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INFLUENCE OF SURFACE SCATTERING ON THE MAGNETORESISTANCE AND HALL EFFECT IN PLATES

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The effect of the plate dimensions and the nature of the electron scattering by the boundary on the kinetic coefficients of monopolar metals and semimetals is studied for the case of a strong magnetic field ($\gamma = r/l \ll 1$, where l is the mean free path and r is the Larmor-orbit radius) parallel to the surface. For diffuse scattering, the Hall field near the surfaces possesses root singularities which result in considerable alteration of the Hall constant compared with its "volume" value. The dependence of the mean electric conductivity on the plate thickness is derived. Electron-hole (intervalley) transitions in the volume and on the surface and redistribution of the carrier densities are taken into account for semi-metals. It is shown that for a small probability of intervalley transitions on the surface, for specular as well as diffuse scattering, the "volume" value of the magnetoresistance $\rho \sim H^2$ is attained at a plate thickness $b > L\gamma^{-2}$ (L is the intervalley diffusion length). For small thicknesses $\rho \sim H$ ($L < b < L\gamma^{-2}$) and is independent of H ($b < L$). The skin effect for a skin-layer depth exceeding r is considered. It is shown that when the relation between L and δ_0 is changed (δ_0 is the skin layer depth at H = 0) the surface impedance may change by a factor γ.

INTRODUCTION

A special role in the determination of the kinetic coefficients of bounded metallic samples is played by the character of the scattering of the electrons from the surfaces. In the case of "smooth" surfaces (when the dimension of the roughnesses is the shortest length) the character of the scattering for the distribution function can be taken into account by the boundary condition (diffuseness denotes here completely isotropic scattering, specularity scattering with conservation of the tangential momentum components; the process of collision with the wall is assumed to be elastic). The boundary condition consists in the requirement that there be no normal surface flux of the particles; by virtue of the continuity equation, this requirement should be satisfied at any point. We shall henceforth confine ourselves to the case of second-order Fermi surfaces and analyze first the situation in a monopolar metal with a singly-connected Fermi surface. In the case of specular scattering, according to the conservation laws, the normal projections of the velocities of the incident and reflected electrons are equal and opposite, and therefore, in the sense of mutual cancellation of the electron fluxes moving from the surface and to the surface, the surfaces themselves are not singled out in any way, i.e., the boundary condition can be satisfied without a spatial variation of the distribution function and without producing an inhomogeneous normal electric field. The situation is different for diffuse scattering: it is clear that the electrons reflected from the surface have a distribution that is in no way similar to that of the electrons moving in the same direction in the interior of the sample. Therefore an inhomogeneous field should arise as well as a spatial dependence of the distribution function and with it a spatial dependence of all the kinetic coefficients.

In Sec. I of the present article we consider a plate of monopolar metal in a strong magnetic field ($\gamma = r/l \ll 1$, r is the Larmor radius and l the mean free path) parallel to the surface, at different ratios of the plate thickness 2b to r. We investigate the character of the spatial inhomogeneity of the Hall field E_H in diffuse scattering. The inhomogeneity of E_H exerts an influence on the Hall constant R_H, particularly strongly in thin samples (when $b < r$). The dimension dependences of the average electric conductivity of the plate are determined.

We now proceed to the case of a semimetal, which is characterized by a multiply-connected Fermi surface and equal electron and hole concentrations. The electron and hole Fermi surfaces (valleys) in semimetals are separated in momentum space by distances that are large compared with their dimensions, as a result of which the times of intervalley transitions T are much longer than the times of intravalley relaxation τ; a similar relaxation can also be expected between the probabilities of the intervalley and intravalley scattering (including diffuse scattering) on the surface. Consequently, in formulating the boundary-value problem we encounter a fundamental difference from the case of a singly-connected Fermi surface. If the intervalley transitions on the surface can be neglected, then the boundary condition that requires the vanishing of the total normal flux breaks up into conditions for the vanishing of the fluxes of the particles of individual valleys, which do not transform into one another upon collision with the surface. These conditions can be satisfied only when account is taken of the change in the particle concentration in the valleys (there is no total change of concentration). Owing to the presence of concentration gradients, diffusion contributions to the fluxes appear. Together with the normal field, the gradients of the concentrations compensate for the Hall drift for each valley (of course, near the boundaries the corresponding fluxes are determined by integral relations). In the general case the diffusion fluxes are determined by the rate of intervalley scattering from the surface.

As a result, scattering from the surface exerts an influence on the conductivity up to distances on the order of the intervalley diffusion length L ($L \gg r$, in a magnetic field parallel to the surface $L \sim r\sqrt{T/\tau}$), over which the concentration gradients decrease as a result of intervalley transitions in the volume, and the current density assumes its volume value. Thus, the current can be concentrated on the surface at a depth considerably larger than the Larmor diameter. The possibility of realizing this case depends on the magnitude of the intervalley scattering. In the case of weak intervalley scattering (a criterion will be derived below) the effect is practically insensitive to the type of the intravalley scattering (the degree of specularity of the reflection). The case of diffuse reflection differs only in the behavior of the transverse field and of the concentration gradients in the immediate vicinity of the boundary.

In Sec. II we consider also the problem of the skin effect in an alternating electromagnetic field at a skin-layer depth greatly exceeding r. Allowance for the indicated singularities that are characteristic of semimetals leads, in the case of weak intervalley scattering, to changes in the value of the surface impedance and the character of its dependence on the magnetic field.

The influence of the surface on the galvanomagnetic characteristics of metallic samples was investigated in a number of papers; the most thorough theoretical investigations were those of Azbel'[1,2] and of Azbel' and Peschanskiĭ[3,4]. In[1,2], correct results for the dimension dependence of the average electric conductivity of metallic samples were obtained for the first time, but no analysis was made of the inhomogeneity of the normal field and of the Hall effect. In[3,4] the semimetal was investigated in the limiting case when the probabilities of the intra- and intervalley transitions coincide and the diffusion processes can be neglected. Therefore the current density is concentrated at distances $\sim r$ from the surface (the so-called static skin effect). Consequently the dimensional relations obtained in[3,4] differ significantly from those derived in the present paper. Analogously, our formulation of the skin-effect problem differs from that used in[5].

For simplicity we confine ourselves to the case of spherical Fermi surfaces (in a semimetal—electron and hole Fermi spheres separated in momentum space); an analysis of other surfaces entails no difficulty and does not result in significant differences in the case of a strong magnetic field. The parameters of surface scattering are assumed to be specified constants.

