# INDUCED SCATTERING OF WAVES AND PLASMA HEATING BY COHERENT RADIATION

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A method is proposed for investigating the nonlinear interaction of quasimonochromatic waves in a plasma. It is used to analyze the character of the induced scattering of coherent high-frequency radiation during the kinetic and hydrodynamic stages. The possibility of using the scattering processes for collisionless heating of a plasma is considered.

NONLINEAR wave interactions in a plasma have been investigated by various methods in a large number of studies. The results of these studies, however, cannot always be used to describe the effects produced by propagation of powerful quasimonochromatic waves with fixed oscillation phase in the plasma. The present article is devoted to problems involved in the theory of interaction of quasimonochromatic waves in a plasma.

The character of the processes of nonlinear transformation of quasimonochromatic waves in a plasma can be clearly illustrated by using the concept of the averaged high-frequency force acting on a charged particle in an alternating electromagnetic field.[1] It is known[1] that in a field of two traveling electromagnetic waves with close frequencies

$$\mathbf{E} = \mathbf{E}_1 e^{i(\omega_1 t - \mathbf{k}_1\mathbf{r})} + \mathbf{E}_2 e^{i(\omega_2 t - \mathbf{k}_2\mathbf{r})}, \quad \omega_1 - \omega_2 \ll \omega_1, \omega_2 \tag{1}$$

the force averaged over the periods of the partial oscillations, acting on a single charged particle, is potential

$$\mathbf{F} = -\nabla\Phi \tag{2}$$

with a traveling high-frequency potential relief

$$\Phi = \frac{e^2(\mathbf{E}_1\mathbf{E}_2^*)}{2m\omega_1\omega_2} e^{i(\Omega t - \varkappa r)}, \quad \Omega = \omega_1 - \omega_2, \quad \varkappa = \mathbf{k}_1 - \mathbf{k}_2, \tag{3}$$

e and m are the charge and mass of the charged particle.

This force produces in the plasma stimulated longitudinal (in the $\varkappa$ direction) wave motions. If $\Omega$ and $\varkappa$ satisfy the dispersion equation of the natural oscillations of the plasma, i.e., if the condition of synchronism is satisfied between the driving force and one of the natural waves of the plasma, then resonant excitation of the plasma oscillations occurs at the difference frequency (the processes of wave decay and coalescence). If the decay conditions are not satisfied, then the main process leading to the transformation of the spectra of the waves in the plasma is the interaction of the averaged force with the resonant particles whose velocity is close to the phase velocity of the low-frequency potential:

$$\omega_1 - \omega_2 = (\mathbf{k}_1 - \mathbf{k}_2, \mathbf{v}). \tag{4}$$

Whereas the decay interaction of waves with fixed phase can be investigated on the basis of the quasihydrodynamic approximation, to describe the processes of nonlinear Landau damping in the general case it is necessary to use the kinetic equations.

In the usual procedure of solving the kinetic equations, the distribution function of the charged particles of a weakly turbulent plasma is represented in the form of a series[3–5] $f = f_0 + f_1 + f_2 + f_3 + \ldots$, in which $f_0$ is the unperturbed distribution function, $f_1$ is a perturbation linear in the field and determining the tensor of the linear dielectric constant of the plasma, $f_2$ is an increment quadratic in the field, responsible for processes of the decay type and contributing to the induced scattering, and $f_3$ is a term with cubic nonlinearity, responsible for the effects of self-action and induced scattering of the waves. To describe the induced scattering of waves with random phase by particles, extensive use is also made of the method of trial particles.[6–8] Introduction of the concept of the averaged high-frequency force acting on a charged particle makes it possible to develop a more compact method that is particularly convenient for the investigation of the interaction of quasimonochromatic waves.

We use this method in the present paper to consider the problem of induced scattering in a plasma of an intense monochromatic wave. Depending on the pump-field intensity, there exist two stages of scattering: kinetic and hydrodynamic.[1)] In the case of small amplitudes, the kinetic stage is realized, during which the field interacts with a small group of resonant plasma particles. In this case, if the plasma is dense enough, predominant heating of the ions takes place. During the hydrodynamic stage, which sets in at pump-wave amplitudes larger than a certain threshold value, all the plasma particles take part in the scattering, and the result is an effective increase of their energy, and at large high-frequency field amplitudes it is the electrons that are predominantly heated. This circumstance makes it possible, in principle, to regulate the heating of the different components of the plasma by changing the amplitude of the electromagnetic wave incident on the plasma.

We determine in this paper the energy dissipated by the coherent electromagnetic field in the plasma, and calculate the nonlinear scattering increments. On the basis of the analogy with the theory of instability of oscillations of a plasma through which a beam of charged particles penetrates, we carry out a qualitative analysis

[1)]Certain aspects of the hydrodynamic stage of induced scattering were considered earlier in [9].

of nonlinear effects of saturation of wave scattering. Estimates are presented, illustrating the possibility of using induced scattering processes to heat a dense plasma by coherent radiation.

