THE THEORY OF μ^+-MESON DEPOLARIZATION WITH ALLOWANCE FOR THE PROCESS OF CHARGE EXCHANGE OR FORMATION OF UNSTABLE CHEMICAL COMPOUNDS

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Submitted January 22, 1971

Zh. Eksp. Teor. Fiz. 60, 1985–2000 (June, 1971)

A consistent theory of the depolarization of μ^+ mesons in magnetic fields parallel to the direction of the initial polarization of the μ^+ mesons is developed, in which the possibility of charge exchange occurring is taken into account. Formulas are derived for the residual polarization and the dependence of the polarization on time. Analysis of the formulas shows that in some cases experiments in longitudinal fields are sufficient to detect charge exchange processes. The results obtained allow the broadening of the class of substances which can be investigated by means of the μ^+-meson technique. The theory can primarily be employed for the study of semiconductors. The formalism developed also describes a different physical situation when two competing chemical reaction channels, one of which leads to the formation of an unstable diamagnetic chemical compound, are open to the μ^+ meson.

1. The possible role of charge exchange in the process of μ^+-meson depolarization has repeatedly been mentioned in the literature[1,2]. However, further analysis is extremely desirable since an erroneous expression for the polarization was obtained in[1], while in[2] the authors consider only the simplest case of absence of electron spin relaxation in the muonium atom, under the conditions that the mean lifetime of a muonium atom with respect to ionization is much larger or much smaller than the characteristic time of hyperfine interaction, and that the muonium atom does not enter into a chemical association.

2. As is well known, the process of charge exchange is one in which a muonium atom (Mu), formed in a material at μ^+-meson velocities close to the Bohr velocity, subsequently and several times successively ionizes and captures an electron. Clearly, the ionization probability is proportional to

$$\exp[-(I-\Delta E_F)/kT],$$

where I is the ionization potential of the atom Mu in the medium, while ΔE_F is the distance from the Fermi level to the conduction band. Naturally, in the general case the ionization potential cannot be calculated in any reliable manner and it is a parameter of the theory. Besides, in solid bodies the ionization potential can be expressed in terms of other thermodynamic characteristics of the Mu atom in the medium. For this purpose we should use the phenomenological formula obtained in[3]:

$$E_{Mu} = 13.54\text{eV} - W_e - W_{\mu^+} + Q_{Mu}. \quad (1)$$

Here, E_{Mu} is the equilibrium binding energy of muonium in the medium, W_e and W_{μ^+} are respectively the electron and μ^+-meson thermodynamic work functions and Q_{Mu} is the heat of solution of muonium. Clearly, the ionization potential I, defined as the distance of the level to the bottom of the conduction band, is related to the muonium binding energy E_{Mu} by the relation

$$I = E_{Mu} + \Delta E_F. \quad (2)$$

3. Before proceeding to the quantitative analysis, let us note the characteristic features of the process. As is well-known, the characteristic time determining the depolarization of the μ^+ meson in a Mu atom is given by $\Delta t_0 = 1/\omega_0$, where ω_0 is the hyperfine splitting constant. If the muonium atom exists during the time $\Delta t \ll \Delta t_0$, then during this time the μ^+-meson spin practically does not feel the influence of the electron spin and is not depolarized. Therefore, if the charge exchange processes proceed so intensely that the span of each "muonic" stage of the life of the μ^+ meson is much smaller than Δt_0, then depolarization scarcely occurs. In vacuum $1/\omega_0 = 3.6 \times 10^{-11}$ sec, but in material media this time may turn out to be considerably longer. For the S-state the hyperfine splitting frequency is determined by the density of the wave function at the origin; for a hydrogen-like atom $|\psi(0)|^2 = 1/\pi a^3$, where a is the Bohr radius. In (and only in) the case when the muonium atom is highly "swollen" in the medium and the corresponding Bohr radius is several times the lattice constant, the muonium can be described as a hydrogen-like atom with an interaction potential $e^2/\epsilon r$ and an effective electron mass m^*. Since for a hydrogen-like atom $I \sim e^2/2\epsilon a$, it follows, for instance, for an ionization potential ~ 0.1 eV and $\epsilon \lesssim 10$, that the radius of the atom in the material medium attains a value of several Angstroms and the corresponding depolarization time attains by order of magnitude a value of 10^{-9}–10^{-8} sec. We emphasize once again that the macroscopic description of the effect of the medium on the Mu atom with the aid of the parameters m^* and $\epsilon(\omega, k)$ is possible only when the dimensions of the muonium atom are large; the impracticability of this approach in the opposite case has been repeatedly noted in the literature[4].

4. Since the time of thermalization of muonium from atomic to Maxwellian velocities is of the order of 10^{-12}–10^{-13} sec, charge exchanges can influence the depolarization only if they continue to occur after thermalization. This is possible only when the binding energy E_{Mu} is small (or negative). For example, in

semiconductors muonium may, under certain conditions, generally turn out to be ionized in the final state[2]. The latter will be the case if the donor impurity level formed by the Mu atom in the forbidden zone turns out to be much higher than the Fermi level.

It is worthwhile, in this connection, to emphasize the difference between the energy characteristics of the muonium atom in a material medium: I and E_{Mu}. In its sense, E_{Mu} stands for the energy expanded in the excitation of a muonium electron to the Fermi level and, practically, it is precisely this quantity that determines whether the muonium will be ionized in the equilibrium state or not. The ionization potential I is suitable for use as a characteristic of the individual properties of the impurity center (the Bohr radius, etc.). Whereas in semiconductors $I > E_{Mu}$ always, in metals $E_{Mu} > 1$.

As has already been noted, if the charge exchanges are very intense and in each stage the Mu atom exists for the time $\Delta t \ll 1/\omega_0$, then the coupling between the electron and μ^+-meson spins does not practically have time to establish itself and depolarization does not occur. As can be seen, qualitatively, the nature of the behavior of the depolarization as a function of the frequency of charge exchange is analogous to the dependence of the depolarization on the mean "flipping" time of the electron spin in a "purely muonic" mechanism of depolarization[2,3,5].

5. Let us proceed to the calculation of the depolarization. In future, we shall take into consideration three possible modes of state of a μ^+ meson in a material medium, namely, a Mu atom which has become part of a diamagnetic chemical compound, a free Mu atom in any excited state, in particular, ionized muonium (a free μ^+-meson), and a free Mu atom in the ground state. To these states are assigned respectively the indices 0, 1 and 2.

Notice that, generally speaking, we could leave out the excited (exciton) states of muonium and assume that the state 1 corresponds to a free μ^+ meson, since, as follows from the well-known formula for the ionization probability (see, for example,[6]), the population of the excited states is extremely small in comparison with the population of the ionized state. However, bearing in mind still another variant of the physical interpretation of the formalism developed below (it will be considered in Secs. 8 and 9 of the present article), we shall write the general equations with allowance for the excited states.