I. MONOPOLAR METALS

1. Distribution Function

Let us consider a plate with surfaces $z = \pm b$ in a homogeneous magnetic field $H = H_y$. In the τ-approximation, the kinetic equation for the non-equilibrium increment $ef\partial F_0/\partial\epsilon$ to the distribution function ($F_0(\epsilon-\mu)$—Fermi distribution) is as follows:

$$v_z\frac{\partial f}{\partial z}+\frac{\partial f}{\partial t}+\frac{f}{\tau}=\mathbf{Ev}. \tag{1}$$

The time of motion along the trajectory t is determined by the equation

$$\frac{d\mathbf{p}}{dt}=-\frac{e}{c}[\mathbf{vH}]. \tag{2*}$$

We consider first the case of all-specular scattering from the surfaces, when the boundary conditions are of the form

$$f_{\gtrless}(\mp b)=f_{\lessgtr}^{*}(\mp b). \tag{3}$$

The symbols $\gtrless$ denote $v_z \gtrless 0$, and the asterisk denotes replacement of v_z by $-v_z$. In order not to write out the cumbersome formulas, we present a solution of (1) satisfying the condition (3), for the case of a sufficiently thick plate, when $b \gg r$. Near the surface $z = -b$, for example, there is no influence of the collisions with the other surface, and f takes the form

$$f=\frac{1}{\Omega}\int_{-\infty}^{\varphi}d\varphi' e^{\gamma(\varphi'-\varphi)}\mathbf{v}'\mathbf{E}(z')-\frac{\Theta_1}{\Omega(1-e^{2\gamma(\lambda_1-\pi)})}\Big[\int_{-\infty}^{\lambda_1}d\varphi' e^{\gamma(\varphi'-\varphi)}\mathbf{v}'\mathbf{E}(z') - e^{2\gamma(\lambda_1-\pi)}\int_{-\infty}^{2\pi-\lambda_1}d\varphi' e^{\gamma(\varphi'-\varphi)}\mathbf{v}'\mathbf{E}(z')\Big]. \tag{4}$$

We have taken into account here the periodicity of f with respect to the time t, and changed over to the phases $\varphi = \Omega(t-t_1) \in [0, 2\pi]$ reckoned for concreteness from the upper vertical diameters (Ω is the cyclotron frequency). According to (2) and the assumed origin,

$$v_x=-v_\perp\cos\varphi,\quad v_z=v_\perp\sin\varphi,\quad v_\perp=v_F\sin\theta,\quad \theta\in[0,\pi].$$

The argument of E in (4) is

$$z'=z-z(\varphi)+z(\varphi'),\quad z(\varphi)=\frac{1}{\Omega}\int^{\varphi} v_z(\varphi')\,d\varphi'=-R\cos\varphi,\quad R=\frac{v_\perp}{\Omega}, \tag{5}$$

The Θ function in (4) takes into account the limitations on the integration interval, connected with the discontinuity of the orbits when they cross the surface; $\lambda_{1,2}(z, \varphi)$ are the phases corresponding to the instants of start from the surfaces $z = \mp b$, and are determined from the equations

$$R\cos\lambda_{1,2}(z,\varphi)=z\pm b+R\cos\varphi, \tag{6}$$

which have solutions for λ if the following conditions are satisfied:

$$-R<z\pm b+R\cos\varphi<R,\ \text{i.e.},\ \Theta_{1,2}=\Theta(R\mp z-b\mp R\cos\varphi). \tag{7}$$

If $0 \le \varphi \le \pi$ and $\varphi^* = 2\pi - \varphi$, then

$$\lambda_1^*=\lambda_1,\quad \lambda_2^*=\lambda_2+2\pi.$$

From (4) follows the already noted independence of f of the coordinate.

Let us consider the case of crossed electric and magnetic fields when the applied field is $E_0 = E_x$. Assuming that the Hall field E_z is constant and putting

$$E_z=\gamma^{-1}E_x, \tag{8}$$

we obtain after carrying out the integration in (4)

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.

$$f = \tau v_x E_x. \tag{9}$$

The distribution function (9) satisfies all the required conditions and coincides with the "volume" value of f, from which the Hall field, equal to (8) for a bulky sample, is excluded. The current density j_x in the entire plate has a constant value

$$j_x = \sigma_0 E_x, \quad \sigma_0 = e^2\tau\langle v_x^2\rangle = e^2 n\tau / m,$$
$$\langle x\rangle = -\int d\tau_p \frac{\partial F_0}{\partial \varepsilon} x, \quad d\tau_p = \frac{2}{h^3} d^3p = \frac{2}{h^3} m\, d\varphi\, d\varepsilon\, dp_y. \tag{10}$$

It is easy to show that (8)–(10) hold for an arbitrary ratio of the plate thickness to the orbit radius. It is also easy to verify that in the case of an arbitrarily oriented Fermi ellipsoid there is likewise no coordinate dependence of f, and the Hall field is equal to its volume value throughout.

In parallel electric and magnetic fields ($E_0 = E_y$) there is likewise no influence of the boundaries, and all of the kinetic characteristics of the plate coincide with the volume ones.

To determine the principal features of the problem, we shall analyze now the case of pure diffuse scattering. We write the boundary conditions in the form

$$f^{\gtrless}(\mp b) = \chi_{1,2}. \tag{11}$$

The constants χ, which have the meaning of the shift of the chemical potential for the incident and reflected electrons, are necessary in order to take into account the fact that the densities of the electrons with $v_z > 0$ and $v_z < 0$ (not the fluxes!) may not coincide, as it turns out in the presence of transverse drift. The importance of taking the parameter χ into account was noted by Azbel' and Peschanskiĭ[3,4] 1). We shall show below that the value of χ plays an important role for all thicknesses of the plate. The values of χ are determined from the condition that there be no transverse flux j_z on the surfaces:

$$\chi_1 = \frac{\langle v_z f^{<}(-b)\rangle_-}{\langle v_z\rangle_-}, \quad \chi_2 = \frac{\langle v_z f^{>}(b)\rangle_+}{\langle v_z\rangle_+}. \tag{12}$$

Here the signs ± at the angle brackets denote integration at $v_z \gtrless 0$. The solution of (1) under conditions (11) is

$$f^{\lessgtr} = \frac{1}{\Omega}\int_{-\infty}^{\varphi} d\varphi'\, e^{\gamma(\varphi'-\varphi)} \mathbf{v}'\mathbf{E}(z') + \Theta_{1,2}\Phi_{1,2} e^{\gamma(\lambda_{1,2}-\varphi)} + \Theta_{2,1}(1-\Theta_{1,2})\Phi_{2,1} e^{\gamma(\lambda_{2,1}-\varphi)},$$
$$\Phi_{1,2} = \chi_{1,2} - \frac{1}{\Omega}\int_{-\infty}^{\lambda_{1,2}} d\varphi'\, e^{\gamma(\varphi'-\lambda_{1,2})} \mathbf{v}'\mathbf{E}(z'). \tag{13}$$

In the case of crossed fields, it follows from (12) and (13), using the obvious property that the Hall field is even ($E_z(z) = E_z(-z)$) that $\chi_1 = -\chi_2 = \chi$. In parallel fields we get from (12) and (13) $\chi_1 = \chi_2 = 0$.

2. Hall Field in Diffuse Reflection

The Hall field E_z should be determined from the electroneutrality condition $\langle f\rangle = 0$, which with the aid of (13) is best represented in the form

$$\left\langle \frac{1}{\Omega}\int_{-\infty}^{\varphi} d\varphi' e^{\gamma(\varphi'-\varphi)} \mathbf{v}'\mathbf{E}(z')\right\rangle + \langle \Theta_1\Phi_1 g_1 + \Theta_2\Phi_2 g_2\rangle_+ = 0,$$
$$g_1 = e^{\gamma\lambda_1}(e^{-\gamma\varphi} + (1-\Theta_2)e^{\gamma(\varphi-2\pi)}), \quad g_2 = e^{\gamma\lambda_2}(e^{\gamma\varphi} + (1-\Theta_1)e^{-\gamma\varphi}). \tag{14}$$

