OSCILLATORY AND RESONANT EFFECTS IN S–N–S JUNCTIONS IN A MAGNETIC FIELD

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Rules are formulated for quasiclassical quantization of the energy of elementary excitations in a S–N–S junction in a magnetic field in terms of areas limited by trajectories of the particles and the holes in p-space. The periods of the de Haas–van Alphen oscillations, which differ from the corresponding periods in the normal metal, are determined. The effect of absorption of sound propagating in a direction perpendicular to the plane of the junction is considered and it is shown that in a weak field the absorption coefficient has singularities at frequencies corresponding to transitions between the Andreev levels.

1. INTRODUCTION

SPATIAL quantization in films of a normal metal making ideal contact with superconductors are characterized by some specific features. The elementary excitations moving from the normal metal are reflected from the superconducting boundary if their energy does not exceed the energy gap $|\Delta|$, which plays the role of the scattering potential. Since this potential is small, it follows that on the boundary of the normal and superconducting metals the momentum of the quasiparticle remains practically unchanged, and the scattering leads to the transition of the excitation from a "particle" type state to a "hole" type state with the velocity **v** replaced by $-\mathbf{v}$[1] 1).

If the film is bounded on both sides by superconductors (S–N–S junction), then the motion of the quasiparticles is finite, and their spectrum is quantized. In the absence of a magnetic field, the distance between the quantized levels depends on the thickness of the normal layer d, and the position of the levels is determined by the phase difference χ of the ordering parameters of the two superconductors[3]:

$$\varepsilon_n(q)=\frac{\pi|v_z|}{d}\left(n+\frac{1}{\pi}\arccos\frac{\varepsilon}{|\Delta|}\pm\frac{\chi}{2\pi}\right), \tag{1}$$

where $v_z = (v_0^2 - (q/m)^2)^{1/2}$ is the velocity of the excitations along the normal to the film boundary, q is the quasimomentum component lying in the plane of the N–S boundary, and v_0 is the Fermi velocity. It must be emphasized that the very existence of discrete levels that depend on the thickness d (Andreev levels) is connected with the coherence of the phases of the superconductors. In the case when the phase difference χ is not fixed, the position of the levels is not determined and the spectrum of the quasiparticles is continuous in this sense.

The purpose of the present paper is to clarify the question of the character of the quantum states of the electron in an S–N–S junction in a magnetic field parallel to the film. It will be shown that the presence of the magnetic field leads to the appearance of energy bands, a fact connected with the quantization of the magnetic flux in the doubly-connected superconducting region. Conditions are derived for the quasiclassical quantization, making it possible to analyze the structure of the energy spectrum. Although the distance between the low-lying levels of the S–N–S junction depend on the applied constant magnetic field, nonetheless the scattering of the quasiparticles by the N–S boundary leads to an appreciable difference between the character of the energy spectrum and the spectrum of the free electrons in the magnetic field. If at specified values of the quantum numbers we change the magnitude of the magnetic field (or the thickness of the normal layer), then at definite values of H the form of the trajectory of the quasiparticle in real space changes significantly because of the additional scattering by the N–S boundary (see below). This leads to the existence of new singularities in the dispersion law, connected with the collision surfaces (type of magnetic surface levels[4]). Thus, the picture of the levels of an S–N–S junction in a magnetic field has a complicated character.

The obtained conditions of quasiclassical quantization make it possible to consider a unique oscillatory effect. An estimate shows that in weak magnetic fields the period of the resultant oscillations greatly exceeds the period of the usual de Haas–van Alphen effect.

The dependence of the Andreev levels on the phase difference (1) has a profound meaning and makes it possible to explain the cause of the "vanishing" of these levels in an arbitrarily weak magnetic field. The explanation of the resultant instability lies in the fact that although one can speak locally of a phase difference of the superconductors (and consequently of Andreev levels), this difference itself varies along the N–S boundary. Therefore in the presence of a magnetic field, in essence, there is no single set of energy levels, dependent on the thickness of the normal layer, for the entire macroscopic volume. In the calculation of the thermodynamic quantities of an S–N–S junction in a magnetic field there is no quantum size effect as a function of the thickness of the normal layer. However, by virtue of the statements made above, the Andreev levels should appear in quantum effects in which the local interaction is significant. To illustrate the foregoing, we present here a calculation of the energy of an acoustic

1) Direct observation of such a reflection of electrons with the aid of the radio-frequency size effect is reported in a recent paper by Krylov and Sharvin [2].

wave absorbed by a normal layer of S—N—S junction in a weak field H. After averaging of the phase, we obtain a non-zero result, and the transparency coefficient of the normal layer reveals singularities whenever the frequency of the sound is a multiple of the distance between the Andreev levels.

