THRESHOLD SINGULARITY IN ONE-DIMENSIONAL PROBLEM AND POLARON EFFECT IN MAGNETOOPTICAL ABSORPTION

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Ordinary perturbation theory for the electron mass operator (in the case of weak electron-phonon coupling) leads to a divergence of the one-dimensional problem near the singularity connected with the singularity of the state density. It is shown how a series in powers of the coupling constant, which converges at arbitrarily close distances from the singularity can be set up for the mass operator. The electron spectrum for the lowest Landau level (in an extremely strong magnetic field) interacting with the optical phonons is calculated by this method. The singularity in the interband optical absorption coefficient, due to the threshold nature of the optical phonon emission by an electron moving parallel to the magnetic field, is also calculated.

INTRODUCTION

MUCH attention has recently been paid to "polaron" singularities of interband magnetooptical absorption, which take place when the cyclotron frequency of the electron $\omega_c = eH/mc$ coincides with the limiting frequency of the optical phonons ω_0[1-3]. In a note by two of us[4] it was indicated that, in distinction from this effect, which is connected with the motion of the electron perpendicular to H, there can exist singularities of interband magnetooptical absorption connected with the motion of the electron parallel to H. They are due to the threshold character of the emission of optical phonons, which is possible only when $p^2/2m > \omega_0$. The role of the magnetic field in this case reduces to the creation of a one-dimensional situation, a fact that greatly intensifies the threshold singularity. Such a polaron effect, according to[4], leads to a dip in the absorption coefficient at the frequency $\omega^* = \Delta E + \omega_0$, where ΔE is the width of the forbidden band with allowance for the shift of the bottom of the conduction band and the top of the valence band in the magnetic field.

However, the calculation of the absorption coefficient $K(\omega)$ in[4], which was based on ordinary perturbation theory for the mass operator, cannot be regarded as satisfactory, for it is seen from this calculation that the perturbation-theory series ceases to converge near the singularity, when $|\omega - \omega^*| \lesssim \alpha\omega_0$, where $\alpha \ll 1$ is the constant of the weak electron-phonon coupling. Such a situation is characteristic of the one-dimensional problem and is connected with the singularity in the state density. This difficulty has arisen many times in a number of problems[2,5].

In the present paper we develop a method which makes it possible to obtain for the mass operator an expansion in powers of α, valid at an arbitrarily close distance to the singularity. This method is a development of the theory of threshold singularities, in which perturbation theory is not used[6]. The developed method is used to calculate (in the lowest order in α) the spectrum of the electron and the absorption coefficient. It turns out here that the absorption coefficient differs little from that calculated by perturbation theory below the threshold ($\omega < \omega^*$), but differs greatly in the region above the threshold ($\omega > \omega^*$).

1. FORMULATION OF PROBLEM

In the calculation of the interband absorption it will be assumed that the crystal temperature is T = 0, the hole band is completely filled, and the electron band is empty. Since the investigated effect is connected with a singularity of the spectrum in the electron band, it can be assumed that the holes do not interact with the phonons and that their mass is infinitely large. Under these assumptions, the absorption coefficient, as shown in[2], is determined completely by the electronic Green's function and is proportional to the density of states in the electron band.

For simplicity, the dispersion of the free electron is assumed to be parabolic, so that its Green's function

$$G_l^0(\varepsilon p_z) = \left\{\varepsilon - \left[\omega_c\left(l+\frac{1}{2}\right) + \frac{p_z^2}{2m}\right] + i\eta\right\}^{-1}. \qquad (1.1)$$

It is assumed that the phonons have no dispersion, so that their Green's function is

$$D(\omega q) = (2\pi)^3 B(q)\left[\frac{1}{\omega - \omega_0 + i\eta} - \frac{1}{\omega + \omega_0 - i\eta}\right], \qquad (1.2)$$

where B(q) is the square of the matrix element of the electron-phonon interaction. Since the electron concentration is equal to zero, D is not altered by the electron-phonon interaction.

We shall henceforth consider the case of strong magnetic fields $H \to \infty$, when we certainly have $\omega_c \gg \omega_0$. It is clear that in this case the singularities of the absorption at $\omega = \Delta E + \omega_0$ are connected only with the Landau level $l = 0$. Also connected with the longitudinal motion at this level are the singularities of the absorption at $\omega = \Delta E + 2\omega_0$, $\Delta E + 3\omega_0$, ..., corresponding to thresholds with emission of two, three, and more phonons. We shall not consider these singularities here, however.

