EFFECT OF INTERNAL DIELECTRIC BOUNDARIES ON THE PROPERTIES OF OPTICAL RESONATORS AND ON GENERATION OF STIMULATED EMISSION

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Some properties of plane optical resonators with external mirrors are investigated. It is shown that the position of the active medium in the resonator is a fairly significant parameter of lasers and should not be neglected in the analysis of stimulated emission generation.

GENERATORS of stimulated emission frequently have resonators with plane external mirrors, i.e., mirrors that are not in contact with the boundaries of a dielectric with positive population inversion (active medium). The presence of dielectric boundaries inside the resonator significantly affects the generation of stimulated emission. This is due to the fact that the electromagnetic energy distribution within the resonator in regions I, II, and III (Fig. 1) depends both on the position of the dielectric and the configuration of the electromagnetic field. Since the number of stimulated transitions depends on the energy in the active medium II, while the mirrors and gaps I and III merely cause losses proportional to the energy in these regions, the most likely to generate are those frequencies (other conditions being equal) whose energy density is large within the active medium and small without.

In the study of optical resonators with external mirrors the problem of energy density distribution as a function of dielectric position is usually put aside. This paper deals with the features of resonators and stimulated emission generation that are due to the presence of dielectric boundaries within the resonator.

Let the system under consideration (Fig. 1) consist of two perpendicular axes z of unbounded plane mirrors with a large coefficient of reflection and parallel plane dielectric layer II. Let L be the distance between the mirrors, a the thickness of the dielectric layer, z_0 the distance from the left mirror to the nearest dielectric boundary, $d = L - a$ the total width of the gaps, and $b = \frac{1}{2}L - (z_0 + \frac{1}{2}a)$ the displacement of the resonator from the symmetric position in the center of the resonator. The index of refraction of the medium filling the gaps is considered equal to unity; the index of refraction of the dielectric is μ.

The analysis is performed in one-dimensional approximation, i.e., we consider those oscillations in the resonator for which the field depends only on the z coordinate. In this case the electromagnetic field inside the resonator can be represented as a superposition of transverse standing waves (axial modes) each of which is assigned a natural oscillation with a definite cyclic frequency ω_i. The amplitudes of the electric field intensity of the i-th mode E_1^i, E_2^i, and E_3^i, for regions I, II, and III respectively, are perpendicular to the z axis and equal

$$E_1^i = 4\sqrt{\pi}\sqrt{W_1^i}\sin k_i z,$$
$$E_2^i = \frac{4\sqrt{\pi}}{\mu}\sqrt{W_2^i}\sin[\mu k_i(z - z_0) + \varphi_i],$$
$$E_3^i = \pm 4\sqrt{\pi}\sqrt{W_3^i}\sin k_i(L - z), \tag{1}$$

Here $k_i = 2\pi/\lambda_i = \omega_i/c$ is the wave vector of the i-th mode, λ_i is the wavelength in regions I and III, W_1^i, W_2^i, and W_3^i is the average electromagnetic energy density of the i-th mode in regions I, II, III respectively, and φ_i is the amplitude phase of the i-th mode in the dielectric. The expressions in (1) reflect the fact that the standing waves of vector E^i have nodes at the mirrors. Analogous expressions hold for the magnetic field of the mode.

Using conditions of continuity of the tangential components of the electric and magnetic fields at the dielectric boundary we can obtain equations determining the possible values of k_i, W_1^i, W_2^i, W_3^i, and φ_i:

$$-\frac{\mu^2+1}{2\mu}\operatorname{tg}\mu k_i a = \left(\cos k_i d - \frac{\mu^2-1}{\mu^2+1}\cos 2k_i b\right)^{-1}\sin k_i d, \tag{2}$$

$$W_1^i = \eta_1^i W_2^i, \qquad W_2^i = \eta_2^i \varepsilon_i / a, \qquad W_3^i = \eta_3^i W_2^i, \tag{3}$$

$$\operatorname{tg}\varphi_i = \mu \operatorname{tg} k_i z_0. \tag{4}$$

Here

$$\eta_1^i = \frac{2}{\mu^2+1}\left(1 - \frac{\mu^2-1}{\mu^2+1}\cos 2k_i z_0\right)^{-1}, \tag{5}$$

$$\eta_2^i = \left(1 + \frac{z_0}{a}\eta_1^i + \frac{d-z_0}{a}\eta_3^i\right)^{-1}, \tag{6}$$

$$\eta_3^i = \frac{2}{\mu^2+1}\left[1 - \frac{\mu^2-1}{\mu^2+1}\cos 2k_i(d - z_0)\right]^{-1}, \tag{7}$$

$$\varepsilon_i = z_0 W_1^i + aW_2^i + (d - z_0)W_3^i,$$

where ϵ_i is the total energy of the i-th mode.

