POLARON ELECTRIC CONDUCTIVITY TENSOR IN A MAGNETIC FIELD

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The method used in [1] to calculate the electric conductivity tensor of polarons in a magnetic field is extended to the case of electrons obeying a nonquadratic dispersion law, where scattering both by lattice vibrations and by ionized and neutral impurities is taken into account. Allowance for the nonquadricity leads to a renormalized temperature-dependent effective mass in the expressions for the shift of the cyclotron resonance (c.r.) frequency ω_c, the polaron effect, and the relaxation times. Impurity scattering is considered; as examples, the longitudinal and transverse magnetoresistances are calculated and also the contributions to the half-widths of c.r. resonance lines. Cyclotron resonance of optical polarons is investigated at $\omega_c = \omega$ (the frequency of longitudinal optical phonons). The derived equations predict a resonant variation of the polaron mass, and describe a resonant increase of the c.r. line half-width at any arbitrary temperature. The theory is consistent with experimental data on c.r. in InSb.

IN our previous article[1] (which will be cited as I) we developed a method for calculating the electric conductivity tensor σ_{ik}[1)] of polarons in a magnetic field H for quadratic dispersion of electrons interacting with the phonon subsystem. In the present work we extend our method to the case of nonquadratic dispersion and to electron interaction with charged and neutral impurities. The chosen Lagrangian contains a nonparabolic term in the zeroth approximation. Electron scattering by impurities has not previously been considered in quantum mechanics within the framework of a Lagrangian formalism.

We must point out one feature of this problem. When the Lagrangian includes the potential of an isolated impurity we are not enabled to consider scattering, because this (dynamic) system does not possess dissipative properties. In this case we obtain localized states. Dissipation arises when we allow for the interaction of an electron with all impurity centers distributed randomly in the interior of a crystal.

We know [I, Eq. (46)] that at low temperatures and low energies polaron corrections to the mass are positive, but become negative at sufficiently high temperatures and energies. This effect occurs for both acoustic and optical interactions. The polaron correction associated with quadratic electron dispersion is given by Eq. (46) of I. The indicated sign change of the polaron correction has been observed experimentally[2] for electrons in InSb at $\omega_c = \omega$. In the present work this correction is obtained for the nonparabolic case and is compared with experiment.

When the optical vibration frequency ω equals ω_c we observe resonant broadening of cyclotron lines. A theory that allows only for the emission of optical phonons has been discussed in [3-5]. We shall here develop a theory for the temperature dependence of this effect and shall obtain the cyclotron line shape at an arbitrary temperature. (A temperature limitation arises only when powers of momentum higher than the fourth are neglected in the electron dispersion.)

I. POLARON MASS WITH ALLOWANCE FOR NONPARABOLICITY

$A_{III}B_V$ compounds, of which InSb is a typical representative, possess a narrow forbidden band (bandgap) and the Hamiltonian of the electron must include higher powers of its momentum than the second:[6]

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m^*} + \frac{K_2}{E_g}\frac{\mathbf{p}^4}{4m^{*2}} \pm K_0\frac{\hbar\omega_c^{(0)}}{E_g}\frac{\mathbf{p}_\perp^2}{2m^*} \pm K_1\frac{\hbar\omega_c^{(0)}}{E_g}\frac{p_3^2}{2m^*} + V(r) \qquad (1)$$

Here $\mathbf{p} = -i\hbar\nabla - e\mathbf{A}/c$, $\hbar\omega_c^{(0)} = eH/m^*c$, E_g is the width of the forbidden band, and m^* is the effective mass at $H = 0$ and $p = 0$. The values of the coefficients K_0, K_1, K_2 for the two-band approximation are given in [6]; the symbol "±" corresponds to spin $\pm\frac{1}{2}$. Interaction between the spin and magnetic field is omitted as unimportant in cyclotron resonance; $V(r)$ is the energy of the interaction with impurities.

The Lagrangian corresponding to (1) is

$$\mathcal{L}(\mathbf{r}, t) = \frac{m_\perp}{2}\dot{\mathbf{r}}_\perp^2 + \frac{m_\parallel}{2}\dot{r}_3^2 - \frac{e}{c}\dot{r}_i A_i - \frac{K_2 m^{*2}}{4E_g}\dot{\mathbf{r}}^4 - V(r). \qquad (2)$$

Here

$$m_\perp = m^*\left(1 \mp K_0\frac{\hbar\omega_c^{(0)}}{E_g}\right), \quad m_\parallel = m^*\left(1 \mp K_1\frac{\hbar\omega_c^{(0)}}{E_g}\right).$$

The trial action $S_0(\mathbf{r})$ is selected as in Eq. (17) of I; now, in addition to the polaron effect and dissipation, nonparabolicity is taken into account in the effective masses. Therefore Eqs. (27)–(29) of I are again obtained for σ_{ik}, except that the trace $g(\tau - \sigma)$ is here expanded using a difference between functionals:

$$F(\mathbf{r}, \mathbf{r}') - F_0(\mathbf{r}, \mathbf{r}') = \Phi(\mathbf{r}, \mathbf{r}') + \frac{m_\parallel - m_{33}}{2}\int_{-\infty}^{\infty} dt\,[\dot{r}_3^2(t) - \dot{r}_3'^2(t)]$$
$$+ \sum_{i=1}^{2}\frac{m_\perp - m_{ii}}{2}\int_{-\infty}^{\infty} dt\,[\dot{r}_i^2(t) - \dot{r}_i'^2(t)] + \frac{K_2 m^{*2}}{4E_g}\int_{-\infty}^{\infty} dt\,[\dot{\mathbf{r}}^4(t) - \dot{\mathbf{r}}'^4(t)]. \qquad (3)$$

