RESONANT SHIFT AND BROADENING OF FMR LINES, DUE TO FINE MAGNETIC STRUCTURE[1)]

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It is found that the shift in broadening of the FMR lines in thin films, due to the fine magnetic structure, have a resonant character. The importance of taking into account some nonlinear terms in the Landau–Lifshitz equation, which lead to effects of the same order of magnitude as the linear terms, is demonstrated. An iteration process is found, which leads to an expansion of the denominator of the high-frequency magnetic susceptibility in series in terms of the correlation functions.

INTRODUCTION

It is known that the smallness of the dimensions of crystallites compared with the effective radius of manifestation of exchange interaction leads to the unique phenomenon in thin magnetic films, namely, the formation of a quasi-periodic wave-like structure of magnetization with the characteristic wavelength $\sim 10^{-4}$ cm. This fine magnetic structure ("ripple" of magnetization) is presently under intense experimental study; the main premises of the theory of such a structure have been developed in [1–4].

A dynamic theory of the fine magnetic structure was developed for the first time in [3], where it was shown that such a structure leads to a shift and asymmetric broadening of the FMR line. The mechanism of this phenomenon is the excitation of spin waves in the linear regime by a homogeneous external high-frequency field owing to the inhomogeneity of the internal fields due to the fine structure. Since small oscillations of the magnetization were studied, the usual procedure for linearizing the Landau-Lifshitz equation was used in [3]. The system of integro-differential equations derived in that paper was solved by using an iteration process corresponding to a series expansion of the high-frequency susceptibility in powers of the crystallographic anisotropy β:[2)]

$$\chi = \chi_0 + \beta\chi_1 + \beta^2\chi_2 + \ldots,$$

where χ_0 is the susceptibility describing the homogeneous FMR, and χ_i are corrections due to the excitation of spin waves.

In [5] the same method was used to calculate the statistical and dynamic properties of the fine structure in films magnetized normally to the surface.

Zlochesvkiĭ[6] found a better method for approximately solving the system obtained in [3], based on the use of the Green's function and the hypothesis of local independence of the correlations. The results of [6] agree qualitatively with the results of [3], but the frequency dependences of the shift and broadening of the FMR line, obtained in these papers, are different.

Further analysis has shown that complete linearization of the Landau-Lifshitz equation in the solution of this problem is incorrect (for arbitrarily small magnetization oscillations) for reasons that will be explained in detail below. Therefore the solutions in both [3] and [6] are true in such a narrow frequency interval, that it is meaningless to speak of a frequency dependence. In addition, an iteration process peculiar to random functions was found, corresponding to a series expansion of the denominator of χ. Thus, the present paper is the development of the ideas of [3] both from the physical point of view (allowance for certain nonlinear terms of the Landau-Lifshitz equation) and from the mathematical one (the use of a more perfect iteration process). As a result, expressions are obtained for the shift and broadening of the FMR line; these expressions are valid in a broader frequency interval. An analysis of these expressions has shown that both the shift and the broadening of the line have a unique resonance character.

1. EQUATIONS OF MOTION. ITERATION PROCESS

The equation of motion of the system is the Landau-Lifshitz equation

$$\dot{\mathbf{M}} = g[\mathbf{M}\mathbf{H}^{(e)}] - \frac{\xi}{M}[\mathbf{M}\dot{\mathbf{M}}] \qquad (1.1)^*$$

with an effective field containing the random function $\rho_{ij}(\mathbf{r})$—the orientation of the anisotropy axis, which is different in each crystallite.

For a film magnetized in its own plane, representing the normalized magnetization $\mathbf{m}(\mathbf{r}, t) = \mathbf{M}(\mathbf{r}, t)/M$ in the form

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r}) + \boldsymbol{\mu}(\mathbf{r}, t), \qquad (1.2)$$

changing over to Fourier components, and determining the demagnetizing fields, we obtain for the static parts of the magnetization in the linear approximation the system (1.9) of [3], and for the dynamic part the following system of equations:

$$[B(\mathbf{k}) + i\xi\sigma]\mu_y + i\sigma\mu_z + \beta[g(\rho_{yy}\mu_y) - g(\rho_{yz}\mu_z) + g(\rho_{xz}m_z\mu_y) + g(\rho_{xz}m_y\mu_z) + 4g(\rho_{xy}m_y\mu_y) + 2g(\rho_{xy}m_z\mu_z)] = a\delta(\mathbf{k}),$$

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1) A summary of this paper was published in [9].