1. FREQUENCIES OF AXIAL MODES

Equation (2) determines the frequency spectrum of axial modes in the resonator. Unfortunately the trans-

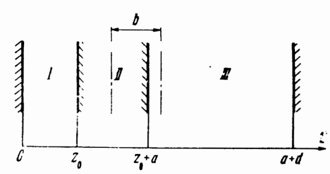

FIG. 1. Resonator with external mirrors. I and III—gaps, II—active medium.

cendental equation cannot be solved for k_i; some results concerning the k_i spectrum can be obtained however without solving the equation:

a. The frequencies of axial modes in the resonator vary when the dielectric is moved periodically with the period $\Delta b_i = \lambda_i/2$.

b) The frequency of each axial mode in certain positions of the dielectric coincides with the axial mode frequency in a homogeneous (without dielectric boundaries) resonator having an optical length $L_0 = \mu a + d$. In a homogeneous resonator $k_i = i\delta k$; $i = 1, 2, \ldots$ where $\delta k = \pi/L_0$.

c. Δk_i is the range of variation of k_i due to dielectric movement and, depending on i, it can assume various values within the interval

$$0 \leqslant \Delta k_i < \varkappa\delta k, \quad \varkappa = \frac{2}{\pi}\frac{\mu^2-1}{\mu^2+1}\left(1+\mu\frac{a}{d}\right)\Big/\left(1+\frac{2\mu^2}{\mu^2+1}\frac{a}{d}\right); \tag{8}$$

for ruby $\mu = 1.76$ and $\kappa \approx 1/3$.

d. The majority of modes vary their frequencies upon the motion of the dielectric; we call them "creeping" modes. However there are modes whose frequency does not depend on b and we call them "fixed" modes. The frequency variation mechanism of "creeping" mode photons is based on the Doppler effect at the dielectric boundaries due to motion of the dielectric[1].

e. The frequency variation of axial modes is associated with an interesting effect that consists of the following: not all positions of the dielectric layer inside the resonator are equilibrium positions. To prove this we note that the total energy of the electromagnetic field in the resonator is $U(b) = \hbar c \sum_i (N_i + 1/2) k_i(b)$ (here N_i is the number of photons of the i-th mode). Since U depends on b the dielectric is subject to the force

$$F = -\frac{\partial U}{\partial b} = -\hbar c \sum_i \left(N_i + \frac{1}{2}\right)\frac{\partial k_i}{\partial b}.$$

Equilibrium position occurs for $F = 0$. Force F is due to the difference of light pressure on the dielectric from the right- and the left-hand sides.

2. ENERGY DENSITY DISTRIBUTION IN THE RESONATOR

The mode energy density distribution in gaps I and III and in the dielectric II is defined by (3), (5)–(7). Energy density W_1^i, W_2^i, and W_3^i as a function of the dielectric position is determined by the type of functions $\eta_1^i(k_i, z_0)$, $\eta_2^i(k_i, z_0)$, and $\eta_3^i(k_i, z_0)$, where η_1^i and η_3^i are periodic functions of z_0 and the values of η_1^i and η_3^i repeat as the dielectric moves through $\lambda_i/2$. As to η_2^i, it is a periodic function of z_0 with a period of $\lambda_i/2$ only for "fixed" modes while for "creeping" modes its periodicity is of an approximate nature.

Thus small movements of the dielectric strongly change the energy distribution in the resonator; for example for $\mu = 1.76$ the displacement of dielectric boundaries by 0.1λ can change the ratio of energy densities in the dielectric and in the gap 1.7 times.

The cause of the strong effect exerted by the position of dielectric boundaries on the energy distribution in the resonator is illustrated in Fig. 2 that shows the electric field of a "fixed" mode ($i = 8$). We see that

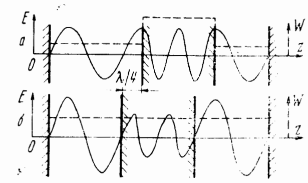

FIG. 2. Distribution of standing wave amplitude E and energy density W as a function of the position of the active medium. a–anti-node at the boundary of the active medium, b–node of vector E at the boundary of the active medium.

the distribution of standing wave amplitudes in regions I, II, and III depends on the amplitude phase at the dielectric boundary: in Fig. 2a an anti-node and in Fig. 2b a node of the standing wave appear at the dielectric boundaries. Dashed lines indicate the distribution of energy density in the resonator. Inside the dielectric the greatest energy density occurs at frequencies for which standing-wave anti-nodes fall on the boundaries and the least energy is associated with frequencies with nodes at the dielectric boundary. With varying z_0 the ratio of energy density in the dielectric and in the gaps oscillates within the limits from 1 to μ^2 (a factor of 3 for ruby).

The dependence of W_1^i, W_2^i, and W_3^i on k_i also has periodic nature determined by the periodicity of the functions η_1^i, η_2^i, and η_3^i in wihch k_i appears as a cosine argument. The dependence of energy density distribution in the resonator on the axial mode number i was noted in[2-4].

The strong variation of the longitudinal energy distribution in the resonator due to small displacements of the dielectric such as $\sim 0.1\lambda$ is caused by the fact that the dielectric boundaries coincide with the constant phase surface of the standing wave amplitude. If this is not true then a mode field inhomogeneity arises in the transverse direction. For example a resonator with spherical mirrors and plane dielectric boundaries has a fairly significant variation in the energy density distribution over the cross section. When the dielectric moves along the resonator axis the nodes and anti-nodes of the spherical standing wave amplitudes move transversely along the dielectric boundary.

Thus the plane resonator with external mirrors is a system that is fairly sensitive to dielectric displacement in terms of longitudinal (along the z axis) energy distribution.