2. DERIVATION OF THE CONDITIONS OF QUASI-CLASSICAL QUANTIZATION

We proceed to consider the problem of the structure of the energy spectrum of excitations of an S—N—S junction placed in a constant magnetic field. We start from the Bogolyubov—de Gennes equations[5] for a two-component wave function $\Psi = \begin{pmatrix}\psi \\ \varphi\end{pmatrix}$, describing the motion of the quasiparticles in a superconductor

$$\hat{T}(\mathbf{H})\psi + \Delta(\mathbf{r})\varphi = \varepsilon\psi, \quad -\hat{T}(-\mathbf{H})\varphi + \Delta^*(\mathbf{r})\psi = \varepsilon\varphi, \tag{2}$$

where $\hat{T}(\mathbf{H})$ is the kinetic-energy operator, equal to $\hat{T}(\mathbf{H}) = -(2m)^{-1}\cdot(\nabla - iec^{-1}\mathbf{A})^2 - \mu$, $\hat{T}(-\mathbf{H}) = \hat{T}^*(\mathbf{H})$, and μ is the chemical potential. The normal to the surface of the N layer is directed along the z axis, the magnetic field along the y axis, and we choose a gauge in which the vector potential is $A = A_x$.

The ordering parameter $\Delta(\mathbf{r})$ that enters in (2) depends on the coordinates. It vanishes inside the normal layer $|z| < d/2$. For simplicity we consider a model in which $\Delta(z)$ is a step function

$$|\Delta| = \begin{cases}\Delta_0, & |z| > d/2, \\ 0, & |z| < d/2.\end{cases} \tag{3}$$

The phase of the ordering parameter in the presence of a magnetic field cannot be eliminated by a gauge calibration. This important circumstance was pointed out by Galaĭko[6]. To find the phase, we assume that the magnetic field changes jumpwise the interface with the superconductor. In this case we have for the vector potential ($A_\infty = Hd/2$):

$$A = \begin{cases} Hz, & |z| < d/2, \\ A_\infty \operatorname{sign} z, & |z| > d/2, \end{cases} \tag{4}$$

which gives for the phase χ, in accordance with the equation $\nabla\chi = 2ec^{-1}\mathbf{A}$,

$$\chi = \begin{cases} gx, & z > d/2 \\ -gx, & z < -d/2, \end{cases} \tag{5}$$

where $g = 2eA_\infty/c = eHd/c$.

We seek the solution of Eqs. (2) in the form

$$e^{ipy}\begin{Bmatrix}\psi(x,z) \\ \varphi(x,z)\end{Bmatrix},$$

where p is the component of the quasimomentum along the direction of the magnetic field. We then obtain for the functions $\psi(x, z)$ and $\varphi(x, z)$ the system of equations

$$\left[-\frac{1}{2m}\left(\frac{\partial}{\partial x} - i\frac{e}{c}A(z)\right)^2 - \frac{1}{2m}\frac{\partial^2}{\partial z^2} - \tilde{\mu}\right]\psi + \Delta(z)e^{igx\operatorname{sign}z}\varphi = \varepsilon\psi,$$
$$-\left[-\frac{1}{2m}\left(\frac{\partial}{\partial x} + i\frac{e}{c}A(z)\right)^2 - \frac{1}{2m}\frac{\partial^2}{\partial z^2} - \tilde{\mu}\right]\varphi + \Delta(z)e^{-igx\operatorname{sign}z}\psi = \varepsilon\varphi, \tag{6}$$

where $\tilde{\mu} = \mu - p^2/2m = k_0^2/2m$, and $\Delta(z)$ is given by the formula (3).

On the basis of the form of Eqs. (6), we can make several general statements concerning the character of the quantum states of such a problem. Considering first only the dependence on the coordinate x, we see that (6) represents a system of differential equations with periodic coefficients. This allows us to state, on the basis of the well known Floquet theorem, that the solution of (6) takes the form

$$\psi = e^{ikx}u(x, z), \quad \varphi = e^{ikx}v(x, z), \tag{7}$$

where k is the quasimomentum component along the N—S boundary perpendicular to H, and u and v are periodic functions of x with a period $a = 2\pi/2g = \pi c/eHd$. The value of the period is determined from the flux quantization condition: the magnetic-field flux penetrating into the junction on a length a is equal to the flux quantum $\Phi_0 = hc/2e$. The quasimomentum k can be regarded as varying within the limits of the one-dimensional Brillouin zone ($-g/2 < k < g/2$). Substituting (7) in (6) and solving the corresponding equations, we find for each value of k a set of discrete energy levels $\epsilon = \epsilon^{(s)}(k_0, k)$ corresponding to different zones s.

Writing u(x, z) and v(x, z) in the form of series

$$\psi = e^{ikx}\sum_{n=-\infty}^{\infty}A_n(z)e^{ingx}, \quad \varphi = e^{ikx}\sum_{n=-\infty}^{\infty}B_n(z)e^{ingx} \tag{8}$$

and substituting (8) in (6), we have an infinite system of equations for the quantities $A_n(z)$ and $B_n(z)$

$$\frac{1}{2m}\left[\left(k + ng - g\frac{z}{d}\right)^2 - \frac{d^2}{dz^2} - k_0^2\right]A_n = \varepsilon A_n,$$
$$-\frac{1}{2m}\left[\left(k + ng + g\frac{z}{d}\right)^2 - \frac{d^2}{dz^2} - k_0^2\right]B_n = \varepsilon B_n,$$
$$|z| < \frac{d}{2}; \tag{9}$$