2) The entire notation of this paper corresponds to the notation of [3], with the exeption of the crystallographic-anisotropy constant, which is denoted here β and not β_C.

*$[\mathbf{MH}] \equiv \mathbf{M} \times \mathbf{H}$.

$$[A(\mathbf{k})+i\xi\sigma]\mu_z-i\sigma\mu_y+\beta[g(\rho_{zz}\mu_z)-g(\rho_{yz}\mu_y)+g(\rho_{xy}m_z\mu_y)+g(\rho_{xy}m_y\mu_z)+4g(\rho_{xz}m_z\mu_z)+2g(\rho_{xz}m_y\mu_y)]=0. \quad (1.3)$$

This system differs from the corresponding system (2.6) of [3] in that account is taken of nonlinear terms of the type $\mathbf{m}(\mathbf{r})\,\mu(\mathbf{r},t)$. The physical meaning of these terms is as follows. If a convolution of the type $g(\rho\mu)$ describes the perturbations (spin waves) produced by direct interaction of the dynamic part of the magnetization $\mu(\mathbf{r},t)$ with the driving random function $\rho(\mathbf{r})$, then convolutions with nonlinear terms describe the excitation of spin waves due to the interaction of $\mu(\mathbf{r},t)$ with the static fine structure $\mathbf{m}(\mathbf{r})$ due to the driving random function $\rho(\mathbf{r})$. It will be shown below that the interactions of both types are of the same order of magnitude. Naturally, allowance for such nonlinear terms leaves the system linear in terms of the dynamic variables.

Owing to the strong demagnetizing field perpendicular to the plane of the film (for wavelengths larger than the thickness of the film) we have $m_z \ll m_y$ and $\mu_z \ll \mu_y$, and the system (1.3) can be strongly simplified:

$$[B(\mathbf{k})+i\sigma\xi]\mu_y+i\sigma\mu_z+\beta g(\rho_{yy}\mu_y)+4\beta g(\rho_{xy}m_y\mu_y)=a\delta(\mathbf{k}),\qquad [A(\mathbf{k})+i\sigma\xi]\mu_z-i\sigma\mu_y=0. \quad (1.4)$$

Substituting μ_z from the second equation into the first and introducing the notation

$$m_y=m,\quad \mu_y=\mu,\quad \rho_{yy}=\rho,\quad \rho_{xy}=\rho_1, \quad (1.5)$$

we obtain one dynamic equation

$$L(\mathbf{k})\mu+\beta g(\rho\mu)+4\beta g(\rho_1 m\mu)=a\delta(\mathbf{k}), \quad (1.6)$$

where

$$L(\mathbf{k})=\alpha\varkappa^2+h+4\pi\frac{k_z^2}{\varkappa^2}(1-V)+i\xi\sigma-\frac{\sigma^2}{\alpha\varkappa^2+h+4\pi V}. \quad (1.7)$$

After substituting the static solution (1.10) of [3] in the triple convolution, the latter takes the form

$$g(\rho_1 m\mu)=\iint\frac{d\mathbf{k}_1 d\mathbf{k}_2}{B(\mathbf{k}_1)}\mu(\mathbf{k}_2)\rho_1(\mathbf{k}_1)\rho_1(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2). \quad (1.8)$$

For the iteration process, which we plan to develop here, it is necessary that the quantities under the integral signs in the convolutions have the meaning of the mutual correlators of the function μ and a function that is the product of the functions ρ. We therefore make use of the fact that real $\rho(\mathbf{r})$ satisfy the relation $\rho^*(\mathbf{k}) = \rho(-\mathbf{k})$, and we write the dynamic equation in the final form

$$L(\mathbf{k})\mu(\mathbf{k})+\beta\int d\mathbf{k}_1\,\mu(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})+4\beta\iint\frac{d\mathbf{k}_1 d\mathbf{k}_2}{B(\mathbf{k}_1)}\mu(\mathbf{k}_2)\rho_1^*(-\mathbf{k}_1)\rho_1^*(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})=a\delta(\mathbf{k}). \quad (1.9)$$

