)}+\left(\frac{b_n}{\tilde{\nu}_{0n}}\right)^{1/(r+q)}\right]^{-1}$$
$$\times\left(\frac{|F_0|^2}{N_0T}\right)^{-1/(r+q)}\frac{k}{\xi(0)}\left[\left(1-\frac{2(1+q)}{r+q}\xi(0)z\right)^{q/(1+q)}-1\right]-\frac{\xi(0)}{2k},$$

$$\vartheta_{e,n}=\vartheta_{0e,n}\left[1-\frac{2(1+q)}{r+q}\xi(0)z\right]^{1/(1+q)}; \tag{10}$$

in the region where $\vartheta_{e,n} - 1 \ll 1$

$$F=|F_0|S_Fe^{ikz-\xi_0z}, \quad \vartheta_{e,n}=1+\frac{b_{e,n}}{\tilde{\nu}_{0e,n}}\frac{|F_0|^2}{N_0T}|S_F|^2e^{-2\xi_0z}. \tag{11}$$

Here and throughout

$$\xi(0)=k\frac{c_e\vartheta_{0e}^{-q}+c_n\vartheta_{0n}^{-q}}{\vartheta_{0e}+\vartheta_{0n}},$$

$$\vartheta_{0e,n}=\left(\frac{b_{e,n}}{\tilde{\nu}_{0e,n}}\frac{|F_0|^2}{N_0T}\right)^{1/(r+q)} \tag{12}$$

$$F=\begin{cases}E_x\pm iE_y - \text{for longitudinal propagation,}\\ E_x - \text{for transverse propagation with polarization along x,}\\ E_y - \text{for transverse propagation with polarization along y:}\end{cases}$$

are the values of the temperatures of the electrons and holes on the plane z = 0, S_F is the so-called self-action factor for the field (see[2,3], $\xi_0 = k(c_e + c_n)/2$ is the damping coefficient in the linear theory. The values of $a_{e,n}$, $b_{e,n}$, and $c_{e,n}$ are tabulated in[2], where $c_0 = 1$. In the derivation of (10) and (11) it was assumed for simplicity that the electrons and holes scatter their momentum by the same objects. This assumption is valid if their effective masses are of the same order. If the masses differ significantly, then it can be assumed that the heavier carriers are not heated, and the results will coincide with the formulas for a weakly-ionized plasma.

2. MODERATE ELECTRON-ELECTRON (HOLE-HOLE) INTERACTION

An investigation of the propagation of strong electromagnetic waves in semiconductors with satisfaction of the inequalities $\nu_e \gtrsim \nu_{ee} \gg \tilde{\nu}_e$ and $\nu_n \gtrsim \nu_{nn} \gg \tilde{\nu}_n$ was carried out in[2]. We therefore proceed directly to a consideration of this case in a weakly-ionized plasma ($r_{e,n} = 3/2$, $q_{e,n} = -1/2$). From the kinetic equations for the electron and ion distribution functions we obtain for the density of the static electric currents of the ions $j_c^{(i)}$ and of the electrons $j_c^{(e)}$ (the static currents and fields are connected with the carrier temperature gradients resulting from the damping of the wave)

$$dj_c^{(e,i)}/dz=0. \tag{13}$$

If we require that there be no electron and ion currents through the boundary z = 0, then, owing to the one-dimensional character of the problem (there is no static electric field in a plane perpendicular to z: $E_x = E_y = 0$), we get from (13)

$$j_{cz}^{(e)}=j_{cz}^{(i)}=0. \tag{14}$$

Taking into account the expression for the electronic static current[2], the condition $j_{cz}^{(e)} = 0$ yields

$$E_{cz}-\frac{T\vartheta_e}{eN}\frac{dN}{dz}-\left(1\mp\frac{1}{2}\right)\frac{T}{e}\frac{d\vartheta_e}{dz}=0. \tag{15}$$

Here and throughout the upper sign pertains to longitudinal propagation at an arbitrary value of the magnetic field, and the lower sign to the case of transverse propagation in a strong magnetic field $(\omega_{He,n}/\nu_e)^2 \gg 1$.

The value of $j_{cz}^{(i)}$ is determined by the anisotropic part of the distribution function of the ions $f_1^{(i)}$. As indicated above, owing to the fact that $m_i/M \sim 1$, $\nu_i \sim \tilde{\nu}_i$, and $|f_1^{(i)}| \ll f_0^{(i)}$ only in weak electric fields (which do not heat the ions, $\vartheta_i = 1$). We note immediately that the same fields can heat the electrons effectively ($\vartheta_e \gg 1$).