Although this analysis was performed for resonators without limits in the direction normal to the z axis, the obtained results are applicable (with some limitations) also to axial modes of resonators with limited transverse dimensions.

If the mode field depends not only on z (transverse indices are non zero), the longitudinal energy density distribution (as for "purely" axial modes) also depends on the position of the dielectric boundaries.

If the dielectric boundaries are not ideal planes, the boundary unevenness (steps) with a height of $\sim 0.1\lambda$ can markedly affect the energy distribution in regions I, II, and III. Different energy densities then exist at different points of the cross section. Consequently surface inhomogeneities in the given resonator give rise to volume inhomogeneities (if only because of light absorption by the dielectric) and the transverse inhomogeneity of the resonator increases when mode energy increases.

If the light falls on the dielectric boundary at the

Brewster angle α_B (Fig. 3), the energy density distribution in the resonator does not depend on the position of the dielectric.

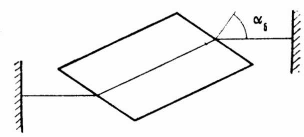

FIG. 3. Active medium boundary at the Brewster angle. The same energy density in the gaps and in the active medium.

3. GENERATION OF STIMULATED EMISSION

We now turn to the process of generating stimulated emission in a plane resonator with external mirrors. Generation is possible if a sufficient population inversion exists in the dielectric. The generation process taking spatial inhomogeneity of the mode field into account is described by kinetic equations[5] that however are subject to the following considerations:

a. The amount of energy radiated into the i-th mode per unit time due to stimulated transitions is proportional to mode energy in the active medium, i.e., it is equal to $Dg_i(k)aW_2^i \int n\varphi_i(z)dz$, where $Dg_i(k)\varphi_i(z)$ is the probability of spontaneous transition of the active center situated at point z with the emission of a photon into the i-th mode, and n is the linear density of inverted population, i.e., the difference between excited and unexcited centers per unit length.

b. The losses of the i-th mode (per unit time) for non-active absorption and scattering in the active medium equal $\gamma_{II}aW_2^i$.

c. The losses of the i-th mode (per unit time) in gaps and at the mirrors equal $\gamma_I' z_0 W_1^i$ and $\gamma_{III}'(d - z_0)W_3^i$.

Taking all this into account and remembering that $\epsilon_i = \hbar\omega_i N_i$ we obtain kinetic equations

$$\dot{N}_i = [-\gamma_i(k_i, z_0) + Dag_i(k)\eta_2^i \bar{n}_i]N_i, \tag{9}$$

$$\dot{n} = \frac{n_0 - n}{\tau_1} - snD\sum_i g_i(k)\Phi_i(z)\eta_2^i N_i, \tag{10}$$

where

$$\begin{gathered}\gamma_i(k_i, z_0) = \eta_2^i(\gamma_I\eta_1^i + \gamma_{II} + \gamma_{III}\eta_3^i),\\ a\gamma_I = z_0\gamma_I', \quad a\gamma_{III} = (d - z_0)\gamma_{III};\end{gathered} \tag{11}$$

$$\bar{n}_i = \frac{1}{a}\int n\Phi_i(z)\,dz. \tag{12}$$

Here $\bar{n}_i$ is the average linear population inversion density for the i-th mode, γ_I, γ_{II}, and γ_{III} usually are independent of the mode number, $g_i(k)$ is the probability of spontaneous transition depending on k, i.e., the shape of the luminescence line, n_0 is the population inversion density for constant pump intensity and absence of the resonator, τ_1 is the time in which stationary population inversion is established in the absence of the resonator, the factor s equals s = 2 for a three-level generation system and s = 1 for a four-level system, and the quantity $\varphi_i(z) = 1 - \Theta\cos 2[\mu k_i(z - z_0) + \varphi_i]$ takes the spatial inhomogeneity of the mode field into account. The factor Θ is unity for a standing wave if the working transition is dipole electric. If along with the dipole electric transition there is an appropriately oriented magnetic dipole radiation then $\Theta < 1$. If the probabilities of electric and magnetic dipole emissions are equal then $\Theta = 0$, i.e., the spatial inhomogeneity of the mode field plays no role in generation.

We first consider the case of $\theta = 0$ for which the spatial mode field distribution is insignificant. We consider only stationary generation for which $\dot{N}_i = 0$ and $\dot{n} = 0$ for generating modes and $\dot{N}_i < 0$ for modes not participating in generation. For the sake of simplicity we consider that energy losses occur only in reflection from the left-hand mirror. As pump intensity slowly increases population inversion increases until threshold $n_{i_1}^n$ is reached whereupon the i_1-th mode is generated for which $n_{i_1}^n = f(i_1, z_0)$, $f(i, z_0) = \gamma_i/Dag_i\eta_2^i$ has the lowest value as compared with all other axial modes. As pump intensity increases further the population inversion density $\bar{n}_{i_1}^n$ does not change, but the number of photons N_{i_1} of the i_1-th mode increases and generation is limited to this mode alone.