To present the subsequent computational procedure with maximum clarity, we omit for the time being the third term of (1.9) and solve first the simpler equation which was solved in [3]:

$$L(\mathbf{k})\mu(\mathbf{k})+\beta\int d\mathbf{k}_1\,\mu(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})=a\delta(\mathbf{k}). \quad (1.10)$$

1. We average the equation over the random realizations

$$L(\mathbf{k})\langle\mu(\mathbf{k})\rangle+\beta\int d\mathbf{k}_1\langle\mu(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle=a\delta(\mathbf{k}). \quad (1.11)$$

Since $\mu(\mathbf{r})$ is a stationary random function, i.e., $\langle\mu(\mathbf{r})\rangle = \langle\mu\rangle$ is independent of $\mathbf{r}$, it follows that

$$\langle\mu(\mathbf{k})\rangle=\langle\mu\rangle\delta(\mathbf{k}). \quad (1.12)$$

Let us consider the integrand; according to the general rules

$$\langle\mu(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle=\langle\mu(\mathbf{k}_1)\rangle\langle\rho(\mathbf{k}_1-\mathbf{k})\rangle+\langle\mu_c(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle. \quad (1.13)$$

The subscript c will henceforth denote a centered random function; since all the functions ρ are centered by definition, we shall not mark them with this index, so as not to clutter up the expressions. Since ρ is centered, the product of the mathematical expectation values in the right-hand side of (1.13) vanishes and there remains one correlator: (1.11) takes the form

$$L(\mathbf{k})\langle\mu\rangle\delta(\mathbf{k})+\beta\int d\mathbf{k}_1\langle\mu_c(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle=a\delta(\mathbf{k}). \quad (1.14)$$

2. The μ_c which enters in (1.14) is determined from (1.10):

$$\mu_c(\mathbf{k})=-\frac{\beta}{L(\mathbf{k})}\int d\mathbf{k}_1\{\mu(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})-\langle\mu(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle\}. \quad (1.15)$$

We substitute this expression in (1.14) and obtain

$$L(\mathbf{k})\langle\mu\rangle\delta(\mathbf{k})-\beta^2\iint\frac{d\mathbf{k}_1 d\mathbf{k}_2}{L(\mathbf{k}_1)}\langle\mu(\mathbf{k}_2)\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle=a\delta(\mathbf{k}). \quad (1.16)$$

Let us consider the integrand. According to the general rules, assuming the product ρ^* to be one function, we have

$$\langle\mu(\mathbf{k}_2)\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle=\langle\mu(\mathbf{k}_2)\rangle\langle\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle+\langle\mu_c(\mathbf{k}_2)\{\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\}_c\rangle. \quad (1.17)$$

Substituting this expression into (1.16) with allowance for (1.12), we obtain

$$L(\mathbf{k})\langle\mu\rangle\delta(\mathbf{k})-\beta^2\langle\mu\rangle\int\frac{d\mathbf{k}_1}{L(\mathbf{k}_1)}\langle\rho^*(-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle-\beta^2\iint\frac{d\mathbf{k}_1 d\mathbf{k}_2}{L(\mathbf{k}_1)}\langle\mu_c(\mathbf{k}_2)\{\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\}_c\rangle=a\delta(\mathbf{k}). \quad (1.18)$$

3. We substitute here (1.15); then the third term of (1.18) takes the form

$$\beta^3\iiint\frac{d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3}{L(\mathbf{k}_1)L(\mathbf{k}_2)}\langle\mu(\mathbf{k}_3)\rho^*(\mathbf{k}_3-\mathbf{k}_2)\{\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\}_c\rangle. \quad (1.19)$$

We again break up the integrand, in analogy with (1.13) and (1.17), into a product of mathematical expectations and a mutual correlator of the functions μ and a function that is the product of the functions ρ, etc.