Notice that in our case the combination of the excited states with the ionized state is connected with two assumptions. First, all the excited states are, in so far as depolarization of a μ^+ meson is concerned, equivalent to the ionization state and, secondly, all the excited and ionized states are highly "mixed up" and enter into the kinetic equations as a single level. The first assertion is quite natural since even in a 2S-state of a hydrogen-like atom the hyperfine splitting characteristic time is larger by a factor of 8 than in the ground state, and, remembering that in investigations into charge exchanges the case of the "swollen" muonium is the most interesting, we see that depolarization in the excited states proceeds so slowly that we may identify them with the ionization states. The second assertion is also realistic enough; however, bearing in mind the relatively small population of the excited levels, we shall not discuss it.

Let us introduce the transition probability $\alpha_{ik} = 1/\tau_{ik}$, where τ_{ik} is the mean decay time of the state i into the state k. Let us now define $P_i(t)$ as the contribution of the μ^+ mesons in the i-th state to the general polarization:

$$P_i(t) = P_i'(t) N_i(t) / N(t). \tag{3}$$

Here, $P_i'(t) = |N_i^+(t) - N_i^-(t)|/N_i(t)$ is the polarization of the μ^+ mesons in the corresponding state; $N_i = N_i^+ + N_i^-$ is the number of μ^+ mesons in this state; N_i^+ and N_i^- are the number of mesons with spin orientation along and opposite the initial direction of momentum; and N(t) is the total number of μ^+ mesons. Then the polarization of the whole ensemble of μ^+ mesons is

$$P = P_1 + P_2 + P_0$$

Let us write the system of differential equations for the case when the external magnetic field is directed along the initial polarization:

$$\begin{aligned} \frac{dP_1(t)}{dt} &= -(\alpha_{12} + \alpha_{10}) P_1(t) + \alpha_{21} P_2(t), \\ \frac{dP_2(t)}{dt} &= -(\alpha_{21} + \alpha_{20}) P_2(t) + \alpha_{12} P_1(t) + \left(\frac{\partial P_2}{\partial t}\right)_{N_2(t)/N(t)}, \\ \frac{dP_0(t)}{dt} &= \alpha_{20} P_2(t) + \alpha_{10} P_1(t). \end{aligned} \tag{4}$$

The term $(\partial P_2(t)/\partial t) N_2/N$ reflects the fact that the contribution to the general polarization of the μ^+ mesons in the Mu atoms decreases even if the relative fraction of these μ^+ mesons does not change. In accord with[2], we suppose that depolarization does not occur in the states 1 and 0. In other words, we are not considering here "slow depolarization" in a chemical compound caused, as a rule, by interaction with the nuclear magnetic moments of the atoms of the medium and having characteristic times of the order of 10^{-5}—10^{-4} sec. We may assume that α_{ik} does not depend on time beginning from the instant when thermalization is accomplished. In contrast, during the process of thermalization the probabilities of capture and ionization significantly depend on the μ^+-meson and muonium velocities and, hence, on the time. Clearly, it is not possible to establish, in the general case, the form of this dependence and therefore we shall consider the system (4), beginning from the moment of completion of thermalization, with the initial conditions:

$$P_1(0) = 1 - r - \beta, \quad P_2(0) = r, \quad P_0(0) = \beta. \tag{5}$$

Here, r is the fraction of the μ^+ mesons which form muonium towards the end of the thermalization, while β is the fraction of the μ^+-mesons which form during this time a diamagnetic chemical compound ("hot chemistry"). As can be seen from (5) and in accord with what has been said earlier, we assume that the μ^+ mesons are not depolarized during the time of thermalization. Clearly, r and β are parameters of the theory.

In order to find $(\partial P_2/\partial t)_{N_2/N}$, let us introduce the function q(t), which defines the law according to which the polarization of the μ^+ mesons in muonium decreases from the moment of formation of the muonium:

$$P_2'(t) = q(t-t')P_2'(t') \tag{6}$$

In conformity with[2,3], we set $q(t) = \rho_{10}(t)$, where $\rho_{10}(t)$ is the component of the muonium density matrix which determines the polarization of the μ^+ meson. To determine $(\partial P_2/\partial t)_{N_2/N}$, we first consider the contribution to this term from the mesons produced at the moment of time t'. The number of such mesons remaining up to the time t in the state 2 is evidently equal to

$$\delta N_2(t, t') = \alpha_{12} N_1(t') \exp\{-(\alpha_{21}+\alpha_{20})(t-t')\}dt'.$$

On the average, the polarization of each such meson is equal to $P_2'(t) = q(t-t')P_1'(t)$, from which it is easy to see that

$$\delta\left(\frac{\partial P_2}{\partial t}\right)_{N_2} = \alpha_{12} P_1(t') \exp\{-(\alpha_{21}+\alpha_{20})(t-t')\}\frac{dq(t-t')}{dt}dt'. \tag{7}$$

In order to obtain $(\partial P_2/\partial t)_{N_2/N}$, we must integrate (7) and add the change in polarization in that portion of the Mu atoms which was produced just before the initial moment:

$$\left(\frac{\partial P_2}{\partial t}\right)_{N_2/N} = \alpha_{12}\int_0^t P_1(t')\exp\{-(\alpha_{21}+\alpha_{20})(t-t')\}\frac{dq(t-t')}{dt}dt' + r\exp\{-(\alpha_{21}+\alpha_{20})t\}\frac{dq(t)}{dt}. \tag{8}$$

By formally integrating it and taking into account (5), (6) and (8), we easily reduce the system (4) to a system of Volterra type integral equations with a difference kernel:

$$\begin{aligned} P_1(t) &= (1-r-\beta)\exp\{-(\alpha_{12}+\alpha_{10})t\} + \alpha_{21}\int_0^t P_2(t')\exp\{-(\alpha_{12}+\alpha_{10})(t-t')\}dt', \\ P_2(t) &= r\exp\{-(\alpha_{21}+\alpha_{20})t\}q(t) + \alpha_{12}\int_0^t P_1(t')\exp\{-(\alpha_{21}+\alpha_{20})(t-t')\}q(t-t')dt', \\ P_0(t) &= \beta + \alpha_{20}\int_0^t P_2(t')dt' + \alpha_{10}\int_0^t P_1(t')dt'. \end{aligned} \tag{9}$$

The system of equations (9) can, of course, be obtained directly from physical considerations.

Remembering now that the role of the excited states is small in comparison with the ionized state and, also, assuming that a two-stage reaction of the type $e + \mu^+ + A \rightarrow (\mu A^+) + e \rightarrow (\mu A)$, which leads to the same stable dimagnetic compound as the reaction $Mu + A \rightarrow MuA$, does not occur in matter, we shall henceforth assume $\alpha_{10} = 0$. The case when $\alpha_{10} \neq 0$ will be considered in Sec. 9.