From (14) it is easy to reveal the character of the spatial behavior of the function E_z. It is more convenient to investigate it with the aid of the integral equation obtained by differentiating (14) with respect to z. After simple transformations we obtain

$$E_z(z)\langle 1\rangle = -\langle \Theta_1 g_1 F_1 + \Theta_2 g_2 F_2\rangle_+ + \langle \delta_1[\Phi_1 g_1 - \Theta_2\Phi_2 e^{\gamma(\lambda_2-\varphi)}]$$
$$-\delta_2[\Phi_2 g_2 - \Theta_1\Phi_1 e^{\gamma(\lambda_1+\varphi-2\pi)}]\rangle_+ + E_1, \tag{15}$$
$$F_{1,2} = E_x\frac{v_x(\lambda_{1,2})}{v_z(\lambda_{1,2})} \mp \frac{\gamma\Omega\chi}{v_z(\lambda_{1,2})}, \quad \delta_{1,2} = \delta(R - b \mp z \mp R\cos\varphi).$$

The last term (15) combines the integrals of E_z:

$$E_1 = \langle X(\varphi) + X(2\pi-\varphi) - \Theta_1 X(\lambda_1) g_1 - \Theta_2 X(\lambda_2) g_2\rangle_+,$$
$$X(\varphi) = \gamma\int_{-\infty}^{\varphi} d\varphi' e^{\gamma(\varphi'-\varphi)} E_z(z').$$

Let us consider separately in (15) the terms $E_{z\delta}$, which contain δ functions. Taking into account the relations

$$\delta(R - b \mp z \mp R\cos\varphi) = \frac{\delta(\varphi - \varphi_{1,2})}{R|\sin\varphi_{1,2}|}\Theta(2R - b \mp z), \tag{16}$$
$$\cos\varphi_{1,2} = \pm 1 - (z \pm b)/R, \quad R|\sin\varphi_{1,2}| = \sqrt{(b\pm z)(2R - b \mp z)},$$

we note that these terms lead to root divergences for E_z on the surfaces of the plate. Let us write out the expressions for $E_{z\delta}$ in the limiting case $\gamma \ll 1$, when the exponentials can be omitted:

$$E_{z\delta}^0 = \frac{1}{\pi\sqrt{r}}\left[\frac{A_1}{\sqrt{z+b}} + \frac{A_2}{\sqrt{b-z}}\right],$$
$$A_{1,2} = \sqrt{r}\int_0^{\pi/2}\frac{d\theta\sin\theta}{\sqrt{2R - b\mp z}}\Theta(2R - b \mp z)\Big\{\chi - Y(0, R-b)$$
$$+\Theta(R-b)\Big[\sqrt{b(R-b)}E_x + \frac{1}{2}Y(0, R-b) + \frac{1}{2}Y(\lambda_2', R-b)\Big]\Big\},$$
$$Y(\varphi, x) = R\int_{-\infty}^{\varphi} d\varphi' \sin\varphi' E_z(x - R\cos\varphi')e^{\gamma(\varphi'-\varphi)},$$
$$\lambda_2' = -\arccos(\cos\varphi - 2b/R). \tag{17}$$

It follows from (15), furthermore, that when b ± z = 2r the function E_z has discontinuities. It is easy to verify that the expressions connected with the Θ functions do not vanish at b ± z = 2r, while the corresponding terms vanish abruptly at distances from the surfaces exceeding the orbit diameter. The presence of singularities and jumps denotes that the electroneutrality condition ⟨f⟩ = 0, used to obtain (14) and (15), is violated near the singular points. Therefore the expressions (17) are meaningful only up to distances on the order of the Debye radius from the singular points z = ±b and ±(2r − b). A more accurate analysis calls for the use of the Poisson equation

$$dE_z/dz = -4\pi\delta\rho, \quad \delta\rho = -e^2\langle f(z)\rangle; \tag{18}$$

$\delta\rho$ is the density of the uncompensated charge. It is necessary here to take into account the deviation from the equilibrium density in the collision integral in (1), i.e., to use it in the form $(f - \bar f)/\tau$, $\bar f = \langle f\rangle/\langle 1\rangle$. It is easy to show that the latter is equivalent to replacement of E_z

1) The boundary condition for diffuse reflection should have the form (11) also in the absence of a magnetic field, if there exists in the system a transverse drift connected, for example, with the inclination of the conduction ellipsoid relative to the surface [6].

by $\mathcal{E}_Z = E_Z - d\bar{f}/dz$ in all the formulas (with the exception of (18)). The solution of (18) then shows that when the distance to the singular points becomes smaller than the Debye radius $R_D = (4\pi e^2\langle 1\rangle)^{-1/2}$, the divergences in $\mathcal{E}_Z$ are compensated by the concentration gradient $d\bar{f}/dz$, and the quantity $\bar{f}$ has in this case, like the field E_Z, a finite value.

Thus, a space charge is produced in the vicinity of the singular points (the charges have opposite signs at different boundaries). The increase of the transverse field at the boundary and its jump at $|z - b| = 2r$, accompanied by the occurrence of a layer of a space charge of thickness R_D, constitute a specific feature of the case of diffuse scattering in a strong magnetic field parallel to the surface[2)].

The quantity $E^0_{Z\delta}$ is determined by the values of χ and Y. The latter depend strongly on the ratio of the plate thickness to the orbit diameter.

3. Thick Plate ($b \gg r$).

Let us proceed to determine χ. Were we to know the exact solution for E_Z, which expresses E_Z in terms of the parameters χ and E_X, then by using this solution in (12) we could then express χ in terms of E_X. It is impossible to obtain an exact solution of (15). However, to determine χ we can employ the very useful relation between the boundary values of the current densities j_X and j_Z and the parameters χ and E_X. It can be shown that in a sufficiently thick plate ($b \gg r$) a generalization of the method of invariant embeddings developed for problems of diffuse scattering of particles[7,8] to the case of the presence of a magnetic field, makes it possible to obtain the following exact relation[3)]:

$$\frac{\tilde{j}_z^2}{\sigma} + E_x(\sigma E_x - 2j_x) = \frac{3\sigma_0}{l^2}\chi^2; \quad \tilde{j}_z = j_z + \frac{\sigma}{\gamma}E_x, \quad \sigma = \frac{\sigma_0\gamma^2}{1+\gamma^2} \quad (19)$$

Here $l = v_F\tau$ is the mean free path, and the values of the current density are taken on the boundary $z = \pm b$. We note that as $\gamma \to \infty$ (absence of magnetic field, when $j_X(\pm b) = (1/2)\sigma_0 E_X$), Eq. (19) goes over into the relation between χ and $j_Z(\pm b)$, obtained in[6] from an exact solution of the problem. When the exact solution is used for E_Z, formula (19) should be satisfied identically. We note that this should take place for an arbitrary ratio of χ to E_X, since the boundary condition (12), which establishes a connection between χ and E_X, was not used in the derivation of (19).

The subsequent analysis consists in the following. We represent the exact solution for E_Z in the form of a linear function of the parameters χ and E_X:

$$E_z = \chi l^{-1} U(z) + E_x V(z), \quad (20)$$

where U and V satisfy the equations that follow from (15) and do not contain external parameters. Then substitution in (19) of the expressions for the currents written out, using (13) and (20), also in the form of a linear combination of χ and E_X, should identically satisfy relation (19) for all χ and E_X. These conditions make it possible to estimate a number of integrals containing the functions U and V.