During each such step, the product of the expectation values forms the next term of the iteration series. The approximate solution corresponding to this step can be obtained by discarding the correlator of the same order as the last term of the series. Continuing the procedure to infinity and assuming that the correlation between μ and $(\rho^*)^n$ decreases with increasing n, we obtain after integrating the iteration series with respect to $\mathbf{k}$

$$\langle\mu\rangle\{L(0)-\beta^2G_1+\beta^3G_2-\beta^4G_3+\ldots\}=a. \quad (1.20)$$

Accordingly the sought high-frequency susceptibility is given by the expression

$$\chi=\Big[L(0)-\sum_{j=1}^{\infty}(-\beta)^{j+1}G_j\Big]^{-1}. \quad (1.21)$$

Here

$$G_1=\iint\frac{d\mathbf{k}\,d\mathbf{k}_1}{L(\mathbf{k}_1)}\langle\rho(\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\rangle,$$
$$G_2=\iiint\frac{d\mathbf{k}\,d\mathbf{k}_1 d\mathbf{k}_2}{L(\mathbf{k}_1)L(\mathbf{k}_2)}\langle\rho(\mathbf{k}_2)\{\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\}_c\rangle, \quad (1.22)$$
$$G_3=\iiiint\frac{d\mathbf{k}\,d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3}{L(\mathbf{k}_1)L(\mathbf{k}_2)L(\mathbf{k}_3)}\langle\rho(\mathbf{k}_3)\{\rho^*(\mathbf{k}_3-\mathbf{k}_2)\cdot\times\{\rho^*(\mathbf{k}_2-\mathbf{k}_1)\rho^*(\mathbf{k}_1-\mathbf{k})\}_c\}_c\rangle,\ldots.$$

Comparing this expression with the solution obtained by Zlochevskiĭ[6] by another method, we verify that the solution of [6] corresponds to inclusion of only the first term of the series in (1.21).

We now turn to the solution of the complete equation (1.9). Performing for it the iteration process in accordance with the scheme presented above, we obtain, besides the series of autocorrelators of the function ρ (1.20), a series of the autocorrelators of the function ρ_1, and also, starting with β^3, a mixed series of mutual correlators of the functions ρ and ρ_1. We shall not present here the complete expression, since it is too cumbersome, and confine ourselves to terms not higher than β^2, just as in [3]. The susceptibility has in this case the form

$$\chi = [L(0) - \beta^2 G_1 + 4\beta^2 G_1']^{-1}, \tag{1.23}$$

where

$$G_1 = \iint \frac{dk\,dk_1}{L(k_1)} \langle \rho(k_1)\rho^*(k_1 - k)\rangle = \int \frac{S(k)}{L(k)} dk,$$

$$G_1' = \iint \frac{dk\,dk_1}{B(k_1)} \langle \rho_1(k_1)\rho_1^*(k_1 - k)\rangle = \int \frac{S_1(k)}{B(k)} dk.$$

According to formulas (1.6) of [3], $S_1 = S/4$, and we have ultimately

$$\chi = \left[L(0) - \beta^2 \int \frac{S(k)\,dk}{L(k)} + \beta^2 \int \frac{S(k)\,dk}{B(k)} \right]^{-1}. \tag{1.24}$$

Since $L(k) \to B(k)$ as $\sigma \to 0$, we can easily see from this expression that the effect due to the direct action of the random function ρ on the dynamic variable μ (the second term in the denominator of χ), and the effect due to the $\rho \to m \to \mu$ coupling (the third term) are quantities of the same order.

As already indicated,[3] the exact form of the autocorrelation function for the function ρ is immaterial. It can be approximated by any expression that cuts off the correlation at a distance on the order of the crystallite dimension 2b. Therefore in a Cartesian coordinate system we can assume

$$K = D \exp\left(-\frac{|\xi|}{b} - \frac{|\eta|}{b}\right), \tag{1.25}$$

and in a polar system

$$K = D \exp(-r/b). \tag{1.25a}$$

Accordingly, the spectral density of the function ρ will be represented in the form

$$S(k_1 k_2) = D\left(\frac{b}{\pi}\right)^2 \frac{1}{(1 + k_1^2 b^2)(1 + k_2^2 b^2)} \tag{1.26}$$

or in the form

$$S(\varkappa) = D \frac{b^2}{2\pi} \frac{1}{(1 + b^2\varkappa^2)^{3/2}}. \tag{1.26a}$$

Here $D \approx 4b/15d$ is the dispersion of the function ρ averaged over the thickness of the film.