Since the relaxation-time approximation does not hold for ions ($m_i/M \sim 1$), we must proceed as follows to determine $f_1^{(i)}$. The equation for $f_1^{(i)}$ in weak electric fields is written in the form (see[6]):

$$\frac{\partial \mathbf{f}_1^{(i)}}{\partial t}-e\sqrt{\frac{2\varepsilon}{m_i}}\mathbf{E}\frac{\partial f_0^{(i)}}{\partial\varepsilon}+\sqrt{\frac{2\varepsilon}{m_i}}\nabla_r f_0^{(i)}-\omega_{H_i}(\mathbf{h}\mathbf{f}_1^{(i)})+\mathbf{S}\{\mathbf{f}_1^{(i)}\}=0.$$

Here $\mathbf{S}\{\mathbf{f}_1^{(i)}\}$ is the integral of the ion-molecular collisions, which, as can be shown, coincides with the direction of $\mathbf{f}_1^{(i)}$, and it can be written in the form $\mathbf{S}\{\mathbf{f}_1^{(i)}\} = \hat{S}\mathbf{f}_1^{(i)}$.

The electric field in the plasma is given by

$$\mathbf{E}=\mathbf{E}_c+\mathbf{E}e^{-i\omega t}+\mathbf{E}^*e^{i\omega t}.$$

Representing $\mathbf{f}_1^{(i)}$ in the form

$$\mathbf{f}_1^{(i)}=\mathbf{f}_{1c}^{(i)}+\mathbf{f}_{1v}^{(i)}e^{-i\omega t}+\mathbf{f}_{1v}^{(i)*}e^{i\omega t},$$

we obtain for $\mathbf{f}_{1c}^{(i)}$ and $\mathbf{f}_{1v}^{(i)}$ the following equations:

$$\sqrt{\frac{2\varepsilon}{m_i}}\nabla_r f_{0c}^{(i)}-\sqrt{\frac{2\varepsilon}{m_i}}e\mathbf{E}_c\frac{\partial f_{0c}^{(i)}}{\partial\varepsilon}-\omega_{H_i}[\mathbf{h}\mathbf{f}_{1c}^{(i)}]+\hat{S}\mathbf{f}_{1c}^{(i)}=0, \tag{17*}$$

$$-i\omega\mathbf{f}_{1v}^{(i)}-\sqrt{\frac{2\varepsilon}{m_i}}e\mathbf{E}_v\frac{\partial f_{0c}^{(i)}}{\partial\varepsilon}-\omega_{H_i}[\mathbf{h}\mathbf{f}_{1v}^{(i)}]+\hat{S}\mathbf{f}_{1v}^{(i)}=0. \tag{18}$$

From (17) we determine formally $f_{1cz}^{(i)}$:

$$f_{1cz}^{(i)}=\hat{L}\left\{\sqrt{\frac{2\varepsilon}{M}}\left(\frac{\partial f_{0c}^{(i)}}{\partial z}-eE_{cz}\frac{\partial f_{0c}^{(i)}}{\partial\varepsilon}\right)\right\},$$

where

$$\hat{L}=\hat{K}^{-1}\hat{S}^{-1}(\omega_{H_i}h_z^2+\hat{S}^2/\omega_{H_i}),$$
$$\hat{K}=\omega_{H_i}+\hat{S}^2/\omega_{H_i}, \qquad h_z=H_z/H,$$

meaning that

$$j_{cz}^{(i)}=\frac{8\sqrt{2}\pi m_i^{1/2}e}{3}\left(\frac{1}{N}\frac{dN}{dz}+e\frac{E_{cz}}{T}\right)\int_0^\infty\varepsilon\hat{L}\{\varepsilon^{1/2}f_{0c}^{(i)}\}d\varepsilon.$$

*$[\mathbf{hf}_{1c}] \equiv \mathbf{h}\times\mathbf{f}_{1c}$.

In the determination of $j_{cz}^{(i)}$ it was assumed that the increment to the ion temperature as a result of the heating, $\Theta_i' \ll T$, such that

$$\frac{1}{T}\frac{d\Theta_i'}{dz} \ll \frac{1}{N}\frac{dN}{dz}.$$

Simple estimates show that this condition is always satisfied.