Let us first consider the simplest case, when electron spin relaxation is absent in the muonium atom and the characteristic times for ionization and chemical reaction is considerably larger than the characteristic time of hyperfine interaction in the medium, which determines the depolarization of the μ^+ meson in the muonium atom:

$$1/\omega_0 \ll 1/\alpha_{20} + 1/\alpha_{21}. \tag{10}$$

Then, in conformity with[1-3], we may assume that in a longitudinal magnetic field $q(t)$ is equal to

$$q(t) = \begin{cases} 1 & \text{for } t = 0, \\ (1/2 + x^2)/(1+x^2) = q_\infty & \text{for } t \neq 0. \end{cases} \tag{11}$$

From which

$$\frac{\partial q}{\partial t} = -\frac{1}{2(1+x^2)}\delta(t).$$

Here the dimensionless field is

$$x = \frac{H(1+m_e/m_\mu)}{H_0} = \frac{H_{2c}\omega_{0\,\text{vacuum}}}{1587\omega_0}, \tag{12}$$

m_e, m_μ are the electron and μ^+-meson masses, and $\omega_0 = eH_0/m_e c$. In accordance with (12), the system (4) takes the form

$$\begin{aligned} dP_1/dt &= -\alpha_{12}P_1 + \alpha_{21}P_2, \\ dP_2/dt &= -(\alpha_{21}+\alpha_{20})P_2 + \alpha_{12}q_\infty P_1(t), \\ dP_0/dt &= \alpha_{20}P_2. \end{aligned} \tag{13}$$

When (12) is taken into account the initial conditions (5) take the form

$$P_1(0) = (1-r-\beta), \quad P_2(0) = rq_\infty, \quad P_0(0) = \beta. \tag{14}$$

The solution of the system (13) is elementary, but in the general case the expression for $P(t) = P_1(t) + P_2(t) + P_0(t)$ is quite unwieldly. Therefore we only give the answer for the special case when there is no chemical reaction ($\alpha_{20} = 0$, $\beta = 0$):

$$\begin{aligned} P(t) &= e^{-(\alpha_{12}+\alpha_{21})t/2}\left\{A\,\mathrm{sh}\frac{\sigma t}{2} + B\,\mathrm{ch}\frac{\sigma t}{2}\right\}, \\ \sigma &= \sqrt{(\alpha_{12}-\alpha_{21})^2 + 4\alpha_{12}\alpha_{21}q_\infty}, \\ A &= \sigma^{-1}\{(\alpha_{12}+\alpha_{21})[1-r(1-q_\infty)] - \alpha_{12}(1-r)(1-q_\infty)\}, \\ B &= 1 - r(1-q_\infty). \end{aligned} \tag{15}$$

If $r = 0$ (which, generally speaking, is hardly realistic), the formula (15) coincides with the solution obtained in[2] by a different method.

6. In the general case it is more convenient to obtain the solution using the notation of the system in the form of (9). To begin with, notice that in an experiment, besides the polarization $P(t)$ at a given instant of time, we can measure the mean polarization of the ensemble:

$$\bar{P} = \frac{1}{\tau_\mu}\int_0^\infty P(t)e^{-t/\tau_\mu}dt,$$

where $\tau_\mu = 2.2 \times 10^{-6}$ sec is the mean lifetime of the μ^+ meson and P_∞ is its residual polarization.

As can be seen from the system (4), we neglect in this paper the direct interaction of the μ^+-meson spin with the random magnetic fields existing in the medium. Accordingly, we assume that if a muonium formed a diamagnetic chemical compound with saturated bonding, its depolarization ceases (see (4)). Then, evidently, $P_\infty = \bar{P}$ only when, after a time $t \ll \tau_\mu$, the whole ensemble of μ^+ mesons finds itself in chemical combination. To solve the system (9), let us use the Laplace transformation. The Laplace transform of the function $f(t)$, as is well known, is

$$L(f, \sigma) = \int_0^\infty e^{-\sigma t}f(t)dt$$

and we see at once that

$$P_\infty = P_0(t=\infty) = \alpha_{20}\int_0^\infty P_2(t')dt' = \alpha_{20}L(P_2; \sigma = 0) \tag{16}$$

and the mean polarization is

$$\bar{P}=\frac{1}{\tau_\mu}\sum_{i=0}^{2}L\left(P_i;\ \sigma=\frac{1}{\tau_\mu}\right). \tag{17}$$

In accordance with the well-known convolution theorem, the Laplace transformation for the system (9) yields for us

$$L(P_1,\sigma)=\frac{1-r-\beta}{\alpha_{12}+\sigma}+\frac{\alpha_{21}L(P_2,\sigma)}{\alpha_{12}+\sigma},$$
$$L(P_2,\sigma)=rL(q,\ \sigma+\alpha_{21}+\alpha_{20})+\alpha_{12}L(P_1,\ \sigma)L(q,\ \sigma+\alpha_{21}+\alpha_{20}),$$
$$L(P_0,\sigma)=\frac{\beta}{\sigma}+\frac{\alpha_{20}}{\sigma}L(P_2,\sigma). \tag{18}$$

As has been previously shown[3], we have for L(q, u)

$$L(q,u)=\frac{1}{u}q_\infty(u)=\frac{1}{u}\frac{(1+2\nu/u)^2+(\omega_0/u)^2(1/2+x^2)}{(1+2\nu/u)^2+(\omega_0/u)^2(1+x^2+\nu/u)}. \tag{19}$$

Here, 2ν is a parameter characterizing the attenuation of the polarization of the electron in the Mu atom[2,3].

Solving the system (18), we obtain

$$L(P_1,\sigma)=\frac{(1-r-\beta)+\alpha_{21}rL(q,\sigma+\alpha_{21}+\alpha_{20})}{\alpha_{21}+\sigma-\alpha_{12}\alpha_{21}L(q,\sigma+\alpha_{21}+\alpha_{20})}$$
$$L(P_2,\sigma)=\frac{[\alpha_{12}(1-\beta)+\sigma r]L(q,\sigma+\alpha_{21}+\alpha_{20})}{\alpha_{12}+\sigma-\alpha_{21}\alpha_{12}L(q,\sigma+\alpha_{21}+\alpha_{20})} \tag{20}$$

In accord with (16), we have for the residual polarization

$$P_\infty=\beta+\frac{\alpha_{20}(1-\beta)q_\infty(\alpha_{21}+\alpha_{20})}{\alpha_{21}+\alpha_{20}-\alpha_{21}q_\infty(\alpha_{21}+\alpha_{20})}, \tag{21}$$

from which

$$\frac{P_\infty-\beta}{1-\beta}=1-\frac{[1/2+(\nu+1/2\alpha_{21})/\alpha_{20}]\omega_0^2}{[\alpha_{20}+\alpha_{21}+2\nu]^2+\omega_0^2[1+x^2+(\nu+1/2\alpha_{21})/\alpha_{20}]}. \tag{22}$$

It can be seen from formula (22) that in case of charge exchanges the residual polarization P_∞ is described exactly by the same formula as in the "purely muonic" case[3,7], but now the role of the relaxation parameter is played by the quantity $\nu' = \nu + 1/2\alpha_{21}$. For this reason it is not possible to reveal the presence of charge exchanges if we restrict ourselves to the analysis of P_∞.