## 1. FUNDAMENTAL EQUATIONS

1. In the investigation of the behavior of a plasma in a weakly inhomogeneous high-frequency field, we start from a system of kinetic equations for the distribution functions of the electrons and ions of a collisionless plasma

$$\frac{\partial f_\alpha}{\partial t}+(\mathbf{v}\nabla)f+e_\alpha\left\{\mathbf{E}+\frac{1}{c}[\mathbf{vB}]\right\}\frac{\partial f_\alpha}{\partial \mathbf{p}}=0, \qquad (5)^*$$

where $\alpha$ is the species of the particles, with e and i standing for electrons and ions. Following [1], we represent the motion of charged particles in electromagnetic fields as a superposition of oscillator motion $\rho_\alpha$ and averaged drift motion $R_\alpha$:

$$\mathbf{r}^\alpha=\mathbf{R}^\alpha+\boldsymbol{\rho}^\alpha, \quad \mathbf{v}^\alpha=\mathbf{V}^\alpha+\dot{\boldsymbol{\rho}}^\alpha. \qquad (6)$$

Such a representation is valid if the criteria of weak inhomogeneity of the field are satisfied

$$|\rho|/L_E\ll 1, \quad |\dot{\rho}|/c\ll 1, \qquad (7)$$

where $L_E = |E/\nabla E|$ is the characteristic dimension of the field inhomogeneity (in a traveling wave, $L_E \sim 1/k$).

We perform in the kinetic equation (5) the change of variables (6), which is equivalent to a transition to an oscillating system of coordinates. As a result we obtain the equations [10]

$$\frac{\partial f_\alpha}{\partial t}+V_j\frac{\partial f_\alpha}{\partial R_j}+\frac{1}{m}\langle F_j^\alpha\rangle\frac{\partial f_\alpha}{\partial V_j}=0, \qquad (8)$$

in which

$$\langle F_j^\alpha\rangle=F_j^\alpha-m\ddot{\rho}_j=e_\alpha\tilde{E}_j+\frac{e}{c}[V_\alpha\tilde{B}]_j+(\rho_\alpha\nabla)F_{\sim j}^\alpha+(\rho_\alpha\nabla_\alpha)F_{\sim j}^\alpha \qquad (9)$$

is the total force averaged over the period of the high-frequency oscillation; $\mathbf{F}_\sim$ is the Lorentz force acting along the unperturbed (in the absence of a high-frequency field) trajectory $r_0(t)$; $\rho(t)$ and $\dot{\rho}(t)$ represent a solution of the equation of the oscillating motion (in first approximation)

$$\ddot{\rho}=\mathbf{F}_\sim(r_0(t),t), \qquad (10)$$

$\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are low-frequency or static ($\tilde{\mathbf{B}} = 0$) fields arising in the plasma under the influence of the averaged high-frequency force.

2. We use the kinetic equations (8) to investigate the processes of induced scattering of electromagnetic waves in an isotropic plasma. In the case of the high-frequency field (1), Eq. (8) can be represented in the form

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{V}\frac{\partial f_\alpha}{\partial \mathbf{R}}+e_\alpha\left\{\tilde{\mathbf{E}}-\frac{1}{e_\alpha}\nabla\Phi_\alpha\right\}\frac{\partial f_\alpha}{\partial \mathbf{p}}=0, \qquad (11)$$

where the high-frequency potential is determined by expression (3). We shall solve the system (11) under the assumption that the perturbations are small, i.e., representing $f_\alpha$ in the form

$$f_\alpha=f_{\alpha 0}+f_{\alpha 2}, \qquad (12)$$

where $f_{\alpha 2}$ is the perturbation produced by the averaged force.

In addition, we change over to the Lagrangian coordinates $\mathbf{R}_0$, $\mathbf{V}_0$, and t, and determine the connection between them by means of the law of unperturbed motion

$$\mathbf{R}(t)=\mathbf{R}_0(t)+\int_0^t\mathbf{V}_0(t')\,dt'$$

We then obtain for the perturbation $f_{\alpha 2}$ the equation

$$\frac{\partial f_{\alpha 2}}{\partial t}=-e_\alpha\left\{\tilde{\mathbf{E}}(\mathbf{R}(t),t)-\frac{1}{e_\alpha}\nabla\Phi_\alpha(\mathbf{R}(t),t)\right\}\frac{\partial f_{0\alpha}}{\partial \mathbf{p}}. \qquad (13)$$

A solution of (13) satisfying the condition $f_{\alpha 2} = 1$ as $t \to \infty$ is

$$f_{\alpha 2}=-e_\alpha\int_{-\infty}^{t}\left\{\tilde{\mathbf{E}}(t')-\frac{1}{e_\alpha}\nabla\Phi_\alpha(t')\right\}\frac{\partial f_{0\alpha}}{\partial \mathbf{p}_\alpha(t')}\,dt'. \qquad (14)$$

Further, using the analogy with the linear theory of electric conductivity of a plasma, [11] we can write with the aid of (14) an expression for the total current in the plasma at the difference frequency

$$j_m=\sigma_{mn}\tilde{E}_n-i\sum_\alpha\frac{\Phi_\alpha}{e_\alpha}k_m\sigma_{mn}{}^\alpha=\sigma_{mn}\tilde{E}_n+j_m^{\text{extr}}. \qquad (15)$$

The field $\tilde{\mathbf{E}}$ entering in (15) is the field that excites in the plasma the extraneous current $j_{\text{extr}}$ due to the action of the averaged high-frequency force $\nabla\Phi$. In an isotropic plasma, this is a purely longitudinal electric field resulting from the separation of the charges

$$\tilde{\mathrm{E}}(R,t)=\frac{4\pi i}{\Omega}\frac{j^{\text{st}}(\Omega,\varkappa)}{\varepsilon_e(\Omega,\varkappa)}e^{i(\Omega t-\varkappa\mathrm{R})}=i\varkappa\frac{\varepsilon_e}{1+\varepsilon_e+\varepsilon_i}\frac{\Phi_e}{e}e^{i(\Omega t-\varkappa\mathrm{R})}. \qquad (16)$$