Since the integral with respect to $d\epsilon$ does not vanish, the condition $j_{cz}^{(i)} = 0$ reduces to

$$E_{cz} + \frac{T}{eN}\frac{dN}{dz} = 0 \tag{19}$$

Solving the system (15) and (19), we determine N and E_{cz}:

$$N = N_0\left(\frac{2}{\vartheta_e+1}\right)^{1\mp 1/2}, \quad E_{cz} = \frac{(1\mp 1/2)T}{e(\vartheta_e+1)}\frac{d\vartheta_e}{dz}. \tag{20}$$

The formula (20) obtained by us for the concentration differs from the formula (8) for the concentration. This is connected with the fact that, owing to the dependence of ν_e on ϵ, the friction force exerted on the electronic subsystem by the molecules differs from zero even when $j_{cz}^{(e)} = 0$ (the average electron velocity is equal to zero). Therefore the condition for hydrodynamic equilibrium, which was used in[1] (**R** = 0 at **j** = 0, where **R** is the friction force), is not valid when account is taken of the dependence of ν_e on ϵ, whereas this condition is identical with the kinetic approach when the inequalities of the case of the strong electron-electron interaction are satisfied.

Maxwell's equation for the electromagnetic wave has the usual form

$$d^2F/dz^2 + k^2(\varepsilon_r(\vartheta_e) + i\varepsilon_i(\vartheta_e))F = 0. \tag{21}$$

Here ϵ_r and ϵ_i are the real and imaginary parts of the dielectric constant, which in the case of weak damping is represented in the form

$$\varepsilon_r = 1 + \frac{N}{N_0}(a_e + a_n), \quad \varepsilon_i = \frac{N}{N_0}(c_e\vartheta_e^{1/2} + c_i), \tag{22}$$

where a_e and c_e describe the electronic contribution to the dielectric constant (their values are given in[2]), while a_i and c_i describe the ionic contribution. To calculate the latter it is necessary to solve Eq. (18). When the condition

$$|\omega - \omega_{H_i}||f_{1v}^{(i)}| \gg |\hat{S}f_{1v}^{(i)}|$$

is satisfied $f_{1v}^{(i)}$ can be sought by the method of successive approximations. As a result we obtain $f_{1v}^{(i)}$, after which we easily obtain a_i and c_i. They coincide with the expressions a_n and c_n of[2] with $c_0 = 1$, and

$$\nu_{0i}(T) = \frac{4}{3\sqrt{\pi}T^{5/2}}\int_0^\infty \varepsilon\hat{S}(\varepsilon^{1/2}e^{-\varepsilon/T})\,d\varepsilon. \tag{23}$$

It is necessary to add to Eq. (21) the boundary condition of the plane z = 0 and as $z \to \infty$:

$$F(-0) = F(+0), \quad \frac{\partial F(-0)}{\partial z} = \frac{\partial F(+0)}{\partial z}, \quad F(z)\underset{z\to\infty}{\to} 0. \tag{24}$$

Using a procedure similar to that of[2], we can write out expressions for the fields and the temperatures for arbitrary frequencies of the electromagnetic wave and for arbitrary magnetic fields. For simplicity we confine ourselves here to the propagation of weakly damped waves with $\epsilon_r \approx 1$ and cyclotron resonance with large damping ($\omega = \omega_{He}$, $\omega_{0e}^2\vartheta^{-1}/\omega_{He}\nu_{0e} \gg 1$). In the case of weak damping

$$\vartheta_e = 1 + \gamma_e u^2, \quad F = ue^{is}, \tag{25}$$

and for longitudinal propagation we have[2)]

$$-2\xi_0 z = \frac{1}{\sqrt{2}}\ln\frac{\sqrt{\rho}+1}{\sqrt{\rho_0}+1}\frac{\sqrt{\rho_0}-1}{\sqrt{\rho}-1} + \ln\frac{\sqrt{\rho}-\sqrt{2}}{\sqrt{\rho_0}-\sqrt{2}}\frac{\sqrt{\rho_0}+\sqrt{2}}{\sqrt{\rho}+\sqrt{2}},$$
$$s = kz - \frac{\xi_0}{2k}\left(\frac{2\vartheta_{0e}}{1+\vartheta_{0e}}\right)^{1/2} + \frac{k}{4\xi_0}a_e\ln\frac{\sqrt{\rho-1}-1}{\sqrt{\rho_0-1}-1}\frac{\sqrt{\rho_0-1}+1}{\sqrt{\rho-1}-1} \tag{26}$$

where

$$\rho = \frac{2+\gamma_e u^2}{1+\gamma_e u^2}, \quad \rho_0 = \frac{2+\gamma_e|F_0|^2}{1+\gamma_e|F_0|^2}, \quad \gamma_e = \frac{b_e}{\tilde{\nu}_{0e}N_0T}.$$