We shall now consider the calculation of the contribution to

1)The notation of I is used.

$$G_{ik}^{(1)}=\frac{i}{2e^2}\frac{\partial^2}{\partial\zeta_i\partial\varepsilon_k}g_0\langle F-F_0\rangle_0$$

that is derived from the last, nonparabolic, term in (3), where the fourth power of $\dot{r}$ will be represented by

$$\int_{-\infty}^{\infty}dt\,\dot r_i^2(t)\dot r_k^2(t)=\int_{-\infty}^{\infty}dt\,ds\,dt'\,ds'\,\delta(t-t')\delta(t-s)\delta(t'-s')\times\frac{\partial^4}{\partial s^2\partial s'^2}r_i(t)r_i(s)r_k(t')r_k(s'). \quad (4)$$

Here we must first differentiate with respect to s and s', and then integrate taking the δ functions into account. Continual averaging of (4) is achieved by means of the generating function

$$\Psi^{(1)}_{\varkappa_i\varkappa_k}(\xi,\eta,\xi',\eta')=\langle\exp[i\varkappa_i(\xi r_i(t)-\eta r_i(s))+i\varkappa_k(\xi' r_k(t')-\eta' r_k(s'))]\rangle_0 \quad (5)$$

and the formula

$$\langle r(t)r_i(s)r_k(t')r_k(s')\rangle_0=\frac{1}{\varkappa_i^2\varkappa_k^2}\frac{\partial^4}{\partial\xi\,\partial\eta\,\partial\xi'\,\partial\eta'}\Psi^{(1)}_{\varkappa_i\varkappa_k}(\xi,\eta,\xi'\eta')\,|_{\xi=\eta=\xi'=\eta'=0}. \quad (6)$$

Calculating $\Psi^{(1)}_{\varkappa_i\varkappa_k}$ like $\Psi^{(1)}_{\varkappa_i}$ in I, we obtain for i ≠ k

$$\Psi^{(1)}_{\varkappa_i\varkappa_k}(\xi,\eta,\xi',\eta')=\Psi^{(1)}_{\varkappa_i}(\xi,\eta)\Psi^{(1)}_{\varkappa_k}(\xi',\eta'), \quad (7)$$

where $\Psi^{(1)}_{\varkappa_i}(\xi,\eta)$ is given in I [Eqs. (37)–(39)]; for i = k

$$\Psi^{(1)}_{\varkappa_i\varkappa_k}(\xi,\eta,\xi'\eta')=\Psi^{(1)}_{\varkappa_i}(\xi,\eta)\Psi^{(1)}_{\varkappa_i}(\xi'\eta')\prod_{i=1}\exp\Big(\frac{i\hbar\varkappa_i^2}{2\pi m^*}\times\int_{-\infty}^{\infty}d\nu\{[\xi\xi'\cos\nu(t-t')-\xi\eta'\cos\nu(t-s')-\xi'\eta\cos\nu(t'-s)+\eta\eta'\cos\nu(s-s')][J(\omega_{ii},\nu)+iA(\omega_{ii},\nu)]\}\Big). \quad (8)$$

Calculating $G_{ik}^{(1)1}(\nu)$ with the aid of (4)–(8), from the condition $G_{ik}^{(1)}(\nu)=0$ [I, Eq. (20)] we obtain the components of the effective mass tensor:

$$\frac{M_{ii}}{m^*}=1+\frac{\Delta m_{ii}^*}{m^*}-\mathrm{Im}\int_0^{\infty}dt(1-\cos\nu t)S_{ii}(t), \quad (9)$$

where $\Delta m_{ii}^*/m^*$, which describes the contribution from the nonparabolic term, has the values

$$\frac{\Delta m_\perp^*}{m^*}=\frac{\hbar\omega_c^{(0)}}{E_g}\Big[-2K_2\Big(\mathrm{cth}\frac{\lambda\hbar\omega_c^{(0)}}{2}+\frac{1}{2\lambda\hbar\omega_c^{(0)}}\Big)\mp K_0\Big], \quad (10a)$$

$$\frac{\Delta m_\parallel^*}{m^*}=\frac{\hbar\omega_c^{(0)}}{E_g}\Big[-K_2\Big(\mathrm{cth}\frac{\lambda\hbar\omega_c^{(0)}}{2}+\frac{3}{2\lambda\hbar\omega_c^{(0)}}\Big)\mp K_1\Big]. \quad (10b)$$

The last term in (9) describes the contribution from the interaction with lattice vibrations and differs from (43) of I by taking nonparabolicity into account in the effective masses that appear in the function D(t).

We note that at high electron energies sixth-order terms begin to play a role in the dispersion law. However, for most $A_{III}B_V$ compounds, except InSb, the utilized approximation is quite adequate when considering cyclotron resonance.

2. INTERACTION WITH THE IMPURITY SUBSYSTEM

The interaction of an electron with impurities leads to impurity contributions in the parameters M_{ii} and Δ_{ii} of the trial action.