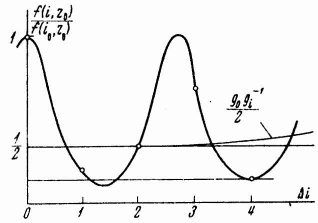

FIG. 4

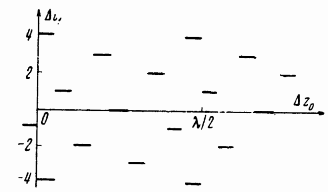

FIG. 5

FIG. 4. Dependence of $f(i, z_0)$ on $\Delta i = i - i_0$ for $\mu = 1.73$, $z_0/L_0 = 1/3(1 + 1/8)$, $\cos 2\pi(z_0/L_0)i_0 = 1$. Stationary generation occurs in a mode with a minimal value of $f(i, z_0)$.

FIG. 5. Number of generating mode Δi_1 as a function of the position of dielectric for $z_0/L_0 \approx 1/3(1 + 1/8)$.

According to Sec. 1 above the frequency spectrum of axial modes of the given resonator is similar to that of a homogeneous resonator and therefore we consider that $k_i = i\delta k$. Of course, exact values of $k_i(b)$ will change the quantitative results somewhat.

For a resonator with external mirrors the number i_1 of the generating mode significantly depends on the position of the active medium and varies strongly when the medium moves through small distances of $\sim\lambda/4$. Let the luminescence line have a Lorentz shape $g_i = g_0[1 + \beta(\Delta i)^2]^{-1}$, where $\Delta i = i - i_0$, i_0 is the mode number corresponding to the maximum of the luminescence line and $\beta = (L_0\Gamma)^{-2}$, where $\Gamma(\text{cm}^{-1})$ is the half-width of the luminescence line; then

$$f(i, z_0) = \frac{2\gamma_I(1 + \beta(\Delta i)^2]}{Dg_0a(\mu^2 + 1)} \Big/ \left[1 - \frac{\mu^2 - 1}{\mu^2 + 1}\cos 2\pi\frac{z_0}{L_0}(i_0 + \Delta i)\right]. \tag{13}$$

Figure 4 shows the dependence of $f(i, z_0)$ on Δi that represents a product of the periodic function γ_i/η_2^i with the period L_0/z_0 and the parabola g_i^{-1}. The circles denote values of $f(i, z_0)$ for various Δi and a given z_0. Figure 4 illustrates the case of $\mu = 1.73$, $z_0/L_0 = 3/8$, $\beta < 6 \times 10^{-3}$ and $f(i_0, z_0)$ with a maximum in the center of the luminescence line when $\Delta i_1 = 4$.

Figure 5 shows the variation of the number of generating modes following a displacement of the active medium for the same parameters as in Fig. 4.

We evaluate the range of variation of i_1 for a few cases.

a. Let $L_0/z_0 = q$, where $q \geq 2$ is a small integer. Then for a small change of z_0 ($\sim\lambda/2$) generation involves modes for which $|\Delta i_1| \leq q$. Small change of z_0

also leads to variation of the generation threshold n_i^n by the approximate amount

$$\frac{\mu^2-1}{\mu^2}\sin^2\frac{\pi}{2q}\cdot 100\%.$$

For q = 2 and μ = 1.76 (from now on in our computations we assume that the parameters of the active medium are close to those of ruby at T = 300°K) threshold $n_{i_1}^n$ increases by a factor of 1.5.

Optical pumping not only increases population inversion but also heats the active medium. The heat-induced change of the geometric and optical parameters of the resonator also leads to a change in the threshold $n_{i_1}^n$. Since the process of increasing population inversion by optical pumping is relatively slow, the rate of growth of n can be lower than the rate of growth of the threshold value $n_{i_1}^n$, and generation in such a case can occur only for such values of z_0 for which the threshold $n_{i_1}^n$ is close to the minimum. Increased pumping intensity increases both the rate of population inversion and the rate of threshold variation so that termination and initiation of generation occur more often. This mechanism can be associated with one of the possible causes of "spiking" in generation.

b. Let $L_0/z_0 = q$, where $q \gg 1$; then the frequency interval $\Delta\nu$ occupied by the generating modes is $\Delta\nu \sim \sqrt{\mu^2-1}\,\Gamma$ for small changes of z_0. The threshold value $n_{i_1}^n$ here changes μ^2 times in comparison to the minimum value.

c. Generation of modes that are far removed from the maximum of the luminescence line (with a small change of the threshold n_i^n by $(\pi\sqrt{\mu^2-1}/2\mu L_0\Gamma) \times 100\%$) is possible if $z_0/L_0 = (1 \pm 1/p)/q$, where $p \gg q$. For q = 2 the range of variation of the numbers of generating modes Δi_p and the spectral region $\Delta\nu$ occupied by these modes with changing z_0 are

$$|\Delta i_p| \sim \left(\frac{\pi\sqrt{\mu^2-1}}{2\mu}\right)^{1/2}\sqrt{L_0\Gamma},\quad \Delta\nu \sim \left(\frac{\pi\sqrt{\mu^2-1}}{2\mu}\right)^{1/2}\sqrt{\frac{\Gamma}{L_0}}. \tag{14}$$

When z_0 changes within the interval $\lambda/2$ stationary generation is possible for any mode within the interval $i_0 \pm |\Delta i_p|$ without a significant increase of population inversion. For L_0 = 60 cm and $\Gamma = 10\ \text{cm}^{-1}$ we obtain $\Delta i_p = 25$ and $\Delta\nu = 0.4\ \text{cm}^{-1}$. Thus a displacement of the dielectric by $\sim 0.01\lambda$ can terminate generation at one frequency and initiate it at another whose threshold is nearby.