To calculate the corresponding Green's function of the electron $G_l(\epsilon p_z)$, it is convenient to use a diagram technique in the form proposed in[7], where there are no diagrams with loops, since the electron concentration is equal to zero. Since we are interested in $G_0(\epsilon p_z)$, the free ends of the diagrams correspond to $l = 0$. Further, according to[7], the nodes correspond to the matrix elements $\Lambda_{ll'}$, from the expressions for which we see that $\Lambda_{ll'} \to \delta_{ll'}$ as $H \to \infty$. Therefore the summation over l drops out and all the included Green's functions correspond to $l = 0$. The phase factor, which enters in accordance with[7], also vanishes when $H \to \infty$. Since $\Lambda_{ll'}$ ceases to depend on the transverse components of the phonon momenta in the asymptotic approximation considered here, the integration with respect to them couples only the phonon Green's function, and it is natural to put

$$\int \frac{dq_\perp}{(2\pi)^2} D(\omega \mathbf{q}) = D(\omega q_z). \tag{1.3}$$

We see thus that the problem becomes one-dimensional.

Let us now find the criterion for one-dimensionality of the problem (the inequality $\omega_c \gg \omega_0$ will be shown to be insufficient). To this end we note that the elimination of the summation over l in the calculation of $G_0(\epsilon p_z)$ is based on discarding Λ_{0l} with $l \neq 0$ compared with Λ_{00}. Therefore a criterion is obtained from a comparison of these quantities at the actual $q_\perp$, which is attained by averaging them with the aid of $B(\mathbf{q})$. It should be borne in mind here that the actual momentum q_z is of the order of the threshold longitudinal momentum of the electron $p_0 = \sqrt{2m\omega_0}$. Using the expressions for Λ_{ll}, we obtain the criterion

$$\int_0^\infty dq_\perp q_\perp (aq_\perp)^l B(\mathbf{q}) e^{-1/4 a^2 q_\perp^2} \ll \int_0^\infty dq_\perp q_\perp B(\mathbf{q}) e^{-1/4 a^2 q_\perp^2}, \tag{1.4}$$

where $a = (c/|e|H)^{1/2}$ is the magnetic length. The explicit form of the criterion depends on the form of $B(\mathbf{q})$, i.e., on the electron-phonon interaction mechanism. For the polarization (long-range) interaction PO it is customarily assumed that

$$B(\mathbf{q}) = B_0/q^2, \tag{1.5}$$

while the deformation (short-range) interaction DO

$$B(\mathbf{q}) = B_0 = \text{const}. \tag{1.6}$$

This form is actually valid only for long-wave phonons, and therefore when necessary we shall introduce a cutoff momentum $q_0 \gg p_0$ and assume that

$$B(\mathbf{q}) = 0 \quad \text{for} \quad q > q_0. \tag{1.7}$$

Let us return to the criterion (1.4), in which both integrals are functions of the parameter $aq_z \approx ap_0$. Assuming that $\omega_c \gg \omega_0$, we have $ap_0 \ll 1$. In the case of PO the integral on the right diverges logarithmically at $aq_z = 0$, whereas the integral on the left (with $l \neq 0$) converges. Therefore the criterion of one-dimensionality takes the following form:

$$|\ln ap_0| \approx \ln|\omega_c/\omega_0| \gg 1 \quad \text{(PO)}. \tag{1.8}$$

It is obvious that this is a much more stringent criterion than $\omega_c \gg \omega_0$. In the case of DO (with allowance for cutoff) the integrals converge and we can put in them $aq_z = 0$. Then the one-dimensionality criterion takes the form

$$aq_0 \ll 1 \quad \text{for} \quad \omega_c \gg q_0^2/2m \quad \text{(DO)}, \tag{1.9}$$

which is also more stringent than $\omega_c \gg \omega_0$.

The phonon Green's function involved in the one-dimensional problem can be represented in the form

$$D(\omega q_z) = B(q_z)\left[\frac{1}{\omega - \omega_0 + i\eta} - \frac{1}{\omega + \omega_0 - i\eta}\right], \tag{1.10}$$

where

$$B(q_z) = 2\pi \int (dq_\perp) B(\mathbf{q}). \tag{1.11}$$

We have

$$B(q_z) = 2\pi^2 B_0 \ln(q_0/q_z)^2 \quad \text{(PO)}, \tag{1.12}$$

$$B(q_z) = 2\pi^2 B_0 (q_0^2 - q_z^2) \quad \text{(DO)}. \tag{1.13}$$

We see therefore that when $q_z \approx p_0$ the function $B(q_z)$ is practically independent of q_z. We shall therefore assume henceforth in the consideration of the one-dimensional problem that

$$B(q_z) = B = \text{const}. \tag{1.14}$$

In going over to the one-dimensional problem, we shall drop the subscript z of p_z and q_z and the subscript $l = 0$ of G_l; we shall also reckon the electron energy from the bottom of the Landau band $l = 0$, i.e., we make the substitution $\epsilon - \frac{1}{2}\omega_c \to \epsilon$. It is also convenient to carry out beforehand the integration with respect to the frequency parameters of the phonon lines in the diagrams for G. To this end we note that