$$\frac{1}{2m}\left[\left(k + \left(n - \frac{1}{2}\right)g\right)^2 - \frac{d^2}{dz^2} - k_0^2\right]A_n + \Delta_0 B_{n-1} = \varepsilon A_n,$$
$$-\frac{1}{2m}\left[\left(k + \left(n - \frac{1}{2}\right)g\right)^2 - \frac{d^2}{dz^2} - k_0^2\right]B_{n-1} + \Delta_0 A_n = \varepsilon B_{n-1},$$
$$z > \frac{d}{2}; \tag{10}$$

$$\frac{1}{2m}\left[\left(k + \left(n + \frac{1}{2}\right)g\right)^2 - \frac{d^2}{dz^2} - k_0^2\right]A_n + \Delta_0 B_{n+1} = \varepsilon A_n,$$
$$-\frac{1}{2m}\left[\left(k + \left(n + \frac{1}{2}\right)g\right)^2 - \frac{d^2}{dz^2} - k_0^2\right]B_{n+1} + \Delta_0 A_n = \varepsilon B_{n+1},$$
$$z < -\frac{d}{2}. \tag{11}$$

In solving Eqs. (9)—(11), we are interested only in excitations whose energies do not exceed Δ_0. Being localized in the normal film, these excitations execute finite motion. Their two-component wave function (8) will contain a finite number of terms. Indeed, in Eqs. (10) or (11), which contain a classical turning point, it is possible to put $\Delta_0 = 0$. This gives a "closure" condition, as a result of which we obtain a system of a finite number of equations for the coefficients A_n and B_n.

The quasiclassical solution of Eqs. (9)—(11) (valid under condition $\omega_H \ll \mu$, where $\omega_H = eH/mc$ is the classical frequency of revolution of the electron in the magnetic field) leads to quantization conditions analogous to the Lifshitz-Onsager conditions[7] for an electron with an arbitrary dispersion law. The distinguishing feature of the present problem is that owing to the alternating transition from states of the "particle" type to

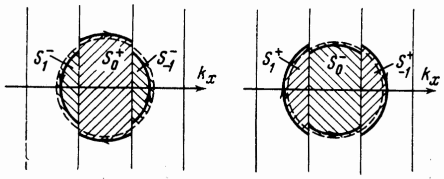

FIG. 1

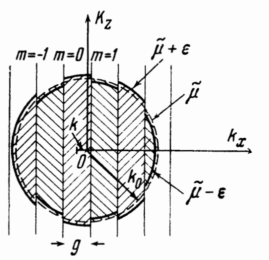

FIG. 2

states of the "hole" type, the areas on the corresponding sections of the quasiclassical trajectories are taken with alternating signs.

Let us consider for simplicity the case k = 0. In a strong magnetic field the trajectory of the electron (circle) lies entirely inside the normal layer. In this case all the expansion coefficients in (8) vanish with the exception of A_0 and B_0. The quasiclassical solution of (9) gives the well known quantization rule

$$S = 2\pi eHc^{-1}(n + 1/2), \tag{12}$$

where S is the area described by the trajectory of the quasiparticle in momentum space. This condition holds both for an electron (A_0) and a "hole" (B_0).

With decreasing field, trajectories shown on Fig. 1 arise. Now the coefficients that differ from zero are A_n and B_n with n = 0 and ±1. We put $B_{-2} = 0$ in Eq. (10) with n = −1, and accordingly $B_2 = 0$ in Eq. (11) with n = 1. As a result, the system (9)—(11) breaks up into two independent systems for the functions (A_1, B_0, A_{-1}) and (B_1, A_0, B_{-1}). Joining together the quasiclassical wave functions at the turning points, we obtain the quantization conditions

$$S_0^+ - S_1^- - S_{-1}^- = 2\pi eHc^{-1}(n + 1/2), \tag{13a}$$

$$-S_0^- + S_1^+ + S_{-1}^+ = 2\pi eHc^{-1}(n - 1/2), \tag{13b}$$

where the areas $S_0^\pm$, $S_1^\pm$, and $S_{-1}^\pm$ are shown in Fig. 1. Actually the conditions (13a) and (13b) are equivalent. They can be combined by assigning a plus sign to the areas bounded by the quasiclassical trajectories of the "particles," and a minus sign for "holes."

It can be shown (see the Appendix) that the general condition for quasiclassical quantization in an S—N—S junction at k ≠ 0 and at an arbitrary value of the magnetic field is given by

$$S(\varepsilon, k_0, k) = \sum_m (-1)^m S_m(\varepsilon, k_0, k) = 2\pi eHc^{-1}(n + \gamma), \tag{14}$$

where the areas S_m shown in Fig. 2 are taken alternately with alternating signs, and n are integers including negative ones. The wave vector k characterizes the shift of the centers of all the circles relative to the center of the band (−g/2, +g/2), and γ is a quantity on the order of unity, equal to 1/2 when the number of bands to the left and to the right of the central one (m = 0) coincide, and to the $\pm\pi^{-1}\cos^{-1}(\epsilon/\Delta_0)$ when these numbers differ ($m_1 \neq m_2$). Equations (14) include as limiting cases the quantization conditions (12) and (13) given above.