We have neglected here the averaged force exerted by the high-frequency field on the ions, and introduced the notation $\epsilon_\alpha \approx 4\pi i\sigma_{\alpha e}/\Omega$, $\sigma_{\alpha e}$—the longitudinal conductivity due to particles of species $\alpha$. From (16) in particular, there follows a condition for the decays

$$\varepsilon_e(\Omega,\varkappa)=1+\varepsilon_e+\varepsilon_i=0.$$

The total current produced by the particles of species $\alpha$, is equal to $\mathbf{j}_\alpha = \sigma_{\alpha e}\tilde{\mathbf{E}}^\alpha_{\text{eff}}$, where $\tilde{\mathbf{E}}^\alpha_{\text{eff}}$ is the effective electric field acting in the plasma on the particle of species $\alpha$:

$$\tilde{\mathrm{E}}^e_{\text{eff}}=-i\varkappa\frac{\Phi_e}{e}\left(\frac{1+\varepsilon_i}{1+\varepsilon_e+\varepsilon_i}\right)e^{i(\Omega t-\varkappa\mathrm{R})}, \quad \tilde{\mathrm{E}}^i_{\text{eff}}=\tilde{\mathrm{E}}. \qquad (17)$$

Expressions (16) and (17) contain the quantities $\epsilon_e$ and $\epsilon_i$, which are well known from linear plasma theory (see [11]):

$$\varepsilon_\alpha=\frac{4\pi e_\alpha^2N_\alpha}{m_\alpha\Omega}\int_{-\infty}^{+\infty}\frac{m_\alpha v_z^2}{T_\alpha}\xi(\omega-kv_z)f_0(\mathbf{p})\,d\mathbf{p},$$

$$\xi(x)=i\int_0^\infty e^{-ixt}\,dt=\frac{P}{x}+i\pi\delta(x). \qquad (18)$$

To determine the power dissipated by the high frequency field in the plasma, we find the work, averaged over the period $2\pi/\Omega$, performed by the electromagnetic field on the electrons and ions of the plasma:

$$Q=\frac{1}{2}\mathrm{Re}\sum_\alpha\mathbf{j}_\alpha\tilde{\mathbf{E}}_\alpha^*=\frac{1}{2}\mathrm{Re}(\mathbf{j}_e\tilde{\mathbf{E}}_e^*{}_{\text{eff}}+\mathbf{j}_i\tilde{\mathbf{E}}_i^*{}_{\text{eff}}) \qquad (19)$$

$$=\frac{1}{2}\mathrm{Re}(\sigma_l^e|\tilde{\mathbf{E}}^e_{\text{eff}}|^2+\sigma_l^i|\tilde{\mathbf{E}}^i_{\text{eff}}|^2)=$$

*$(\mathbf{v}\nabla) = \mathbf{v}\cdot\nabla$; $[\mathbf{vB}] = \mathbf{v}\times\mathbf{B}$.

$$= \frac{1}{2}\left|\frac{\varkappa\Phi_e}{e}\right|^2\left(\left|\frac{1+\varepsilon_i}{1+\varepsilon_e+\varepsilon_i}\right|^2 \operatorname{Re}\sigma_i^e + \left|\frac{\varepsilon_e}{1+\varepsilon_e+\varepsilon_i}\right|^2 \operatorname{Re}\sigma_i^i\right).$$

It follows from (19), in particular, that in the case of scattering by the electrons the contributions of the nonlinear and Compton scatterings cancel each other,[6] and this leads to a considerable decrease of the cross section for scattering by electrons surrounded by Debye space-charge clouds (the Coulomb field due to the separation of the charges offsets almost completely the averaged force acting on the electrons). Formally this cancellation is expressed in terms of the factor K $= |(1 + \epsilon_i)/(1 + \epsilon_i + \epsilon_e)|^2$, which can be a very small quantity. The cancellation disappears near the plasma resonance $1 + \epsilon_e + \epsilon_i \approx 0$ and at $\epsilon_e \ll 1$. The former case corresponds to an approach to the condition of the decay of three-wave interaction, when decays as well as scattering must be taken into account in the investigation of the nonlinear processes. The latter case of Compton scattering by free electrons ($\epsilon_e < 1$) is realized if the spatial period is small compared with the Debye radius of the electrons $\kappa r_{de} \gg 1$, or else the beat frequency is large compared with the reciprocal relaxation time of the screening charge $\Omega \gg \omega_{0e}$. In the remaining cases the main contribution to the induced scattering is made by ions situated in a strong Coulomb field.

Equations for the energy densities of the interacting waves can be obtained by using the conservation laws. In scattering of electromagnetic waves at positive energies, the law of conservation of the total number of quanta is satisfied:[2)]

$$d(N_1+N_2)\,/\,dt = 0 \tag{20}$$

as well as the law of conservation of the total energy of the plasma-plus-field system

$$\frac{d(W_1+W_2)}{dt} = -Q, \quad W_i = \left(\frac{\partial[\varepsilon(\omega)\omega^2]}{\omega\partial\omega}\right)_{\omega=\omega_i}\frac{E_i^2}{16\pi}. \tag{21}$$

As a result we have

$$\frac{dW_1}{dt} = -\frac{\omega_1}{\Omega}Q = -\alpha_1 W_1 W_2, \qquad \frac{dW_2}{dt} = \frac{\omega_2}{\Omega}Q = \alpha_2 W_1 W_2. \tag{22}$$

3. The results obtained for the case of interaction of two quasimonochromatic waves are valid either if the initial amplitudes of the interacting waves greatly exceed the noise level in the plasma, or if the pump wave has a narrow spectrum and the scattering increment has a sharp maximum in a small frequency region, for example during the hydrodynamic stage of induced scattering (see Sec. 2). In the general case, when considering induced scattering of waves in a plasma, it is necessary to take into account the interaction of waves with broad spectra. A generalization of the theory to include this case does not entail any fundamental difficulties.