Leaving out the simple intermediate steps, we present expressions for j_Z and j_X at $z = -b$, obtained in the indicated manner for $\gamma \ll 1$:

$$j_z = \sigma\left[Z_{zU}\frac{\chi}{l} + Z_{zV}E_x\right], \quad j_x = \sigma\left[Z_{xU}\frac{\chi}{l} + Z_{xV}E_x\right], \quad (21)$$

$$Z_{zU} = \frac{\sqrt{3}}{\gamma}, \quad Z_{zV} = -\frac{1}{\gamma} + a_1, \quad Z_{xU} = \frac{a_2}{\gamma}, \quad Z_{xV} = a_3, \quad a_i \sim 1.$$

It is now possible to find the connection between χ and E_X with the aid of the boundary condition (12) or the identical condition $j_Z(-b) = 0$. As a net result, the principal part of χ is equal to

$$\chi = E_x l / \sqrt{3} \quad (22)$$

We proceed further to calculate the current density.

a) Crossed fields. The average current is given by

$$\bar{j}_x = \frac{1}{2b}\int_{-b} dz\, j_x(z), \quad j(z) = e^2\langle vf\rangle. \quad (23)$$

Using here the distribution function (13), we can verify that it is important to take into account the terms connected with the parameter χ. We do not present the cumbersome intermediate steps (we note that the integral terms that depend on E_Z are similar to those contained in (21) and are estimated in a similar manner), but write out the final results:

$$\bar{j}_x = \sigma_0 E_x(1 - a'r/b), \quad a' \sim 1. \quad (24)$$

In the same approximation, the Hall constant R_H turns out to be

$$R_H = \frac{\bar{E}_z}{\bar{j}_x H} = R_H^\infty\left(1 - \frac{z}{b\sqrt{3}}\right), \quad R_H^\infty = \frac{1}{enc},$$

$$\bar{E}_z = \frac{1}{2b}\int_{-b}^{b} dz\, E_z(z). \quad (25)$$

With the aid of the distribution function (13) we can also investigate the character of the current-density distribution over the plate. According to (21) and (22), j_X on the surfaces is proportional to $\sigma_0\gamma E_X$. Analyzing the expression for the current density $j_X(z)$, we can show that at distances $\sim r$ from the surfaces the current density increases strongly: $j_X \sim \sigma_0 E_X$. However, it is impossible to obtain an estimate of $j_X(z)$ at large distances but still prior to the assumption of the asymptotic value $\sigma_0 E_X$.

b) Parallel fields. When $\gamma \ll 1$, the average current density $\bar{j}_y$ and the current density on the surface are equal to

$$\bar{j}_y = \sigma_0 E_y\left[1 - \frac{3\pi}{16}\frac{r}{b}\right], \quad j_y(\pm b) = \frac{\pi\gamma}{2}\sigma_0 E_y. \quad (26)$$

A density on the order of the asymptotic value $\sigma_0 E_y$ is reached at distances $\sim r$ from the surfaces.

4. Thin Plate in a Strong Field ($b \ll r \ll l$)

The analysis in a thin plate is facilitated by the fact that in this case Eq. (17) is a sufficiently good approximation to the exact solution (15), as can easily be veri-

2) In a vanishingly weak magnetic field it can be deduced from (15) that the inhomogeneous term goes over into $\chi l^{-1}\mathrm{Ei}(z/l)$ (Ei is the integral exponential function) in accord with the results of [6]. In this case, however, $\chi \neq 0$ only in the multivalley case, in the presence of intervalley scattering.

3) For lack of space we do not present here the derivation of (19).

fied by iteration. Equation (12) for χ reduces, in the zeroth approximation in γ, to the form

$$\left\langle (R-b)\Theta(R-b)\left\{\chi^0+\frac{W_0}{2}-\frac{RE_x}{4(1-b/R)}\left[\arccos\left(1-\frac{2b}{R}\right)-2\left(\frac{b}{R}\right)^{1/2}\left(1-\frac{b}{R}\right)^{1/2}\left(1-\frac{2b}{R}\right)\right]\right\}\right\rangle=0. \quad (27)$$

Here W^0 denotes the integral term

$$W^0(\varphi)=R\int_{\varphi}^{-\lambda_2}d\varphi'\sin\varphi' E_z^0(-b+R\cos\varphi-R\cos\varphi')=W^0(0)=\frac{4A^0}{\pi}\sqrt{\frac{2b}{r}}. \quad (28)$$

We have used (17) with $\gamma = 0$. As a result, for $b \ll r$, we have

$$\chi^0\sim-\frac{W^0}{2}=-\frac{2}{\pi}bE_x. \quad (29)$$

We present the result for the average current density $\bar{j}_x$ in crossed fields, obtained by straightforward but cumbersome calculations using (29):

$$\bar{j}_x=\frac{3}{4}\frac{b}{l}\ln\frac{r}{b}\sigma_0E_x. \quad (30)$$

Analogous calculations lead to the following values of $\bar{E}_z$ and of the Hall constant:

$$\bar{E}_z\sim-\frac{1}{\pi}E_x\ln\frac{r}{b},\quad R_H=-R_{H^\infty}\frac{4}{3\pi}\frac{r}{b}. \quad (31)$$

In parallel fields we have

$$\bar{j}_y=\frac{3b}{4l}\left(\ln\frac{r}{b}+\frac{2b}{3\gamma r}\right)\sigma_0E_y. \quad (32)$$

II. SEMIMETALS

1. General Equations and Solution of Static Problem

External magnetic and electric fields $H = H_y$ and $E = E_x$ are applied to a plate (surfaces $z = \pm b$). We assume that the field H is strong ($\gamma \ll 1$). The case of parallel fields **E** and **H** does not differ in any way from the corresponding above-considered case of a monopolar metal, and will not be discussed further.

We shall assume the plate to be "thick," i.e., $b \gg r$; it is precisely this case which is of fundamental interest for semimetals. The results for a "thin" plate ($b \ll r$) practically coincide with those obtained in Sec. I.

Let us write down equations for the distribution functions of the conduction electrons (c-electrons) and for the valence band (v-electrons). (We shall sometimes omit the indices c and v.) The nonequilibrium additions f to the distribution functions satisfy linearized kinetic equations that differ from the equations used in[6] in that account is taken of the field H:

$$v_z\frac{\partial f_{c,v}}{\partial z}+\frac{\partial f_{c,v}}{\partial t}+\frac{f_{c,v}-\bar{f}_{c,v}}{\tau^0_{c,v}}+\frac{f_{c,v}-\bar{f}_{v,c}}{T_{c,v}}=\mathrm{Ev}. \quad (33)$$

The arrival terms in the collision integrals are determined by the average quantities

$$\bar{f}=\langle f\rangle/\langle 1\rangle, \quad (34)$$

with the integration carried out over the Fermi surface corresponding to the index. The changes in the concentrations are

$$\delta n=n-n_0=-e\langle f\rangle,$$
$$\delta n_c+\delta n_v=0,\ \text{i.e.},\ \bar{f}_c+\alpha\bar{f}_v=0. \quad (35)$$

In formula (33), $\tau^0_{c,v}$ are the times of intravalley relaxation, $T_{c,v}$ are the times of the intervalley relaxation (transitions from the c-states to the v-states and vice versa);

$$\frac{T_v}{T_c}=\alpha,\quad \alpha=\frac{\langle 1\rangle_v}{\langle 1\rangle_c}=\frac{m_v}{m_c}.$$

We rewrite (33) for the functions

$$\psi=f-\bar{f}+G,\quad G_{c,v}=\frac{\tau_{c,v}}{T_{c,v}}(\bar{f}_{c,v}-\bar{f}_{v,c}),\quad \frac{1}{\tau}=\frac{1}{\tau_0}+\frac{1}{T}, \quad (36)$$

where the quantities G are small by virtue of the proposed smallness of the ratios τ/T. We obtain in place of (33) the equation

$$v_z\frac{\partial\psi}{\partial z}+\frac{\partial\psi}{\partial t}+\frac{\psi}{\tau}=\mathscr{E}_iv_i,$$
$$\mathscr{E}_x=E_x,\quad \mathscr{E}_y=0,\quad \mathscr{E}_z=E_z-\frac{d}{dz}(\bar{f}-G), \quad (37)$$

which is analogous to (1).