1. Let us consider the bound states of an electron. In (2) we insert

$$V(r)=-e^2/\varepsilon_0 r \quad (11)$$

and into the trial Lagrangian we introduce a linear oscillator with frequency w along the z axis, to simulate a localized electronic state in a high magnetic field. The frequency w can be found from the condition

$$G^{(1)}_{33,\,\mathrm{imp}}(\tau-\sigma)=\frac{i}{2e^2}\frac{\partial^2}{\partial\zeta_3\partial\varepsilon_3}g_0(\tau-\sigma)\times\Big\langle\frac{m^*w^2}{2}\int_{-\infty}^{\infty}dt[r_3^2(t)-r_3'^2(t)]+\frac{e^2}{\varepsilon_0}\int_{-\infty}^{\infty}dt\Big[\frac{1}{r(t)}-\frac{1}{r'(t)}\Big]\Big\rangle_0. \quad (12)$$

The requisite calculations yield

$$w^2=\frac{4e^2m^{*1/2}}{\pi^{1/2}\varepsilon_0\hbar^{3/2}}\int_0^{\pi/2}d\vartheta\sin\vartheta\cos^2\vartheta\eta^{-3/2}, \quad (13)$$

where

$$\eta=\frac{1}{\omega_c}\sin^2\vartheta\,\mathrm{cth}\frac{\lambda\hbar\omega_c}{2}+\frac{m_\perp^*}{m_\parallel^*}\frac{1}{w}\cos^2\vartheta\,\mathrm{cth}\frac{\lambda\hbar w}{2},\qquad \omega_c=eH/m_\perp^*c.$$

For H = 0, ω_c = w (changing to a three-dimensional oscillator), and $\lambda\to\infty$ we obtain $\hbar w = 32E_0/9\pi$, where $E_0 = m^*e^4/2\hbar^2\epsilon_0^2$ is the ionization energy of an impurity center.

We note that w can be obtained by maximizing the trial sum of impurity center states, using the model of Coulomb interaction represented by an oscillator with the frequency w and regarding w as a variational parameter.[7] Both procedures yield the same result for w.

The impurity effective mass is determined from the relation $G^{(1)}_{11,\mathrm{imp}}(\nu)=0$, which gives

$$\Big(\frac{\Delta m_\perp}{m^*}\Big)_{\mathrm{imp}}=-\frac{2m^{*1/2}e^2}{\pi^{1/2}\varepsilon_0\hbar^{3/2}\omega_c^2}\int_0^{\pi/2}d\vartheta\sin^3\vartheta\eta^{-3/2}. \quad (14)$$

2. For a free electron the interaction with all impurities is

$$V(r)=\sum_i V_i(r),\qquad V_i(r)=V_{1i}(r)+V_{2i}(r)+V_{3i}(r). \quad (15)$$

Here

$$V_{1i}(r)=-\frac{e^2}{\varepsilon_0|\mathbf{R}_i-\mathbf{r}|}e^{-qr} \quad (16a)$$

is the energy of the interaction between the electron and a screened Coulomb impurity; $\mathbf{R}_i$ is the radius vector of the impurity center; q is the reciprocal Debye radius;

$$V_{2i}(r)=-\frac{e^2}{\varepsilon_0|\mathbf{R}_i-\mathbf{r}|}+\frac{e^2}{\varepsilon_0|\mathbf{r}_i-\mathbf{r}|} \quad (16b)$$

is the energy of interaction with a hydrogen-like impurity center; $\mathbf{r}_i$ is the radius vector of a bound electron;

$$V_{3i}(r)=-V_0\exp\Big(-\frac{|\mathbf{r}-\mathbf{R}_i|^2}{4r_0^2}\Big) \quad (16c)$$

is the energy of interaction with a deep neutral impurity center, where V_0 and R_0 are parameters of the impurity potential.

Regarding V(r) as a perturbation, we add to $F-F_0$ in (3) the term

$$F_{\mathrm{imp}}(r,r')=-\int_{-\infty}^{\infty}dt[V(r)-V(r')]. \quad (17)$$

Since this functional is real, the first-order correction describes certain mass and energy changes that result

from the shift of the bottom of the conduction band but does not contribute to scattering. To obtain τ_{imp} we must continue the expansion (18) in I:

$$G_{ik}(\tau-\sigma)=G_{ik}^{0}(\tau-\sigma)+G_{ik}^{(1)}(\tau-\sigma)+G_{ik}^{(2)}(\tau-\sigma)+\dots \quad (18)$$

and calculate

$$G_{ik}^{(2)}(\tau-\sigma)=\frac{\hbar}{2e^2}\frac{\partial^2}{\partial\xi_i\partial\varepsilon_k}g_0(\tau-\sigma)\frac{1}{2}\left(\frac{i}{\hbar}\right)^2\langle(F-F_0)^2\rangle_0. \quad (19)$$

where only F_{imp} (17) is retained in the difference $F-F_0$.