Computation of dielectric temperature increment ΔT necessary to shift the left-hand boundary of the active medium from a node to an anti-node of the standing wave (with the right-hand boundary rigidly fixed), taking the change of the optical length of the resonator into account (for a ruby the standing wave anti-node moves toward the dielectric boundary), yields

$$\Delta T = \frac{\lambda L_0}{4az_0}\left[\left(\frac{L_0}{z_0}+\mu-1\right)d_T+\frac{d\mu}{dT}\right]^{-1}. \tag{15}$$

For $\lambda = 7\times10^{-5}$ cm, L_0 = 60 cm, a = 12 cm, z_0 = 20 cm, μ = 1.76, $\alpha_T = 0.5\times10^{-5}\ \text{deg}^{-1}$ (α_T is linear expansion coefficient), and $d\mu/dT = 1.5\times10^{-5}\ \text{deg}^{-1}$, we obtain $\Delta T \cong 0.1$°C. To change conditions at the boundary corresponding to a displacement of $\sim 0.01\lambda$ the dielectric heating amounts to $\sim 4\times10^{-3}$°C.

When considering the effect of dielectric position on the stimulated emission spectrum we can identify two relationships: first "rapid", approximately periodic, changes of i_1 (within the limits of Δi_p) resulting from an active medium displacement by $\sim\lambda/2$, and second, a "slow" change of $\Delta i_p(z_0)$ for a displacement of $\Delta z_0 \gg \lambda/2$. The larger the ratio of losses at the mirrors and gaps to losses in the active medium the stronger the effect of dielectric position on the generation frequency.

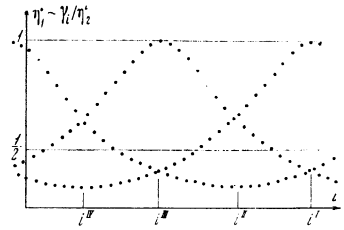

FIG. 6. Relative losses γ_1/η_2^i as a function of the mode number for $z_0/L_0 = 1/3(1 + 1/50)$.

In solid state lasers with plane external mirrors generation usually has a "spiking" character where the time sequence of "spikes" and their frequency content have an accidental random form. We can assume that it is at least partly due to the redistribution of energy caused by a change in resonator parameters during generation (change in the position of the boundaries, in optical length, etc., induced by heating). The random frequency distribution in various spikes may prove to be unreal and merely corresponding to the minima of $f[i, z_0(t)]$ whose position varies with time due to the increased temperature of the active medium, its displacement, etc.

If the luminescence line involved in generation remains approximately unchanged in shape and moves along the frequency scale (drift), a step-wise change in the generating mode frequencies can be observed when the remaining parameters of the resonator remain constant. To verify this we consider the case when

$$\frac{z_0}{L_0}=\frac{1}{q}\left(1\pm\frac{1}{p}\right),$$

where $p \gg q$. Modes with a low value of γ_i/η_2^i that happen to have a low threshold n_i^n (with a corresponding position of maximum g_i) fall into groups. The difference in mode numbers corresponding to the centers of neighboring groups is equal to p. Within each group the difference between the numbers of neighboring modes with a low threshold is equal to q.

Figure 6 shows the dependence of γ_i/η_2^i on i for the case of q = 3, p = 50, and μ = 1.76. Let i_0 decrease slowly with time. When the drift makes i_0 equal to i^{I} (see Fig. 6) generation commences in a mode that has the largest number in the group located near i^{II}; generation then shifts to modes of this group until i_0 becomes equal to i^{III} whereupon generation commences in a group near i^{IV}. This is accompanied by a generation frequency jump of $\sim p\delta k/2\pi\ \text{cm}^{-1}$. For p = 50 and L_0 = 50 cm the generation frequency jump thus is $\sim 0.5\ \text{cm}^{-1}$. Figure 7 shows the number i_1 of the generating mode as a function of position of the luminescence line.

Therefore a smooth drift of the luminescence line can lead to frequency jumps in laser generation. If the luminescence line drift is due to heating of the ac-

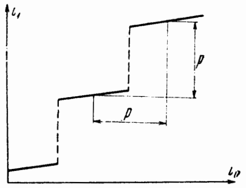

FIG. 7. Number of generating mode i_j as a function of the position of the luminescence line maximum i_0.

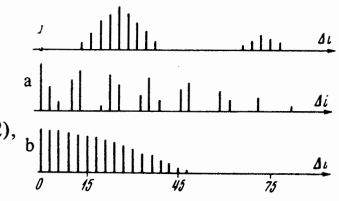

FIG. 8. Stationary generation spectra as a function of the active medium position. a–$z_0/L_0 = 1/3$ $(1 + 1/50)$, b–$z_0/L_0 = 1/3(1-1/12)$, c–$z_0/L_0 = 1/3$.

tive medium other resonator parameters are also affected at the same time; as a result the position of mode groups for which generation is possible at the given pump level is displaced along the frequency scale. This in turn may lead to variations of generating mode frequencies between the jumps in a direction opposite to that of the luminescence line drift.