We represent the electromagnetic field in the form of a Fourier integral

$$\mathbf{E}(r,t) = \int d\mathbf{k}\,d\omega\mathbf{E}(\mathbf{k},\omega)\exp(i\omega t - i\mathbf{k}\mathbf{r}) \tag{23}$$

and consider for simplicity[3)] the case when the correlation function of the electric field in the zeroth approximation is of the form

$$\langle E_m(\omega_1,\mathbf{k}_1)E_n(\omega_2,\mathbf{k}_2)\rangle = I_0(\mathbf{k},\omega)\delta_{mn}\delta(\omega_1-\omega_2)\delta(\mathbf{k}_1-\mathbf{k}_2). \tag{24}$$

The averaged force acting on the charged particle in the field (23) can be represented in the form

$$\mathbf{F} = i\int \varkappa\Phi_0 dk_1\,dk_2\,\exp(i\Omega t - i\varkappa\mathbf{r}). \tag{25}$$

We have introduced here the notation $dk_i = d\mathbf{k}_i d\omega_i$, $\Phi_0 = e^2\mathbf{E}_1\mathbf{E}_2^*/2m\omega_1\omega_2$.

Acting in analogy with the case of quasimonochromatic waves, we can determine an expression for the extraneous current due to the action of the averaged force, calculate the fields excited by this current, and determine the work, averaged over the time and over the macroscopic volume, performed by the high-frequency field of the plasma. By simple transformations we obtain

$$Q_\Sigma = \int Q(k_1,k_2)\,dk_1\,dk_2, \tag{26}$$

where $Q(k_1, k_2)$ is the energy dissipated in the plasma by a pair of monochromatic waves with frequencies $\omega_1$ and $\omega_2$ (see (19)). To find the sought equations for the spectral energy density of the interacting waves, we use, as before, the conservation laws. As the result we obtain an equation describing the transfer of the energy of the spectrum in induced scattering of waves by particles in an isotropic plasma

$$\frac{dW_{k_1}}{dt} = W_{k_1}\int G(k_1,k_2)W_{k_2}\,dk_2, \tag{27}$$

where the kernel G is defined by

$$G(k_1,k_2) = \frac{16\pi^2e^2(k_1-k_2)^2}{m^2\omega_1^2\omega_2\Omega}\left\{\left|\frac{1+\varepsilon_i(\Omega,\varkappa)}{1+\varepsilon_i(\Omega,\varkappa)+\varepsilon_e(\Omega,\varkappa)}\right|^2 \operatorname{Re}\sigma_i^e(\Omega,\varkappa)\right.$$
$$\left.+\left|\frac{\varepsilon_e(\Omega,\varkappa)}{1+\varepsilon_i(\Omega,\varkappa)+\varepsilon_e(\Omega,\varkappa)}\right|^2 \operatorname{Re}\sigma_i^i(\Omega,\varkappa)\right\}\left[\frac{\partial(\omega^2\varepsilon)}{\omega\partial\omega}\right]^{-1}_{\omega=\omega_1}\left[\frac{\partial(\omega^2\varepsilon)}{\omega\partial\omega}\right]^{-1}_{\omega=\omega_2}. \tag{28}$$

Relations (27) and (28) coincide with equations obtained in [6,7] by other methods and describing the induced scattering of waves with random phases.

## 2. KINETIC AND HYDRODYNAMIC STAGES OF INDUCED SCATTERING

Let us illustrate the application of the resultant relations by means of several examples that make it possible to explain the features of the process of induced scattering of coherent radiation in the plasma.

1. As the simplest example, we consider the scattering of a quasimonochromatic electromagnetic wave by an electron beam with low electron density (the condition $\Omega \gg \omega_{0s}$, where $\omega_{0s}$ is the Langmuir frequency of the beam electrons). In this case we can neglect the effects of compensation of the scattering by the occurrence of a strong Coulomb space-charge field, and for a beam with a Maxwellian distribution function shifted by the average velocity $V_0$, the expression for the energy dissipated by the field takes the form

$$Q = -\left(\frac{\varkappa\Phi_0}{e}\right)^2\frac{\omega_{0s}^2}{8\pi}\frac{\Omega}{\varkappa^2V_{Ts}^2}\operatorname{Im}\left\{Z\left(\frac{\Omega-\varkappa V_0}{\varkappa V_{Ts}}\right)-1\right\}, \tag{29}$$

$$Z(x) = X(x) - iY(x), \quad X(x) = 2xe^{-x^2}\int_0^x e^{t^2}dt, \quad Y(x) = \sqrt{\pi}xe^{-x^2}. \tag{30}$$

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[2)]The conservation of the total number of quanta in induced scattering follows from the laws of conservation of the total energies and of the momentum of the plasma-plus-field system.

[3)]We can obtain analogously equations for the energy density of the interacting waves when account is taken of the finite correlation time of the field.