Using the formulas (17)–(20) we obtain for the mean polarization $\overline{P}$.

$$(\bar{P}-\beta)/(1-\beta)=1-1/2b'\omega_0^2[1+r/(1-\beta)\alpha_{12}\tau_\mu]\{\alpha_{20}'[b'^2+\omega_0^2(1+x^2+(\nu+1/2\alpha_{21})/\alpha_{20}]+(\alpha_{20}'+\alpha_{21})R/\alpha_{20}'\alpha_{12}\tau_\mu\}^{-1}, \tag{23}$$

where

$$\alpha_{20}'=\alpha_{20}+1/\tau_\mu,\quad b'=(\alpha_{20}'+\alpha_{21}+2\nu),$$
$$R=b'^2+\omega_0^2(1+x^2+\nu/(\alpha_{20}'+\alpha_{21})).$$

It can be seen from formula (23) that, again for the mean polarization in the event of charge exchanges as well as in the "purely muonic" mechanism, the characteristic three-point relation

$$\left[\frac{\bar{P}(H_1)}{1-\bar{P}(H_1)}-\frac{\bar{P}(H_2)}{1-\bar{P}(H_2)}\right]\frac{1}{H_1^2-H_2^2}=\text{const}$$

is fulfilled. It is clear from obvious physical considerations that $\overline{P}$ differs from P_∞ only when a "highly prolonged" time dependence of the polarization is observed. From formula (23) follow at once sufficient conditions for $\overline{P}$ to coincide with P whenever they are simultaneously fulfilled. These conditions are

$$\frac{1}{\tau_\mu}\ll\alpha_{12},\qquad\frac{1}{\tau_\mu}\ll\alpha_{20},\qquad\frac{1}{\tau_\mu}\ll\frac{\alpha_{20}\alpha_{12}}{\alpha_{21}+\alpha_{20}}.$$

The physical meaning of these requirements are clear enough: all the μ^+ mesons should attain a chemically bound state within a time much shorter than the lifetime. These requirements can, of course, be obtained by using obvious kinetic considerations. We emphasize that these are not necessary conditions.

By letting in formula (23) $\alpha_{12}\to\infty$ and $\alpha_{21}\to 0$, we obtain $\overline{P}$ for the "purely muonic" case. Taking into account the three-point relation, we see that if for the "purely muonic" case $\overline{P}$ differs from P_∞, then we have four equations for four unknown parameters: ν, ω_0, α_{20} and β. Consequently, we can, in principle, determine all the parameters using only P_∞ and $\overline{P}$. Notice, however, that in the "purely muonic" case $\overline{P}$ differs from P_∞ only if $\alpha_{20}\lesssim 1/\tau_\mu = 4.54\times10^5\ \text{sec}^{-1}$. Such small reaction rates are, generally speaking, seldom met with.

7. Let us turn now to the analysis of the time dependence of the polarization in a longitudinal field. Using (18), (19) and (20), we obtain for the Laplace transform of the polarization $P(t) = P_1(t) + P_2(t) + P_0(t)$ the expression

$$L(P,\sigma)=\frac{\beta}{\sigma}+\frac{(1-\beta-r)\sigma R(\sigma)+Q(\sigma)\{\alpha_{21}r\sigma+(\sigma+\alpha_{20})[\alpha_{12}(1-\beta)+\sigma r]\}}{\sigma[(\alpha_{12}+\sigma)R(\sigma)-\alpha_{12}\alpha_{21}Q(\sigma)]}, \tag{24}$$

where $Q(\sigma)\equiv(\alpha_{21}+\alpha_{20}+\sigma+2\nu)^2+\omega_0^2(1/2+x^2)$,

$$R(\sigma)\equiv(\sigma+\alpha_{21}+\alpha_{20})(\alpha_{21}+\alpha_{20}+\sigma+2\nu)^2+\omega_0^2[(\alpha_{20}+\alpha_{21}+\sigma)\times(1+x^2)+\nu].$$

To find P(t) we must take the inverse Laplace transform of (24). For this purpose, as is well known, we must break the expression (24) up into partial fractions, which, generally speaking, requires our finding the roots of the equation

$$(\alpha_{12}+\sigma)R(\sigma)-\alpha_{12}\alpha_{21}Q(\sigma)=0. \tag{25}$$

Since this is quartic equation in σ, its roots can, generally speaking, be obtained by a standard method. We shall not, however, do this, bearing in mind that in reality, the dependence of the polarization on time may be observed only if Eq. (25) has roots which are small compared with ω_0. In fact, it follows from (24) that the polarization has the form

$$P(t)=\sum_{i=1}^{4}A_ie^{\sigma_it}+P_\infty. \tag{26}$$

At the present level of experimental technique, the time dependence can be observed only if there are roots $|\sigma|\le 10^8\ \text{sec}^{-1}$, which is equivalent to the requirement $|\sigma|\ll\omega_0$.

In its expanded form Eq. (25) has the form

$$\sigma^4+\sigma^3[3(\alpha_{21}+\alpha_{20})+\alpha_{12}+4\nu]+\sigma^2[\alpha_{12}\alpha_{20}+(\alpha_{21}+\alpha_{20}+2\nu)\times(2\alpha_{12}+3\alpha_{21}+3\alpha_{20}+2\nu)+\omega_0^2(1+x^2)]$$
$$+\sigma[(\alpha_{21}+\alpha_{20}+2\nu)^2(\alpha_{21}+\alpha_{20}+\alpha_{12})+2\alpha_{12}\alpha_{20}(\alpha_{21}+\alpha_{20}+2\nu)+\omega_0^2\{(\alpha_{21}+\alpha_{12}+\alpha_{20})(1+x^2)+\nu\}]$$
$$+[\alpha_{12}\alpha_{20}(\alpha_{21}+\alpha_{20}+2\nu)^2+\omega_0^2\alpha_{12}\{\alpha_{20}(1+x^2)+\nu+1/2\alpha_{21}\}]. \tag{27}$$

It can be verified that for small roots $|\sigma|/\omega_0\ll 1$, for all values of the parameters: α_{12}, α_{20}, α_{21}, ν, the σ^4-term is small in comparison with the term quadratic in σ while the term cubic in σ is small compared with the term which is linear to within

$|\sigma|^2/\omega_0^2$. Therefore, the small roots may be found by solving the quadratic equation obtained from (27) by neglecting terms depending on σ^4 and σ^3.

