STRIPPING OF ELECTRONS FROM NEGATIVE IONS IN A CONSTANT MAGNETIC FIELD

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The differential probability is obtained for the stripping of an electron from a negative ion passing through a magnetic field. In addition, the influence is considered of the change of the statistical weight of the final state of the electron on electron detachment due to some forces in a strong magnetic field.

1. INTRODUCTION

IN view of improvements in the technique of obtaining strong magnetic fields, it is of interest to investigate different processes in such fields. So far, experimental studies were made of the stripping of electrons from the ions H^-[1,2] and D^-[3] on passing through a magnetic field. The most accurate results, obtained in[2], agree with the theory[4,5] that takes into account only the action of the electric field on the detached electron, if the unknown parameter of the theory is taken to be approximately three times larger than the value corresponding to a potential with zero radius of action of the forces. This conclusion agrees with the information given in[2] and[3] with respect to the theoretical results of Hiske and Vogt (although the author did not succeed in establishing whether the papers of the latter were published). The numerical results for the probability of extracting an electron from H^- by an electric field are given in[6].

In Sec. 2 of the present paper we consider the stripping of electrons with account taken of the specific features of the constant magnetic field, thereby obtaining a more complete description of the phenomenon. As expected, for a nonrelativistic system of the H^- type, the total probability of separating the electron is determined by the formula for the tunnel extraction of the electron by an electric field[4,5], if the ion velocity is much larger than the velocity of the bound electron.

In Sec. 3 there is discussed a unique kinematic effect of intensification of the stripping of electrons by a strong magnetic field when $eH \gg p_{\text{eff}}^2$, where p_{eff} is the effective momentum of the outgoing electrons. This is in essence an extrapolation to atomic systems of the following result by Schwinger: the presence of a strong magnetic field collinear with the electric field increases (decreases) the probability of production of spinor (scalar) pairs by the electric field[7].

2. STRIPPING OF ELECTRON FROM A NEGATIVE ION

A. Matrix element and probability. The process of separating an electron from a negative ion in an electromagnetic field is conveniently regarded as the transformation of elementary particles[5]. This means that the decay is described by a Feynman diagram with three ends and the entire influence of the short-range forces of the coupling of the external electron reduces to an "interaction constant" at the vertex.

Such an approach is justified if the distances effective in the decay are large compared with the atomic distances. Then the formula for the probability contains an exponentially small factor. If in addition we are not interested in spin effects, then we can regard all the quantities as scalar. Thus, we arrive at the matrix element

$$\mathfrak{M}_{p'p} = f\int \Psi_{p'}^*(x)e^{-ikx}\Psi_p(x)\,d^4x\,[2k_0L^3]^{-1/2}. \tag{1}$$

Here p, p′, and k are respectively the 4-momenta of the ion, electron, and atom, f is the interaction constant, the order of magnitude of which is given by[5]

$$f^2/32\pi m^2 \sim \sqrt{(M+m'-m)/2m'}, \tag{2}$$

$p^2 = -m^2$, $p'^2 = -m'^2$, $k^2 = -M^2$, L^3 is the normalization volume; $\Psi_p(x)$ is a solution of the Klein-Gordon equation for a charged particle in a field.

Choosing the vector potential of the magnetic field directed along the x_3 axis in the form

$$A_1 = A_3 = A_0 = 0, \quad A_2 = Hx_1, \tag{3}$$

we obtain

$$\begin{gathered}\Psi_p(x) = [2p_0L^2]^{-1/2}C_l\exp\{i[p_2x_2+p_3x_3-p_0x_0]\}\psi_l(\eta),\\ C_l = \frac{(eH)^{1/4}}{(\sqrt{\pi}\,l!\,2^l)^{1/2}} \quad \eta = \sqrt{eH}\left(x_1 - \frac{p_2}{eH}\right),\\ p_0 = \sqrt{m^2+p_\perp^2+p_3^2}, \quad p_\perp^2 = eH(2l+1), \quad l = 0, 1, 2, \ldots,\end{gathered} \tag{4}$$

$\psi_l(\eta) = [\exp(-\eta^{l/2})]H_l(\eta)$ is the Hermite function.