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} iD(\omega) F(\omega) = BF(\omega_0 - i\eta), \tag{1.15}$$

if $F(\omega)$ is analytic when $\text{Im}\ \omega < 0$. In the considered case of zero concentration, the Green's functions are retarded. The energy parameters of the Green's functions G^0 in the diagrams for $G(\epsilon p)$ have the form $\epsilon - \omega_1 - \omega_2 - \ldots$. Since $G^0(\epsilon p)$ is analytic when $\text{Im}\ \epsilon > 0$, the product of all the G^0 containing ω_1 is analytic when $\text{Im}\ \omega_1 < 0$ and it is possible to integrate with respect to ω_1 with the aid of (1.15). This is followed by the appearance of G^0 with parameters $\epsilon + i\eta - \omega_0 - \omega_2 - \ldots$. The product of all the G^0 containing ω_2 is again analytic when $\text{Im}\ \omega_2 < 0$, etc. We can therefore finally assume that the internal phonon line corresponds only to the factor B and $\int_{-\infty}^{+\infty} dq/2\pi$.

2. CALCULATION OF THE MASS OPERATOR

The simplest approximation for the mass operator is given by the loop of Fig. 1, the contribution from which is

$$M_1(\varepsilon p) = -i\alpha\omega_0 \frac{p_0}{s(\varepsilon)}, \qquad s^2(\varepsilon) = 2m(\varepsilon - \omega_0) + i\eta, \quad \text{Im}\, s(\varepsilon) > 0, \quad \alpha = 2Bm^2/p_0^3. \tag{2.1}$$

We have introduced here the dimensionless weak-coupling constant α. We see that M_1 is small only far from the threshold $\epsilon = \omega_0$. This suggests that perturbation theory does not hold near the threshold. This can be verified by estimating the contributions from the diagrams of higher order, shown in Fig. 2:

$$M_2^1 \approx \alpha^2\omega_0^{3/2}|\varepsilon - \omega_0|^{-3/2}, \quad M_2^2 \approx \alpha^2\omega_0^2|\varepsilon - \omega_0|^{-1}. \tag{2.2}$$

From a comparison of these quantities with M_1 we see that perturbation theory ceases to work at distances on

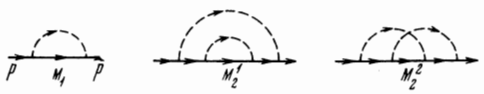


FIG. 1 FIG. 2

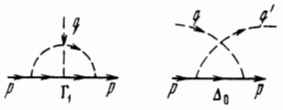


FIG. 3

the order of $\alpha\omega_0$ from the threshold, where a diagram without intersection of the phonon lines becomes significant. At even closer distances on the order of $\alpha^2\omega_0$, a diagram with intersection of phonon lines becomes significant.

Since the usual expansion of M in terms of α is impossible near the threshold, it is natural to use the procedure of[6] for the consideration of threshold singularities. The only condition for the applicability of these methods is proximity to the threshold, i.e., the region of applicability is $|\epsilon - \omega_0| \ll \omega_0$. Since this region overlaps the region of applicability of perturbation theory, it should be possible to join these methods together.

To obtain an idea of the character of the threshold, let us calculate the contributions from the simplest diagrams for the vertices Γ and Δ shown in Fig. 3. We have

$$\Gamma_1(\varepsilon p, \varepsilon' p'; \omega q) = i\alpha p_0^3\left[\frac{1}{s(\varepsilon)}+\frac{1}{s(\varepsilon')}\right]\cdot\frac{1}{q^2-[s(\varepsilon)+s(\varepsilon')]^2}, \quad (2.3)$$

$$\Delta_0(\varepsilon p, \varepsilon' p'; \omega q, \omega' q') = [(\varepsilon' - \omega) - (p'-q)^2/2m + i\eta]^{-1}. \quad (2.4)$$

It will be seen from (2.5) and (2.6) below that near the threshold an important role is played by the behavior of the vertex Γ when ϵ and ϵ' approach ω_0, and by that of the vertex Δ when ϵ and ϵ' approach zero. From expressions (2.3) and (2.4) we see that M_1 and Γ_1 become infinite near the threshold, whereas Δ_0 has no singularities. It can be verified that an analogous property is possessed also by the contributions from the diagrams of higher orders. From this point of view, the singularity in question is close to case "c" of[6].