In the case when $a \equiv \alpha_{12} + \alpha_{20} + \alpha_{21} \gtrsim \omega_0$, we can, to within $|\sigma|/\omega_0$, neglect the term quadratic in σ and, hence, verify that Eq. (27) has the only small root

$$\sigma_1 = -\alpha_{12}\frac{[\alpha_{20}b^2 + \omega_0^2\{\alpha_{20}(1+x^2)+\nu+{}^1/_2\alpha_{21}\}]}{[b^2a + 2\alpha_{12}\alpha_{20}b + \omega_0^2\{a(1+x^2)+\nu\}]}, \quad (28)$$

where $b \equiv \alpha_{20} + \alpha_{21} + 2\nu$. It is easy to see that for $\alpha_{12} \ll \omega_0$ there is a small root for all values of the remaining parameters.

If $\alpha_{12} \gtrsim \omega_0$, analysis of formula (28) shows that in the case when $\omega_0/b \gg 1$, there is a small root if $\alpha_{20} \ll \omega_0$. When $\omega_0/b \sim 1$, there is a small root if $\alpha_{20} \ll \omega_0$ and the magnetic field is strong ($x^2 \gg 1$). If, however, $\omega_0/b \ll 1$, then the conditions for the existence of a small root are: $\alpha_{20} \ll \omega_0$, if $\alpha_{12} > \alpha_{21}$, and $\alpha_{20}/\alpha_{21} \ll \omega_0/\alpha_{12}$ if $\alpha_{12} > \alpha_{12}$.

Notice, however, that there exists another very general restriction on the parameter α_{20}. It is clear that any observation of the time dependence is possible only in the case when P_∞ differs so much from unity that this fact can reliably be fixed by experiment. Estimates which may be obtained on the basis of formula (22) show that at the present standard of experiments the condition $\alpha_{20} \lesssim \omega_0$ should always be fulfilled. Notice, finally, that in all the cases when a small root exists, the term $2\alpha_{12}\alpha_{20}b$ in the denominator of formula (28) turns out to be negligibly small in comparison with the rest and is consequently dropped in future.

Let us begin the analysis with the case $\alpha_{12} \gtrsim \omega_0$. If at the same time $\omega_0/b \ll 1$, then, calculating in the conventional way the coefficient A_1 which corresponds to a small root[1], we obtain for the time dependence of the polarization

$$P(t) = P_\infty + (1 - P_\infty)e^{\sigma_1 t}. \quad (29)$$

The formula (28) which determines the root σ_1, is in this case insignificantly simplified. The physical meaning of the formula (29) is clear: a rapid depolarization of the μ^+ meson does not occur in the case being considered. When $\omega_0/b \sim 1$, a small root exists only at high fields ($x^2 \gg 1$). In this case we have again $P(t) = P_\infty + (1 - P_\infty)e^{\sigma_1 t}$, but the formula (28) is considerably simplified:

$$\sigma_1 = -\frac{\alpha_{12}}{\alpha_{21}+\alpha_{12}}\left[\alpha_{20} + \frac{\nu + {}^1/_2\alpha_{21}}{1+x^2}\right]. \quad (30)$$

Finally, if $\omega_0/b \gg 1$, we have

$$P(t) = P_\infty + \left[\frac{1+2x^2}{2(1+x^2)} + \frac{6}{2(1+x^2)} - P_\infty\right]e^{\sigma_1 t}, \quad (31)$$

and

$$\sigma_1 = -\left[\alpha_{20} + \frac{\nu + {}^1/_2\alpha_{21}}{1+x^2}\right]. \quad (32)$$

When $\alpha_{12} \ll \omega_0$ it is sufficient to consider the cases $\omega_0/b \sim 1$ and $\omega_0/b \ll 1$ since the condition $a \gtrsim \omega_0$, which has been taken as the basis of our classification, excludes the case $b \ll \omega_0$. Generally speaking, when $a \ll \omega_0$ there can also be a case when only one small root exists, but we shall consider it later.

When $\omega_0/b \ll 1$ we again have for the polarization formula (29). From (28) we obtain for σ_1[2].

$$\sigma_1 = -\frac{\alpha_{12}}{a}\left[\alpha_{20} + \frac{1}{2}\frac{\omega_0^2 b}{b^2 + \omega_0^2(1+x^2)}\right]. \quad (33)$$

If $\omega_0/b \sim 1$, the coefficient A_1 is most easily obtained from physical considerations. At the initial moment of time we have according to formula (26)

$$1 = P_\infty + A_1 + \sum_{i=2}^{4} A_i. \quad (34)$$

Let us note, in order to find the sum of the coefficients for "fast" roots, that during an interval of time of the order of $1/\omega_0$ the loss in polarization can only be due to the depolarization of those mesons which found muonium from the very outset. Since $\alpha_{12} \ll \omega_0$, no appreciable pumping into the "muonic state" will occur during this time. In order to determine the polarization lost in the "muonic state," it is useful to remember that the decay rate of the second state $(\alpha_{21} + \alpha_{20})$ in our case is of the order of ω_0. Therefore, the loss in polarization after a time Δt, satisfying the condition $1/\sigma_1 \gg \Delta t \gg 1/\omega_0$, will be determined by the formula

$$\Delta P = \sum_{i=2}^{4} A_i = r[1 - q_\infty(\alpha_{21} + \alpha_{20}, \nu)]. \quad (35)$$

We obtain from the formulas (34) and (35)

$$P(t) = P_\infty + \{1 - P_\infty - r[1 - q_\infty(\alpha_{21} + \alpha_{20},\ \nu)]\}e^{\sigma_1 t}. \quad (36)$$

The quantity $q_\infty(u)$ is determined by formula (19). The formula for the small root takes the form

$$\sigma_1 = -\alpha_{12}\left[1 - \frac{\alpha_{21}[b^2 + \omega_0^2({}^1/_2 + x^2)]}{(\alpha_{21}+\alpha_{20})[b^2 + \omega_0^2(1+x^2)] + \omega_0^2\nu}\right] \quad (37)$$

When $\alpha_{12} \ll \omega_0$ we can obtain a further simplification of the formulas (36) and (37), namely: $\sigma_1 = -\alpha_{12}$ and

$$P(t) = P_\infty + \frac{(1-r-\beta)(1-P_\infty)}{(1-\beta)}e^{-\alpha_{12}t}. \quad (38)$$

Let us now consider the case when $a \ll \omega_0$. As has already been noted, we must seek the small roots, neglecting in Eq. (27) the terms of the order of σ^4 and σ^3. Then we have

$$\sigma_{1,2} = -\frac{1}{2}\left[a + \frac{\omega_0^2\nu}{4\nu^2 + \omega_0^2(1+x^2)}\right] \pm \frac{1}{2}\left\{\left[a + \frac{\omega_0^2\nu}{4\nu^2 + \omega_0^2(1+x^2)}\right]^2 - 4\alpha_{12}\left[\alpha_{20} + \frac{(\nu + {}^1/_2\alpha_{21})\omega_0^2}{4\nu^2 + \omega_0^2(1+x^2)}\right]\right\}^{1/2}. \quad (39)$$

As can be seen from (39), there exist two small roots if the condition

$$\left(\frac{2\nu}{\omega_0}\right) + \frac{\omega_0}{2\nu}(1+x^2) \gg 1. \quad (40)$$

is fulfilled. It is clear from this that only one small root exists if $\nu \sim \omega_0$ and the external field is not too

[1] Here, as below, we omit the simple but long computations and give only the final results, which, moreover, almost always can be obtained from physical consideration.