The impurity relaxation times are obtained from

$$G_{ik}^{(1)}(\nu)+G_{ik}^{(2)}(\nu)=0. \quad (20)$$

a) In a Fourier representation $F^2_{imp}(\mathbf{r},\mathbf{r}')$ for screened Coulomb impurities is given by

$$F^2_{imp}(\mathbf{r},\mathbf{r}')=\frac{2N_ie^4}{\pi\varepsilon_0^2}\int_{-\infty}^{\infty}dt\,ds\int\frac{d\varkappa}{(\varkappa^2+q^2)^2}[\exp(i\varkappa,\mathbf{r}_t-\mathbf{r}_s)-\exp(i\varkappa,\mathbf{r}_t'-\mathbf{r}_s)-\exp(i\varkappa,\mathbf{r}_t-\mathbf{r}_s')+\exp(i\varkappa,\mathbf{r}_t'-\mathbf{r}_s')]. \quad (21)$$

Continual averages of $\exp(i\kappa,\ \mathbf{r_t}-\mathbf{r_s})$ and the other exponents are calculated by means of the generating functions $\Psi_K^{(i)}(\xi,\ \eta)$ given in I. From (20), using (21), we obtain

$$\frac{1}{\tau_{ii}}=\frac{4N_ie^4}{\pi\hbar\varepsilon_0^2m^*\nu}\operatorname{sh}\frac{\lambda\hbar\nu}{2}\int d\varkappa\frac{\varkappa_i^2}{(\varkappa^2+q^2)^2}\times\int_0^{\infty}dt\cos\nu t\exp\left[-\frac{\hbar\varkappa^2}{m_\perp^*\omega_c}D\left(t+\frac{i\lambda\hbar}{2}\right)\right], \quad (22)$$

where

$$D(t)=\frac{\sin^2\vartheta}{2}\left[\operatorname{cth}\frac{\lambda\hbar\omega_c}{2}-\frac{\cos\omega_c(t-i\lambda\hbar/2)}{\operatorname{sh}^{1}/_{2}\lambda\hbar\omega_c}\right]+\frac{m_\perp^*}{m_\parallel^*}\frac{\cos^2\vartheta}{2}\left[\frac{\lambda\hbar\omega_c}{4}+\frac{\omega_c}{\lambda\hbar}\left(t-\frac{i\lambda\hbar}{2}\right)^2\right],$$

and N_i is the concentration of ionized impurities.

Equation (18) determines the relaxation time at arbitrary temperatures and magnetic field strengths. For example, with $\nu = 0$ and in the absence of a magnetic field we have

$$\frac{1}{\tau(0)}=\frac{2}{3}\frac{(2\pi)^{1/2}N_ie^4\lambda^{3/2}}{\varepsilon_0^2m^{*1/2}}\ln\left(\frac{8m^*}{\hbar^2q^2\lambda\gamma}\right), \quad (23)$$

where $\gamma = e^C$ is Euler's constant. Equation (19) agrees with a quantum mechanical calculation averaged over a Maxwellian distribution.

In high magnetic fields at low temperatures we have $\lambda\hbar\omega_c/2 \gg 1$, and with $\nu = 0$ we obtain from (22) for the transverse magnetoresistance [I, Eq. (57)]

$$\frac{\rho_\perp(H)}{\rho(0)}=\frac{3}{4}\left(\frac{m_\parallel^*}{m^*}\right)^{1/2}\frac{\ln(2/q^2l^2\gamma)}{\ln(8m^*/\hbar^2q^2\lambda\gamma)}K_0\left(\frac{\lambda E}{2}\right). \quad (24)$$

The dependence of $\rho_\perp$ on H and T agrees with the result obtained by Adams and Holstein for this case.[8] Methods of cutting off Born divergences of the energy E are given in [8-10]. Within the framework of our present method it is also possible to consider a non-Born approximation in the scattering. We shall describe a method of determining E without showing details of the calculation. In zeroth approximation the cutting-off interaction is simulated by an oscillator with frequency w_0. For small w_0 we represent D(t) by a series in $(w_0t)^2$, which leads to a cutoff at characteristic energies E $\sim \hbar w_0$. For example, when the electron-electron interaction is taken into account we have $w_0 = \omega_p$ (the plasma frequency).

Under the same conditions as in (24) the longitudinal magnetoresistance is given by

$$\frac{\rho_\parallel(H)}{\rho(0)}=\frac{3}{4}\left(\frac{m_\parallel^*}{m^*}\right)^{1/2}\frac{1}{1-a}\left[1-a\frac{\ln(2/q^2l^2\gamma)}{\ln(8m^*/\hbar^2q^2\lambda\gamma)}\right],\quad a=\frac{4m_\parallel^*c}{\lambda\hbar eH}. \quad (25)$$

When $a < 1$ this equation gives negative magnetoresistance.[11] The temperature and field dependences of $\rho_\parallel(H)$ agree with those given in [8]. The value of $(\omega_c\tau_\perp)^{-1}$ that is required for calculating the c.r. line halfwidth when $\nu = \omega_c$, $\lambda\hbar\omega_c/2 > 1$, $l^2q^2/2 \ll 1$ is

$$\frac{1}{\omega_c\tau_\perp}=(2\pi)^{1/2}\frac{N_ie^4\lambda^{1/2}m_\parallel^{*1/2}}{\varepsilon_0^2\omega_c^2\hbar m^*}K_0\left(\frac{\lambda E}{2}\right). \quad (26)$$

b) The Fourier component $F^2_{imp}(\mathbf{r},\mathbf{r}')$ for the interaction of an electron with hydrogen-like centers (16b) has the form

