RADIATIVE FREQUENCY SHIFT OF RADIATION IN A RESONANT MEDIUM

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The radiative (Lamb) frequency shift of radiation of an atomic system in a resonant medium is investigated. It is shown that the resultant corrections are quite important for standards with frequency reproducibility 10^{13} or higher.

THE energy lost by a dipole per unit time in a dispersive medium[1]

$$E = {}^2/{}_3 d^2 / \varepsilon' v^3 \tag{1}$$

differs by a factor $\sqrt{\epsilon'}$ from the energy lost by the same dipole in vacuum. Near the absorption band, the dielectric constant $\epsilon(\omega) = \epsilon'(\omega) - i\epsilon''(\omega)$ changes sharply, and this leads, by virtue of (1), to a resonant dependence of the damping constant on the frequency, and as a consequence to a change in the value of the Lamb shift. The resultant corrections of the order of $\epsilon''\gamma_{mn}$ (γ_{mn} is the Einstein coefficient of the transition) are small ($\epsilon'' \ll 1$) and do not play a noticeable role in linear spectroscopy. The situation changes radically in connection with the problem of developing length standards and superstabilized lasers with frequency reproducibility 10^{13} and higher. In this case the radiation frequency can be "tied in" with the frequency of the corresponding atomic resonance, in principle, with much higher accuracy than the aforementioned corrections. In such a situation they simply must be known. The purpose of the present note is to call attention of researchers to this phenomenon and to its role in the operation of the laser.

We expand the field in the medium in Fourier integral

$$\mathbf{E}(\mathbf{x}, t) = \int d\omega \int d\mathbf{k} \{\mathbf{E}^+(\mathbf{k}, \omega) e^{-i(\omega t - \mathbf{kx})} + \mathbf{E}^-(\mathbf{k}, \omega) e^{i(\omega t - \mathbf{kx})}\} \tag{2}$$

and assume the following value for the average, over the ground state, of the commutator[2]:

$$\langle [E_i^+(\mathbf{k}, \omega) E_j^-(\mathbf{k}', \omega')] \rangle = \frac{\hbar}{(2\pi)^2} \frac{2}{\varepsilon''} \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \tag{3}$$
$$\times \delta(\mathbf{k} - \mathbf{k}') \delta(\omega - \omega') \frac{1}{\pi} \frac{1}{(k - \omega c^{-1} \sqrt{\varepsilon'})^2 / \alpha^2(k) + 1},$$

where $\alpha(k) = k\epsilon''/2\epsilon'$.

The radiative corrections are determined by solving the Schrödinger equation for the atom in the field (2):

$$i\hbar \frac{\partial \Phi}{\partial t} = \{H_a - \mathbf{DE}\} \Phi, \tag{4}$$

where H_a is the atomic Hamiltonian. The width and shift of the level n in first approximation are determined by the functions[3]

$$\Sigma_n(t' - t'') = -2\pi i \langle n \,|$$  $$|\, n \rangle \tag{5}$$

The solid line corresponds to the propagation function of the atomic excitations, and the wavy line to the propagation function of the photon in the medium.

Using the standard procedure[3], we can obtain for the Fourier transform of the function (5)

$$\Sigma_n(t - t') = \int_{-\infty}^{+\infty} \Sigma_n(\omega) e^{-i\omega(t-t')} d\omega$$

the expression

$$\Sigma_n(\tilde{\omega}) = -2\pi i \left(\frac{i}{\hbar}\right)^2 \sum_{m<n} \int d\mathbf{k} D_i^{nm} D_j^{mn} \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \tag{6}$$
$$\times \int d\omega \frac{\hbar}{(2\pi)^2} \frac{k}{\varepsilon'} \frac{1}{\pi} \frac{\tilde{\gamma}}{(k - \omega c^{-1} \sqrt{\varepsilon'})^2 + \tilde{\gamma}^2} \left(-\frac{1}{2\pi i}\right) \frac{1}{\tilde{\omega} - \omega_m - \omega + i\delta'}.$$

Here D^{nm} is the matrix element of the dipole moment of the atom between the states m, n, $\gamma = k\epsilon''/2\epsilon'$. The summation in (6) is over all the states m lying below the level n; δ' fixes the rule of going around the pole. Integrating (6) with respect to **k** and representing $\Sigma_n(\omega)$ in the form

$$\Sigma_n(\tilde{\omega}) = \Delta_n(\tilde{\omega}) + {}^1/{}_2 i\gamma_n(\tilde{\omega}) + \delta\Sigma_n(\tilde{\omega}), \tag{7}$$

where Δ_n and γ_n are respectively the shift and width of the level n in vacuum, we obtain the following expression for the correction connected with the deviation of ϵ' from unity:

$$\delta\Sigma_n(\tilde{\omega}) = \sum_{m<n} \frac{\gamma_{nm}}{4\pi} \int \frac{\varepsilon'(\omega) - 1}{\tilde{\omega} - \omega_m - \omega + i\delta'} d\omega. \tag{8}$$

Formula (8) can be made more concrete only within the framework of a definite model. For example, in classical dispersion theory[1]

$$\varepsilon(\omega) - 1 = 4\pi N e^2 m^{-1} (\omega_0^2 - \omega^2 + i\omega\gamma_0)^{-1},$$

where N is the number of oscillators per unit volume, ω_0 is the frequency, and γ_0 is the damping of the oscillator. Assuming that the frequency ω_0 is close to the frequency of the transition ω_{nm} of the atomic system, we find

$$\delta\Sigma_n(\omega) = {}^1/{}_4 i\gamma_{nm} [\varepsilon(\tilde{\omega} - \omega_m) - 1].$$

Thus, the shift of the center of the radiation line turns out to be

$$\delta\Delta = {}^1/{}_4 \gamma_{nm} \varepsilon''(\omega_{nm}). \tag{9}$$

From formula (9) we see that the position of the atomic resonance depends on the number N of the oscillators per unit volume, and thus on the operating regime of the laser. If γ_0 is set equal in order of magnitude to the Doppler width of the radiation line ($\gamma_0 \sim k\bar{v}$), $\omega \approx 10^{15}$ sec^{-1}, then $\delta\Delta \approx (\gamma_{mn}/k\bar{v}) N \times 10^{-6}$ sec^{-1}. Assuming for N the value[4] 10^9 cm^{-3}, we obtain $\delta\Delta \approx 10$–100 sec^{-1}. This indicates that the considered phenomenon turns out to be quite important in systems with frequency reproducibility $10^{13}-10^{14}$.

[1] D. V. Sivukhin, Lektsii po fizicheskoĭ optike (Lectures on Physical Optics), Part I, NGU, 1968.
[2] A. A. Abrikosov, A. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].
[3] S. S. Schweber, Introduction to Relativistic Quantum Theory, Harper, 1961.
[4] L. Allen and D. Jones, Fundamentals of Gas-laser Physics (Russ. transl.), Nauka, 1970.

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