Here $\xi_0 = kc_e/2$, and ϑ_{0e} is the electron temperature on the plane z = 0:

$$\vartheta_{0e} = 1 + \gamma_e|F_0|^2$$

Assuming that $\gamma_e|F_0|^2 \gg 1$ in the region where $\gamma_e u^2 \gg 1$, we get from (26)

$$u = |F_0|e^{-\sqrt{2}\xi_0 z}, \quad s = kz - \xi_0/k\sqrt{2}.$$

As $z \to \infty$ ($u \to 0$) we get

$$u = 1{,}12(\gamma_e|F_0|^2)^{(\sqrt{2}-2)/4}|F_0|e^{-\xi_0 z},$$
$$s = k\left(1+\frac{a_e}{2}\right)z - \frac{ka_e}{4\xi_0}\ln 0{,}53(\gamma_e|F_0|^2)^{\sqrt{2}/2}.$$

In transverse propagation in a strong magnetic field we have

$$-2\xi_0 z = \frac{5}{4\sqrt{2}}\ln\frac{\sqrt{\rho}+1}{\sqrt{\rho_0}+1}\frac{\sqrt{\rho_0}-1}{\sqrt{\rho}-1} + \ln\frac{\sqrt{\rho}-\sqrt{2}}{\sqrt{\rho_0}-\sqrt{2}}\frac{\sqrt{\rho_0}+\sqrt{2}}{\sqrt{\rho}+\sqrt{2}}$$
$$+\frac{1}{2\sqrt{2}}\left(\frac{\sqrt{\rho}}{\rho-1} - \frac{\sqrt{\rho_0}}{\rho_0-1}\right), \tag{27}$$

$$s = kz - \frac{\xi_0}{2k}\left(\frac{2\vartheta_{0e}^{1/3}}{1+\vartheta_{0e}}\right)^{3/2} - \frac{k}{4\xi_0}a_e\ln\frac{\sqrt{\rho-1}-1}{\sqrt{\rho_0-1}-1}\frac{\sqrt{\rho_0-1}+1}{\sqrt{\rho-1}+1}.$$

Assuming as before that $\gamma_e|F_0|^2 \gg 1$ in the region where $\gamma_e u^2 \gg 1$ we have

$$u = F_0 - \sqrt{2}\xi_0 z/\gamma_e|F_0|,$$
$$s = kz - \sqrt{2}\,\xi_0/k.$$

As $z \to \infty$

$$u = 165(\gamma_e|F_0|^2)^{(5-4\sqrt{2})/8\sqrt{2}}e^{\gamma_e|F_0|^2/4\sqrt{2}}|F_0|e^{-\xi_0 z},$$
$$s = k\left(1+\frac{a_e}{2}\right)z + \frac{ka_e}{8\sqrt{2}\xi_0}\gamma_e|F_0|^2.$$

In cyclotron resonance, in the region where $\vartheta_e \gg 1$:

$$\vartheta_e = \sqrt{\gamma_e|F|},$$
$$F = 2\zeta F_0\left[1 - \frac{1}{\sqrt{6}}\frac{k}{|\zeta|}z\right]^{2-i\sqrt{2}}. \tag{28}$$

Here ζ is the surface impedance, defined by the formula

$$\zeta = \left(\frac{1}{24\pi^{3/2}}\right)^{1/2}\left(\frac{k}{\xi_0}\right)^2\left(\frac{\omega_{0e}^2|F_0|^2}{\nu_{0e}\tilde{\nu}_{0e}N_0T}\right)^{4/5}e^{-0{,}94i},$$

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2)We note that in the calculation of s below one cannot assume $\epsilon_r = 1$ even in the case when $\epsilon_r - 1 \gg 1$ [2].

where

$$\gamma_e = \frac{1}{3\pi^{1/2}} \frac{\omega_{0e}^2}{\nu_{0e}\tilde{\nu}_{0e}} \frac{1}{N_0 T}, \qquad \xi_0{}^2 = \frac{2}{3\pi^{1/2}} \frac{\omega_{H_e}\omega_{0e}^2}{c^2 \nu_{0e}}.$$