2. FMR CONDITIONS

Separating from (1.24) the imaginary part of the susceptibility, we obtain

$$\chi'' = \frac{\varepsilon(1 + r)}{(x - p + q)^2 + \varepsilon^2(1 + r)^2}. \tag{2.1}$$

Here $\epsilon = \sigma\xi$, $x = h - \sigma^2/4\pi$ is the detuning reckoned from the frequency of the homogeneous FMR which takes place at $\beta = 0$; p, q, and r are corrections due to the fine magnetic structure and are proportional to β^2:

$$p = \beta^2 \int \frac{\Phi S\,dk}{\Phi^2 + \varepsilon^2}, \quad q = \beta^2 \int \frac{S\,dk}{B}, \quad r = \beta^2 \int \frac{S\,dk}{\Phi^2 + \varepsilon^2}, \tag{2.2}$$

where $\Phi = L(k) - i\epsilon$ is the real part of L(k).

It is obvious that in the first approximation the position and the width of the FMR line can be described by the expressions

$$x \approx (p - q)_{x=0}, \quad \Delta h \approx 2\varepsilon(1 + r)_{x=0}. \tag{2.3}$$

The problem thus reduces to a calculation of the double integrals (2.2) at $x = 0$. Changing over to a polar coordinate system (κ, φ), we calculate exactly all the integrals with respect to φ. The cumbersome expressions obtained thereby simplify greatly if it is recognized that in the essential region of integration the following inequalities are valid:

$$4\pi(1 - V) \gg \varepsilon, \quad 4\pi(1 - V) \gg |t|, \tag{2.4}$$

where

$$t = \frac{\alpha}{d^2}u^2 + h_0 - \frac{\sigma^2}{\alpha d^{-2}u^2 + h_0 + 4\pi V}, \quad u = \varkappa d, \quad h_0 = \frac{\sigma^2}{4\pi}.$$

In this notation (at $x = 0$)

$$p = \sqrt{\frac{\pi}{2}} \left(\frac{\beta}{d}\right)^2 \int_0^\infty \frac{[t + (t^2 + \varepsilon^2)^{1/2}]^{1/2} S\,u\,du}{(1 - V)^{1/2}(t^2 + \varepsilon^2)^{1/2}},$$

$$q = \sqrt{\pi} \left(\frac{\beta}{d}\right)^2 \int_0^\infty \frac{S\,u\,du}{(1 - V)^{1/2}(\alpha d^{-2}u^2 + h_0)^{1/2}}, \tag{2.5}$$

$$r = \sqrt{\frac{\pi}{2}} \left(\frac{\beta}{d}\right)^2 \int_0^\infty \frac{S\,u\,du}{(1 - V)^{1/2}(t^2 + \varepsilon^2)^{1/2}[t + (t^2 + \varepsilon^2)^{1/2}]^{1/2}},$$

where S is determined by the expression (1.26a).

The integration was carried out approximately in the following manner. The function V(u) was approximated by the expression

$$V = \begin{cases} 1/(1 + u), & 0 \leqslant u \leqslant 1 \\ 1/2u, & u > 1. \end{cases} \tag{2.6}$$

At different values of the parameter $u_1 = h_0 d^2/\alpha$ there are possible different simplifications in the integrands of (2.5); when these are used we obtain

$$x \approx \frac{\beta^2 D b^2}{2(\pi\alpha)^{1/2} d} f_x(u_1), \tag{2.7}$$

where $f_x(u_1)$ is a very complicated function of u_1, i.e., of the frequency. Thus, for $u_1 \leq 1 - \epsilon d^2/\alpha$ we have

$$f_x \approx \frac{3}{2} u_2^{3/2} \left(\frac{\alpha}{2\varepsilon d^2}\right)^{1/2} + 2\sqrt{1 - u_1} - u_1 - 2\left(\frac{\varepsilon d^2}{\alpha u_2}\right)^{1/2}$$
$$+ u_1 \ln \frac{2(u_0 - u_1)}{\sqrt{1 - u_1} + 1 - u_1} + \frac{4}{3}(u_1)^{1/4} - 2, \tag{2.8}$$

where $u_2 = u_1/2 + \sqrt{u_1^2/4 + \epsilon d^2/\alpha}$, $u_0 = d/b$; f_x increases with increasing frequency.

