THE PROPAGATION OF A SHORT ACOUSTIC PULSE IN A MEDIUM CONTAINING PARAMAGNETIC CENTERS

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Submitted October 20, 1970

Zh. Eksp. Teor. Fiz. 60, 2269–2273 (June, 1971)

The propagation of an acoustic pulse with slightly varying parameters in a medium containing paramagnetic centers with an effective spin $S = \frac{1}{2}$ is considered. It is pointed out that pulses of constant shape exist when the pulse duration is much shorter than the dissipative transverse relaxation time.

1. INTRODUCTION

THE behavior of a pulse of light as it propagates in a resonantly absorbing (or enhancing) medium has been sufficiently well studied.[1, 2] It has been shown that the evolution of a pulse whose duration is much less than the relaxation time T_2 of the polarization of the medium has a number of essentially new features, among which, for example, is the possibility of formation of stationary pulses (pulses whose shape does not change during propagation) in both enhancing and absorbing media.

A certain analogy between the behavior of electromagnetic and elastic waves has been repeatedly noted (e.g., [3]). The study of the passage of a short acoustic pulse in a medium resonantly interacting with it is therefore also of interest. A paramagnetic crystal usually serves as such a medium.

The change in the form of an acoustic wave as it propagates in a medium containing paramagnetic centers was considered in papers [4, 5]. The use of dispersion relations, as has been clearly shown in [6], presupposes linearization of the system of equations describing the behavior of the wave. But the phenomenon of the formation of a stationary light pulse is a consequence of the inclusion of precisely the nonlinear terms in the equations of motion for the polarization pseudovector. We should also expect a similar connection in our case.

In this paper, we consider the propagation of a plane transverse acoustic wave pulse in a cubic crystal containing paramagnetic centers with effective spin $S = \frac{1}{2}$, with the wave-vector of the oscillations and the magnetic field both directed along a four-fold symmetry axis. In this case, the analogy between electromagnetic and elastic waves is in its most complete form. The acoustic wavelength is assumed to be significantly greater than the lattice constant of the crystal, i.e., the crystal is regarded as a continuous medium.

A system of equations is found which describes the propagation of an acoustic pulse with weakly varying pulse characteristics. We study in detail the case when the pulse duration is significantly less than the dissipative transverse relaxation time.

2. DERIVATION OF THE CONTRACTED EQUATIONS

We shall find a system of equations describing the propagation of an acoustic pulse in a medium containing paramagnetic centers.

The Hamiltonian for the interaction of an effective spin $S = \frac{1}{2}$ with the lattice vibrations has the form[7]

$$\mathcal{H}_{s-p} = \beta \sum F_{ijkm} H_i S_j u_{km}, \tag{1}$$

where F_{ijkm} are the components of the coupling tensor, H is the intensity of the constant magnetic field, S is the effective spin, β is the Bohr magneton, and u_{km} are the components of the deformation tensor.

We choose the four-fold symmetry axes as the coordinate axes. We shall assume the constant magnetic field $\mathbf{H}_0$ and the wave-vector of the acoustic wave to be directed along one of these axes (the z-axis). We shall consider only the transverse vibrations. For cubic symmetry, we have the relations

$$F_{zxzx} = F_{zxxz} = F_{zyzy} = F_{zyyz}. \tag{2}$$

between the required coupling-tensor components. We shall denote this quantity by F. We shall assume the acoustic wavelength to be sufficiently large: $a \ll \lambda \ll L$, where a is the lattice constant and L is the length of the crystal. We can then regard the crystal as a continuous medium and characterize the paramagnetic centers by a spin density $S = S(r, t)$, the spin moment per unit volume.

Since we shall confine ourselves to studying the behavior of a plane wave, below we shall take only the dependence of quantities on the variables z and t into consideration.

The Hamiltonian for unit volume of such a medium, with allowance for the elastic vibrations of the volume elements as the transverse acoustic wave passes through, has the form

$$\mathcal{H} = \mathcal{H}_p + \mathcal{H}_s + \mathcal{H}_{s-p}, \tag{3}$$

where $\mathcal{H}_p$ is the part of the Hamiltonian density associated with the elastic vibrations of the volume element, $\mathcal{H}_S = \hbar\omega_0 S_z$ is the spin-energy density, ω_0 is the resonance transition frequency, and

$$\mathcal{H}_{s-p} = F\beta H_0 \left(\frac{\partial R_x}{\partial z} S_x + \frac{\partial R_y}{\partial z} S_y \right),$$

where $R_{x(y)}$ are the components of the transverse displacement of the volume element. In writing $\mathcal{H}_{S-p}$, we have used the relations (2).