The temperature on the boundary is given by

$$\vartheta_{0e} = 2|\zeta|\,|\sqrt{\gamma_e}|F_c|. \tag{29}$$

Let us estimate the region of applicability of the theory constructed above for a weakly-ionized plasma in the case of a normal skin effect, using as an example weak damping in the absence of a magnetic field. Assuming that $m_e \sim 10^{-27}$ g, $m_i \sim 10^{-23}$ g, $T \sim 10^{-13}$ erg, $N \sim 10^{12}\ \text{cm}^{-3}$, we have $\nu_e \sim 10^{10}\ \text{sec}^{-1}$, $\tilde{\nu}_e \sim 10^6\ \text{sec}^{-1}$, $\nu_i \sim \tilde{\nu}_i \approx 10^8\ \text{sec}^{-1}$, $\tilde{l}_e \sim 1$ cm, $\tilde{l}_i \sim 10^{-2}$ cm, and $L \sim 10$ cm. In order for the electron temperature Θ_e to be much larger than T ($\Theta_e \sim 10^{-12}$ erg), and that of the ions $\Theta_i \approx T$, the field amplitude should lie in the range $1\ \text{V/cm} < |F_0| < 10\ \text{V/cm}$. The last estimate can be easily obtained from formula (25) and the expression obtained for ϑ_i from (25) by replacing γ_e with γ_i.

3. ABSENCE OF ELECTRON-ELECTRON (HOLE-HOLE) INTERACTION

The distribution function of the electrons in the absence of electron-electron interaction was calculated in[4]. The distribution function of the holes is calculated analogously. For the symmetrical part of the distribution function of the electrons and holes we have

$$f_0^{(e,n)} = \frac{N}{K_{e,n}(u)} \mathcal{F}_{e,n}\left(\frac{\varepsilon}{T}, u\right). \tag{30}$$

Here

$$K_{e,n}(u) = 4\sqrt{2}\,\pi m_{e,n}^{3/2} T^{3/2} \int_0^\infty x^{1/2} \mathcal{F}_{e,n}(x,u)\,dx,$$

$$\vartheta_{e,n}(x,u) = 1 + \gamma_{e,n} u^2 x^{1-r_{e,n} \mp q_{e,n}},$$

$$\mathcal{F}_{e,n}(x,u) = \exp\left\{-\int_0^x \frac{dx}{\vartheta_{e,n}(x,u)}\right\}.$$

The values of $\gamma_{e,n}$ for different cases are listed in the table. The upper sign in the expression for $\vartheta_{e,n}$ pertains to the case of weak damping, and the lower one to the case of cyclotron resonance.

After calculating the electron and hole static currents, after equating them to zero and solving the resultant system of equations with respect to N and E_{cz}, we obtain for N

$$N = N_0 \exp\left\{-\int_0^u \frac{\Psi_n(u)\,d\Phi_e(u)/du + \Psi_e(u)\,d\Phi_n/du}{\Psi_n(u)\Phi_e(u) + \Psi_e(u)\Phi_n(u)}\,du\right\}. \tag{31}$$

Here

$$\Psi_{e,n}(u) = \frac{T^{3/2}}{K_{e,n}(u)} \int_0^\infty \frac{x^{3/2}\,dx}{\vartheta_{e,n}(x,u)} \frac{\nu_{e,n}^2(x) + \omega_{H_{e,n}}^2 h_z{}^2}{\nu_{e,n}(x)\left(\nu_{e,n}^2(x) + \omega_{H_{e,n}}^2\right)} \mathcal{F}_{e,n}(x,u),$$

$$\Phi_{e,n}(u) = \frac{T^{3/2}}{K_{e,n}(u)} \int_0^\infty x^{3/2}\,dx \frac{\nu_{e,n}^2(x) + \omega_{H_{e,n}}^2 h_z{}^2}{\nu_{e,n}(x)\left(\nu_{e,n}^2(x) + \omega_{H_{e,n}}^2\right)} \mathcal{F}_{e,n}(x,u).$$

Formula (31) simplifies greatly if it is assumed that the electrons and the holes are scattered by the same scattering centers. If the scattering of both carriers is by acoustic phonons, then in weak damping

$$N = N_0\left(\frac{2}{2 + \gamma_e u^2 + \gamma_n u^2}\right)^{1 \mp 1/2} \tag{32}$$