[2] In the case $x = 0$, $\alpha_{20} = 0$ and $\nu \gg \alpha_{21}$, the formula (33) goes over into the Nosov-Yakovleva formula [2].

strong. We obtain at once this root from (39); $\sigma_1 = -\alpha_{12}$. Accordingly, we have in this case the physically obvious expression for the polarization

$$P(t) = \beta + (1 - r - \beta)e^{-\alpha_{12}t}. \quad (41)$$

For the case of two small roots, a convenient formula may be obtained for the sum $A_1 + A_2$ of the coefficients before the small roots. It is clear that when $\nu \gg \omega_0$, or when $\nu \sim \omega_0$ and $x^2 \gg 1$, there is, generally, no rapid loss in the polarization of the μ^+ mesons. Therefore

$$A_1 + A_2 = 1 - P_\infty. \quad (42)$$

The rapid losses in the case when $\nu \ll \omega_0$ are connected only with the loss in the polarization of the μ^+ mesons which formed muonium at the initial moment of time (after thermalization). We obtain in analogous fashion to formula (35)

$$A_1 + A_2 = 1 - P_\infty - r/2(1 + x^2). \quad (43)$$

Formula (43) is very important since it affords us the possibility of a direct experimental determination of the parameter r—the fraction of μ^+ mesons which form free muonium towards the end of thermalization—and also of the hyperfine splitting frequency ω_0, provided the condition $\nu \ll \omega_0$ is fulfilled.

The coefficients A_1 and A_2 must be found by conventional means. For the case $\nu \ll \omega_0$

$$A_k = \frac{(\sigma_k^2 + \sigma_k a + \alpha_{12}\alpha_{20})[(1-\beta)\alpha_{12} + r\sigma_k]}{\sigma_k\{\alpha_{12}\alpha_{21} + 2(\sigma_k + \alpha_{12})^2(1+x^2)/(1+2x^2)\}}, \quad (44)$$

$$\sigma_{1,2} = -\frac{1}{2}\left(a + \frac{\nu}{1+x^2}\right) \pm \frac{1}{2}\left\{\left(a + \frac{\nu}{1+x^2}\right)^2 - 4\alpha_{12}\left(\alpha_{20} + \frac{\nu + {}^1/_2\alpha_{21}}{1+x^2}\right)\right\}^{1/2}$$

For the case $\nu \gg \omega_0$

$$A_k = \frac{(\sigma_k^2 + a\sigma_k + \alpha_{12}\alpha_{20})[(1-\beta)\alpha_{12} + r\sigma_k]}{\sigma_k[\alpha_{12}\alpha_{21} + (\sigma_k + \alpha_{12})^2]}, \quad (45)$$

$$\sigma_{1,2} = -\frac{1}{2}(a+c) \pm \sqrt{(a+c)^2 - 4\alpha_{12}(\alpha_{20}+c)},$$

where $c \equiv \omega_0^2\nu/[4\nu^2 + \omega_0^2(1 + x^2)]$. With the exception of the case of ultrastrong fields we have $c \approx \omega_0^2/4\nu$.

The formulas (44) and (45) become considerably simplified if the radicand can be expanded in a series. Let us classify the cases which are of interest for the interpretation of experiments.

a) Let $\omega_0 \gg \nu$; $\alpha_{12} \gg \{\alpha_{21}, \alpha_{20}, \nu/(1 + x^2)\}$. Then we have

$$P(t) = \left[1 - P_\infty - \frac{(1-\beta)}{2(1+x^2)}\right]\exp\left\{-\left(\alpha_{20} + \frac{\nu + {}^1/_2\alpha_{21}}{1+x^2}\right)t\right\} + \frac{(1-\beta-r)}{2(1+x^2)}e^{-\alpha_{12}t} + P_\infty. \quad (46)$$

As can be seen, if an experimental P(t) dependence is available and the parameters of the problem satisfy the set conditions, then using the formula (46), we can determine the parameters β, α_{12}, α_{20} and $\nu + {}^1/_2\alpha_{21}$. As has already been noted, the parameters r and ω_0 are determined, in the general case, with the aid of formula (43). Thus, in the case under consideration we can extract practically all the information about the nature of the charge exchange process.

b) Let $\alpha_{12} \sim \alpha_{21} \gg (\alpha_{20}, \nu/(1 + x^2)$ and $x \gg 1$, $\omega_0 \gg \nu$.

Then

$$P(t) = \frac{1-\beta}{1 + 2\alpha_{20}(1+x^2)/\alpha_{21}}\exp\left\{-\frac{\alpha_{12}\alpha_{20}[1 + \alpha_{21}/2\alpha_{20}(1+x^2)]t}{\alpha_{21} + \alpha_{12}}\right\} + \left[1 - P_\infty - \frac{(1-\beta)}{1 + 2\alpha_{20}(1+x^2)/\alpha_{21}}\right]\exp[-(\alpha_{12} + \alpha_{21})t] + P_\infty. \quad (47)$$

It can be verified that we can, with the aid of formula (47), extract all the information about the parameters of the problem with the exception of ν, which we can find from the residual polarization P_∞ provided it does not turn out that $\nu \ll \alpha_{21}$.

c) Let $\alpha_{21} + \alpha_{20} + \nu/(1 + x^2) \gg \alpha_{12}$, and let, as before, $\omega_0 \gg \nu$. In that case

$$P(t) = \frac{(1+2x^2)r\nu}{2(1+x^2)[a(1+x^2) + \nu]}\exp\left[-\left(a + \frac{\nu}{1+x^2}\right)t\right] + \left[1 - P_\infty - r + \frac{(1+2x^2)ra}{2[(1+x^2)a + \nu]}\right] \times \exp\left[\frac{-\alpha_{12}(\alpha_{20} + (\nu + {}^1/_2\alpha_{21})/(1+x^2))t}{a + \nu/[1+x^2]}\right]. \quad (48)$$

As a simple analysis shows, we can, when we have available the experimental P(t, x) dependence (for different values of the field), extract with the aid of formula (48) all the necessary information except the parameter β, which is determined with the aid of the formula (22) for P_∞, the formula assuming a simple form in this case.

d) Let us proceed now to the cases when $\nu \gg \omega_0$. There arise here especially simple relations. If $\alpha_{12} + \alpha_{20} + c \gg \alpha_{12}$, then

$$P(t) = P_\infty + \left(1 - P_\infty - \frac{rc}{a+c}\right)\exp\left[-\frac{\alpha_{12}(\alpha_{20}+c)}{a+c}t\right] + \frac{rc}{a+c}\exp[-(a+c)t]. \quad (49)$$

If we do not make any additional assumptions about the parameters of the problem, then experiments in ultrastrong fields—$\omega_0^2(1 + x^2) \sim \nu^2$—are necessary in this case for the extraction of the complete information. If, however, the condition $\alpha_{21} + \alpha_{12} \gg \alpha_{20} + c$ is fulfilled, then

$$P(t) = P_\infty + (1 - P_\infty)\exp\left\{-\frac{\alpha_{12}(\alpha_{20}+c)t}{\alpha_{12} + \alpha_{21}}\right\}. \quad (50)$$

In this case it is not possible to extract the complete information from experiments in a longitudinal field only.