Carrying out the integrations in (1), we obtain

$$\begin{aligned}\mathfrak{M}_{p'p} = f[8p_0'p_0k_0L^3]^{-1/2}M(l',l)(2\pi)^3\delta(p_2-p_2'-k_2)\delta(p_3-p_3'-k_3)\\ \times\delta(p_0-p_0'-k_0),\end{aligned} \tag{5}$$

where $M(l', l)$ can be written in accordance with[8] in the form

$$\begin{gathered}M(l',l) = C_{l'}C_l\int_{-\infty}^{\infty}dx_1\psi_{l'}(\eta')\psi_l(\eta)e^{-ik_1x_1}\\ = \frac{(-1)^{l'}}{\sqrt{l'!\,l!}}\exp\left\{-\frac{ik_1(p_2+p_2')}{2eH} + i\varphi(l-l') - \frac{\zeta}{2}\right\}\\ \times\zeta^{(l+l')/2}\,{}_2F_0(-l', -l; -1/\zeta),\\ \zeta = \frac{(p_2'-p_2)^2+k_1^2}{2eH}, \quad p_2'-p_2-ik_1 = \sqrt{2eH\zeta}e^{i\varphi}\\ \eta' = \sqrt{eH}(x_1 - p_2'/eH).\end{gathered} \tag{6}$$

Starting from the integral representation for $M(l', l)$ and using the completeness of the system of functions $\psi_l(\eta)$, we can easily obtain the following relation:

$$\sum_{l=0}^{\infty}|M(l',l)|^2 = 1. \tag{7}$$

Further, without loss of generality it can be assumed that in our system $p_3 = 0$. (Obviously, the probability can be expressed in terms of invariant quantities.) Then we obtain in the usual manner the differential probability of ion disintegration per unit time:

$$dW = \frac{f^2}{8p_0}|M(l', l)|^2 \frac{|\mathbf{k}|}{p_0' + k_0\cos^2\vartheta}\frac{d\cos\vartheta}{2\pi}. \qquad (8)$$

The total probability of disintegration per unit time is obtained by integrating (8) with respect to the angle ϑ between the vectors **k** and **H** and by summing over l' from 0 to l'_{max}, determined from the energy conservation law

$$p_0 = M + [m'^2 + eH(2l'_{max} + 1)]^{1/2}. \qquad (9)$$

B. Approximation of weak field and high velocities. For $l' \sim 1$ the probability W is obtained directly from (8) and (6), since ${}_2F_0(-l', -l; -1/\zeta)$ consists of a small number of terms. For $l' \gg 1$, we can find an approximate expression for $M(l', l)$. In this case

$$M + m' - m = I, \quad I > 0 \qquad (10)$$

and, as will be shown below, $n = l - l' > 0$. Then

$$\frac{(l-l')!}{l!}(-\zeta)^{l'}{}_2F_0\left(-l', -l; -\frac{1}{\zeta}\right) = \Phi(-l', l - l' + 1; \zeta), \quad (11)$$

where $\Phi(\alpha, \beta; \zeta)$ is the confluent hypergeometric function, with

$$\Phi(-l', n+1; \zeta) = \frac{1}{2\pi i}\frac{\Gamma(1+l')\Gamma(1+n)}{\Gamma(1+l)}\oint\frac{dt}{t}e^{f(t)},$$
$$f(t) = t\zeta - l'\ln(-t) + l\ln(1-t). \qquad (12)$$

The asymptotic form of $\Phi(-l', n+1; \zeta)$ for $n, l', \zeta \gg 1$ was investigated in[8] (see also[9]). The results are used here also to the extent that they are needed for the present problem.

The saddle points of the function f(t) are determined from the condition

$$f'(t_0) = 0, \quad t_0 = 1/2 - n/2\zeta \pm \delta, \quad n = l - l',$$
$$\delta = \left[\left(\frac{1}{2} - \frac{n}{2\zeta}\right)^2 - \frac{l'}{\zeta}\right]^{1/2} = \left[\left(\frac{1}{2} + \frac{n}{2\zeta}\right)^2 - \frac{l}{\zeta}\right]^{1/2}, \qquad (13)$$
$$\delta^2 = (\zeta - \zeta_1)(\zeta - \zeta_2)/4\zeta^2, \quad \zeta_{1,2} = (\sqrt{l} \mp \sqrt{l'})^2.$$

The conservation laws

$$p_3' + k_3 = p_3 = 0, \quad p_0' + k_0 = p_0 \qquad (14)$$

guarantee that $\zeta < \zeta_1$, i.e., $\delta^2 > 0$ if $I > 0$.