If the initial energy densities of the interacting waves satisfy the relation $W_{10} \gg W_{20}$, then we can linearize Eqs. (22) and obtain a relation for the growth increment $\gamma$ of the wave with frequency $\omega_2$, under the assumption $W_1 = W_{10} = \text{const}$:

$$\gamma = \alpha_2 W_{10}. \tag{31}$$

It is necessary here to distinguish between two limiting stages (with respect to the magnitude of the increment) of the induced scattering. In the limit of small increments ($\gamma \ll \kappa V_{TS}$) the value of the increment is determined by the imaginary part of the function Z(x), calculated for real $\Omega$. In analogy with the theory of ordinary two-stream instability, we call this case the kinetic stage of the process of induced wave scattering. In this case Q is given by

$$Q = \left(\frac{\varkappa\Phi_0}{e}\right)^2 \frac{\omega_{0s}^2}{8\sqrt{\pi}} \frac{(\Omega - \varkappa V_0)\Omega}{\varkappa^3 V_{Ts}^3} \exp\left[-\left(\frac{\Omega - \varkappa V_0}{\varkappa V_{Ts}}\right)^2\right]. \tag{32}$$

It follows from (32) that scattering by the beam can be accompanied either by a decrease or by an increase of the frequency, depending on the relation between $\Omega/\kappa$ and $V_0$. The maximum increment under the condition $\omega_1, \omega_2 \gg \omega_{0S}$ is

$$\gamma_{\text{kin}} = \sqrt{\frac{2\pi}{e}} \frac{\omega_{0s}^2}{\omega_1} \frac{V_\sim^2}{V_{Ts}^2} \tag{33}$$

and is reached if $|\Omega - \kappa V_0| \approx \kappa V_{TS}/\sqrt{2}$. Here $V_\sim = eE_{10}/\sqrt{2} m\omega_1$ is the rms velocity of the oscillations of the electrons in the pump wave.

Let us consider further the case of large increments $\gamma \gg V_{TS}$, when we can neglect the imaginary part of the function Z and the thermal spread of the electron velocities. The dissipated power is determined in this case by the expression[4)]

$$Q = \left(\frac{\varkappa\Phi_0}{e}\right)^2 \frac{\omega_{0s}^2}{8\pi} \operatorname{Im}\left\{\frac{\Omega}{(\Omega - \varkappa V_0)^2}\right\}. \tag{34}$$

It follows from (34), in particular, that the work performed by the high-frequency field on the electron beam differs from zero if $\Omega$ has an imaginary part, i.e., if the interacting waves grow or attenuate.

Leaving out the details, we present an expression for the maximum growth increment of a weak wave during the hydrodynamic stage of scattering

$$\gamma_{\text{hyd}} = \frac{\sqrt{3}}{2} \omega_1 \left(\frac{2V_\sim}{c} \frac{\omega_s}{\omega_1}\right)^{2/3}; \tag{35}$$

The maximum is attained at $|\Omega - \kappa V_0| \approx \gamma_{\text{hyd}}/\sqrt{3}$, $\gamma_{\text{hyd}} \gg \omega_{0S}$. It is interesting that, just as in the ordinary two-stream instability of plasma oscillations, we have during the kinetic stage an increment $\gamma_{\text{kin}} \sim W_{10}N_S$, and during the hydrodynamic stage $\gamma_{\text{hyd}} \sim (W_{10}N_S)^{1/3}$.

2. Let us investigate with the aid of relations (22) and (23) the dependence of the scattering increments in an isotropic plasma on the intensity of the quasimonochromatic high-frequency radiation ($\omega_1 \gg \omega_{0e}$). Unlike in linear induced processes, in induced scattering and in an equilibrium plasma there are two possible interaction stages: hydrodynamic, when the increment is $\gamma \gg \kappa V_{T\alpha}$, and kinetic, when $\gamma \ll \kappa V_{T\alpha}$. In the equilibrium plasma, the scattering of the waves is accompanied by lowering of the frequency (it follows from (22) that in the case of Maxwellian distribution functions of the electrons and of the ions $Q > 0$). Under the condition $\Omega/\kappa V_{Te} \ll \Omega/\kappa V_{Ti} \approx 1/\sqrt{2}$ the scattering is determined mainly by the ions, and the growth increment of a weak wave with frequency $\omega_2$ during the kinetic stage of the scattering is equal to[6]

$$\gamma_{ik} = \sqrt{\frac{2\pi}{e}} \frac{\omega_{0i}^2}{\omega_2} \left(\frac{V_\sim}{V_{Ti}}\right)^2 a^2, \tag{36}$$

where

$$a = \begin{cases} 4(\varkappa r_{de})^{-2} & \text{for } \varkappa r_{de} \gg 1, \\ (1 + T_e/T_i)^{-1} & \text{for } \varkappa r_{de} \ll 1. \end{cases}$$

During the hydrodynamic stage, in scattering by ions $\gamma \gg \kappa V_{Ti}$ the maximum increment is given by

$$\gamma_{i2} = \frac{\sqrt{3}}{2} \left(\frac{2V_\sim}{V_{\text{ph}}} \frac{\omega_{0i}}{\omega_1} b\right)^{2/3} \omega_1, \tag{37}$$

$$b = \begin{cases} 4(\varkappa r_{de})^{-2} & \text{for } \varkappa r_{de} \gg 1, \\ [1 + (T_e/T_i)\varkappa^2 V_{Ti}^2/\Omega^2]^{-1} & \text{for } \varkappa r_{de} \ll 1 \end{cases}$$

and is reached at $\Omega \approx \gamma$. For transverse waves with frequency $\omega_1 \gg \omega_{0e}$, the phase velocity of the wave is $V_{\text{ph}} \approx c$, but the expressions for the growth increments are also valid for other types of waves (with allowance for the coefficient $[\partial(\epsilon\omega^2)/\omega\, \partial\omega]_{\omega=\omega_1}[\partial(\epsilon\omega^2)/\omega\, \partial\omega]_{\omega=\omega_2}$), provided the velocity of the particles from which the scattering takes place does not exceed $V_{\text{ph}}$. In the opposite case (for example, in the scattering of ion-acoustic waves by electrons or in the scattering of waves by relativistic beams) it is necessary to take into account the Doppler corrections.