In analogy with this case, we start from the exact equations

$$M(\varepsilon p) = B\int_{-\infty}^{+\infty}\frac{dq}{2\pi}G(\varepsilon-\omega_0, p-q)\Gamma(\varepsilon-\omega_0 p-q, \varepsilon p; \omega_0 q), \quad (2.5)$$

$$\Gamma(\varepsilon-\omega p-q, \varepsilon p; \omega q) = 1+\int_{-\infty}^{\infty}\frac{dq'}{2\pi}G(\varepsilon-\omega_0 p-q')$$
$$\times\Gamma(\varepsilon-\omega_0 p-q', \varepsilon p; \omega_0 q')\Delta(\varepsilon-\omega p-q, \varepsilon'-\omega_0 p-q'; \omega q, \omega_0 q'). \quad (2.6)$$

In the second equation we put $\omega = \omega_0$, make the substitutions $q = p - k$ and $q' = p - k'$, and put

$$\Gamma(\varepsilon-\omega_0 k, \varepsilon p; \omega_0 p-k) = \Gamma(\varepsilon p, k), \quad (2.7)$$

$$\Delta(\varepsilon-\omega_0 k, \varepsilon-\omega_0 k'; \omega_0 p-k, \omega_0 p-k') = \Delta(\varepsilon p, kk'). \quad (2.8)$$

Then the equations take the form

$$M(\varepsilon p) = B\int_{-\infty}^{\infty}\frac{dk}{2\pi}G(\varepsilon-\omega_0 k)\Gamma(\varepsilon p, k), \quad (2.9)$$

$$\Gamma(\varepsilon p, k) = 1+B\int_{-\infty}^{\infty}\frac{dk'}{2\pi}G(\varepsilon-\omega_0 k')\Gamma(\varepsilon p, k')\Delta(\varepsilon p, kk'). \quad (2.10)$$

We shall seek a solution of these equations near the threshold, where perturbation theory does not hold, i.e., at $|\epsilon - \omega_0| \lesssim \alpha\omega_0$. The energy parameter of the Green's function that enters in these equations is $\epsilon - \omega_0 \approx 0$, i.e., it is far from the threshold. The perturbation theory works in this region and makes it possible to calculate G^{-1} by means of an ordinary expansion of the mass operator in powers of α. It will be convenient in what follows to write

$$G(\varepsilon-\omega_0 k)^{-1} = E(\varepsilon)-\frac{k^2}{2m}-K(\varepsilon, k)+i\eta, \quad (2.11)$$

where

$$E(\varepsilon) - \omega_0 - M(\varepsilon-\omega_0, 0) \quad (2.12)$$

$$K(\varepsilon, k) = M(\varepsilon-\omega_0, k) - M(\varepsilon-\omega_0, 0). \quad (2.13)$$

The quantity $E(\epsilon)$ determines the proximity to the true (renormalized) threshold. This is seen from the following considerations. The equation $E(\epsilon) = 0$ obviously gives $\epsilon - \omega_0 = \tilde{\epsilon}(0)$, where $\tilde{\epsilon}(p)$ is the true (renormalized) spectrum, i.e., $\epsilon = \tilde{\epsilon}(0) + \omega_0 \equiv \tilde{\omega}_0$. The quantity $\tilde{\omega}_0$ which lies on ω_0 above the true bottom of the spectrum, indeed determines the true threshold of emission of a phonon of energy ω_0.

Let us consider now the singularities of the integrands in (2.9) and (2.10) in the complex k plane. The factor G has poles at k determined from the equation

$$G(\varepsilon-\omega_0, k)^{-1} = 0. \quad (2.14)$$

Since $G(\epsilon p)$ is even in p, the roots of these equations can be written in the form

$$k = \pm k_0(\varepsilon), \ \mathrm{Im}\, k_0(\varepsilon) > 0. \quad (2.15)$$

It is obvious further from (2.11) that $k_0(\epsilon) \to 0$ as $E(\epsilon) \to 0$, i.e., as $\epsilon \to \tilde{\omega}_0$. The order of magnitude of k_0 can be estimated by noting that $E(\epsilon) \approx \alpha\omega_0$ and discarding K, which yields $k_0 \approx \alpha^{1/2}p_0$. Indeed, since the expansion of K begins with k^2 terms, it follows that $K \approx \alpha\omega_0(k/p_0)^2 \approx \alpha^2\omega_0$, which is much smaller than the retained terms. We note that if K is retained, then G also has singularities in the region of large k, such that $k_0 \to \infty$ as $\alpha \to 0$. These singularities, however, are of no interest in what follows.