Proceeding to an analysis of the spectrum with the aid of Eq. (14), let us calculate the distance between the energy levels at fixed values of k_0 and k. Neglecting the dependence of γ on the energy, we obtain

$$\Delta\varepsilon = \frac{2\pi eH}{c} \Big/ \frac{\partial S(\varepsilon, k_0, k)}{\partial \varepsilon}. \tag{15}$$

It is clear from the geometrical construction that as $\epsilon \to 0$ the sum of the areas S_m is equal in magnitude to the area of a circle of radius k_0. The areas S_m, which enter in the sum (14) with negative sign, decrease with increasing ϵ, and those entering with positive sign increase with increasing excitation energy. Therefore in differentiating with respect to the energy, the alternating-sign sum (14) goes over into a sum of terms having the same sign. As a result we find that the quantity $\partial S(\epsilon, k_0, k)/\partial\epsilon$ in (15) coincides with the derivative of the total area of the circle with respect to the energy, equal to $2\pi m$. Thus,

$$\Delta\varepsilon = eH / mc = \omega_H. \tag{16}$$

Consequently, the distance between the low-lying energy levels of an S—N—S junction in a magnetic field does not depend on the thickness d and coincides with the distance between the Landau levels of an unbounded metal. An analogous conclusion was arrived at in[6]. This is closely connected with the change of the phase of the parameter Δ in a magnetic field and with the dependence of the position of the Andreev levels on χ[3] (for more details see Sec. 3).

With increasing energy ϵ or with changing parameters k_0 and k, the spectrum becomes complicated, owing to the change of the structure of the sum (14), due to the vanishing or to the appearance of new intersections of the electron trajectories with the N—S boundaries.

In spite of the already mentioned equality of the quasiparticle spectrum of the S—N—S junction (at given k_0 and k) to the Landau spectrum (16), the manifestation of quantization in an S—N—S junction may differ from its manifestation in the case of an unbounded metal. In particular, the de Haas—van Alphen oscillations will have a period that does not coincide with the quantity $eh/mc\mu$ characteristic of the normal metal. The magnitude of the period, as follows directly from the quantization condition (14), is determined by the usual formula[8]

$$\Delta(S_{ext} / H) = 2\pi e\hbar / c, \tag{17}$$

where S_{ext} is the extremal value of the area S(ϵ, k_0, k). Owing to the alternating signs of the terms in the sum (14), the latter quantity will itself be a function of H, i.e., the period in the reciprocal field $\Delta(1/H)$ is no longer constant. For example, in the case of a strong field satisfying the condition $g/2 < k_0 < 3g/2$, the period is determined by the area shown in Fig. 3.

In the case of weak magnetic fields, when the number of areas S_m entering in the quantization condition (14)

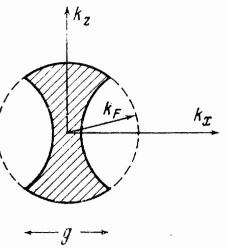

FIG. 3

is large, an important role is assumed by another type of singularity of the quasiparticle spectrum, leading to low-frequency oscillations. Recognizing that $k_0 \approx \approx mg$ ($m \gg 1$), let us find the period of the resultant singularities from the condition $\Delta m = 1$. For fixed d this yields

$$\tilde{\Delta}(1/H) = ed/c\hbar k_0. \tag{18}$$

Comparing the periods (18) and (17), we obtain the estimate

$$\frac{\tilde{\Delta}(1/H)}{\Delta(1/H)} \approx k_0 d \gg 1, \tag{19}$$

i.e., the oscillations in weak magnetic fields have a considerable period. It can be stated that the aforementioned singularities arise whenever a change takes place in the number of bands encompassed by the trajectories of the quasiparticles in momentum space. The reason for the oscillations is the appreciable change in the form of the trajectory of the quasiparticle following additional coherent scattering by the N—S boundary.

3. THERMODYNAMIC CHARACTERISTICS OF S—N—S JUNCTION IN A WEAK MAGNETIC FIELD

Let us consider the second-order equation with periodic potential

$$-{}^1/_2\Psi'' + U\Psi = \varepsilon\Psi. \tag{20}$$

Here Ψ is the vector of state, the matrix U is Hermitian and given by

$$U = \begin{pmatrix} U_{11} & U_{12}\ldots \\ U_{21} & U_{22}\ldots \end{pmatrix}, \qquad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix},$$

with $U(x + a) = U(x)$, where $a = 2\pi/k$. If the potential U varies sufficiently slowly ($k \to 0$), then the result of the calculation of the integral characteristics (of the type of the spectral density) does not depend on whether we solve Eq. (20) exactly or in the adiabatic approximation. For example, in the case of one-dimensional motion, the density of states in the quasiclassical approximation is equal to[9]

$$\begin{aligned} \rho(\varepsilon) &= \frac{d}{d\varepsilon}\int\frac{dx}{2\pi}2[2(\varepsilon - U(x))]^{1/2} \\ &= \frac{L}{a}\int_0^a dx\int\frac{dq}{2\pi}\delta\left(\varepsilon - \frac{q^2}{2} - U(x)\right), \end{aligned} \tag{21}$$

which corresponds to calculation with the aid of the local "dispersion law" $\epsilon(q, x) = (1/2)q^2 + U(x)$ at a certain point x.