$$F^2_{imp}(\mathbf{r},\mathbf{r}')=\frac{2N_de^4}{\pi\varepsilon_0^2}\int\frac{d\varkappa}{\varkappa^4}\langle\{\exp(i\varkappa,\mathbf{r}_t-\mathbf{r}_s)[1-2\exp(-i\varkappa\rho_{ti})+\exp(-i\varkappa,\rho_{ti}-\rho_{si})]-\exp(i\varkappa,\mathbf{r}_t'-\mathbf{r}_s)[1-2\exp(-i\varkappa\rho_{ti})+\exp(-i\varkappa,\rho_{ti}'-\rho_{si})]-\exp(i\varkappa,\mathbf{r}_t-\mathbf{r}_s')[1-2\exp(-i\varkappa\rho_{ti})+\exp(-i\varkappa,\rho_{ti}-\rho_{si}')]+\exp(i\varkappa,\mathbf{r}_t'-\mathbf{r}_s')[1-2\exp(-i\varkappa\rho_{ti})+\exp(-i\varkappa,\rho_{ti}'-\rho_{si}')]\}\rangle_{F_0(\rho_i,\rho_i')}, \quad (27)$$

where $\rho_i = \mathbf{R}_i - \mathbf{r}_i$, and N_d is the donor concentration.

Impurity electron states are taken into account in the zeroth order functional $F_0(\rho_i, \rho_i')$ by following the procedure described in Par. 1 of this Section. Calculating τ_{ii}^{-1} from (20) and using (27), we obtain

$$\frac{1}{\tau_{ii}}=\frac{4N_de^4}{\pi\hbar\varepsilon_0^2m^*\nu}\operatorname{sh}\frac{\lambda\hbar\nu}{2}\int d\varkappa\frac{\varkappa_i^2}{\varkappa^4}\int_0^{\infty}dt\cos(\nu t)\exp\left[-\frac{\hbar\varkappa^2}{m_\perp^*\omega_c}\times D\left(t+\frac{i\lambda\hbar}{2}\right)\left\{1-2\exp\left(-\frac{\hbar\varkappa^2}{4m_\perp^*}\eta\right)+\exp\left[-\frac{\hbar\varkappa^2}{m_\perp^*\omega_c}D_{\text{пр}}\left(t+\frac{i\lambda\hbar}{2}\right)\right]\right\}\right], \quad (28)$$

where

$$D_{imp}(t)=\frac{\sin^2\vartheta}{2}\left[\operatorname{cth}\frac{\lambda\hbar\omega_c}{2}-\frac{\cos\omega_c(t-i\lambda\hbar/2)}{\operatorname{sh}^{1}/_{2}\lambda\hbar\omega_c}\right]+\frac{m_\perp^*}{m_\parallel^*}\frac{\omega_c}{w}\frac{\cos^2\vartheta}{2}\left[\operatorname{cth}\frac{\lambda\hbar w}{2}-\frac{\cos w(t-i\lambda\hbar/2)}{\operatorname{sh}^{1}/_{2}\lambda\hbar w}\right]. \quad (29)$$

At very low temperatures, when $\lambda\hbar\omega_c/2 \gg 1$ and $\lambda\hbar w/2 \gg 1$, by neglecting terms with cos $\omega_c t$ and cos wt in $D_{imp}(t + i\lambda\hbar/2)$ we obtain an equation for scattering by neutral hydrogen-like impurities with an interaction potential that is averaged over the motion of a bound electron.

At higher temperatures, when $\lambda\hbar w/2 \lesssim 1$, the process of scattering with ionization of a center (inelastic collision) becomes effective. Expanding in this case with respect to η and D_{imp}, we obtain [$\nu = 0$, $H = 0$, $\omega_c = w$ (three-dimensional oscillator)]

$$\frac{1}{\tau}=\frac{8}{3}\frac{(2\pi)^{1/2}N_de^4\lambda^{3/2}}{\varepsilon_0^2m^{*1/2}(\lambda\hbar w)^{1/2}}e^{-\lambda^2w}. \quad (30)$$

With $\lambda\hbar w/2 < 1$, when the exponential terms inside the curly brackets in (28) are smaller than unity and can be neglected, only scattering by an ion remains in (28).

We also give the equation for $(\omega_c\tau_\perp)^{-1}$ subject to the limitations $\frac{1}{2}\lambda\hbar\,|\,\nu - w\,| > 1$, $\nu = \omega_c$:

$$\frac{1}{\omega_c\tau_\perp}=\frac{2^{1/2}\pi N_de^4}{m^{*1/2}\varepsilon_0^2\omega_c^{3/2}w\hbar^{1/2}}\int_0^{\infty}\frac{dy}{\sqrt{1+y}}\frac{y^2+2y}{(y^2-2y+a)^2}. \quad (31)$$

$$\times \exp\left\{-\frac{\lambda\hbar|\omega_c - w|}{2} y\right\}.$$

c) Performing calculations similar to those in cases a) and b), we obtain the following result for the components of the relaxation time tensor:

$$\frac{1}{\tau_{ii}} = \frac{16NV_0^2 r_0^6}{\hbar m^* \nu} \operatorname{sh}\frac{\lambda\hbar\nu}{2} \int dx\, x_i^2 e^{-2x^2 r_0^2} \int_0^\infty dt \cos(\nu t) \qquad (32)$$

$$\times \exp\left[-\frac{\hbar x^2}{m_\perp^* \omega_c} D\left(t + \frac{i\lambda\hbar}{2}\right)\right].$$

When $\nu = 0$, $H = 0$, $\omega_c = w$ we obtain

$$\frac{1}{\tau(0)} = \frac{4\pi^{3/2}}{3} \frac{NV_0^2 \lambda^2 r_0 \hbar}{m^*} \left(\frac{8m^* r_0^2}{\hbar^2\lambda}\right)^{5/2} \left(1 + \frac{16m^* r_0^2}{\hbar^2\lambda}\right)^{-2}. \qquad (33)$$

We note that with $16m^* r_0^2/\hbar^2\lambda \ll 1$, $1/\tau \sim T^{1/2}$ the temperature dependence is the same as for piezoelectric scattering.