We now proceed to the singularities of the factors Γ and Δ. It is useful to note that near the threshold it follows from (2.3) and (2.4) that

$$\Gamma_1(\varepsilon p, k) = i\alpha p_0^3/s(\varepsilon)[(p-k)^2+p_0^2], \quad (2.16)$$

$$\Delta_0(\varepsilon p, kk') = -2m/[(p-k-k')^2+p_0^2]. \quad (2.17)$$

We see that the singularities of Δ_0 and Γ_1 in the k plane lie at distances on the order of p_0 from the real axis. This is valid also for the exact Δ, since it can be expanded in a series in α. For the exact Γ this can be assumed and confirmed by the solution.

We now deform the integration contour in Eqs. (2.9) and (2.10) in the manner shown in Fig. 4, and obtain

$$\int_{-\infty}^{+\infty}\ldots = \oint_C\ldots+\int_\Omega\ldots, \quad (2.18)$$

with the closed contour C including only the singularity $k_0(\epsilon)$ in the region of small k, and the contour Ω passing below all the singularities of Δ and Γ in the upper half-

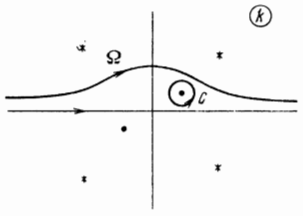


FIG. 4. Deformation of the contour in the equations for the mass operator and the vertex; ●–singularities of G, *–singularities of Γ and Δ.

plane. Then Eqs. (2.9) and (2.10) assume the following form:

$$M(\varepsilon p)=\Gamma(\varepsilon p, k_0(\varepsilon))\tilde{M}(\varepsilon)+B\int_\Omega \frac{dk}{2\pi}G(\varepsilon-\omega_0,k)\Gamma(\varepsilon p,k) \tag{2.19}$$

$$\Gamma(\varepsilon p,k)=1+\Gamma(\varepsilon p,k_0(\varepsilon))\Delta(\varepsilon p,kk_0(\varepsilon))\tilde{M}(\varepsilon) + B\int_\Omega \frac{dk}{2\pi}G(\varepsilon-\omega_0,k')\Gamma(\varepsilon p,k')\Delta(\varepsilon p,kk') \tag{2.20}$$

where

$$\tilde{M}(\varepsilon)=B\oint_C \frac{dk}{2\pi}G(\varepsilon-\omega_0,k). \tag{2.21}$$

In this system of equations, the functions $k_0(\epsilon)$ and $\tilde{M}(\epsilon)$ can be regarded as known, since they are determined by the function $G(\epsilon-\omega_0, k)$, which can be calculated by perturbation theory. On the other hand, as will be shown below, the integrals along Ω (unlike the integrals along the real k axis) can be regarded as small in terms of α and we can iterate in terms of these integrals. This is connected with the fact that since Ω leaves aside the region of small k, the smallness of $\epsilon-\omega_0$ does not lead to large values of $G(\epsilon-\omega_0, k)$.

Let us consider the lowest approximation, discarding the integrals along Ω. In the non-integral terms we can put in this approximation $k_0(\epsilon)=0$ and $\Delta=\Delta_0$. In addition, we can put $\epsilon=\omega_0$ in Δ_0. We then have the equations

$$M(\varepsilon p)=\Gamma(\varepsilon p,\,0)\tilde{M}(\varepsilon), \tag{2.22}$$

$$\Gamma(\varepsilon p,\,k)=1+\Gamma(\varepsilon p,\,0)\Delta_0(\omega_0 p,\,k0)\tilde{M}(\varepsilon). \tag{2.23}$$

Substituting in (2.23) k = 0, we get

$$\Gamma(\varepsilon p,\,0)=[1-\Delta_0(p)\tilde{M}(\varepsilon)]^{-1} \tag{2.24}$$

and then

$$\Gamma(\varepsilon p,\,k)=1+\Delta_0(\omega_0 p,\,k0)[\tilde{M}(\varepsilon)^{-1}-\Delta_0(p)]^{-1}, \tag{2.25}$$

$$M(\varepsilon p)=[\tilde{M}(\varepsilon)^{-1}-\Delta_0(p)]^{-1}, \tag{2.26}$$

where

$$\Delta_0(p)=\Delta_0(\omega_0 p,\,00)=-(\omega_0+p^2/2m-i\eta)^{-1}. \tag{2.27}$$