The boundary conditions at $z = \pm b$ are of the form (compare with[6])

formula $f(\mp b)=qf^*(\mp b)+\chi_{1,2}$

or

$$\psi(\mp b)=q\psi^*(\mp b)+\tilde{\chi}_{1,2},\quad \tilde{\chi}_{1,2}=\chi_{1,2}+(q-1)[\bar{f}(\mp b)-G(\mp b)]. \quad (38)$$

Here q is the fraction of the specular reflection upon collision with the surface. The parameters χ are determined from the balance equation of the incoming and outgoing fluxes, ensuring the absence of a total particle flux through the surface[6]

$$\langle v_zf_c\rangle_++(q_c+d_c)\langle v_zf_c\rangle_-+\tilde{d}\langle v_zf_v\rangle_-=0,$$
$$\langle v_zf_v\rangle_++(q_v+d_v)\langle v_zf_v\rangle_-+\tilde{d}\langle v_zf_c\rangle_-=0, \quad (39)$$
$$q_{c,v}+d_{c,v}+\tilde{d}=1;$$

$d_{c,v}$ are the fractions of the diffuse intravalley scattering on the surface, and $\tilde{d}$ is the fraction of the intervalley scattering (by virtue of the detailed balancing principle, the probabilities of the transitions from c to v and vice versa are equal).

In what follows we shall need the relation between χ_c and χ_v:

$$\tilde{\chi}^c_{1,2}(1-q_v)-\tilde{\chi}^v_{1,2}(1-q_c)=\frac{j_z^c(\mp b)}{e^2\langle v_z\rangle_+}\left[1-q_c-(1-q_v)\frac{d_c}{\tilde{d}}\right]-\beta\bar{f}_c(\mp b)(1-q_c)(1-q_v),$$
$$\beta=\frac{\alpha+1}{\alpha}\left[1-\frac{\tau_c}{T_c}-\frac{\tau_v}{T_v}\right]. \quad (40)$$

This relation was obtained by substituting (38) in (39) and using the definition of the current density and the condition $j_z^c + j_z^v = 0$. When $q_{c,v} = 1$ (all $d = 0$) the parameters χ and the currents $j_z^{c,v}(\pm b)$ are equal to zero; when $\tilde{d} = 0$ we have $j_z^{c,v} = 0$ but $\chi \neq 0$.

We write down the system of continuity equations that follows from (37)

$$\frac{dj_z^{c,v}}{dz}=-e^2\langle 1\rangle_{c,v}\frac{G_{c,v}}{\tau_{c,v}} \quad (41)$$

or the equivalent equations

$$\bar{\psi}_{c,v} = G_{c,v}, \quad (42)$$

from which we can, in principle, determine $\mathscr{E}_z^{c,v}$ and $\bar{f}^{c,v}$. The problem can be simplified by using the presence of the small parameter τ/T in the expression for G. Since the terms with G in the right-hand sides of (41) limit the spatial growths of the concentrations, it follows from the smallness of τ/T that the characteristic distances L, over which this limitation takes place, can be regarded as sufficiently large compared with r. We can therefore consider separately the boundary regions (at a distance from the boundary $\sim \Delta$, with $r \ll \Delta \ll L$), where the terms with G can be neglected, and the remaining volume of the plate (with $|b-z| > \Delta$), where the diffusion approximation for the currents is well satisfied. The matching of the solutions at $|b-z| \sim \Delta$ gives the boundary conditions for the diffusion approximation.

2. Current Density in Plate

We consider first the interior region of the plate, where by virtue of $|b-z| > 2r$ the distribution function has the same form as in an unbounded space, and the expressions for the currents are

$$j_z = \sigma(\mathscr{E}_z \mp \gamma^{-1}E_x), \quad j_x = \sigma(E_x \pm \gamma^{-1}\mathscr{E}_z). \quad (43)$$

Eliminating the field E_z, we get

$$\mathscr{E}_z^{c,v} = \frac{1}{\sigma_c+\sigma_v}\left[E_x\left(\frac{\sigma_c}{\gamma_c}-\frac{\sigma_v}{\gamma_v}\right) \mp \sigma_{v,c}\beta\frac{d\bar{f}_c}{dz}\right]. \quad (44)$$

Solving the continuity equations (41), we obtain

$$\bar{f}_c = C_1e^{-(z+b)/L} + C_2e^{(z-b)/L}, \quad L^2 = \frac{\sigma_c\sigma_v}{\sigma_c+\sigma_v}\frac{T_c}{e^2\langle 1\rangle_c} = \frac{r^2}{3(1+\gamma_c/\gamma_v)}\frac{T_c}{\tau_c}. \quad (45)$$

The constants C_1 and C_2 are determined by the boundary conditions at $z = \Delta - b$ and $z = b - \Delta$. To find these conditions, we use the following relation, which is obtained after averaging the kinetic equation multiplied by $v_z \pm \gamma^{-1}v_x$:

$$\mathscr{E}_z = \frac{\tilde{j}_z}{\sigma} + \frac{\langle v_z(v_z \mp \gamma^{-1}v_x)\, d\psi/dz\rangle}{\langle v_z^2\rangle}, \quad \tilde{j}_z = j_z \pm \frac{\sigma}{\gamma}E_x. \quad (46)$$

After integrating (46), from which we eliminate the field E_z, from $-b$ to $-b+\Delta$ (or from b to $b-\Delta$), we obtain

$$\bar{f}_c(\Delta-b) = \bar{f}_c(-b) + \frac{3}{4\pi}\frac{1}{\beta}\sum_{c,v} \pm\langle\langle \nu^2\sin\varphi(\sin\varphi+\gamma^{-1}\cos\varphi)\psi(-b)\rangle\rangle,$$

$$\nu = \sin\theta, \quad \langle\langle f\rangle\rangle = \int_0^\pi \sin\theta\, d\theta\int_0^{2\pi} d\varphi f(\theta,\varphi),$$

and analogously for $\bar{f}_c(b-\Delta)$. We have used here Eqs. (43)–(45), omitted the small term $\sim \bar{f}_c(\Delta)\Delta/L$, and assumed $j_z(-b) = j_z(\Delta - b)$. Using the relation (40), we eliminate $\bar{f}_c(-b)$ (for $q \neq 1$):

$$\beta\bar{f}_c(\Delta-b) = \frac{\tilde{\chi}_v}{1-q_v} - \frac{\tilde{\chi}_c}{1-q_c} + \frac{4}{3}\frac{l_cj_z^c(0)}{\sigma_0^c}\left[\frac{1}{1-q_v} - \frac{d_c}{d}\frac{1}{1-q_c}\right] + \frac{3}{4\pi}\sum_{c,v} \pm\langle\langle \nu^2\sin\varphi(\sin\varphi+\gamma^{-1}\cos\varphi)\psi(-b)\rangle\rangle. \quad (47)$$

The solution of the problem in the boundary regions should give the connection between the parameters $\tilde{\chi}$ and the values of the current j_x^c and the field E_x, and also determine the last term of (47) in terms of the same parameters.

The cases of specular and diffuse reflections will be considered separately.