If the dielectric boundaries form a small angle α with the mirrors the transverse periodicity of mode structure (due to various conditions at the dielectric boundary) has a period $h_1 = \lambda/2\alpha$. This periodicity can appear in generation if h_1 is smaller than the transverse dimensions h_2 of the inhomogeneities of the active medium. In particular this periodicity increases the diffraction losses of radiation. A diaphragm whose aperture is smaller than h_1 introduced into the resonator separates a region in which the axial energy density distribution varies little in the transverse direction. On the other hand, if the diaphragm diameter is larger than h_1, kinematic modulation, for example (see below), can no longer terminate generation at the given frequency over the entire cross section at once and the generation channel then moves across the cross section of the active medium. Diaphragm diameters capable of separating homogeneous regions (for the same α) are proportional to λ so that the diaphragm diameter is 1.5 times smaller for ruby than for Nd^{3+}.

We now turn to the consideration of generation taking the spatial inhomogeneity of the mode field $\theta \neq 0$ into account. In this case the spectral composition of stimulated emission in stationary generation is determined by the equations

$$-\gamma_i + Dag_i\eta_2{}^i\bar{n}_i \leqslant 0, \tag{16}$$

$$n = n_0\Big(1 + sD\tau_1 \sum_i g_i\Phi_i\eta_2{}^iN_i\Big)^{-1}. \tag{17}$$

The equality sign in (16) applies to modes participating in generation, while the "less than" sign denotes nongenerating modes.

Before the start of generation $\bar{n}_i$ increases with increasing pump intensity. All expressions in (16) are less than zero. Finally for the i_1 mode the losses per single photon are comparable with emission stimulated by a single photon: $Dag_{i_1}\eta_2^{i}\bar{n}_{i_1}$ this is the mode that is first to generate. We see thus that the number of the first-to-generate mode is determined by the minimum of the expression $f(i, z_0)$ that coincides with the condition for $n_{i_1}^{n}$. Consequently the frequency at which generation commences depends on the position of the active medium also in this case. In contrast to the spatially homogeneous case stationary generation can also involve other modes as pumping increases further. After the first-mode generation commences $\bar{n}_{i_1}$ no longer increases, however as pumping increases other $\bar{n}_i$ continue to grow until generation involves mode i_2 for which $\bar{n}_{i_2} = f(i_2, z_0)$. As pumping increases further the values of $\bar{n}_{i_1}$ and $\bar{n}_{i_2}$ remain constant, while the remaining $\bar{n}_i$ continue to grow until the third mode i_3 generates, etc. Neglecting the dependence of frequency on z_0 we can readily find the values of the function $f(i, z_0)$ for any value of the argument i and fixed position of the dielectric (see Fig. 4). The computation of $\bar{n}_i$ is more difficult since the integral in (12) can be obtained only approximately in analytic form. Furthermore in our case it is necessary to know φ_i ($\varphi_i \approx k_iz_0$) to compute this integral.

An earlier paper[2] presented a detailed analysis of the case of a dielectric contiguous with one of the mirrors; it was shown there that the numbers of axial modes $\{i\}$ that are successively involved in generation as pump power level increases can differ strongly from one another.

To evaluate $\{\Delta i\}$ of the numbers of modes that are successively involved in stationary generation we consider that the value of $\bar{n}_i$ is the same for all modes not involved in generation. The stationary generation then successively extends to modes whose numbers correspond to a monotonically increasing sequence of values of $f(i, z_0)$[2,6]. We compute these sequences of $\{\Delta i\}$ for a few cases.

a. Let $a = 12$ cm, $d = 38$ cm, $\mu = 1.76$, $\Gamma = 10\ \text{cm}^{-1}$, and $z_0 = 10.1$ cm. We assume that $\gamma_{III} \ll \gamma_{II} = \gamma_I$; in this case

$$f(i, z_0) = \frac{\gamma_{II}[1+\beta(\Delta i)^2]}{Dag_0}\Big[1 + 2(\mu^2+1)^{-1}\Big(1 - \frac{\mu^2-1}{\mu^2+1} \times \cos 2\pi\frac{z_0}{L_0}(i_0+\Delta i)\Big)^{-1}\Big]. \tag{18}$$

Let i_0, the number of the mode corresponding to the maximum of the luminescence line, be such that $\cos(2\pi z_0 i_0/L_0) = 1$. Omitting the complicated intermediate steps we obtain the sequence $\pm\{\Delta i\} = 25, 22, 28, 19, 31, 16, 34, 13, 37, 72, 75, 69, 10, 78, 66\ldots$ The sign $\pm$ shows that modes enter generation in pairs arranged symmetrically about i_0. The position of generating modes in the spectrum is shown in Fig. 8 a. In the figure the vertical scale was selected so as to show the earlier generating frequencies by longer line segments. As we see modes commence generation in groups whose centers are separated from one another by approximately 50 modes; furthermore the numbers of generating modes are also not spaced in a row but separated by three-mode intervals. If $\cos(2\pi z_0 i_0/L_0) \neq 1$, neither the structure nor relative position of the groups change much. However the position of the groups relative to the maximum of the luminescence line is then different; for example for $\cos(2\pi z_0 i_0/L_0) = -1$ the centers of generating mode groups are situated at $\Delta i = 0, \pm 50, \ldots$