8. Let us now note an extremely important circumstance. The entire formalism of the theory of μ^+-meson depolarization in which charge exchange is taken into account, without a single alteration, describes a completely different physical situation, namely, that case when no charge exchanges occur but muonium may enter into a stable as well as an unstable diamagnetic chemical combination. Then the state 1 should be understood as an unstable chemical compound. The principal equations of the theory completely characterize this situation with the only assumption that no free μ^+ mesons remain after the completion of thermalization. The generalization of the results to this case is, in principle, trivial. However, the corresponding computation is quite tedious and we shall not consider this variant, especially as it is somewhat

exoteric. Notice that the quantity $(1 - r - \beta)$, which earlier determined the number of free μ^+ mesons towards the end of thermalization, but now determines the fraction of μ^+ mesons which form an unstable diamagnetic compound towards the end of thermalization, may be different from zero in the new interpretation of the equations ("hot chemistry").

In view of the possibility of a new interpretation of the equations of the theory, the special case $\beta = 0$, $\alpha_{20} = 0$ is of considerable interest. This case now corresponds to the formation of only unstable chemical compounds. It is obvious, in the first place, that the residual polarization is $P_\infty = 0$ when $\beta = 0$ and $\alpha_{20} = 0$ On the other hand, all the formulas determining the time dependence of the polarization are considerably simplified. Thus, instead of the formula (28) we now have

$$\sigma_1 = -\alpha_{12}\omega_0^2 b / 2\{b^2 a + \omega_0^2[a(1 + x^2) + \nu]\}, \tag{51}$$

Where $b = (\alpha_{21} + 2\nu)$ and $a = (\alpha_{12} + \alpha_{21})$.

In conformity with the foregoing, let us begin the analysis with the case $a \gtrsim \omega_0$, and in it, with the case $\alpha_{12} \gtrsim \omega_0$. Formula (29) now takes the form

$$P(t) = e^{\sigma_1 t}, \tag{52}$$

the root σ_1 being especially simple here if $\nu \ll b^2 a/\omega_0^2$ and $\omega_0\sqrt{1 + x^2} \ll b$. In this case

$$P(t) = \exp\{-\alpha_{12}\omega_0^2 t / 2ab\}. \tag{53}$$

In the case $\omega_0/b \sim 1$ and for strong fields, we obtain

$$P(t) = \exp\{-\alpha_{12} bt / 2a(1 + x^2)\}. \tag{54}$$

When $\omega_0/b \gg 1$ we obtain instead of the formulas (31) and (32)

$$P(t) = \frac{1 + 2x^2}{2(1 + x^2)} \exp\left\{-\frac{bt}{2(1 + x^2)}\right\}. \tag{55}$$

When $\alpha_{12} \ll \omega_0$ and $\omega_0/b \ll \sqrt{1 + x^2}$ the quantity P(t) is determined by the formula (53). When $\alpha_{12} \ll \omega_0$ and $\omega_0/b \sim 1$ the formulas (37) and consequently (36) are insignificantly simplified.

Let us consider now the simplification which arises when $a \ll \omega_0$. In the case of a single small root, we must set $\beta = 0$ in formula (41). If there are two small roots, substantial simplifications arise in those cases when the formulas (46)—(50) work.

Let us consider the case when $\omega_0 \gg \nu$. We now have, when $\alpha_{12} \gg \{\alpha_{21}, \nu/(1 + x^2)\}$,

$$P(t) = \frac{1 + 2x^2}{2(1 + x^2)} \exp\left\{-\frac{(\nu + {}^1/_2\alpha_{21})t}{1 + x^2}\right\} + \frac{1 - r}{2(1 + x^2)} e^{-\alpha_{12}t}. \tag{56}$$

Consequently, it is possible to determine the quantities ω_0, α_{12}, r and $(\nu + {}^1/_2\alpha_{12})$ in the case described by the formula (56). Therefore, the rate of production of the unstable chemical compound α_{21} can be determined in this case only if $\nu \ll \alpha_{21}$ whereas the decay rate α_{12} can be determined at once provided there exists a canal for the production of the unstable compound by means of "hot chemistry." If $\alpha_{12} \sim \alpha_{21} \gg \nu/(1 + x^2)$ and $x^2 \gg 1$, then we obtain instead of the formula (47),

$$P(t) = \exp\left[-\frac{\alpha_{12}\alpha_{21}}{2(\alpha_{12} + \alpha_{21})} \frac{t}{(1 + x^2)}\right]. \tag{57}$$

Evidently, in this case ω_0 and $(\tau_{12} + \tau_{21})$ are determinable, the last expression being the sum of the formation and decay times of the chemical compound.

If $\alpha_{21} + \nu/(1 + x^2) \gg \alpha_{12}$, we have

$$P(t) = \frac{(1 + 2x^2)r}{2(1 + x^2)[1 + (1 + x^2)(\alpha_{12} + \alpha_{21})/\nu]} \exp\left\{-\left(\alpha_{21} + \frac{\nu}{1 + x^2}\right)t\right\} + \left[1 - r + \frac{r(1 + 2x^2)(\alpha_{12} + \alpha_{21})}{2(1 + x^2)[\alpha_{21} + \nu/(1 + x^2)]}\right] \times \exp\left\{-\frac{\alpha_{12}(\nu + {}^1/_2\alpha_{21})t}{(1 + x^2)(\alpha_{12} + \alpha_{21}) + \nu}\right\}. \tag{58}$$

In this case, it is possible to determine all the parameters of the problem with the exception of special situations. Indeed, imposing the more stringent condition: $\alpha_{21} \gg (\alpha_{12}, \nu/(1 + x^2))$, we obtain from formula (58)

$$P(t) = \left[1 - \frac{r}{2(1 + x^2)}\right] e^{-\alpha_{21}t}. \tag{59}$$

In the opposite case: $\nu/(1 + x^2) \gg (\alpha_{12}, \alpha_{21})$ we have

$$P(t) = \frac{(1 + 2x^2)r}{2(1 + x^2)} \exp\left\{-\frac{\nu t}{1 + x^2}\right\} + (1 + r)\exp\{-\alpha_{12}t\}. \tag{60}$$

For a rapid relaxation of the electron spin in muonium $(\nu \gg \omega_0)$ in the case $\alpha_{21} + c \gg \alpha_{12}$ we obtain instead of formula (49)

$$P(t) = \left[1 - \frac{r}{1 + \alpha_{21}/c}\right] \exp\left[-\frac{\alpha_{12}t}{1 + \alpha_{21}/c}\right] + \frac{r}{1 + \alpha_{21}/c} \exp[-(\alpha_{21} + c)t]. \tag{61}$$

In this case the number of unknown parameters is one more than the number of equations which may be obtained with the aid of the formula (61), provided we do not employ ultrastrong fields.