1. The case $q = 1$. In the boundary regions, putting in (4) $f = \psi$ and

$$\mathscr{E}_z^{c,v} = \pm\gamma_{c,v}^{-1}E_x, \quad (48)$$

we obtain for the functions ψ the values

$$\psi_{c,v} = \tau_{c,v}v_xE_x, \quad (49)$$

which satisfy the conditions $\bar{\psi} = 0$, i.e., the continuity equations. It follows from (48) that the concentration at the boundaries varies linearly and the field E_z is constant.

We can now find the current density in the plate. For specular scattering, the boundary conditions are the relations $j_z^c = 0$ both on the surfaces $z = \pm b$ and on the boundaries of the region of the diffusion approximation. Using (43)–(45), we find

$$\bar{f}_c = \left(\frac{1}{\gamma_c}+\frac{1}{\gamma_v}\right)\frac{E_xL}{\beta}\frac{\mathrm{sh}(-z/L)}{\mathrm{ch}(b/L)} \quad (50)$$

(the small, i.e., $\sim \Delta/L$, difference between $\bar{f}_c(-b)$ and $\bar{f}_c(\Delta-b)$ is neglected). For the average current density $\bar{j}_x = \bar{j}_x^c + \bar{j}_x^v$ we obtain with the aid of (49), (43), (44), and (50)

$$\bar{j}_x = E_x\left[\sigma_c+\sigma_v+(\sigma_0^c+\sigma_0^v)\frac{L}{b}\mathrm{th}\frac{b}{L}\right]. \quad (51)$$

2. The case $q = 0$. It is required to solve the system of equations $\bar{\psi} = 0$. It is impossible to find the solutions of these equations, but we shall use the same method of determining χ as in Sec. I. In the case of a multiply-connected Fermi surface, there is also a relation (19) which holds for each group of carriers, except that χ in (19) should be replaced by $\tilde{\chi}$, and it is necessary to use the definition (46) for $\tilde{j}_z$. Proceeding just as in the derivation of (22), we obtain expressions connecting $\tilde{\chi}$ with $j_z(0)$:

$$\tilde{\chi} = \frac{r}{\sqrt{3}}\left[\frac{\tilde{j}_z(0)}{\sigma} \pm aE_x\right], \quad a \sim 1. \quad (52)$$

Formulas (52) should be used in the boundary condition (47), and also in the calculation of the average current. We confine ourselves for simplicity to the case of a symmetrical plate (identical scattering conditions on the surfaces), when

$$\bar{f}(-b) = -\bar{f}(b), \quad \tilde{\chi}(-b) = -\tilde{\chi}(b), \quad j_z(-b) = j_z(b).$$

Calculations with $\gamma \ll 1$ give for the last term of (47) the estimate

$$a_1(\tilde{\chi}_c - \tilde{\chi}_v) + a_2E_xr, \quad a_i \sim 1.$$

As a result, the boundary condition (47) for diffuse scattering takes the form

$$\frac{\beta}{l_c}\bar{f}_c(\Delta-b) = \frac{4}{3}\frac{j_z^c(-b)}{\sigma_0^c}\left[1-\frac{d_c}{d}\right] + \frac{a}{\sigma_0^c\gamma_c}\tilde{j}_z^c(-b) + a'\gamma_cE_x. \quad (53)$$

From (43)–(45) we have for $j_z^c(-b)$ in this case

$$j_z^c(-b) = -\gamma_c\sigma_0^c E_x + \frac{\sigma_c}{1+\gamma_c/\gamma_v}\frac{\beta}{L}\bar{f}_c(\Delta - b)\,\mathrm{cth}\frac{b}{L}. \quad (54)$$

From (53) and (54) it follows that

$$\bar{f}_c(\Delta - b) = \frac{E_x}{\beta}\left(1+\frac{\gamma_c}{\gamma_v}\right)\frac{L}{\gamma_c}\frac{1+a\tilde{d}}{1+Q}\,\mathrm{th}\frac{b}{L},$$

$$Q = \frac{3}{4}\tilde{d}\left(1+\frac{\gamma_c}{\gamma_v}\right)\frac{L}{\gamma_c r}\,\mathrm{th}\frac{b}{L}. \quad (55)$$

In writing down (55) the only term with $\bar{f}_c(\Delta - b)$ retained in the right-hand side of (53) is the one with the coefficient $\tilde{d}^{-1}$, which is capable of competing with the left-hand side. It turns out to be significant for small values of $\tilde{d}$, when $Q \lesssim 1$.

A cumbersome calculation, which is not presented here, yields for the average current density

$$\bar{j}_x = \left[\sigma_c + \sigma_v + (\sigma_0^c + \sigma_0^v)\frac{L}{b}\frac{1-2\tilde{d}}{1+Q}\,\mathrm{th}\frac{b}{L} + 3\sigma_c\frac{l_c}{2b}\right]E_x. \quad (56)$$

In the calculation we used the value resulting from (52)–(55):

$$\tilde{\chi} = \pm\frac{r}{\sqrt{3}}\left(\frac{1}{\gamma(1+Q)} + a\right)E_x. \quad (57)$$

An analysis of the behavior of the current density at the boundary shows that as $z \to r - b$ we have

$$j_x^{c,v(1)} \sim \sigma_0\left(\frac{1}{1+Q} + \gamma a\right)E_x, \quad \gamma \ll 1.$$

Subsequently, on approaching the region of the diffusion approximation, j_X assumes a value that follows from (43)–(45) and (55):

$$j_x(z) = E_x\left[\sigma_c + \sigma_v + (\sigma_c^0 + \sigma_v^0)\frac{1+a\tilde{d}}{1+Q}\frac{\mathrm{ch}(z/L)}{\mathrm{ch}(b/L)}\right]. \quad (58)$$

In the case of specular scattering it is possible to describe the current density by formula (58) with $\tilde{d} = Q = 0$ for all z. An investigation of the behavior of the functions $\mathscr{E}_Z(z)$ at the boundaries in diffuse scattering shows that $\mathscr{E}_Z$ increases in proportion to $z^{-1/2}$ on approaching the surface (of course, we are dealing with an approach to the boundary of the quasi-neutral region, which is located at a distance on the order of the Debye radius from the surface). The transverse field E_Z and the concentration gradients show similar behavior. The corresponding expression is analogous to (17), i.e., the coefficient of $z^{-1/2}$ is determined by the values of E_X and $\tilde{\chi}$ (57).

After a number of transformations we can obtain the following expression for the average Hall field in the case of diffuse scattering:

$$\bar{E}_z = (\gamma_v - \gamma_c)E_x + \frac{1}{\sigma_0^c\gamma_c}\frac{\gamma_v - a\gamma_c}{\gamma_v+\gamma_c}\frac{\bar{j}_{x1}}{1+a}$$
$$-\frac{rE_x}{2b}\left[a + \frac{2}{\sqrt{3}}\left(\frac{1}{\gamma_c} - \frac{a}{\gamma_v}\right)\frac{1}{(1+Q)\cdot(1+a)}\right] \quad (59)$$

where the numerical constant is $a \sim 1$ and $\bar{j}_{X1}$ is equal to the last two terms of (56). In the case of specular scattering, E_Z consists of the first two terms of (59), where $\bar{j}_{X1}$ is the last part of (51). The values of the Hall constant R in samples where the influence of the dimensions is appreciable (i.e., $Q \lesssim 1$, $L > b\gamma^2$) are as follows:

$$R = R_\infty\frac{\gamma_v - a\gamma_c}{(\gamma_v - \gamma_c)(1+a)}\left(1 - \frac{r}{\sqrt{3}L}\,\mathrm{cth}\frac{b}{L}\right), R_\infty = \frac{1}{enc}\frac{\gamma_v - \gamma_c}{\gamma_v + \gamma_c}; \quad (60)$$

R_∞ is the Hall constant of a bulky sample (in our case for $L < b\gamma^2$).