The system of equations of motion, obtained from (3) in the usual way, has the form

$$\left(\frac{\partial^2}{\partial t^2} - v_0^2 \frac{\partial^2}{\partial z^2} \right) \mathcal{E}_+(z, t) = \frac{F\beta H_0}{\rho} \frac{\partial^2}{\partial z^2} S_+(z, t), \tag{4}$$

$$\frac{\partial}{\partial t}S_+ = i\omega_0 S_+ - i\frac{F\beta H_0}{\hbar}\mathscr{E}_+ S_z,$$

$$\frac{\partial}{\partial t}S_z = i\frac{F\beta H_0}{2\hbar}(S_-\mathscr{E}_+ - S_+\mathscr{E}_-),$$

where $S_\pm = S_x \pm iS_y$, $\mathscr{E}_\pm = \mathscr{E}_x \pm i\mathscr{E}_y$, $\mathscr{E}_{x(y)} = \partial R_{x(y)}/\partial z$ are the strain components, ρ is the density of the crystal, and v_0 is the velocity of propagation of the transverse wave. In the following account, we shall assume, for simplicity, that the wave is circularly polarized.

If the parameters of the acoustic wave vary slowly over distances of the order of the wavelength and times of the order of its period, from (4) one can find a system of contracted equations. Since below we shall consider in detail the propagation of pulses whose duration is appreciably shorter than the transverse relaxation time T_2, we shall write down the system of contracted equations with allowance for the inhomogeneous broadening of the absorption (enhancement) line. We shall take $\mathscr{E}_+$, S_+ and S_z in the form

$$\mathscr{E}_+ = \mathscr{E}(z, t)\exp[i(\omega t - kz) + \varphi(z, t)],$$

$$S_+ = \int_{-\infty}^{\infty} g(\Delta\omega)[u(\Delta\omega, z, t) + iw(\Delta\omega, z, t)]\exp[i(\omega t - kz) + \varphi(z, t)]d(\Delta\omega),$$

$$S_z = \int_{-\infty}^{\infty} g(\Delta\omega)\sigma(\Delta\omega, z, t)d(\Delta\omega), \quad (5)$$

where $\mathscr{E}$, u, w and σ are slowly varying amplitudes, φ is a weakly varying phase, and $k = \omega/v_0$; $g(\Delta\omega)$ is a symmetric function characterizing the inhomogeneous broadening of the line, such that

$$\int_{-\infty}^{\infty} g(\Delta\omega)d(\Delta\omega) = 1, \quad \Delta\omega = \omega_0 - \omega,$$

(ω_0 is the resonance frequency of the particles). The frequency of the wave coincides with the central frequency of the spectral function $g(\Delta\omega)$. The small extent of the variations in amplitude and phase of the wave is expressed by the relations

$$\left|\frac{\partial\mathscr{E}}{\partial z}\right| \ll k|\mathscr{E}|, \quad \left|\frac{\partial\mathscr{E}}{\partial t}\right| \ll \omega|\mathscr{E}|, \quad \left|\frac{\partial\varphi}{\partial z}\right| \ll k, \quad \left|\frac{\partial\varphi}{\partial t}\right| \ll \omega; \quad (6)$$

the relations for u, w and σ are similar to the relations for $\mathscr{E}$.

Putting (5) into (4), using (6) and retaining the principal term for the second derivative of S_+

$$\frac{\partial^2}{\partial z^2}S_+ \approx -k^2 S_+,$$

we obtain the system of contracted equations

$$\left(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial z} + \frac{\beta_0}{2}v_0\right)\mathscr{E} = -\frac{F\beta H_0\omega}{2\rho v_0^2}\int_{-\infty}^{\infty} g(\Delta\omega)w(\Delta\omega, z, t)d(\Delta\omega),$$

$$\mathscr{E}\left(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial z}\right)\varphi = \frac{F\beta H_0\omega}{2\rho v_0^2}\int_{-\infty}^{\infty} g(\Delta\omega)u(\Delta\omega, z, t)d(\Delta\omega),$$

$$\frac{\partial u}{\partial t} + \frac{u}{T_2} = -w\Delta\omega + w\frac{\partial\varphi}{\partial t}, \quad (7)$$

$$\frac{\partial w}{\partial t} + \frac{w}{T_2} = u\Delta\omega - u\frac{\partial\varphi}{\partial t} - \frac{F\beta H_0}{\hbar}\mathscr{E}\sigma,$$

$$\frac{\partial\sigma}{\partial t} + \frac{\sigma - \sigma_0}{T_1} = \frac{F\beta H_0}{\hbar}\mathscr{E}w,$$

where we have in addition introduced relaxation terms and a term characterizing the non-resonant losses in the medium. Here, $\sigma_0 = \sigma(\Delta\omega, z, -\infty)$, T_1 is the longitudinal relaxation time, and β_0 is the non-resonant loss factor. Below we shall assume that σ_0 does not depend on z.