In scattering by electrons during the kinetic stage ($\gamma \ll \kappa V_{Te}$) in a dense plasma, the scattering increment is small compared with the "ionic" increment (36), owing to the compensation effects. The hydrodynamic stage occurs if $\gamma \gg \kappa V_{Te}$, and under the condition $\gamma \gg \omega_{0e}$ there is no compensation even in a plasma with $\kappa r_{de} \ll 1$, and the induced scattering by the electrons becomes decisive. The corresponding nonlinear increment is

$$\gamma_e \approx \frac{\sqrt{3}}{2} \left(\frac{4V_\sim^2}{V_{\text{ph}}^2} \frac{\omega_{0e}^2}{\omega_1^2}\right)^{1/3} \omega_1. \tag{38}$$

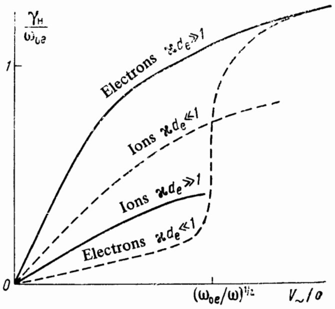
In the weak-turbulence approximation $V_\sim/V_{\text{ph}} \approx V_\sim/c \ll 1$ relation (38) is valid under the condition

$$\frac{\omega_{0e}}{\omega_1} \ll \left(\frac{V_\sim}{c}\right)^2 \ll 1. \tag{39}$$

The qualitative dependence of $\gamma_{i,e}$ on the amplitude of the incident wave is shown in the figure (the solid lines correspond to the case $\kappa r_{de} \gg 1$, and the dashed ones to $\kappa r_{de} \ll 1$).

To compare the roles of the decay effects and induced wave-scattering processes from the point of view of their use for collisionless plasma heating, it is necessary to solve the problem with simultaneous allowance for both processes. Obviously, in strong fields under the condition $\gamma \gg \kappa V_{TS}$ the ion-acoustic oscillations are not a resonant state of the system (the increment is of the order of or larger than the frequency) and the hydrody-

4) Relation (34) can also be obtained on the basis of the quasihydrodynamic equations. In such a formulation, the problem of the interaction of a given quasimonochromatic field with a beam of charged particles was first considered in [12].




namic induced scattering turns out to be the only type of nonlinear wave interaction.

In particular, such considerations were used in [13] to determine the increment for scattering by ions (37) in an analysis of the limiting case of decay instability.

We note that similar processes can take place in stimulated scattering of light in nonlinear dielectrics, if the pump intensity is such that the scattering increment exceeds the sound frequency, $\gamma \gg \kappa V_s$. To obtain an expression for the increment we can use the results of [14], where scattering of ultrashort light pulses was considered:

$$\gamma = \frac{\sqrt{3}}{2} k_1 v_{\text{gr}} \left[ \frac{E^2}{4\pi v_{\text{gr}}^2} \rho \left( \frac{\partial \varepsilon}{\partial \rho} \right)_s^2 \right]^{1/3},$$

$v_{gr}$ is the group velocity of the pump wave.

The aforementioned analogy with the theory of unstable oscillations of a plasma through which a beam of charged particles penetrates can be illustrated most clearly by writing down the dispersion equation obtained in the hydrodynamic approximation for a weak wave of frequency $\omega_2$ in the presence of a pump wave at a frequency $\omega_1$. If both waves are transverse, and the scattering is by the ions, then this equation, accurate to terms $\sim O(\Omega/\omega_1)$, takes the form

$$1 - \frac{\omega_{0e}^2 + k_2^2 c^2}{\omega_2^2} - \frac{\omega_{0i}^2 \varkappa^2 V_\sim^2 \operatorname{Re}(a_1 a_2^*)}{2\omega_2^2 (\omega_2 - \omega_1)^2} = 0, \tag{40}$$

which is perfectly analogous to the well-known dispersion relation for oscillations in a plasma with a beam, the role of the beam being played by the pump wave (the photon beam).

The foregoing expressions for the increments are valid for the description of induced scattering of waves of arbitrary type. In a rarefied plasma the scattering is accompanied by transformation of the transverse wave into a transverse one, the increment being maximal in the case of back scattering. On the other hand, if the pump frequency $\omega_1$ is close to $\omega_{0e}$, then the major role is assumed by the transformation of a transverse wave into a longitudinal plasma wave, and the increment is maximal for scattering angles close to $\pi/2$,[5)] and is equal to

$$\gamma \approx \frac{\sqrt{3}}{2} \left( \frac{V_\sim^2}{V_{\text{ph.p}}^2} \frac{m}{M} \right)^{1/3} \omega_{0e}, \tag{41}$$

where $V_{ph.p}$ is the phase velocity of the plasma wave. As expected, in this limiting case expression (41) coincides with the increment describing the parametric instability in a homogeneous high-frequency field.[15]