In the case of thin samples ($b \ll r \ll l$) a calculation similar to that carried out in Sec. I yields

$$R = -\frac{4}{3\pi}\frac{1}{enc}\frac{r}{b}\frac{1-a}{1+a}. \quad (61)$$

3. Skin Effect

It is obvious that the inhomogeneous distribution of the current density at the surface, analyzed above for the static case, should also take place in an alternating field the depth of penetration of which is large compared with r. We can then use formulas (43) and (44) of the diffusion approximation for the currents. The problem is described by Maxwell's equation

$$\frac{d^2E_x}{dz^2} = -\frac{4\pi i\omega}{c^2}j_x \quad (62)$$

and by the continuity equation (41)

$$\frac{dj_z^c}{dz} = -e^2\frac{\langle 1\rangle_c\bar{f}_c\beta}{T_1} \quad \frac{1}{T_1} = \frac{1}{T} - \frac{i\omega}{\beta}, \quad \beta = \frac{a+1}{a}; \quad (63)$$

ω is the frequency of the electromagnetic field. All the quantities are proportional to $\exp(kz/L - i\omega t)$, and $kr \ll L$. From (62) and (63) we obtain the dispersion equation

$$k^4 + k^2\left[\frac{iL^2}{\delta_0^2} + \frac{iL^2}{\delta^2} - 1\right] - \frac{iL^2}{\delta^2} = 0,$$
$$c^2\delta^{-2} = 4\pi\omega(\sigma_c + \sigma_v), \quad c^2\delta_0^{-2} = 4\pi\omega(\sigma_c^0 + \sigma_v^0). \quad (64)$$

We obtain the value of L by replacing T_c by T_1 in (45). By virtue of $\delta^2/\delta_0^2 \sim \gamma^{-2} \gg 1$ we can write the solutions of (64) in the form

$$k_1^2 \sim 1 - \frac{iL^2}{\delta_0^2}, \quad k_2^2 \sim \frac{L^2/\delta^2}{i + L^2/\delta_0^2}. \quad (65)$$

We confine ourselves to consideration of a half-space and represent E_X and $\bar{f}_c$ in the form

$$E_x = E_1e^{k_1z/L} + E_2e^{k_2z/L}, \quad \bar{f}_c = f_1e^{k_1z/L} + f_2e^{k_2z/L},$$
$$f_{1,2} = \frac{k_{1,2}L}{k_{1,2}^2 - 1}\left(\frac{1}{\gamma_c} + \frac{1}{\gamma_v}\right)\frac{E_{1,2}}{\beta}. \quad (66)$$

We shall assume that on the boundary of the region of the diffusion approximation $E_X(\Delta) = E_X(0)$, i.e., $E_1 + E_2 = E_X(0) = E_0$. The boundary condition for the concentration $\bar{f}$ is given by formula (53); after substituting in (53) the expression for the current $j_Z^c(0)$ and taking (66) into account we obtain a second equation relating E_1 and E_2 with E_0. The solution takes the form

$$E_{1,2} = \pm\frac{E_0}{D}\left[aE + \frac{k_{2,1} + Q_\infty^{-1} - arL^{-1}}{1 - k_{2,1}^2}\right], \quad Q_\infty = \frac{3}{4}\left(1+\frac{\gamma_c}{\gamma_v}\right)\frac{\tilde{d}L}{\gamma_c r},$$

$$D = \frac{k_2 - k_1}{(1-k_1^2)(1-k_2^2)}\left[1 + k_1k_2 + \left(\frac{1}{Q_\infty} - \frac{ar}{L}\right)(k_1 + k_2)\right]. \quad (67)$$

To determine the surface impedance

$$Z = E_0/J_x, \quad J_x = \int_0^\infty dz\,j_x(z), \quad (68)$$

it is necessary to find the total current J_x. To this end we perform a calculation similar to that made above in the static case. After a number of simple transformations we obtain

$$J_x = (\sigma_c + \sigma_v) L \left(\frac{E_1}{k_1} + \frac{E_2}{k_2}\right) + \frac{L}{Q_\infty}\left(\frac{1}{\gamma_c} + \frac{1}{\gamma_v}\right)\sigma_0^c\gamma_c\left(\frac{E_1}{1-k_1^2} + \frac{E_2}{1-k_2^2}\right) + \frac{3}{2}E_0\sigma_0^c\gamma_c r. \quad (69)$$

Calculations using (67) yield

$$\frac{J_x}{E_0} = L\left[1 + k_1k_2 + \frac{k_1+k_2}{Q_\infty}\right]^{-1}\left\{\frac{\sigma_c+\sigma_v}{k_1k_2}\left[k_1+k_2 + (1+k_1k_2)\left(\frac{1}{Q_\infty} + \frac{ark_1k_2}{L}\right)\right] + \left(\frac{1}{\gamma_c}+\frac{1}{\gamma_v}\right)\frac{\sigma_0^c\gamma_c}{Q_\infty}\right\}. \quad (70)$$

We have omitted here a number of terms $\sim rkL^{-1} \ll 1$. It is assumed that $Q \lesssim 1$; actually the case $Q \gg 1$ is not covered by our analysis. Indeed, the current density j_x is then concentrated mainly at the boundaries and it cannot be assumed that $E_x(\Delta) \sim E_x(0)$.

We present expressions for Z at two limiting ratios of L and δ_0:

$$L \gg \delta_0, \quad k_1 = \frac{i^{3/2}L}{\delta_0}, \quad k_2 = \frac{\delta_0}{\delta}, \quad Z_1 \sim i\left(\frac{4\pi i\omega}{c^2(\sigma_0^c + \sigma_0^v)}\right)^{1/2};$$

$$L \ll \delta_0, \quad k_1 = 1, \quad k_2 = i^{-3/2}\frac{L}{\delta},$$

$$Z_2 \sim i\left(\frac{4\pi i\omega}{c^2(\sigma_c+\sigma_v)}\right)^{1/2}\left[1 + \frac{i^{3/2}}{1+Q}\frac{L_0}{\delta_0}\right]^{-1}. \quad (71)$$

Here $L_0 = L/\gamma$ is the diffusion length at H = 0. The realization of the considered limiting cases can be attained by varying the frequency; we note that the surface impedance changes in this case from its value at H = 0 (Z_1) to the value determined by the volume conductivity (Z_2).

DISCUSSION

Let us analyze first the results obtained for the monopolar metal. The main difference between diffuse and specular scattering is, as already noted, the dependence of the kinetic characteristics of the plate on the dimensions. A strong magnetic field parallel to the surface causes a much sharper inhomogeneity of the current density at the boundaries than in the absence of a field. Thus, at H = 0 (or $\gamma \gg 1$) in a thick plate ($b \gg l$) the value of j on the boundaries is only half its volume value; in the case when $\gamma \ll 1$, on the other hand, the current density in a surface layer of thickness $\sim r$ decreases by a factor γ. Thus, in a plate of a monopolar metal (or when $n_e \neq n_h$) with diffusely-scattering walls the current flows mainly in the internal region of the sample—in contrast to the well known Azbel' static skin effect[1], which takes place for metals with different numbers of electrons n_e and holes n_h, and also in contrast to our results for semimetals, according to which the current can be concentrated at the surfaces within a depth $\sim L \gg r$. The results (24), (26), (30), and (32) for the average conductivity were obtained earlier by Azbel'[2].