Indeed, for the maximum value of $k_\perp^2$ for a specified $p'_\perp$ we obtain

$$k_{\perp\,max}^2 = p_\perp^2 + p_\perp'^2 - 2[(m^2 + p_\perp^2)(m'^2 + p_\perp'^2)]^{1/2} + m^2 + m'^2 - M^2, \qquad (15)$$

and since

$$(p_\perp^2 + m^2)(p_\perp'^2 + m'^2) - (p_\perp p_\perp' + mm')^2 = (mp_\perp' - m'p_\perp)^2 > 0, \quad (16)$$

it follows that

$$k_{\perp\,max}^2 < (p_\perp - p_\perp')^2, \quad \zeta < \zeta_1. \qquad (17)$$

Analogously, from (14) it is readily seen that

$$p_\perp^2 - p_\perp'^2 - k_\perp^2 = m'^2 + M^2 - m^2 + 2p_0'k_0 > (m' + M)^2 - m^2 > 0.$$

Taking (10) into account, this yields

$$n = l - l' > \zeta \quad (\zeta > 0). \qquad (18)$$

By the same method as in[8,9] we can verify that for the process in question the integration contour in formula (12) should be drawn through the point

$$t_1 = \frac{1}{2} - \frac{n}{2\zeta} + \delta = -\sqrt{\frac{l'}{\zeta} + \delta^2} + \delta \qquad (19)$$

and

$$f''(t_1) = 2\zeta\delta\left[\sqrt{\frac{l'}{\zeta} + \delta^2} - \delta\right]^{-1}\left[\sqrt{\frac{l}{\zeta} + \delta^2} - \delta\right]^{-1},$$
$$f'''(t_1) = -\frac{2l'}{t_1^3} - \frac{2l}{(1-t_1)^3}. \qquad (20)$$

For our purposes it suffices to confine ourselves to the simple saddle-point method, without taking into account the term with $f'''(t_1)$. We thus obtain

$$M(l', l) = \left[\frac{l'}{l}\right]^{1/4}(-t_1)^{-1}[2\pi f''(t_1)]^{-1/2}\exp\left\{in\varphi - \frac{ik_1(p_2 + p_2')}{2eH} + Q\right\},$$
$$Q = \zeta\delta + l'\operatorname{Arsh}\delta\sqrt{\zeta/l'} - l\operatorname{Arsh}\delta\sqrt{\zeta/l}, \quad l, l' \gg 1. \qquad (21)$$

The condition for the applicability of formula (21) is satisfaction of the inequality $|f'''(t_1)(t - t_1)^3_{\rm eff}| \ll 1$, i.e.,

$$|f'''(t_1)f''^{-3/2}(t_1)| \ll 1. \qquad (22)$$

The approximation (21) solves our problem. It is still quite complicated, however, and it is difficult to use it to integrate formula (8). Fortunately, at sufficiently high ion velocities the effective values of l' and ζ are such that

$$\delta\sqrt{\zeta/l'} \ll 1. \qquad (23)$$

Then formula (21) becomes much simpler. Indeed, from (13) we have

$$\delta_{min}^2 = \frac{(\zeta_1 - \zeta_{max})(\zeta_2 - \zeta_{max})}{4\zeta_{max}^2}, \qquad (24)$$

and from (15), (16), and (10) we get

$$\zeta_{max} = \frac{k_{\perp max}^2}{2eH} = (\sqrt{l} - \sqrt{l'})^2(1 - \varepsilon),$$
$$\varepsilon \approx \frac{I(2M - I)}{(p_\perp - p_\perp')^2} + \frac{(mp_\perp' - m'p_\perp)^2}{(p_\perp - p_\perp')^2(p_\perp p_\perp' + mm')}. \qquad (25)$$

Here ϵ is obtained by expanding with respect to the parameter

$$\left(\frac{p_\perp m' - p_\perp' m}{p_\perp p_\perp' + mm'}\right)^2 \ll 1. \qquad (26)$$

The value ζ_{max} corresponds to spreading of the particles in a plane perpendicular to the magnetic field. We now have for δ^2_{min}

$$\delta_{min}^2 \approx \frac{\varepsilon(\sqrt{l} - \sqrt{l'})^2[4\sqrt{ll'} + \varepsilon(\sqrt{l} - \sqrt{l'})^2]}{4\zeta_{max}^2}. \qquad (27)$$