To calculate $\tilde{M}(\epsilon)$ in the lowest order in α we can discard K, after which we get

$$k_0(\varepsilon)^2=2mE(\varepsilon)+i\eta, \tag{2.28}$$

$$\tilde{M}(\varepsilon)=-i\alpha\omega_0 p_0/k_0(\varepsilon). \tag{2.29}$$

In turn, in the lowest order in α we have

$$E(\varepsilon)=\varepsilon-\omega_0-M_1(\omega_0)=\varepsilon-\bar{\omega}_0,\quad \bar{\omega}_0=\omega_0-\alpha\omega_0. \tag{2.30}$$

A comparison of $\tilde{M}^{-1}$ and Δ_0 determines two regions of closeness to the renormalized threshold:

I $\alpha^2\omega_0 \ll |\varepsilon-\bar{\omega}_0| \lesssim \alpha\omega_0$, where $\alpha^{1/2}\omega_0 \lesssim \tilde{M}(\varepsilon) \ll \omega_0$;

II $|\varepsilon-\bar{\omega}_0| \lesssim \alpha^2\omega_0$, where $\omega_0 \lesssim \tilde{M}(\varepsilon)$.

In region I we have from (2.24)–(2.26)

$$\Gamma(\varepsilon p,\,0)=\Gamma(\,p,\,k)=1, \tag{2.31}$$

$$M(\varepsilon p)=\tilde{M}(\varepsilon), \tag{2.32}$$

i.e., in this region the result for M differs from perturbation theory only in a renormalization of the threshold. It is obvious that the result for Γ corresponds to the fact that in the iteration we regard as small not only the integral term but also the second, nonintegral term. Let us estimate now the individual terms in (2.19) and (2.20), noting that in the estimates

$$G\approx 2m/p_0^2,\quad \int dk\approx p_0,\quad \Delta_0\approx 1/\omega_0. \tag{2.33}$$

Then it turns out that

	Discarded terms	Retained terms
Equation for Γ:	$\Gamma\Delta\tilde{M}\approx\frac{\tilde{M}}{\omega_0}$, $B\int dk G\Gamma\Delta\approx\alpha$	1
Equation for M:	$B\int dk\,G\Gamma\approx\alpha\omega_0$	$\Gamma\tilde{M}\approx\tilde{M}$

A comparison of the retained and discarded terms justifies the iterations made in region I.

Let us proceed to region II. Here there are essential differences between the estimates for Γ at k = 0 and $k\neq 0$; from (2.24) and (2.25) we have

$$\Gamma(\varepsilon p,\,0)\approx\omega_0/\tilde{M}(\varepsilon),\quad \Gamma(\varepsilon p,\,k)\approx 1. \tag{2.34}$$

Estimating again the individual terms of Eqs. (2.19) and (2.20), we find

	Discarded terms	Retained terms
Equation for Γ:	$B\int dk\,G\Gamma(k)\Delta\approx\alpha$	1, $\Gamma(0)\Delta\tilde{M}\approx 1$
Equation for M:	$B\int dk\,G\Gamma(k)\approx\alpha\omega_0$	$\Gamma(0)\tilde{M}\approx\omega_0$

from which we see the justification of the iteration in region II. The system of equations (2.19) and (2.20) can be used for repeated iterations and to obtain results in higher orders in α.

The result (2.26) is not valid in its derivation far from the threshold, when $|\epsilon-\omega_0| \gg \alpha\omega_0$. In this region, however, as can be seen from (2.30), we have $\tilde{E}(\epsilon)=\epsilon-\omega_0$; it follows therefore from (2.29) that $\tilde{M}(\epsilon)=M_1(\epsilon)$, with $M_1(\epsilon)\ll\omega_0$. Therefore (2.26) yields $M(\epsilon p)=M_1(\epsilon)$, i.e., a correct result of perturbation theory, which is valid in this region. As a result, formula (2.26) turns out to be valid in the lower order in the entire energy region.