3. Let us consider the transition from the hydrodynamic scattering stage into the kinetic one in the case of induced scattering of a wave packet of finite width $\Delta\omega$. We start from Eq. (27). We assume that $\Omega = \omega_2 - \omega_1 \gg \kappa V_{T\alpha}$. Then

$$\operatorname{Re} \sigma_l^\alpha = -\operatorname{Im} \frac{\Omega}{4\pi} \frac{\omega_{0\alpha}^2}{\varkappa^2 V_{T\alpha}^2} \left[ Z\left( \frac{\Omega - \varkappa V_0}{\varkappa V_{T\alpha}} \right) - 1 \right] \approx \frac{\omega_{0\alpha}^2}{4\pi} \operatorname{Im} \frac{\Omega}{(\Omega - \varkappa V_0)^2} \tag{42}$$

In the limit as $\Delta\omega \to 0$ we get from (27) and (42) expressions for the increments during the hydrodynamic scattering stage. In the case $\Delta\omega \gg \gamma$, $\kappa V_{T\alpha}$ we have

$$\frac{1}{\Delta\omega} \operatorname{Im} \frac{\Delta\omega + i\gamma}{(\Omega - \varkappa V_0)^2} = -\pi\delta'(\Omega - \varkappa V_0). \tag{43}$$

Substituting (43) in (27), we obtain an equation describing the differential redistribution over the spectrum[16]

$$\frac{dW_{k_1}}{dt} = W_{k_1} \int \frac{\partial}{\partial \omega_2} [\tilde{G} W_{k_2}(\omega_2, \varphi, \theta)]_{\omega_2 = \omega_1 - \varkappa V_0} \sin\theta \, d\theta \, d\varphi, \tag{44}$$

where

$$\tilde{G} = \frac{16\pi^2 e^2 \varkappa^2}{m^2 \omega_1 \omega_2^2} \left\{ \left| \frac{1 + \varepsilon_i}{1 + \varepsilon_e + \varepsilon_i} \right|^2 \omega_{0e}^2 + \left| \frac{\varepsilon_e}{1 + \varepsilon_e + \varepsilon_i} \right|^2 \omega_{0i}^2 \right\} \left( \frac{\partial(\omega^2 \varepsilon)}{\omega \partial \omega} \right)_{\omega=\omega_1}^{-1} \left( \frac{\partial(\omega^2 \varepsilon)}{\omega \partial \omega} \right)_{\omega=\omega_2}^{-1}.$$

Thus, we can separate three stages of induced scattering:

1) Hydrodynamic stage with increments determined by expressions (37) and (38), which sets in at sufficiently large wave amplitudes, when $\gamma_H(W_k) > \max(\Delta\omega, \kappa V_0 \Delta \cos\theta, \kappa V_{T\alpha})$.

2) Kinetic stage (integral redistribution) (see [6]), when $\gamma$, $\Delta\omega < \kappa V_{T\alpha}$ with increment (36).

3) Kinetic stage (differential redistribution), when $\Delta\omega > \gamma$, $\kappa V_{T\alpha}$.

4. The possibility of using the processes of induced wave scattering for collisionless[6)] plasma heating was discussed in [17,18], where plasma heating in a given field of opposing electromagnetic waves with a broad frequency spectrum was considered. An important feature of the processes of induced scattering is that their increments increase appreciably with increasing pump power (the transition from the kinetic stage to the hydrodynamic one). Even in the case of a narrow pump-wave spectrum, all the plasma particles take part in the interaction with the high-frequency radiation, unlike the kinetic stage, during which the high-frequency field accelerates a small group of particles. This circumstance makes it possible to use coherent high-frequency radiation for plasma heating.

To determine the efficiency of the given method of plasma heating, it is necessary to consider the nonlinear stage of the induced scattering. In this case, too, the analogy with the theory of two-stream instability is very useful. During the hydrodynamic stage, the most important saturation effect, leading to a decrease in the velocity of the induced scattering, is connected with the oscillations of the particles captured in the high-frequency potential well. It becomes appreciable if the oscillation

[5)]In the case of scattering of a transverse wave into a longitudinal one we have $k_2/k_1 \approx c/V_{Te} \gg 1$.

[6)]The collisions in the plasma can be neglected if the effective collision frequencies are small compared with the scattering increments $\gamma \approx \nu_{eff}$.

frequency of the particle in the well is comparable with the growth increment. The condition under which this nonlinear effect can be neglected ($\Omega_{capt} \ll \gamma$) has in the case of scattering by ions the form

$$V_\sim / c \ll (\omega_{0e} / \omega)^2 (M / m)^{1/2} \tag{45}$$

and is satisfied for practically all values of the plasma concentration. In scattering by electrons, it is impossible to satisfy the inequality

$$V_\sim / c \ll \omega_{0e}^2 / \omega^2 \tag{46}$$

analogous to (45) (by virtue of relation (39)). Consequently, the indicated saturation effect should lead to a limitation of the amplification of the scattered radiation at the levels $W_2 \ll W_{10}$ and it is possible to disregard the change of the pump amplitude during the hydrodynamic stage. The system of equations describing the induced scattering then becomes mathematically identical with the system of equations of the nonlinear theory of two-stream instability,[19] and it is possible to use certain results of this theory. In particular, the monotonic growth of the intensity of the scattering wave should give way to periodic oscillations above a certain mean value.