Without presenting the even more complicated expressions, we indicate only that in the interval $u_1 \geq 1 + \epsilon d^2/\alpha$ the function f_x decreases with increasing frequency and can become negative. With further increase of the frequency (i.e., at $u_1 \gg 1$) expressions (2.5) cease to be valid, for the waves coming into play

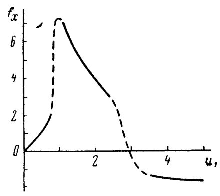

FIG. 1. Shift $x \sim f_x(u_1)$ of the maximum of the FMR curve (relative to the position of the maximum of the homogeneous FMR) vs. the frequency $\omega \sim u_1^{1/2}$.

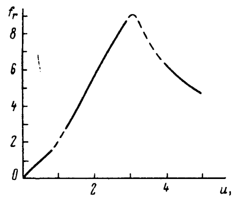

FIG. 2. Frequency dependence of the correction $\delta h \sim f_r(u_1)$ to the line width of the homogeneous FMR.

are so short that the approximation (2.4) is no longer satisfied. From the more exact expressions (2.2) we see that as $\omega \to 0$ both p and q tend to zero.

Figure 1 shows a plot of $f_x(u_1)$, constructed for the following values of the parameters: $\alpha/(2d)^2 \sim 10^{-2}$, $d/b \sim 5$, $\xi \sim 10^{-2}$. The dashed lines show the proposed course of $f_x(u_1)$ outside the intervals for which the approximate integration was carried out. Of course, with changing parameters, the numerical characteristics f_x can change significantly, but its characteristic features are retained when the parameters are considerably varied. Such features include the presence of a resonance in the vicinity of $u_1 = 1$, i.e., in the vicinity of the frequency

$$\omega_1 \sim 2gM(\pi\alpha)^{1/2}/d, \tag{2.9}$$

and the possibility of sign reversal (if $d/2b > 1$) at $u_1 \approx d/2b$, i.e., in the vicinity of the frequency

$$\omega_2 \sim \omega_1(d/2b)^{1/2}. \tag{2.10}$$

A similar approximate integration in the function r leads to the following result. The FMR line width is expressed by the relation

$$\Delta h \approx 2\sigma\xi + \frac{\beta^2 D b^2}{(\pi\alpha)^{1/2} d} f_r(u_1). \tag{2.11}$$

The dependence of the correction to the line width on the frequency is expressed by the function $f_r(u_1)$, which for the parameters chosen above takes the form shown in Fig. 2. The characteristic features of this function are the maximum at $\omega = \omega_2$ and the decrease to zero for both increasing and decreasing frequency.

The expressions obtained above are valid for $u_0 = d/b > 1$. In sufficiently thin films subjected to special treatment, the crystallites can be so large that the inverse inequality is satisfied. When $u_0 < 1$, a more important role in the cutoff of the integrand expressions (2.5) is assumed by S(u).

We can separate here two cases.[3)] The simplest case is that of completely non-interacting crystallites, for which simultaneous satisfaction of two inequalities is required: $u_0 \ll h_0 d^2/\alpha$ (the radius of the crystallite is much larger than the effective radius of the exchange interaction), and $u_0 \ll h_0/2\pi$ (the radius of the crystallite is much larger than the effective radius of the magnetostatic interaction). The approximation (2.5) then turns out to be incorrect, and from the more exact expressions (2.2) we get

$$x \approx \frac{D\beta^2 b}{h_0 d}\left(\frac{u_0}{2\xi^2} - 1\right) \tag{2.12}$$

$$\Delta h \approx 2\sigma\xi + \frac{2D\beta^2 b}{\sigma\xi d}. \tag{2.13}$$

The second—intermediate—case takes place when the exchange interaction can be neglected as before ($u_0 \ll h_0 d^2/\alpha$), but the magnetostatic interaction cannot be neglected ($u_0 \gtrsim h_0/2\pi$). It is then possible to use approximation (2.5) as $\alpha \to 0$. As a result of the numerical integration we obtain

$$x = \frac{4}{3} D\beta^2 \left(\frac{b}{d}\right)^{3/2} \left[\frac{\varphi_x}{(8\pi\xi\sigma)^{1/2}} - \frac{1}{\sigma}\right], \tag{2.14}$$

$$\Delta h = 2\sigma\xi + \frac{2D\beta^2}{3\pi^{1/2}}\left(\frac{b}{d}\right)^{3/2} \frac{\varphi_r}{(\sigma\xi)^{1/2}}. \tag{2.15}$$