The data obtained by us are of interest for another average characteristic of the plate, namely the Hall constant, particularly in thin samples (with $b < r$). Namely, according to (34), the contribution to R_H that depends on the dimension of the sample is much larger than the Hall constant of the bulky sample and has the opposite sign. In the case of diffuse scattering, the Hall field at the surfaces increases sharply compared with the volume value: for thick samples, according to (17) and (29), $E_z \sim \sqrt{r/(b \pm z)}E_x/\gamma$, and in thin plates $E_z \sim E_x\sqrt{b/(b \pm z)}$ (here, as already noted, it is necessary to put at the boundaries $b \pm z = R_D$, and in metals the Debye radius is $R_D \ll r$, b). The space charge connected with the singularities of E_z is, however, negligible, and estimates show that its density in a thick plate is

$$\delta n \sim \frac{neE_x l}{\mu}\left(\frac{R_D}{r}\right)^{1/2}.$$

Direct observation of the inhomogeneities of the Hall field in metals is apparently impossible. Among its indirect manifestations is the effect in which the Hall constant depends on the dimensions, particularly in thin plates. One can expect a sharp increase of the Hall field at the surfaces to affect the period of revolution on the Larmor orbits. In fact, the usual condition of neglecting the electric field E in the equation of motion along the trajectory reduces to the inequality $\gamma eEl \ll \mu$, where μ is the Fermi energy; in our case for $E = E_z^0$ at the boundaries of a thick plate $\gamma eEl \sim \sqrt{r/R_D}eE_x l$, i.e., the influence of the electric field on the trajectory increases strongly. By virtue of the inhomogeneity of E_z, action of the field on the cyclotron frequency is possible.

We did not discuss the experimental data, since our results for the average electric conductivity coincide with the previously obtained data that are in qualitatively good agreement with experiment in parallel fields; for crossed fields, insofar as we know, no detailed investigations have been made, nor are there experiments on the Hall effect and on the inhomogeneities of the currents and of E_z.

We now proceed to the case of a semimetal. We first discuss the results obtained for the average electric conductivity of the plate. As shown in Sec. II, the influence of the surface scattering on the average current $\bar{j}_x$ is connected principally with the fraction of the intervalley (electron-hole) transitions. The magnitude of the intervalley scattering determines the value of the parameter Q (55). If $Q \lesssim 1$, then according to (51) and (56) the type of the intravalley scattering hardly affects the value of the average current $\bar{j}_x$. (In the case of weak diffuseness, i.e., $1 - q < \gamma$, the value of $\bar{j}_x$, as shown by calculation, is determined by formula (56) without the last term.)

For $Q \lesssim 1$ there follow from (51) and (56) the following limiting cases with different types of dependences on b and H:

$b \ll L$, $\bar{j}_x \sim \sigma_0 E_x$ and does not depend on H and b;

$L < b < L\gamma^{-2}$, $\bar{j}_x \sim \sigma_0 L b^{-1} E_x$, i.e., it is proportional to $(Hb)^{-1}$;

$b > L\gamma^{-2}$, $\bar{j}_x \sim \sigma E_x$, i.e., it is proportional to H^{-2}.

A diffusely scattering surface with $Q \gg 1$ leads to different values of $\bar{j}_x$, namely $\bar{j}_x \sim \sigma(1 + alb^{-1})E_x$, with $a \sim H^{-2}$ dependence at all thicknesses. The same result is obtained also for weak diffuseness.

The results of Sec. I remain valid for the case of a thin plate ($b \ll r \ll l$).

The condition $Q \lesssim 1$ calls for a small coefficient of intervalley scattering:

$$\tilde{d} \sim \gamma \sqrt{\frac{\tau}{T}} \operatorname{cth} \frac{b}{L}.$$

It is possible that the above-noted smallness of $\tilde{d}$ takes place in the case when the surface relief is sufficiently smooth, and the collisions cause small changes in the electron momentum compared with its initial value (we recall that in momentum space the distances between the valleys in Bi exceed the dimensions of the valleys by two orders of magnitude).

In such a case the above-noted deviations from the quadratic dependence of $\bar{j}_x$ on H^{-1} with changing thickness b should appear. We note that the quantity Q itself is a function of H (the parameter $\tilde{d}$ and the times τ and T in our model are assumed to be constants independent of H). Therefore when the field increases, the "weak" (in the sense $Q \lesssim 1$) intervalley transitions can become effective ($Q > 1$), and this leads to a quadratic dependence of $\bar{j}_x$ on H^{-1} even in the case of a small share ($\sim \gamma$) of diffuseness.

The diffusion length L is determined by the ratio T/τ. According to the experimental estimates[9,10], $T/\tau > 10^2$ for Bi at helium temperatures. Thus, in Bi the value of L exceeds the Larmor radius by dozens of times, thus justifying the employed method of solution. Therefore deviations from the quadratic dependence of $\bar{j}_x$ on H^{-1} in a field parallel to the surface, as observed, for example, for Bi in[11,12], may be connected with the exceedingly small value of $\tilde{d}$ ($Q \lesssim 1$). In[11], to explain the experimental data, use is made of the diffusion size effect, but its theoretical analysis is carried out in the diffusion approximation using a phenomenological boundary condition $j_z^{c,v}(\pm b) = \pm eS_{c,v}\delta n_{c,v}$, where S are the rates of surface recombination. It was shown in Sec. II that in a strong magnetic field the boundary condition of the diffusion approximation has the form (53). Unfortunately, the experimental data in[11] are given in such a way that it is impossible to extract information on the field dependence of the part of the average conductivity which changes with thickness (corresponding to the last terms in (56)). Apparently in[11] the case $L > b$ was not realized. In[12], to explain deviations from the quadratic dependence of the magnetoresistance on H, they used the static skin effect[2,4], in the theory of which such an explanation calls for the assumption that the scattering by the surface is almost completely specular. Allowance for the diffusion effects makes it possible to explain the deviation from the H^2 law also in diffuse scattering, under the previously assumed condition that the intervalley transitions are weak.

The type of scattering by the surface (intravalley diffuseness or specularity), while hardly affecting the average electric conductivity when $Q \lesssim 1$, does cause a strong difference in the behavior of the transverse electric field E_z at the boundaries, in analogy with the already examined case of a monopolar metal. This difference in the behavior of E_z affects the value of the Hall constant described by formulas (60) and (61).

Information on the scattering properties of the surfaces can also be obtained with the aid of the skin effect. Depending on the ratio of the length L and δ_0, the surface impedance at $Q < 1$ changes in the limiting cases by a factor of γ, and with it a change takes place in the character of the dependence on H. These limiting cases, in principle, are attainable by varying the frequency. When $Q \gg 1$, as already noted, the results of Azbel' and Rakhmanov[5] are valid.

Thus, using a very simple model, we have demonstrated the possibility of a pronounced influence of the surface on the characteristics of the magnetoresistance and on the distribution of the currents and of the transverse field. The experimental data still do not allow us to conclude that diffusion effects are important in the presence of a magnetic field. In this respect, comprehensive measurements of the conductivities, the Hall constant, the surface impedance, and if possible the distributions of the fields and of the currents at the boundaries would be useful.

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