At low temperatures, with $\lambda\hbar\omega_c/2 \gg 1$, $\nu = \omega_c$, we have

$$\frac{1}{\tau_\perp(H)} = \frac{1}{\tau(0)} \frac{3\lambda\hbar\omega_c}{16} \frac{m_\perp^{*2} m_\parallel^{*1/2}}{m^{*5/2}} K_0\left(\frac{\lambda E}{2}\right). \qquad (34)$$

Without presenting the results completely, we note that with $\nu = 0$, $\lambda\hbar\omega_c/2 \gg 1$ we have $\rho_\parallel(H) \sim HT^{-1/2}$, but $\rho_\perp(H) \sim H^2 T^{-3/2}$, in agreement with the results obtained by Adams and Holstein.[8]

Unlike the results given in [8,11], with which our equations have been compared in special cases, the general expressions for τ_{ii}^{-1} [Eqs. (22), (28), and (32)] are valid for arbitrary electric field frequencies ν, magnetic field strengths H, and temperatures T.

3. CYCLOTRON RESONANCE IN InSb

Nonparabolicity, the polaron mass shift at $\omega_c = \omega$, and the resonance broadening of absorption lines were observed in InSb by the authors of [12,2]. Cyclotron line broadening was predicted by Harper[4] and by one of the present authors and Kabisov,[3] and has been discussed in detail by Korovin.[5]

For the real part of $\sigma_{ii}(\nu)$, determined from Eq. (27) of I, we obtain

$$\operatorname{Re}\sigma_{ii}(\nu) = \frac{e^2\tau_{ii}}{M_{ii}} \frac{\nu^2\tau_{ll}^2 + \Omega_c\tau_{ii}\tau_{ll} + 1}{(\Omega_c^2 - \nu^2)^2\tau_{ii}^2\tau_{ll}^2 + \nu^2(\tau_{ii}^2 + \tau_{ll}^2) + 2\Omega_c^2\tau_{ii}\tau_{ll} + 1},$$
$$\Omega_c^2 = \frac{e^2H^2}{c^2 M_{11} M_{22}}; \qquad i,l = 1,2, \quad i \neq l. \qquad (35)$$

In the isotropic approximation, when $\sigma_{11} = \sigma_{22}$, for the half-width (in units of the magnetic field) Eq. (35) gives

$$\Delta H = 2H/\Omega_c\tau_\perp(H). \qquad (36)$$

We obtain the contribution of optical phonons to the half-width by means of Eq. (47) of I, derived for arbitrary fields and temperatures, with the substitution of $|V_{Kj}|^2$ as given by Eq. (54) of I. Confining ourselves to the inequality $\lambda\hbar\omega_c/2 \gtrsim 1$, which is sufficient for the discussion of the experimental work in [2], and using the notation $\Omega = \lambda\hbar\omega/2$, $\Omega_c = \lambda\hbar\omega_c/2$, $b(x) = \sqrt{a + (1-a)x^2}$, we calculate

$$\frac{1}{\tau_\perp(H)} = \frac{a\omega}{2} \frac{m_\perp^* m_\parallel^{*1/2}}{m^{*3/2}} \left(\frac{\lambda\hbar\omega}{\pi}\right)^{1/2} e^{\Omega_c - \Omega} \Big\{ K_0(|\Omega_c - \Omega|) + K_0(\Omega_c + \Omega)$$
$$+ e^{-\Omega_c}[2K_0(\Omega) + K_0(|2\Omega_c - \Omega|)]$$
$$- a\int_0^1 \frac{dx}{bx} \Big[|\Omega_c - \Omega| K_1\left(b(x)\frac{|\Omega_c - \Omega|}{x}\right) \qquad (37)$$
$$+ |\Omega_c + \Omega| K_1\left(b(x)\frac{\Omega_c + \Omega}{x}\right)\Big] - a^2 e^{-\Omega_c} \int_0^1 dx \frac{1-x^2}{xb^2(x)}$$
$$\times \left[\Omega^2 K_2\left(b(x)\frac{\Omega}{x}\right) + \frac{1}{2}(2\Omega_c - \Omega)^2 K_2\left(b(x)\frac{|2\Omega_c - \Omega|}{x}\right)\right]\Big\}.$$

Here we consider the first term, which describes electronic transitions with phonon emission. With $\omega_c < \omega$ and $\frac{1}{2}\lambda\hbar|\omega_c - \omega| > 1$ we have the asymptotic form $K_0(z) \approx \sqrt{\pi/2z}e^{-z}$, which gives

$$\Delta H \sim e^{-\lambda\hbar(\omega - \omega_c)}. \qquad (38)$$

According to (38), to the left of the resonance point the half-width falls off exponentially as H decreases. This is accounted for by a reduction in the number of electrons having energies $p_3^2/2m_\parallel^* > \hbar(\omega - \omega_c)$, which can emit an optical phonon following a cyclotron transition. For $\omega_c > \omega$ and $\lambda\hbar(\omega_c - \omega)/2 > 1$ we have

$$\Delta H \approx H\alpha \frac{\omega}{\omega_c}\left(\frac{\omega}{\omega_c - \omega}\right)^{1/2}. \qquad (39)$$

The limit in (39) agrees with Harper's theoretical result assuming zero temperature. The half-width to the right of $\omega_c = \omega$ in (39) is considerably greater than to the left in (38), because with $\omega_c > \omega$ each electron, following a transition, can emit an optical phonon. The magnitude of ΔH is independent of the temperature. Temperature corrections arising out of K_0 in (37) do not increase, but rather decrease ΔH; this effect is associated with the temperature spread of the electrons in the zeroth Landau band.