Changing over to the case of a S—N—S junction, we break up in the limiting case of a weak magnetic field the layer of normal metal into regions Δx that are small compared with the characteristic period $a = \pi/g$. In each such region we calculate, in analogy with (21), the "local" density of states, and in place of the quasiclassical spectrum obtained by solving Eqs. (14), we use a spectrum of the form (1), corresponding to a fixed phase difference of the superconductors within the limits of a given Δx.

The spectrum of the low-lying levels ($\epsilon \ll \Delta_0$) is given in accordance with (1) by

$$\varepsilon = \varepsilon_n^{\alpha}(q) = \frac{\pi|v_z|}{d}(n + \varphi^{(\alpha)}), \tag{22}$$

where the index α assumes two values corresponding to the signs (±) in (1), and $\varphi^{(\alpha)} = {}^1/_2 \pm \chi/2\pi$. Introducing the notation $E = \epsilon/\Delta\epsilon$, $\Delta\epsilon = \pi v_0/d$, we write down the "local" density of states in the form

$$\begin{aligned} \rho_\varphi &= \frac{1}{d}\sum_{\alpha=1,2}\sum_{n=0}^{\infty}\int\frac{d^2q}{(2\pi)^2}\delta(\varepsilon - \varepsilon_n^{\alpha}(q)) = \frac{mp_0}{\pi^2}F_\varphi(E), \\ F_\varphi(E) &= E\sum_{n=0}^{\infty}\frac{\Theta(n+\varphi-E)}{(n+\varphi)^2}, \end{aligned} \tag{23}$$

where $\Theta(x) = 1$ when $x > 0$ and $\Theta(x) = 0$ when $x < 0$. According to the foregoing, to obtain the true state density it is necessary to average $\rho_\varphi(E)$ over the phase. We have

$$\langle F\rangle = \int_0^1 F_\varphi(E)\,d\varphi = E\sum_{n=0}^{\infty}\int_n^{n+1}dx\frac{\Theta(x-E)}{x^2} = 1.$$

Thus, the density of states of the S—N—S junction in a weak magnetic field ($H \to 0$) coincides exactly with the density of states of the normal metal at $H = 0$[2)]. This does not mean at all, however, that the other integral characteristics of the S—N—S junction in a weak magnetic field will not have singularities when the thickness of the normal layer changes. Among the effects of this type is the effect considered below, that of resonant absorption of sound in an S—N—S junction.

4. RESONANT ABSORPTION OF SOUND IN AN S—N—S JUNCTION

Assume that a sound wave is incident on the S—N—S junction. At very low temperatures, the number of excited carriers in the superconductors is exponentially small, therefore the absorption of sound will occur mainly in the normal layer. We consider an S—N—S junction in so weak a magnetic field, that the Andreev-level concept is valid. This means that we can choose a spectrum of the type (22) for the elementary excitations, by averaging the final expression for the absorbed energy over the phase difference.

Proceeding to the calculation of the absorbed energy, we write down the Hamiltonian of the electron-photon interaction in the Froelich form[10]

$$\hat{H}_{int} = g\int d\mathbf{r}\,\Psi^+(\mathbf{r}\sigma)\Psi(\mathbf{r}\sigma)\Phi(\mathbf{r}); \tag{24}$$

[2)]Yu. V. Sharvin called our attention to the possible appearance of a contact potential difference on the boundary of the N and S layers, and that this may influence the indicated result. To consider the effect it is necessary to take into account the charge-density change connected with the jump of the phase Δ. Since the charge density in the metal should be constant, this means the appearance of a certain electric field, although the gradient of the electrochemical potential at equilibrium, naturally, remains equal to zero.

where g is the interaction constant, $\Psi(r\sigma)$ is the operator of annihilation of an electron at the point **r** with spin σ, and $\Phi(\mathbf{r})$ is the operator of the phonon field, equal to

$$\Phi(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{q}} \left(\frac{\omega}{2}\right)^{1/2} (b_{\mathbf{q}} + b_{-\mathbf{q}}^{+}) e^{i\mathbf{q}\mathbf{r}}. \tag{25}$$

For a spatially inhomogeneous problem it is convenient to change over in the Hamiltonian H_{int} to Fermi operators of quasiparticles γ^+, γ (see[5]):

$$\Psi(\mathbf{r}_\uparrow) = \sum_\nu [\gamma_{\nu\uparrow} u_\nu(\mathbf{r}) - \gamma_{\nu\downarrow}^{+} v_\nu^*(\mathbf{r})],$$

$$\Psi(\mathbf{r}_\downarrow) = \sum_\nu [\gamma_{\nu\downarrow} u_\nu(\mathbf{r}) + \gamma_{\nu\uparrow}^{+} v_\nu^*(\mathbf{r})], \tag{26}$$

where the matrices $\begin{pmatrix} u_\nu \\ v_\nu \end{pmatrix}$ are eigenfunctions of the Bogolyubov equation (2). Substituting (26) in (24), we obtain terms of the type $\gamma^+\gamma$, corresponding to the scattering of quasiparticles, and terms of the type $\gamma^+\gamma^+$, corresponding to the simultaneous production of two excitations. The matrix element of scattering of a quasiparticle from the state n into the state m is equal to

$$M_1 = g\left(\frac{\omega}{2V}\right)^{1/2} \int d\mathbf{r}\,(u_n^* u_m - v_n^* v_m) e^{i\mathbf{k}\mathbf{r}}, \tag{27}$$

and the matrix element for the production of two quasiparticles is

$$M_2 = -g\left(\frac{\omega}{2V}\right)^{1/2} \int d\mathbf{r}\,(u_n^* v_m^* + u_m^* v_n^*) e^{i\mathbf{k}\mathbf{r}}. \tag{28}$$