Finally, if $\alpha_{21} + \alpha_{12} \gg c$, then

$$P(t) = \exp[-\alpha_{12}ct / (\alpha_{12} + \alpha_{21})]. \tag{62}$$

The set of formulas (52)—(62) describes practically all the interesting cases of μ^+-meson depolarization in the formation of unstable diamagnetic chemical compounds. It must also be borne in mind that the phenomenological constants α_{21}, α_{12} and ν could purposely be varied, making those changes in the mode of depolarization which would enable us to obtain sufficiently complete experimental information about all the constants of the theory. We shall not carry out a detailed analysis of the various possibilities. We only note one curious consequence of the formulas (56) and (62). It is easy to see that if the "hot chemistry" channel is absent, these formulas are "degenerate" and, therefore, there arises a possibility, when investigating depolarization, to obtain information about the presence of the "hot chemistry" channel in the production of unstable chemical compounds of hydrogen. Of course, we must then remember that the extrapolation of all the conclusions obtained in the investigation of the chemistry of muonium to hydrogen should be carried out taking into consideration the role of the isotope effect. Especially significant differences may be expected exactly in the investigation of the "hot chemistry" canal since the role of the tunnel effect may be important for this canal. On the other hand, significant differences may arise in the case of unstable compounds with small binding energies as a result of the influence of the isotope effect on the magnitude of the binding energy.

Nevertheless the method proposed appears to be an extremely promising method for the study of the chemical reactions of hydrogen in which unstable compounds are formed. Since unstable diamagnetic compounds of hydrogen are very common (it is sufficient, for example, to mention the group of the metallic hydrides, and also the solutions of acids), the proposed method is of sufficient interest.

9. Let us now consider the case when $\alpha_{10} \neq 0$. Evidently, this means, when the equations are interpreted in the spirit of Sec. 8, that the resulting diamagnetic chemical compound does not necessarily disintegrate into the initial products, but may subsequently be converted into a stable compound. In other words, the final products of the reaction of muonium may be formed in two ways: directly and by means of a two-stage reaction. In this case we obtain for the residual polarization

$$P_\infty = 1 - \frac{[\alpha_{12}(1-\beta)+\alpha_{10}r]\omega_0^2(\nu+{}^1/_2(\alpha_{21}+\alpha_{20}))}{[\alpha_{10}(\alpha_{21}+\alpha_{20})+\alpha_{12}\alpha_{20}][b^2+\omega_0^2(1+x^2)]+ \\ +\omega_0^2[\alpha_{10}\nu+\alpha_{12}(\nu+{}^1/_2\alpha_{21})]}. \quad (63)$$

It is easy to see that with the aid of the substitution

$$1-\beta'' = \frac{(1-\beta)\alpha_{12}+\alpha_{10}r}{\alpha_{12}+\alpha_{10}} = \frac{(1-\beta-r)\alpha_{12}}{\alpha_{12}+\alpha_{10}} + r, \quad (64)$$

$$\alpha_{12}'' = \alpha_{10}+\alpha_{12}, \quad \alpha_{20}'' = \alpha_{20}+\alpha_{21}\alpha_{10}/(\alpha_{10}+\alpha_{12}),$$

$$\nu'' = \nu + \frac{\alpha_{21}}{2} - \alpha_{21}\alpha_{10}/2(\alpha_{12}+\alpha_{10}),$$

we obtain instead of the formula (63) a complete analog of formula (22):

$$P_\infty = 1 - \frac{(1-\beta'')\omega_0^2 b/2}{\alpha_{20}''[b^2+\omega_0^2(1+x^2)]+\omega_0^2\nu''}. \quad (65)$$

Thus, when $\alpha_{10} \neq 0$ the residual polarization is determined by the same formula as in the "purely muonic" case.

As before, it is always possible in the case $\alpha_{10} \neq 0$ to obtain analytic expressions for the time dependence of the polarization P(t). The classification of all the possible variants, as well as the complete analysis, is, generally, absolutely analogous to the case when $\alpha_{10} = 0$.

We shall not give here all the rather unwieldly formulas for P(t). We shall only note a general property characteristic of the structure of these expressions. Experiments in only longitudinal fields do not make it possible for us to recognize the situation when $\alpha_{10} \neq 0$ and for the analysis of this problem we need a complete experiment, i.e., a concurrent investigation of P(t) in longitudinal and transverse magnetic fields. This is quite natural if we remember that experiments in only longitudinal fields are in a number of cases insufficient for depolarization in a charge exchange process to be distinguished from the "purely muonic" mechanism. To illustrate the above statement, we give one of the formulas for P(t) for $\alpha_{10} \neq 0$.

If $\alpha_{12} + \alpha_{10} \gtrsim \omega_0$ and $\omega_0/b \ll 1$, then

$$P(t) = \left[1 - P_\infty - \frac{\alpha_{12}(1-\beta)+r\alpha_{10}}{2(\alpha_{12}+\alpha_{10})(1+x^2)}\right]e^{\sigma t}, \quad (66)$$

with

$$\sigma = -\left[\alpha_{20} + \frac{\alpha_{21}}{\alpha_{12}+\alpha_{20}}\left(\alpha_{10}+\frac{\alpha_{12}}{2(1+x^2)}\right)+\frac{\nu}{1+x^2}\right]. \quad (67)$$

As is easy to see, the formulas (66) and (67) coincide with (31) and (32) when $\alpha_{10} = 0$.

10. It seems to us that the complete theory developed in this paper of μ^+-meson depolarization, in which the process of charge exchange has been taken into account, permits us to significantly broaden the class of solid bodies and chemical reactions for the investigation of which the μ^+-meson method may be employed. However, as is evident from the results, in a number of cases it is impossible to recognize from experiments in longitudinal fields the presence of the charge exchange mechanism and, consequently, the theory should be supplemented by an investigation of μ^+-meson depolarization in magnetic fields perpendicular to the initial polarization.

In conclusion, we express our sincere gratitude to D. P. Grechukhin, I. I. Gurevich, V. G. Nosov, O. B. Firsov, D. Ya. Choporov, and É. G. Chudinov for an extremely useful discussion of the results.

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Translated by A. K. Agyei
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