3. SPECTRUM OF ELECTRON

In the lowest order in α, the spectrum of the electron is determined from the equation

$$\varepsilon-p^2/2m-M(\varepsilon p)=0 \tag{3.1}$$

with M in accordance with (2.26). If we are interested in the spectrum in the complex ϵ plane, then $\tilde{M}(\epsilon)$ should be regarded as specified, in accordance with (2.29), in the ϵ plane with a cut $\mathrm{Re}\,\epsilon\geq\tilde{\omega}_0$. In order to simplify the equation, it is convenient to replace in $\Delta_0(p)$ of (2.27) the quantity ω_0 by $\tilde{\omega}_0$; the error thereby incurred in Δ_0 is of the order of $\alpha\omega_0^{-1}$, i.e., of the same order as the terms discarded in the considered approximation. If we change over to dimensionless variables

$$\varepsilon/\bar{\omega}_0=z,\quad p^2/2m\bar{\omega}_0=t\geq 0, \tag{3.2}$$

then the equation takes the form

$$i\alpha^{-1}(z-1)^{1/2}=1/(z-t)-1/(1+t),\ \mathrm{Im}\,(z-1)^{1/2}>0, \tag{3.3}$$

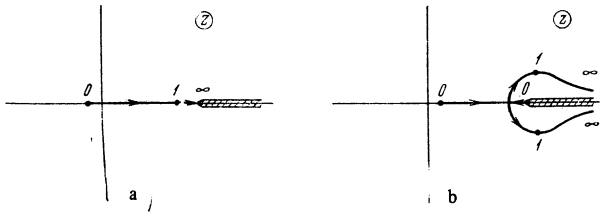


FIG. 5. Trajectories of the roots in the complex plane: a–physical sheet, b–unphysical sheet. The arrows indicate the motion of the roots z(t) when t is varied. The positions of the roots at t = 0, 1, and ∞ are indicated.

FIG. 6. Dispersion law of the electron.

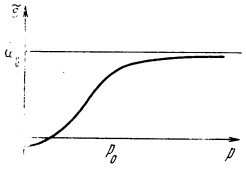


Table I. Roots of spectral equation

	Physical sheet	Unphysical sheet
$t<1$	$z=t-\frac{\alpha}{\sqrt{1-t}}$	$z=t+\frac{\alpha}{\sqrt{1-t}}$ $z=1-\alpha^2\frac{4t^2}{(1-t^2)^2}$
$t=1$	$z=1-\alpha^{2/3}$	$z=1+\alpha^{2/3}e^{\pm i\pi/3}$
$t>1$	$z=1-\alpha^2\frac{4t^2}{(1-t^2)^2}$	$z=t+\alpha^2\left[\frac{1}{2(t-1)^2}+\frac{1}{t^2-1}\right]\pm$ $\pm i\alpha\frac{1}{\sqrt{t-1}}$

or after squaring and multiplying

$$(z-1)(z-t)^2=-\alpha^2\left(1-\frac{z-t}{1+t}\right)^2. \tag{3.4}$$

Equation (3.4) has roots that are not roots of (3.3) or, in other words, lie on the unphysical sheet of the ϵ plane. Using the smallness of α, we can find the roots of (3.4) and pick up those lying on the physical sheet with the aid of the condition

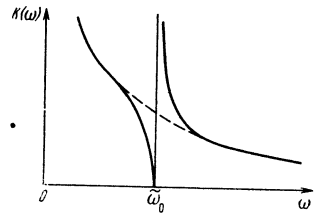
$$\mathrm{Re}\left\{\frac{1}{z-t}-\frac{1}{1+t}\right\}<0. \tag{3.5}$$

The results are summarized in the table. The formulas for $t<1$ and $t>1$ in this table are not valid in the vicinity of $t=1$, since a coalescence of the roots takes place there. Expressions for z(t) in this region can easily be obtained, but they are too cumbersome to present here. The tables suffice to convey an idea of the trajectories z(t) on both sheets of the z plane (Fig. 5).

The true spectrum $\tilde{\epsilon}(p)$, as seen from the table, exists when $z<1$, i.e., below the threshold, $\epsilon<\tilde{\omega}_0$, as should be the case. The dispersion law of $\tilde{\epsilon}(p)$ is shown in Fig. 6. We see that in this case there exists no end point of the spectrum. We note that this in no way contradicts the result of[6], where the existence of an end point was postulated and the behavior of the spectrum near this point was investigated. The very question of the existence of such a point depends on whether the equation

$$[\varepsilon-p^2/2m-M(\varepsilon p)]_{\varepsilon=\tilde{\omega}_0}=0 \tag{3.6}$$

FIG. 7. Dependence of the absorption coefficient on the frequency.



has a solution for finite p. Using (2.26) and noting that $\tilde{M}(\tilde{\omega}_0)^{-1}=0$, we have, accurate to terms of order $\alpha\omega_0$,

$$M(\tilde{\omega}_0 p)=-1/\Delta_0(p)=\omega_0+p^2/2m. \tag{3.7}$$

With the same accuracy $\tilde{\omega}_0=\omega_0$, and therefore Eq. (3.6) has only the solution p = 0, which cannot be regarded as an end point of the spectrum.