We use further the fact that

$$I \ll m' \ll m. \qquad (28)$$

In addition, from physical considerations one should expect the field to have little effect on the particle velocity in the effective region of the values of l' and ζ, i.e.,

$$\left(\frac{v'}{v}\right)_{\rm eff} \approx 1, \quad \left(\frac{p_\perp'}{p_\perp}\right)_{\rm eff} \approx \left(\sqrt{\frac{l'}{l}}\right)_{\rm eff} \approx \frac{m'}{m}. \qquad (29)$$

Let us assume for the time being (until (36) is obtained) that

$$\varepsilon \approx 2MI / p_\perp^2. \tag{30}$$

From the obtained expressions it follows that

$$\delta_{min}^2 \approx \varepsilon[m'/m + \varepsilon/4]. \tag{31}$$

Thus, to satisfy condition (23) it is necessary to have in any case

$$\frac{\zeta}{l'}\delta_{min}^2 \sim \frac{\zeta_{max}m^2\varepsilon}{lm'^2}\left(\frac{m'}{m}+\frac{\varepsilon}{4}\right) \approx \frac{\varepsilon m}{m'}\left(1+\frac{m}{m'}\varepsilon\right) \approx \frac{\varepsilon m}{m'} \ll 1, \tag{32}$$

or, taking (30) into account, $m^2 I/p_\perp^2 m' = \frac{1}{2}(m\eta/p_\perp m')^2 \ll 1$, i.e.,

$$v^2 \gg \frac{I}{m'} = \frac{\eta^2}{2m'^2} = \frac{1}{2}v_{\text{at.}}^2 \tag{33}$$

This condition will be strengthened somewhat below (formula (35)), in order to have a compact expression. We note for the present that the condition (33) means that in the rest system of the ion the action of the electric field on the bound electron is much more important than the action of the magnetic field.

Taking the conditions (23), (28), and (33) into account, we obtain in place of (21)

$$|M(l', l)|^2 \approx \frac{p_\perp}{4\pi\eta l}e^{2Q}, \tag{34}$$

$$Q = -\frac{2^{3/2}}{3}\frac{(ll')^{1/4}}{(\sqrt{l}-\sqrt{l'})^{1/2}}(\sqrt{l}-\sqrt{l'}-\sqrt{\zeta})^{3/2}\left[1+\frac{3}{40}\frac{\zeta\delta^2}{l'}+O\left(\frac{\zeta^2\delta^4}{l'^2}\right)\right].$$

This expression is valid in the effective region of values of l' and ζ if

$$F\left(\frac{m\eta}{p_\perp m'}\right)^4 \ll 1, \quad F = \frac{2}{3}\frac{B_0 m}{Hp_\perp} \quad (F \gg 1). \tag{35}$$

Here $B_0 = \eta^3/em'$ is the characteristic atomic field. Using expression (34), it is easy to integrate formula (8) with respect to ϑ and l':

$$W = \frac{f^2 p_\perp^2}{64\pi^2 p_0 p_0' \eta l}\exp\left\{-F\left[1+\frac{1}{30}\left(\frac{m\eta}{p_\perp m'}\right)^2\right]\right\}$$

$$\times \int_0^{l'_{max}} \exp\left\{-\frac{3eHF}{\eta^2}\frac{(\sqrt{l'}-\sqrt{l_0'})^2}{1+p_\perp^2/m^2}\right\}dl' \int_{-1}^{1}\exp\left\{-\frac{3}{2}\frac{p_\perp^2}{\eta^2}Ft^2\right\}dt$$

$$\approx \frac{f^2}{32\pi m^2}\frac{m\eta}{p_0}\frac{2}{3F}\exp\left\{-F\left[1+\frac{1}{30}\left(\frac{m\eta}{p_\perp m'}\right)^2\right]\right\}$$

$$\approx C'\frac{mI}{p_0}\frac{2}{3F}\exp\left\{-F\left[1+\frac{1}{30}\left(\frac{m\eta}{p_\perp m'}\right)^2\right]\right\},$$

$$t = \cos\vartheta, \quad l_0' = \frac{m'^2}{m^2}l, \quad C' \sim 1. \tag{36}$$

The integrand in (36) gives the differential probability in the effective region of values of l' and ϑ. Details of the calculations that lead to (34) and (36) are given in[10]. Agreement with the experimental data of[2] will occur at $C' \approx 3$. The quantity f^2 (or C') can be regarded as a free parameter. On the other hand, as noted in[5],

$$f^2 / 32\pi m^2 = 2\pi N^2, \tag{37}$$

where N is a coefficient in the asymptotic wave function of the external electron in the ion

$$\psi(r)|_{r\to\infty} = N\frac{e^{-\eta r}}{r}, \quad 2m'\int|\psi(r)|^2 d^3x = 1. \tag{38}$$

In the limiting case of a potential with zero radius of action, the first equation in (38) is valid for any r. Then the second equation gives $N^2 = \eta/4\pi m'$, i.e., $C' = 1$.