Assume that the wave vector of the sound **k** is directed along the normal to the N—S boundary. We can then directly integrate with respect to x and y in (27) and (28), and all that remains to estimate the matrix elements M_1 and M_2 is to integrate with respect to z using the S—N—S-junction single-particle wave functions obtained in[3]. Calculations show that the scattering of the quasi-particles is determined by processes of the type $\gamma_{n,+}^{+}\gamma_{m,+}$ and $\gamma_{n,-}^{+}\gamma_{m,-}$, and the contribution of the processes $\gamma_{n,+}^{+}\gamma_{m,-}$ is small to the extent that the ratio k/k_F is small ($\hbar k_F$ is the Fermi momentum). In the same approximation, the absorption of sound with production of two quasiparticles is connected with the process $\gamma_{n,+}^{+}\gamma_{m,-}^{+}$. The signs of n and m number the branches of the S—N—S-junction spectrum (1). The explicit form of the matrix elements is

$$|M_1|^2 = 2g^2 \frac{\omega}{V} \frac{\delta_{q_x^n, q_x^m}\delta_{q_y^n, q_y^m}}{[\pi^2(n-m)^2 - k^2d^2]^2} \begin{cases} \pi^2(n-m)^2 \sin^2(kd/2), & n-m \text{ — is even}, \\ k^2d^2\cos^2(kd/2); & n-m \text{ — is odd}. \end{cases} \tag{29}$$

$$|M_2|^2 = 2g^2 \frac{\omega}{V} \frac{\delta_{q_x^n, -q_x^m}\delta_{q_y^n, -q_y^m}}{[\pi^2(n+m+1)^2 - k^2d^2]^2}$$
$$\times \begin{cases} \left[\pi^2(n+m+1)^2\cos^2\frac{\chi}{2} + k^2d^2\sin^2\frac{\chi}{2}\right]\cos^2\frac{kd}{2}, & n+m \text{ is even}, \\ \left[\pi^2(n+m+1)^2\sin^2\frac{\chi}{2} + k^2d^2\cos^2\frac{\chi}{2}\right]\sin^2\frac{kd}{2}, & n+m \text{ — is odd}; \end{cases} \tag{30}$$

It is possible to obtain with their aid the number of absorbed sound-field quanta per unit time $n = n_1 + n_2$:

$$n_1 = 2\pi \sum_{\nu,\mu} |M_1|^2 (f(\varepsilon_\mu) - f(\varepsilon_\nu))\delta(\varepsilon_\nu - \varepsilon_\mu - \omega), \tag{31}$$

$$n_2 = 2\pi \sum_{\nu,\mu} |M_2|^2 [1 - f(\varepsilon_\nu)][1 - f(\varepsilon_\mu)]\delta(\varepsilon_\nu + \varepsilon_\mu - \omega). \tag{32}$$

The summation in (31) and (32) is over all the states of the discrete spectrum, specified by the quantum numbers q, n, and α. Integrating with respect to the momenta and assuming the condition $T < \pi v_0/d$ to be satisfied, we obtain the following expressions for n_1 and n_2,

$$n_1(\chi) = 4\omega^2 C F_1(\Omega, \lambda, \chi),$$

$$F_1(\Omega, \lambda, \chi) = \sum_{l=1}^{\infty} \frac{\operatorname{ch}(\lambda\Omega\chi/2\pi l)\Theta(l - \Omega)}{2[\pi^2 l^2 - k^2d^2]^2 l^2 \operatorname{sh}[\lambda\Omega/2l]} \cdot$$
$$\times \begin{cases} \pi^2 l^2 \sin^2\frac{kd}{2}, & l \text{ — is even}, \\ k^2d^2\cos^2\frac{kd}{2}, & l \text{ — is odd}, \end{cases} \tag{33}$$

$$n_2(\chi) = 2\omega^2 C F_2(\Omega, \lambda, \chi),$$

$$F_2(\Omega, \lambda, \chi) = \sum_{m=0}^{\infty}\sum_{l=1}^{\infty} \frac{\Theta(m + l - \Omega)}{[\pi^2(m+l)^2 - k^2d^2]^2 (m+l)^2} \cdot$$
$$\times \begin{cases} \left[\pi^2(m+l)^2\sin^2\frac{\chi}{2} + k^2d^2\cos^2\frac{\chi}{2}\right]\sin^2\frac{kd}{2}, & m+l \text{ — is even}, \\ \left[\pi^2(m+l)^2\cos^2\frac{\chi}{2} + k^2d^2\sin^2\frac{\chi}{2}\right]\cos^2\frac{kd}{2}, & m+l \text{ — is odd}. \end{cases} \tag{34}$$