b. Let all the parameters of the resonator be the same as in (a) except for $z_0 = 18$ cm. The sequence of modes commencing generation with increasing pumping level ($\cos(2\pi z_0 i_0/L_0) = -1$) is $\pm\{\Delta i\} = 0, 13, 23, 36, 10, 49, 3, 46, 59, 33, 72, 39, 62, 6, 20, 82\ldots$ This spectrum is illustrated in Fig. 8 b. We see that the spectrum consists of less clearly expressed line groups (as compared to (a)). The frequency distances between the group components equal triple intermode spacing. This triple period is due to the fact that the ratio of the optical length of the resonator to the gap length is close to three. In the case of (b) a $\lambda/4$ displacement of the active medium results in a situation in which the first to commence generation are modes with $\Delta i_1 = \pm 18$.

c. If all resonator parameters are the same as in (a) except for z_0 = 19.7 cm, then (for $\cos(2\pi z_0 i_0/L_0) = -1$) the sequence of modes commencing generation with increasing pump level is $\pm\{\Delta i\} = 0, 3, 6, 9\ldots$ (see Fig. 8 c). This means that generation involves consecutively every third mode beginning with the mode situated at the center of the luminescence line. A $\lambda/4$ change in z_0 merely changes the spectral generation pattern by one-two modes although this also increases the generation pump threshold.

In stationary generation the generating modes depending on z_0 can either form clearly expressed groups (Fig. 8 a) or can more or less uniformly fill a broad spectral interval provided pump power levels are high enough (Fig. 8 b and c). In any case the modes do not commence generation in a row. Clearly expressed groups of generating modes in the spatially inhomogeneous case occur for z_0 satisfying the condition $z_0/L_0 = (1 \pm 1/p)/q$; $p \gg q$.

Variation of the generation spectrum can be due not only to dielectric displacement but also to temperature drift of the refraction index or the position of luminescence line. The step-like nature of dielectric boundaries results in different spectral composition of stimulated emission of different points of resonator cross section.

In the usual practice of laser experiments if the position of active medium is measured at all it is done with a millimeter rule. Since the laser generation spectrum changes significantly when the active medium is displaced by less than $\lambda/4$, an exact theoretical computation of the generation spectrum in a resonator with plane external mirrors for the purpose of comparing it with experimental data makes no sense as long as the system parameters are not fixed with very high accuracy and their constancy in generation is not guaranteed.

The dependence of the numbers of generating modes on the position of the active medium is twofold. First a displacement over relatively large distances ($\Delta z_0 \gg \lambda/2$) changes the shape (presence or absence of groups, their spacing, etc.) and total width of the spectrum $\sim \Delta i_p$.

For a = 10 cm, L = 50 cm, and $\mu \sim 1.5$ the nature of the spectrum changes significantly when the active medium shifts by several millimeters. Second, a small displacement of $\sim\lambda/4$ of the active medium does not change the shape of the spectrum and its width Δi_p, although this does change the numbers of low-threshold modes that can participate in generation (for $\gamma_{III} \ll \gamma_{II} = \gamma_I$ the threshold value of $\bar{n}_i$ for each mode varies by a factor of $2\mu^2/(\mu^2 + 1)$ and for ruby by a factor of 1.5). This means that generation terminates in some modes and commences in others. This leads to the strong effect these small "shifts" (thermal or mechanical) have on the time behavior of generation.

In the study of generation in a resonator with plane external mirrors it is sensible to evaluate only the general aspects of the spectrum such as its width for a given pump level, the presence of clear groups, etc.

The above features of plane optical resonators are emphasized the lower the losses in the dielectric and the greater the losses at the mirrors and in the gaps. For this reason increasing the length of a resonator of finite cross section (for a fixed length of the active medium) increases the effect of dielectric boundaries on generation; a similar effect is obtained by decreasing the diaphragm diameter in the resonator.

If a spherical rather than plane standing wave is excited in the resonator, the angular distribution of stimulated emission intensity in single mode generation represents an alternation of bright and dark areas. The angular dimensions of bright areas in the case when the active medium with plane boundaries is in the center of the resonator are related by

$$\psi_n^2 - \psi_m^2 = 2\frac{\Delta}{R}(n - m). \tag{19}$$

Here ψ_n is a small angle measured from the resonator axis, R and Δ are the radius of curvature and the distance between the anti-nodes of the standing wave near the dielectric boundary, and n and m are number of bright rings reckoned from the center of the pattern.

The character of nonstationary generation also depends on the position of dielectric boundaries where the resonator with external mirrors is more "inertial" than a resonator with the same active medium but whose mirrors are mounted on the faces of the medium (the gaps act as a kind of damper).

4. REDISTRIBUTION OF ENERGY IN GENERATION

Generation of stimulated emission in an optical resonator with external mirrors is very sensitive to the position of the active medium. Furthermore, due to the variation of the optical parameters of the dielectric upon heating, the redistribution of energy can also occur with a fixed resonator geometry as in the case of a laser with liquid active medium.

Since in real lasers it is usually not possible to eliminate the uncontrollable effect of thermal variation on generation we can affect the latter by an artificially induced redistribution of energy in the resonator at a rate that is much higher than that due to heating of the active medium. Of course this can throw the generation far off the course obtained with a fixed active medium but, in return, thermal effects are no longer important.