	$\lvert\omega_{H_{e,n}}\rvert \ll \omega$	$\lvert\omega_{H_{e,n}}\rvert \gg \omega$
($\parallel$)	$\gamma_{0e,n} \frac{\nu_{0e,n}}{\tilde{\nu}_{0e,n}}$	$\gamma_{0e,n} \frac{\nu_{0e,n}}{\tilde{\nu}_{0e,n}} \frac{\omega^2}{\omega_{H_{e,n}}^2}$
($\perp_x$)	$2\gamma_{0e,n} \frac{\nu_{0e,n}}{\tilde{\nu}_{0e,n}}$	$2\gamma_{0e,n} \frac{\nu_{0e,n}}{\tilde{\nu}_{0e,n}} \frac{\omega^2}{\omega_{H_{e,n}}^2}$
($\perp_y$)	$2\gamma_{0e,n} \frac{\nu_{0e,n}}{\tilde{\nu}_{0e,n}}$	$2\gamma_{0e,n} \frac{\nu_{0e,n}}{\tilde{\nu}_{0e,n}}$

Note. ($\parallel$)—longitudinal propagation, (1_x)—transverse propagation with polarization along x, (1_y)—transverse propagation with polarization along y, $\gamma_{0e,n} = 2e^2/3m_{e,n}T\omega^2$.

In scattering by other objects, assuming that $\gamma_{e,n}u^2 \gg 1$, we have in the case of weak damping

$$N = N_0 A^{1\pm q}\left(\gamma_e^{1/(r+q)} + \gamma_n^{1/(r+q)}\right)^{-(1\pm q)} u^{-2(1\pm q)/(r+q)}. \tag{33}$$

Here

$$A^{1\pm q} = \exp\left\{-\left[\int_0^1 \chi(v)\,dv - \int_1^\infty \left(\chi(v) - 2\frac{1\pm q}{r+q}\frac{1}{v}\right) dv\right]\right\}$$

where

$$\chi(v) = \frac{\Psi_n d\Phi_e/dv + \Psi_e d\Phi_n/dv}{\Psi_n\Phi_e + \Psi_e\Phi_n}, \quad v = \left(\gamma_e^{1/(r+q)} + \gamma_n^{1/(r+q)}\right)^{(r+q)/2} u.$$

It is easy to show that numerically this coefficient is of the order of unity.

Following[4], we can represent the dielectric constant in the case of damping in the following form:

$$\varepsilon_r = 1 + \frac{N}{N_0}(a_e + a_i), \quad \varepsilon_i = \frac{N}{N_0}\left(c_e \frac{C_{0e}(u)}{c_0} + c_n \frac{C_{0n}(u)}{c_0}\right), \tag{34}$$

where

$$C_{0e,n}(u) = \frac{\int_0^\infty x^{3/2+q} \mathcal{F}_{e,n}(x,u)\,dx / \vartheta_{e,n}(x,u)}{\int_0 x^{1/2} \mathcal{F}_{e,n}(x,u)\,dx}.$$

The expressions for the fields in the region of strong heating coincides with formulas (2.8) of[2], where $\xi(0)$ must be taken to mean the following expression:

$$\xi(0) = \frac{k}{2} A^{1\pm q}(\vartheta_{0e} + \vartheta_{0n})^{-(1\pm q)}\left(c_e T_e \vartheta_{0e}^{-q} + c_n T_n \vartheta_{0n}^{-q}\right),$$

$$\vartheta_{0e,n} = \left(\gamma_{e,n}|F_0|^2\right)^{1/(r+q)}, \tag{35}$$

$$T_{e,n} = \frac{2}{3c_0}(r+q)^{r/(r+q)}\Gamma\left(\frac{3/2 + r}{r+q}\right) \Big/ \Gamma\left(\frac{3}{2(r+q)}\right).$$

The case of cyclotron resonance is investigated analogously.

In a weakly-ionized plasma the distribution function of the electrons is described as before by the formulas of[4].

For the distribution function of the ions, it is correct to apply the reasoning used by us for case 2. Therefore, recognizing that in a weakly-ionized plasma in the case of weak damping $r + q = 1$ (the symmetrical part of the distribution function is Maxwellian with a temperature $\vartheta_e = 1 + \gamma_e u^2$, where the values of γ_e are given in the table), the expressions for the fields are described by by formulas (25) and (27) with the corresponding γ_e from the table.

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70