The system (7) and the system of equations describing the propagation of a light pulse[2] are equivalent in form.

3. STATIONARY ACOUSTIC PULSES IN A RESONANTLY ABSORBING MEDIUM

The propagation of a short light pulse of duration $T_2^* < \tau_{pulse} \ll T_2$ ($2/T_2^*$ is the inhomogeneous absorption linewidth) in an initially unexcited medium in the absence of non-resonant losses was considered in [8]. A stationary solution was found for the shape of a pulse propagating without loss of energy (the "self-induced transparency" phenomenon). The area of such a pulse (the "2π-pulse") has a fixed value.

The existence of stationary pulses is also possible in the case of the propagation of short acoustic pulses. We shall not here completely reproduce the calculations, which are performed analogously to those of [8], but shall give the results of them.

Let the duration of the acoustic pulse $T_2^* < \tau_{pulse} \ll T_2$. We shall assume that the system of spins was in the ground state at the initial moment. Then from the system (7), we can find an equation describing the change in the area of the pulse $A(z) = \int_{-\infty}^{\infty}\mathscr{E}(z, t)dt$ as it passes through a paramagnetic crystal

$$\frac{dA}{dz} = -\frac{\alpha_0}{2}\left(\frac{F\beta H_0}{\hbar}\right)^{-1}\sin\left(\frac{F\beta H_0}{\hbar}A\right) - \frac{\beta_0}{2}A, \quad (8)$$

where $\alpha_0 = \pi F^2\beta^2 H_0^2\omega N g(0)/2\hbar v_0^3 M$. Here N is the number of paramagnetic centers in the crystal and M is the mass of the crystal. For a weak pulse, from (8) one can obtain a linear law for the absorption of acoustic energy $E \sim \mathscr{E}^2$

$$\frac{d}{dz}E = -\alpha_0 E - \beta_0 E. \quad (9)$$

Consequently, the coefficient α_0 in Eq. (8) is the usual coefficient of absorption of acoustic energy in unit volume of a crystal. As we should expect, the form of the coefficient coincides with the expression one can obtain from a quantum-mechanical calculation.

In a medium without non-resonant losses ($\beta_0 = 0$), Eq. (8) has the solution

$$A(z) = \frac{2\hbar}{F\beta H_0}\operatorname{arctg}\left[\operatorname{tg}\left(\frac{F\beta H_0}{2\hbar}A(0)\right)\exp\left(-\frac{\alpha_0}{2}z\right)\right]. \quad (10)$$

It follows from (10) that pulses with initial area $A(0) < \pi\hbar/F\beta H_0$ are damped while propagating. If $A(0) > \pi\hbar/F\beta H_0$, the area of the pulse increases up to the value $2\pi\hbar/F\beta H_0$. The resulting pulse propagates without change of shape.

The shape of the stationary pulse of area $A = 2\pi\hbar/F\beta H_0$ has the form

$$\mathscr{E}(z, t) = \frac{2\hbar}{F\beta H_0\tau}\operatorname{sch}\left[\frac{1}{\tau}\left(t - \frac{z}{v}\right)\right], \quad (11)$$

where v is the propagation velocity of the pulse and τ is the pulse duration parameter (the slope of the exponential).

We shall give expressions for the pulse velocity v. In the case of a narrow absorption line, when $g(\Delta\omega) = \delta(\Delta\omega)$,

$$\frac{1}{v}=\frac{1}{v_0}+\frac{F^2\beta^2H_0^2\omega N}{4\hbar v_0^3 M}\tau^2. \quad (12)$$

In the general case,

$$\frac{1}{v}=\frac{1}{v_0}+\frac{\alpha_0\tau^2}{2\pi g(0)}\int_{-\infty}^{\infty}\frac{g(\Delta\omega)d(\Delta\omega)}{1+(\tau\Delta\omega)^2}. \quad (13)$$

If $\tau \gg T_2^*$, then, taking $g(\Delta\omega)$ to be constant for a range of frequencies of width $\approx\tau^{-1}$, from (13) we obtain

$$\frac{1}{v}=\frac{1}{v_0}+\frac{\alpha_0\tau}{2}, \quad (14)$$

i.e., the pulse is delayed by an amount $\Delta t \sim \tau$ in covering a path of length α_0^{-1}.

The attainment of nano-second acoustic pulses of frequency $\omega \sim 10^{10}$ sec^{-1} [9] makes it possible to fulfill the condition $\tau_{\text{pulse}} \ll T_2$ necessary for the observation of the self-induced transparency effect.

The author is grateful to Professor U. Kh. Kopvillem for discussions on the work and comments.

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Translated by P. J. Shepherd
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