4. ABSORPTION COEFFICIENT

The electromagnetic properties of a system of electrons are determined by the polarization loop[8]

$$\Pi(\omega)=g^2\int_{-\infty}^{\infty}dp\left(\omega-\frac{p^2}{2m}-M(\omega p)+i\eta\right)^{-1}, \tag{4.1}$$

where ω is the frequency of the light (reckoned from the absorption edge) and g^2 is the electron-phonon coupling constant and contains various parameters of the crystal (the interband matrix element of the transition, the width of the forbidden band, and others). Usually the observed absorption coefficient is

$$K(\omega)=-\mathrm{Im}\,\Pi(\omega). \tag{4.2}$$

Calculating (4.1) by the residue theorem, we have

$$\Pi(\omega)=g^2\,2\pi i\sum R(p(\omega)), \tag{4.3}$$

where $p(\omega)$ is the position of the pole, $R(p(\omega))$ is the residue, and the summation is carried out over poles for which Im $p(\omega)>0$, or for Im $p(\omega)=0$, Re $p(\omega)>0$.

Again using for brevity the dimensionless variables

$$(\omega-\tilde{\omega}_0)/\omega_0=w,\quad p^2/2m\tilde{\omega}_0=u^2, \tag{4.4}$$

we obtain the positions of the poles

$$u^2=\tfrac{1}{2}[w+2i\alpha/w^{1/2}]\pm\{\tfrac{1}{4}[w+2i\alpha/w^{1/2}]^2+[w+1-i\alpha w^{1/2}]\}^{1/2} \tag{4.5}$$

and the corresponding residues

$$R^{-1}=-\frac{p_0}{m}u\left[1+\frac{(w+1-u^2)^2}{(1+u^2)^2}\right]. \tag{4.6}$$

Far from the singularity ($\alpha\ll|w|\lesssim 1$), obviously, $\Pi(\omega)$ differs little from the corresponding value for the free electron:

$$\Pi(\omega)=\Pi^0(\omega)=|\Pi^0(\omega_0)|(-i)(1+w)^{-1/2},\qquad \Pi^0(\omega_0)=-2\pi g^2 im/p_0. \tag{4.7}$$

We therefore consider the region near the singularity ($|w|\lesssim\alpha$). It is obvious that in region I ($|w|\gg\alpha^2$), when we can use perturbation theory with a renormalized threshold, Eq. (4.7) is valid just as before. It remains to consider region II ($|w|\lesssim\alpha^2$). In this region, it follows from (4.5) that

$$u^2=2\alpha/\sqrt{|w|},\ -\sqrt{|w|}/2\alpha,\ w<0; \tag{4.8}$$

and from (4.6) $u^2=2\alpha i/\sqrt{w},\ i\sqrt{w}/2\alpha,\ w>0;$

$$R=-m/p_0u. \tag{4.9}$$

Gathering the required poles, we ultimately obtain

$$\Pi(\omega)=|\Pi^0(\omega_0)|\{-(2\alpha)^{1/2}/|w|^{1/4}-i|w|^{1/4}/(2\alpha)^{1/2}\},\ w<0,$$
$$\Pi(\omega)=|\Pi^0(\omega_0)|\left\{\frac{1-i}{\sqrt{2}}\frac{(2\alpha)^{1/2}}{(w)^{1/4}}\right\},\quad w>0. \tag{4.10}$$

It is seen from the last formulas that below the threshold the absorption coefficient $K(\omega)$ vanishes, and above the threshold it becomes infinite (Fig. 7). Thus, the behavior above the threshold differs greatly from that obtained with the aid of perturbation theory, when K also vanished above the threshold.

The conditions for experimentally observing the effect, with n-InSb as an example, were discussed in[4]. One must add to this discussion that in realistically attainable magnetic fields it is very difficult to satisfy the criterion of one-dimensionality (1.8) with a strong inequality. One should hope that the influence of other Landau bands does not distort the effect very strongly. These hopes can pertain to a lesser degree to the exciton effects which were not accounted for in this paper, since the widths of the singularity regions $\alpha^2\omega_0$ do not exceed the energy of the Coulomb interaction of the electron and the hole. However, the latter effects can be excluded if one considers not the interband absorption, but the absorption from charged donors.

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