Energy can be redistributed in the resonator in various ways: first, by mechanical means, moving either the active medium or the mirrors along the resonator axis, and second, by electro-optical means, altering the optical length of the gaps. Both methods were tried experimentally by various authors[7-9]. It was expected that this will result in single-mode generation involving the mode with the best Q. Experiments with shifting population inversion density relatively to the nodes and anti-nodes of the standing wave yielded many interesting results[8-10] although they failed to produce single-mode generation. According to the above discussion single-mode generation cannot be expected since the number of the best-Q mode changes when the dielectric moves.

We consider in greater detail the results of dielectric motion during generation. We assume that we have a quasi-stationary approximation where the mode field configuration at a given time is determined by the position of the dielectric at that time. The validity condition of the quasi-stationary approximation is $v \ll v_{cr} = \lambda c/L_0$[11,12]. If the velocity of the dielectric is comparable to or larger than v_{cr} the field configuration in the resonator depends on the previous history of the system.

During the motion of the active medium the right-hand sides of (9) and (10) are explicit functions of time as $z_0 = vt$. Since η_1^i, η_2^i, and η_3^i are periodic functions of time with the period $\tau_i = \lambda/2v$ we should expect that the solution of kinetic equations is modulated by the same period τ_i. Consequently the emission of a laser with a moving medium is modulated with a period τ_i. This phenomenon called "kinematic modulation" was recently discovered and investigated[11,12].

The solution of kinetic equations for the case of a moving medium is more difficult than for a fixed dielectric; furthermore if during the motion each mode undergoes termination and initiation of generation in time τ_i we must take into account spontaneous emission neglected in kinematic equations (9) and (10).

The results obtained for a fixed active medium can be applied also to a moving medium if the generation process is quasi-stationary, i.e., if the relaxation time of the generation process is much less than τ_i and at each point of time generation is determined by the position of active medium in the resonator at that time.

Let the generation emission be uniformly distributed throughout the volume of the active medium, i.e., $\theta = 0$. As the active medium moves in time τ_i the function $f[i, z_0(t)]$ takes on minimal values for various i and the generation successively commences and terminates in modes included in the interval $\sim \Delta i_p(z_0)$.

Spatial distribution of the mode field where $\theta \neq 0$ results in multimode generation. In the quasi-stationary case the motion of the dielectric causes a time-dependent variation of generating mode distribution in the spectrum. The most intense is the mode represented by generation in the spatially homogeneous case. Depending on pumping level other generating modes are arrayed about the most intense mode. As the dielectric moves the spectral pattern of generating mode distribution shifts in frequency. If z_0 is such that generation produces distinct mode groups, these groups shift in frequency retaining their formation. Each group, at first in the form of a single generating mode, appears far from the maximum of the luminescence line; as the active medium moves the pump shifts toward the center of the luminescence line increasing both the intensity of the group and the number of generating modes in the group. Then, leaving the center of the luminescence line the group reduces its intensity (and the number of generating modes) and finally falls out of the generation process altogether. Consequently the motion of the dielectric causes some broadening of the (time) integral spectrum of generation and a uniform mode distribution in the spectrum.

Longitudinal oscillations of the active medium change the conditions at the dielectric boundary and thus strongly affect the time behavior of generation.

If only one mirror moves the i-th mode generation can be terminated by the change in $Q(\sim f[i, z_0(t)]^{-1})$ of the mode due to the redistribution of energy in the resonator and only because of the shift in mode frequency relative to the center of the luminescence line. The motion of a single mirror produces a phenomenon analogous to kinematic modulation as well as a more dense distribution of generating modes in the (time) integral generation spectrum.

The effect of dielectric boundaries in the resonator is not so significant in generation at a spectrally inhomogeneously broadened luminescence line (Nd^{3+} in glass for example). In particular the effect of dielectric boundaries determines both the kinematic modulation during motion of the active medium with inhomogeneously broadened luminescence line[11] and the fact that in a fixed active medium modes can commence generation in groups; these may be large in number if the luminescence line is broad. In the moving active medium the integral generation spectrum is much more densely (contiguously) filled with generating modes due to the shifting groups of generating modes.

Thus the dependence of energy distribution on the position of dielectric boundaries and the configuration of the electromagnetic field of natural oscillations, for a given spatial distribution of losses, leads to an additional thinning out (selection) of the frequency spectrum of high-Q modes in an open optical resonator. The frequency spectrum of high-Q modes of the resonator turns out to be very sensitive to external perturbations (thermal, mechanical, and optical) so that small (and therefore uncontrollable) perturbations of the system cause a sharp variation of the Q of each mode and a significant variation of the frequency spectrum of modes with the highest Q.

We note that the presence of internal dielectric boundaries in the active medium can be the result of longitudinal inhomogeneities of the optical pumping system.

In the generation of stimulated emission the position of the active medium in the resonator proves to be a very significant parameter of the laser; both the spectral and the time behavior of generation are quite critical relative to this hard-to-control parameter.

Many features of laser operation of various designs cannot be understood without an analysis of electromagnetic energy density distribution in the resonator. Therefore in the study of lasers we must take into account the effect of the position of the active medium on generation; there is no justification for neglecting this effect.

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