3. KINEMATIC EFFECT OF INTENSIFICATION OF IONIZATION OF THE ATOMS BY A STRONG MAGNETIC FIELD

Let us consider by way of an example the ionization of a hydrogen atom by a constant electric field E. The probability of extracting the electron from the s state with principal quantum number n is equal to[11] 1)

$$W_n = C_n \int_0^\infty dp_\perp^2 \exp\left\{-\frac{2}{3}\frac{B_n}{E}\left(1+\frac{p_\perp^2}{\eta_n^2}\right)^{3/2}\right\} \approx$$

$$\approx C_n \exp\left\{-\frac{2}{3}\frac{B_n}{E}\right\}\int_0^\infty dp_\perp^2 \exp\left\{-\frac{B_n}{E}\frac{p_\perp^2}{\eta_n^2}\right\} = C_n\exp\left\{-\frac{2}{3}\frac{B_n}{E}\right\}\frac{E\eta_n^2}{B_n},$$

$$C_n = \frac{1}{4[n!]^2 m'}\left[4\frac{B_n}{E}\right]^{2n}, \quad B_n = \frac{\eta_n^3}{em'}, \quad \eta_n^2 = 2m'I_n. \tag{39}$$

Assume now the presence also of a strong magnetic field parallel to the electric field. The question is: how does the probability W_n change? The shift of the levels of the atom in the strong field can be readily included in the change of η_n. It is more difficult to take other changes of the matrix element into account. It can be assumed, however, that they are not too large. In the model of the type considered above, where no account is taken of the long-range character of the Coulomb forces, it is easy to show, by using (7), that the matrix element remains the same, but, of course, $p_\perp^2$ must be taken to mean $eH(2l + 1)$. A similar situation arises in the case of pair production by a constant field[7,13]. If this is true also for the Coulomb forces, then we obtain in lieu of (39) for the probability of emission of an electron in the spin state r

$$W_{n,r} = C_n\exp\left\{-\frac{2}{3}\frac{B_n}{E}\right\}eH\sum_{l=0}^{\infty}\exp\left\{-\frac{B_n}{E}\frac{2eH(l+r-1)}{\eta_n^2}\right\} =$$

$$= C_n\exp\left\{-\frac{2}{3}\frac{B_n}{E}-\frac{B_n}{E}\frac{2eH(r-1)}{\eta_n^2}\right\}eH\left[1-\exp\left\{-\frac{B_n}{E}\frac{2eH}{\eta_n^2}\right\}\right]^{-1}, \tag{40}$$

r = 1 or 2. We see therefore that when $B_n eH/E\eta_n^2 \ll 1$ the dependence of the total probability on η_n does not change when the magnetic field is turned on. When $B_n eH/e\eta_n^2 \gg 1$, only the term with r = 1 and $l = 0$ makes a decisive contribution and the emerging electron is strongly polarized.

Thus, when $eH \gg p_{\perp\,\text{eff}}^2$, assuming that the matrix element is independent of the magnetic field, the ionization probability changes both because of the change of $\eta_n = \eta_n(H)$ and as a result of the change of the statistical weight in proportion to eH. It does not matter here which of the external fields has caused the ionization. This may be in the field of the photon or of the incoming electron. Obviously, the foregoing considerations are even better applicable to negative ions, for in these the external electron is in the field of short-range forces. We note also that for a scalar charged particle $p_\perp^2 = eH(2l + 1)$ increases with increasing H even if $l = 0$. Therefore the law of energy conservation can forbid in general the decay with emission of such a particle, if the magnetic field is sufficiently strong.

1)The total probability W_n, i.e., the right-hand side of (39), was obtained earlier in [12]; it is not important for us to know the distribution with respect to $p_\perp^2$, i.e., the integrand in (39).

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