We have introduced here the notation $\lambda = \pi v_0/Td$, $\Omega = \omega d/\pi v_0$, $C = g^2 m^2 d^2 L_x L_y/\pi^2 V$, where L_x and L_y are the linear dimensions in the plane of the S—N—S junction. Averaging (33) and (34) over the phase χ, we obtain ultimately

$$\langle n_1\rangle = 4\omega T C F_1(\Omega),$$

$$F_1(\Omega) = \sum_{l=1}^{\infty} \frac{\Theta(l-\Omega)}{(\pi^2 l^2 - k^2d^2)^2 l} \begin{cases} \pi^2 l^2 \sin^2(kd/2), & l \text{ — is even}, \\ k^2d^2\cos^2(kd/2), & l \text{ — is odd}; \end{cases}$$

$$\langle n_2\rangle = \omega^2 C F_2(\Omega), \tag{35}$$

$$F_2(\Omega) = \sum_{m=0}^{\infty}\sum_{l=1}^{\infty} \frac{\Omega(m+l-\Omega)[\pi^2(m+l)^2 + k^2d^2]}{[\pi^2(m+l)^2 - k^2d^2]^2 (m+l)^2}$$
$$\times \begin{cases} \sin^2(kd/2), & m+l \text{ — is even}, \\ \cos^2(kd/2), & m+l \text{ — is odd}. \end{cases} \tag{36}$$

As seen from the obtained formulas, $\langle n_2\rangle$, unlike $\langle n_1\rangle$, does not tend to xero with the temperature. In addition, the average number of absorbed phonons $\langle n\rangle$ is not a monotonic function of the frequency ω. At the points $\Omega = n$, n = 1, 2, 3, ... $F_1(\Omega)$ experiences jumps of magnitude

$$F_1(n+0) - F_1(n-0) = -\frac{1}{[\pi^2 n^2 - k^2d^2]^2 n}$$
$$\times \begin{cases} \pi^2 n^2 \sin^2(kd/2), & n \text{ — is even}, \\ k^2d^2\cos^2(kd/2), & n \text{ — is odd}. \end{cases}$$

The function $F_2(\Omega)$ behaves similarly. The condition $\Omega = n$ means that the energy of the sound quantum is a multiple of the distance between the Andreev levels. Thus, whenever the frequency of sound is a multiple of the distance between the Andreev levels $\pi v_0/d$, the average number of absorbed phonons in the S—N—S junction changes jumpwise. We emphasize once more that such a behavior appears in the presence of a weak magnetic field, when the density of states (Sec. 3) does not reveal any singularities as a function of the thickness d.

In conclusion we note that when a sound wave passes through an S—N—S junction, there occurs, besides absorption in the normal layer, also reflection of sound from the N—S boundary. The total transmission coefficient is

$$D = 1 - R - I/I_0,$$

where R is the reflection coefficient of the wave, I_0 is the initial energy flux density normalized to the total volume of the S—N—S junction, and I is the density of the absorbed energy. The non-monotonic character of the variation of the number of absorbed phonons with changing frequency is manifest in the fact that the transmission coefficient D will experience each time jumps whenever $\omega = n\pi v_0/d$. In the case of normal incidence of the sound wave on the N—S boundary, it is easy to estimate the reflection coefficient R. According to[11]

$$R = \left| \frac{Z_n^2 - Z_s^2}{Z_s^2 + Z_n^2 + 2iZ_sZ_n \operatorname{ctg} kd} \right|^2,$$

where $Z = \rho c$ is the impedance of the medium, ρ is the density of the medium, and c is the speed of sound. Assuming $\rho_S = \rho_n$, we get $R \approx |\Delta c/c|^2$, where Δc is the change of the velocity of sound on going over from the normal to the superconducting state; $\Delta c/c \sim 10^{-5}$[12], whence $R \approx 10^{-10}$. For the characteristic values of the parameters of the problem in question, this is much smaller than I/I_0. For example, when $\omega \sim \pi v_0/d$ and $d \sim 10^{-2}$ cm, assuming the dimensionless constant of the electron-phonon interaction to be of the order of unity, we obtain according to (36)

$$I/I_0 \sim \omega^2/cv_0k^2 \sim 10^{-3}.$$

APPENDIX

Assume that the condition $g/2 < k_0 < g$ is satisfied, but $k_0 > k + g/2$ (without loss of generality we assume $k > 0$). The quasi-classical wave functions inside the normal metal are given by

$$A_0(z) = \frac{\alpha_0^-}{\sqrt{k_0^+(z)}} \exp\left(-i\int_0^z k_0^+ dz\right) + \frac{\alpha_0^+}{\sqrt{k_0^+(z)}} \exp\left(i\int_0^z k_0^+ dz\right),$$

$$B_0(z) = \frac{\beta_0^-}{\sqrt{k_0^-(z)}} \exp\left(-i\int_0^z k_0^- dz\right) + \frac{\beta_0^+}{\sqrt{k_0^-(z)}} \exp\left(i\int_0^z k_0^- dz\right),$$