The functions φ_x and φ_r for different frequency intervals have the following form:

$$\varphi_x \approx \begin{cases} 1, & (4\pi\alpha/db)^{1/2} \leqslant \sigma \leqslant 4\pi\xi, \\ 1 - 0{,}32\,(u_0/u_3)^{3/2}, & 4\pi\xi \leqslant \sigma \leqslant 4\pi\xi/u_0, \\ 0{,}68\,(u_3/u_0)^{3/2}(1 + \ln(u_0/u_3)), & 4\pi\xi/u_0 \leqslant \sigma; \end{cases} \tag{2.16}$$

$$= 2\pi^{1/2}\xi/h_0^{1/2},$$

$$\approx \begin{cases} 1, & (4\pi\alpha/db)^{1/2} \leqslant \sigma \leqslant 4\pi\xi, \\ 1 + 0{,}03\,(u_0/u_3)^{3/2} - 0{,}53\,(u_3 u_0^3)^{1/2}, & 4\pi\xi \leqslant \sigma \leqslant 4\pi\xi/u_0, \\ (u_3/u_0)^{1/2}[1{,}56 - 0{,}56 u_3/u_0 - 0{,}53 u_0^2], & 4\pi\xi/u_0 \leqslant \sigma. \end{cases} \tag{2.17}$$

The frequency dependence of the FMR line is in this case similar to the frequency dependence of Fig. 1. The increment to the line width has a monotonically decreasing frequency dependence.

Let us discuss our results. As a rule, the presently investigated polycrystalline films have crystallites subject to exchange and magnetostatic interaction ($b \ll \alpha/h_0 d$, $b \ll 2\pi d/h_0$); in this case the shift and broadening of the FMR line are described by Eqs. (2.7) and (2.11) and have resonant singularities near the characteristic frequencies (2.9) and (2.10).

In some cases it is possible to obtain films with such large crystallites, that only the magnetostatic interaction need to be taken into account ($b \gg \alpha/h_0 d$, $b \lesssim 2\pi d/h_0$), and then the shift and broadening of the FMR line is described by expressions (2.14) and (2.15); the shift has resonant singularities and the broadening depends on the frequency monotonically.

The case of completely non-interacting crystallites ($b \gg \alpha/h_0 d$, $b \gg 2\pi d/h_0$), when formulas (2.12) and (2.13) are valid, is not realistic in practice for the presently obtained films. However, as indicated in [3], the fine magnetic structure may be due not only to the polycrystalline character of the film, but also to the presence in it of some random inhomogeneities, for example internal elastic stresses. In this case 2b is not the dimension of the crystallite, but the characteristic dimension of the inhomogeneity, which can be quite large.

In real films, one apparently encounters quite fre-

[3)]The following stipulation should be made. As soon as the dimension of the crystallites exceeds the effective radius of the exchange interaction, the ordering, due to the exchange, of the magnetizations of the neighboring crystallites ceases. In order for the expressions obtained above to remain valid in this case (corresponding to small deviations of demagnetization from the equilibrium position that is homogeneous for the entire film), the external magnetic field must be strong as to counteract the disordering action of the crystallographic anisotropy. Therefore satisfaction of the inequality $H_0 > \beta M$ is obligatory for both cases considered below.

quently a situation where there exists not one characteristic dimension of the inhomogeneities. For example, in addition to strongly interacting crystallites the same film can contain a unique block structure with practically non-interacting macroscopic region-blocks. Naturally, the frequency dependence of the shift and broadening of the FMR line in such films will be very complicated.

Formulas (2.12) and (2.13) for non-interacting regions were obtained here as the limiting case of a more complicated situation and in the approximation assumed for this situation. The case of non-interacting regions was considered by itself more accurately in [7, 8].

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