For $\omega_c \sim \omega$ and $\lambda\hbar(\omega_c - \omega)/2 < 1$ we can in the Macdonald functions limit ourselves to the first term of the series, $K_0(z) \approx \ln(2/\gamma z)$, thus obtaining

$$\Delta H \approx H\alpha \frac{\omega}{\omega_c}\left(\frac{\lambda\hbar\omega}{\pi}\right)^{1/2} \ln\left(\frac{4}{\gamma\lambda\hbar|\omega_c - \omega|}\right). \qquad (40)$$

For $\omega_c = \omega$ this expression gives a logarithmic divergence, which can be eliminated by allowing for phonon dispersion (at low concentrations of electrons) or renormalization of optical vibrations as a result of electron-electron interactions (at high electron concentrations).[13]

A theory, more complete than Harper's, of cyclotron line resonance broadening has been developed by Korovin[5] for $T \neq 0$, but taking into account the instability of only the n = 1 state; this involves the electronic transition from the first Landau level (n = 1) to the ground level (n = 0) with phonon emission.

The first term in (37) describes processes with phonon emission, as in Korovin's paper. The second term, which is proportional to $K_0(\Omega_c + \Omega)$, makes a contribution $\sim(\omega/(\omega_c + \omega))^{1/2} e^{-\lambda/\hbar\omega}$ to ΔH that is associated with phonon absorption and a transition from the n = 0 to the n = 1 state. The term containing $K_0(\Omega)$ describes intraband (n = 0) absorption of an optical phonon with a change of electron kinetic energy.

At high temperatures, when $\lambda\hbar\omega_c/2 \sim 1$, we must in cyclotron resonance also take into account transitions from n = 1 to n = 2. Instability of the n = 2 level against phonon emission results in a contribution to the cyclotron resonance line width that is described by the term $\sim K_0|2\Omega_c - \Omega|$ and that becomes especially important under the resonance condition $2\omega_c = \omega$.

In [2] the cyclotron resonance line width of electrons in InSb was measured at 15° and 88°K in a broad range

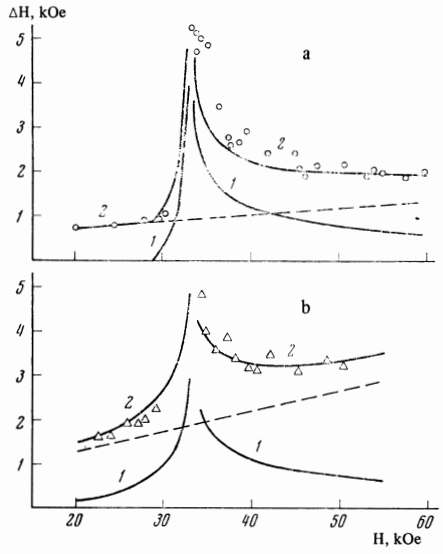

FIG. 1. Half-width ΔH of cyclotron resonance line versus magnetic field strength at (a) T = 15° and (b) T = 88°K. Curve 1—contribution from scattering by optical vibrations, $\alpha = 0.014$; 2—total half-width including the "background" (denoted by a dashed line); O—experimental data [2] for $N = 1.13 \times 10^{14}$ cm^{-3}, Δ—for $N = 5.7 \times 10^{13}$ cm^{-3}.

of magnetic fields from 20 to 60 kG. In Fig. 1a, which shows the magnetic field dependence of the half-width at 15°K, curve 1 represents the contribution of the optical interaction that leads to resonance broadening at $\omega_c = \omega = 3.65 \times 10^{13}$ sec^{-1}. The remaining interactions form a "background" (nonresonance broadening), whose contribution is represented by a dashed line. The resultant curve 2 agrees with the experimental data (represented by small circles). Figure 1a shows that in weak fields the entire half-width results from the "background." Since this broadening has no singularities its asymptote (the dashed line) is easily constructed.

The calculated nonresonance part of the broadening was calculated from (26) with allowance for scattering by ionized impurities, whose concentration is given in the discussed experimental work, and by the deformation potential, for which

$$\frac{1}{\omega_c \tau_\perp} = \frac{a_0^2}{(2\pi)^{3/2}} \frac{m_\parallel^{*1/2} m_\perp^{*2}}{2\rho m^* w^2 \hbar^3 \lambda^{1/2}} \ln\left(\frac{2a}{\lambda m_\parallel^* w^2 \gamma}\right), \qquad (41)$$

where a_0 is the constant of the deformation potential. To these two interactions it was necessary to add scattering by small-radius impurities, whose concentration and other parameters were chosen to obtain agreement with the control measurements of mobility at H = 0 and T = 77°K that are given in [2]. Although the experimental data for different samples do not agree, subtraction of the background yields full agreement, as is shown by curve 1.