$$A_{-1}(z) = \frac{2iA}{\sqrt{k_{-1}^+(z)}} \sin\left(\int_z^{z_1} k_{-1}^+ dz + \frac{\pi}{4}\right), \quad -\frac{d}{2} < z < z_1(k),$$

$$B_{-1}(z) = \frac{2iB}{\sqrt{k_{-1}^-(z)}} \sin\left(\int_{-z_2}^z k_{-1}^- dz + \frac{\pi}{4}\right), \quad -z_2(k) < z < \frac{d}{2}, \tag{A.1}$$

$$A_1(z) = \frac{2iC}{\sqrt{k_1^+(z)}} \sin\left(\int_{-z_3}^z k_1^+ dz + \frac{\pi}{4}\right), \quad -z_3(k) < z < \frac{d}{2},$$

$$B_1(z) = \frac{2iD}{\sqrt{k_1^-(z)}} \sin\left(\int_z^{z_4} k_1^- dz + \frac{\pi}{4}\right), \quad -\frac{d}{2} < z < z_4(k),$$

where $k_n^{\pm}(z) = [k_0^2 - (k + ng \pm gz/d)^2 \pm 2ms]^{1/2}$; $\alpha_i^{\pm}$, $\beta_i^{\pm}$, A, B, C, D are certain constants, and $z_j(k)$ are the turning points in real space.

To the right of $z = d/2$, the solutions of (10) are of the form

$$\begin{pmatrix} A_0(z) \\ B_{-1}(z) \end{pmatrix} = \alpha_+^1 \exp\left\{i\lambda_0^+\left(z - \frac{d}{2}\right)\right\} \begin{pmatrix} 1 \\ \gamma_0 \end{pmatrix} + \alpha_+^2 \exp\left\{-i\lambda_0^-\left(z - \frac{d}{2}\right)\right\} \begin{pmatrix} \gamma_0 \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} A_1(z) \\ B_0(z) \end{pmatrix} = \beta_+^1 \exp\left\{i\lambda_0^+\left(z - \frac{d}{2}\right)\right\} \begin{pmatrix} 1 \\ \gamma_0 \end{pmatrix} + \beta_+^2 \exp\left\{-i\lambda_0^-\left(z - \frac{d}{2}\right)\right\} \begin{pmatrix} \gamma_0 \\ 1 \end{pmatrix}. \tag{A.2}$$

To the left of $z = -d/2$ we write

$$\begin{pmatrix} A_0(z) \\ B_1(z) \end{pmatrix} = \alpha_-^1 \exp\left\{i\lambda_0^-\left(z + \frac{d}{2}\right)\right\} \begin{pmatrix} 1 \\ \gamma_0 \end{pmatrix} + \alpha_-^2 \exp\left\{-i\lambda_0^+\left(z + \frac{d}{2}\right)\right\} \begin{pmatrix} \gamma_0 \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} A_{-1}(z) \\ B_0(z) \end{pmatrix} = \beta_-^1 \exp\left\{i\lambda_0^-\left(z + \frac{d}{2}\right)\right\} \begin{pmatrix} 1 \\ \gamma_0 \end{pmatrix} + \beta_-^2 \exp\left\{-i\lambda_0^+\left(z + \frac{d}{2}\right)\right\} \begin{pmatrix} \gamma_0 \\ 1 \end{pmatrix}, \tag{A.3}$$

with $\gamma_0 = \Delta_0/(\epsilon + i(\Delta_0^2 - \epsilon^2)^{1/2}$. The two-role matrix $\begin{pmatrix} 1 \\ \gamma_0 \end{pmatrix}$ describes the possibility of both ordinary reflection from the N—S boundary with change of the sign of the z component of the quasimomentum, and of scattering with change of the sign of all three velocity components. The parameters $\lambda_0^{\pm}$ characterize the damping of the wave in the superconducting regions. Joining together the wave functions on the boundaries $z = \pm d/2$, we arrive at the quantization conditions:

$$e^{-\pi i} \exp\left\{2i\int_{-d/2}^{d/2} k_0^+ dz\right\} \exp\left\{-2i\int_{-z_2}^{d/2} k_{-1}^- dz\right\} \exp\left\{-2i\int_{-d/2}^{z_4} k_1^- dz\right\} = 1,$$

$$e^{\pi i} \exp\left\{-2i\int_{-d/2}^{d/2} k_0^- dz\right\} \exp\left\{2i\int_{-z_3}^{d/2} k_1^+ dz\right\} \exp\left\{2i\int_{-d/2}^{z_1} k_{-1}^+ dz\right\} = 1. \tag{A.4}$$

The integrals in the argument of the exponential represent, when account is taken of the relation $dk_x = (eH/c)dz$, the areas bounded by the electron trajectory in momentum space. As a result we obtain the quasi-classical quantization condition (13).

Let us consider another possible case for the states with $g/2 < k_0 < g$. Let k, remaining in the first Brillouin zone, satisfy the condition $k_0 < k + g/2$. Then a similar calculation leads to the quantization condition

$$\gamma_0^2 \exp\left\{2i\int_{-z_0}^{d/2} k_0^+ dz\right\} \exp\left\{-2i\int_{-z_1}^{d/2} k_{-1}^- dz\right\} = 1. \tag{A.5}$$

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