In scattering by ions, the hydrodynamic saturation effects do not play any role, and most pump quanta should experience single scattering after a time $\sim\gamma^{-1}$. The plasma then receives an energy $Q \approx W_{10}\gamma/\omega_1$. This is expected to be followed by a sharp decrease of the scattering velocity, since as a result of the first stage of the interaction the width of the radiation spectrum may become comparable with the value of the hydrodynamic increment $\Delta\omega \gtrsim \gamma_{max}$ ($\gamma \sim \gamma_{max}$ in the frequency region $\Delta\omega \sim \gamma_{max}$), and the kinetic stage sets in, in which quasilinear relaxation of the particle and photon energy distributions takes place. It is possible, however, that the nonlinear effects of the interaction of the spectral components of the scattered radiation lead to a narrowing of the line width and to a retention of the integral character of the succeeding transformation of the radiation spectrum. A transition to the kinetic stage of scattering can also occur if the ion heating causes violation of the condition $V_{ti} \ll \gamma/\kappa$.

We note, finally, that the efficiency of the transfer of the coherent-radiation energy to the plasma depends on the relation between $\omega_{0e}$ and $\omega_1$ and is maximal when $\omega_{0e} \approx \omega_1$, for in this case there occurs a transformation of the incident electromagnetic wave into longitudinal Langmuir oscillations which, in turn, can be absorbed by the plasma. The bulk of the radiation energy will then be transferred to the electrons. For ion heating, it is necessary to ensure the possibility of multiple passage of the scattered radiation through the plasma.

Of course, the considerations advanced above can be used only to construct a qualitative picture of the phenomenon, and should be supplemented by a quantitative nonlinear theory of induced scattering.

In conclusion, to illustrate the importance of the conskdered processes for different applications, we present some numerical estimates. Expression (37) for the maximum increment of scattering by ions can be represented in the form (in the case of a hydrogen plasma)

$$\gamma = 10^6 \left[\frac{20 N_e \widetilde{P}[\text{вт}/\text{см}^2]}{\omega_1}\right]^{1/3}, \tag{47}$$

where $\widetilde{P}$[W/cm$^2$] is the energy flux density of the coherent radiation. For example, for optical breakdown at atmospheric pressure, $N_e = 3\times 10^{19}$ cm$^{-3}$ and $\lambda = 10^{-4}$ cm we find that at the presently realistic values $\widetilde{P} = 10^{16}$ W/cm$^2$ the increment is equal to $\gamma_i = 10^{13}$ sec$^{-1}$, and consequently the processes in question turn out to be significant if the plasma-formation length exceeds $c/\gamma = 3\times 10^{-3}$ cm. Analogously, in the microwave band, we obtain for waves with $\lambda = 1$ cm, $\widetilde{P} \approx 10^6$ W/cm$^2$ at $N_e = 10^{12}$ cm$^{-3}$ an increment $\gamma = 3\times 10^8$ sec$^{-1}$.

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[1] A. V. Gaponov and M. A. Miller, Zh. Eksp. Teor. Fiz. **34**, 242 (1958) [Sov. Phys.-JETP **7**, 168 (1958)].

[2] A. G. Litvak, Izv. VUZov, Radiofizika **7**, 562 (1964).

[3] B. B. Kadomtsev and V. I. Petviashvili, Zh. Eksp. Teor. Fiz. **43**, 2234 (1962) [Sov. Phys.-JETP **16**, 1578 (1963)].

[4] V. I. Karpman, Dokl. Akad. Nauk SSSR **152**, 587 (1963) [Sov. Phys.-Doklady **8**, 919 (1964)].

[5] L. M. Gorbunov, V. V. Pustovalov, and V. P. Silin, Zh. Eksp. Teor. Fiz. **47**, 1437 (1964) [Sov. Phys.-JETP **20**, 967 (1965)].

[6] V. N. Tsytovich, Nelineĭnye éffekty v plazme (Nonlinear Effects in Plasma), Nauka, 1967.

[7] L. M. Kovrizhnykh, Trudy FIAN **32**, 173 (1966).

[8] B. Coppi, M. N. Rosenbluth, and R. N. Sudan, Ann. Phys. **55**, 207 (1969).

[9] V. N. Tsytovich, Preprint No. 12, FIAN, 1968.

[10] A. G. Litvak, M. I. Petelin, and E. I. Yakubovich, Zh. Tekh. Fiz. **35**, 108 (1965) [Sov. Phys.-Tech. Phys. **10**, 81 (1965)].

[11] V. V. Shafranov, Voprosy Teorii Plazmy (Problems of Plasma Theory), No. 3, Gosatomizdat, 1964.

[12] M. A. Miller, Izv. VUZov, Radiofizika **5**, 929 (1962).

[13] L. M. Gorbunov, Zh. Eksp. Teor. Fiz. **55**, 2298 (1968) [Sov. Phys.-JETP **28**, 1220 (1969)].

[14] V. I. Bespalov and G. A. Pasmanik, ibid. **58**, 309 (1970) [**31**, 168 (1970)].

[15] V. P. Silin, ibid. **48**, 1679 (1965) [**21**, 1127 (1965)].

[16] A. A. Galeev, V. N. Karpman, and R. Z. Sagdeev, Nuclear Fusion **5**, 20 (1965).

[17] L. M. Kovrizhnykh, ZhETF Pis. Red. **2**, 142 (1965) [JETP Lett. **2**, 89 (1965)].

[18] Ya. B. Zel'dovich and E. V. Levich, ibid. **11**, 497 (1970) [**11**, 339 (1970)].

[19] L. A. Vaĭnshteĭn, Radiotekhnika i élektronika **2**, 1027 (1957).


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