A similar comparison with experiment was carried out at 88°K (Fig. 1b). Although at 15°K the main contribution to the half-width comes from the first term in (37), the remaining terms also begin to play an appreciable role at 88°K and double the width in low fields (H = 20 kG). For comparison with the theory we selected data[2] that had been obtained from a sample with the electron concentration $N = 5.7 \times 10^{13}$ cm^{-3}. We note, however, that the experimental data obtained at 88°K

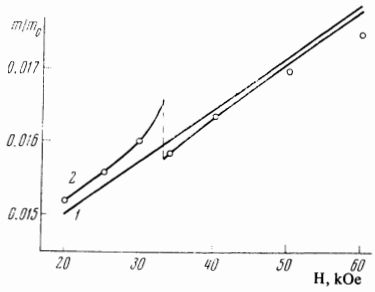

FIG. 2. Cyclotron mass as a function of the magnetic field at 15°K and m* = 0.0135 m_0; curve 1—omitting the polaron effect, curve 2—the polaron effective mass $M_\perp/m_0$; experimental data from [2].

are not very reliable; this is shown by the nonagreement of the optical contributions for different samples.

In [2] a change of sign was also observed in the polaron correction to the mass at $\omega_c \approx \omega$, when, according to (9), for the optical interaction we have

$$\frac{M_\perp - m_\perp^*}{m^*} = \frac{\pi^{1/2}}{4} \alpha \frac{\omega}{\omega_c} (\lambda\hbar\omega)^{1/2} \frac{m_\perp^* m_\parallel^{*1/2}}{m_*^{3/2}} \times \left[\operatorname{sign}(\omega - \omega_c) + 1 - \frac{2/\sqrt{\lambda\hbar\omega_c}}{1 + 2/\sqrt{\lambda\hbar\omega_c}}\right]. \qquad (42)$$

For $\omega > \omega_c$ we have $\operatorname{sign}(\omega - \omega_c) = 1$, while for $\omega < \omega_c$ we have $\operatorname{sign}(\omega - \omega_c) = -1$; (42) is thus discontinuous at $\omega = \omega_c$.

The plot of (9) in Fig. 2 (T = 15°K) agrees well with the experimental points except in very high fields. The break in the curve at resonance is accounted for by phonon emission, which becomes possible for $\omega_c \geq \omega$. The deviation of the theoretical curve from experiment at high fields is attributed to the neglect of sixth order momentum terms in the electron dispersion law.

We note in conclusion that in [2] cyclotron resonance of bound electrons was also observed, whose absorption band peaks are shifted towards lower fields;[14,15] this corresponds to a reduction of the cyclotron mass. The calculation of this effect by means of (14) gives $(\Delta m_\perp/m^*)_{imp} = -0.05$ as the change of the impurity mass at 20 kG and −0.03 at 60 kG, in agreement with experiment.

[1] A. A. Klyukanov and E. P. Pokatilov, Zh. Eksp. Teor. Fiz. 60, 312 (1971) [Sov. Phys.-JETP 33, 170 (1971)].

[2] C. J. Summers, R. B. Dennis, B. S. Wherrett, P. G. Harper, and S. D. Smith, Phys. Rev. 170, 755 (1968).

[3] E. P. Pokatilov and K. S. Kabisov, Uchenye zapiski Kishinevskogo gosudarstvennogo universiteta 90, 27 (1967).

[4] P. G. Harper, Proc. Phys. Soc. (London) 92, 793 (1967).

[5] L. I. Korovin, Zh. Eksp. Teor. Fiz. 58, 1995 (1970) [Sov. Phys.-JETP 31, 1075 (1970)].

[6] E. D. Palik, G. S. Picus, S. Teitler, and R. F. Wallis, Phys. Rev. 122, 475 (1964).

[7] A. A. Kljukanov and E. P. Pokatilov, Phys. stat. sol. 38, 277 (1970).

[8] E. N. Adams and T. D. Holstein, J. Phys. Chem. Sol. 10, 254 (1959) [Russ. transl. in Voprosy kvantovoĭ teorii

neobratimykh protsessov (Problems in the Quantum Theory of Irreversible Processes), IIL, 1961].

[9] V. G. Skobov, Zh. Eksp. Teor. Fiz. **38**, 1304 (1960) [Sov. Phys.-JETP **11**, 941 (1960)].

[10] L. I. Magarill and S. K. Savvinykh, Zh. Eksp. Teor. Fiz. **57**, 2079 (1969) [Sov. Phys.-JETP **30**, 1128 (1970)].

[11] L. S. Dubinskaya, Zh. Eksp. Teor. Fiz. **56**, 801 (1969) [Sov. Phys.-JETP **29**, 436 (1969)].

[12] D. H. Dickey, E. J. Johnson, and D. M. Larsen, Phys. Rev. Lett. **18**, 599 (1967).

[13] Yu. A. Firsov and V. L. Gurevich, Zh. Eksp. Teor. Fiz. **41**, 512 (1961) [Sov. Phys.-JETP **14**, 367 (1961)].

[14] Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Sol. **1**, 137 (1956).

[15] E. P. Pokatilov and M. M. Rusanov, Fiz. Tverd. Tela **10**, 3117 (1968) [Sov. Phys.-Solid State **10**, 2458 (1969)].

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