INCREASE OF THE ELECTROMAGNETIC AND WEAK CORRECTIONS TO THE SCATTERING OF HADRONS WITH ENERGY

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It is shown that if the strong interaction occurs over larger (increasing with energy) distances, then in this connection the electromagnetic and weak corrections to the scattering of hadrons should increase with energy. The effects of the violation of isotopic invariance (due to the electromagnetic corrections) and of the nonconservation of spatial parity (due to the weak interactions) which are expected at high energies are estimated.

INTRODUCTION

IT is well known that the radiative corrections in the quantum electrodynamics of an electron and in the theory of leptonic weak interactions increase with energy as a consequence of the fact that the photon's spin is one (the vector nature of the weak interaction). If it is assumed that all of the hadrons are Reggeized and that the strong interaction occurs over larger (increasing with energy) distances, then according to the same reason the electromagnetic and weak corrections to the scattering of hadrons should also increase with energy. This result means that the strong interaction does not cut off the increase, characteristic for point particles, of the weak and electromagnetic corrections with increasing energy, since it occurs over different distances.

In Sec. 1 of this article the energy dependence of the electromagnetic corrections to the scattering of hadrons is discussed from the point of view of the singularities of the partial wave amplitudes with respect to the angular momentum. If, in the absence of the electromagnetic corrections, the vacuum pole for $t = 0$ is located at the point $j = 1$ or to the left of $j = 1$ by an amount of the order of e^2,[1] then the electromagnetic corrections shift it to the right of $j = 1$. This result means that the electromagnetic corrections to the scattering of hadrons must increase with energy. With the aid of the same concepts, at the end of Sec. 1 it is shown that in order to estimate the weak corrections to the scattering of high-energy hadrons it is correct to use the simplest graphs involving the exchange of one or two W mesons. There, on the basis of the scaling hypothesis for the amplitude for the scattering of a W meson by a hadron,[2] it is shown that the weak corrections of second order to the hadron amplitude must increase quadratically with the energy[3] in exactly the same way as the analogous corrections to the scattering of point particles.

Possible experiments with regard to the verification of the concepts developed in this article are discussed in Sec. 2. The effects, anticipated at high energies, of the violation of isotopic invariance (owing to the electromagnetic corrections) and of the nonconservation of spatial parity (due to the weak corrections) are estimated.

1. ANALYSIS OF THE ENERGY DEPENDENCE OF THE ELECTROMAGNETIC AND WEAK CORRECTIONS TO THE SCATTERING OF HADRONS

In order to simplify the discussion, here it is assumed that the photon has a nonvanishing rest mass. The method of separating out the infinite Coulomb phase and the other specific problems which are related to the masslessness of the photon are investigated in detail in[4,5]; therefore they will not be discussed in the present article.

Let us consider the electromagnetic correction of order e^4 to elastic nucleon-nucleon scattering. In this case one can use the line of reasoning connected with the presence of the third spectral function $\rho(s, u)$ in the Mandelstam representation.[6] Owing to the existence of $\rho(s, u)$, the partial wave amplitudes of purely hadronic processes contain fixed singularities in the angular momentum (coming from graphs of the type shown in Fig. 1), which are related to the fact that the spin of the photon is equal to one.[6,7] As a consequence of the absence of Reggeization of the photon, these singularities do not cancel out, which therefore leads to the growth with energy of the electromagnetic corrections to the scattering of hadrons.

In the present investigation, the presence (to lowest order in e^2) of a fixed singularity with respect to the angular momentum in the t-channel sense-nonsense partial wave amplitude $\varphi_j^{+\gamma N}$ for Compton scattering plays an essential role.[1)] It is known that $\varphi_j^{+\gamma N} \sim e^2/\sqrt{j-1}$.[8] Then the fact that the singularity occurs

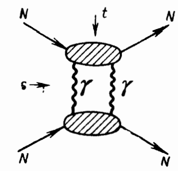

FIG. 1

[1)]The terms "sense" ("nonsense") are introduced in [8] for the purpose of distinguishing states for which the projection of the angular momentum is smaller (larger) than the value of the momentum itself.

at j = 1 is related to the spin of the photon, and the fact that the signature is positive is related to the identity of the quanta.[6,8,9] The following reasons are known for the existence of such a square-root singularity in $\varphi_j^{+\gamma N}$:

1. Within the framework of perturbation theory, the linear condition of unitarity is valid for the partial wave amplitudes of the process $\gamma + \gamma \rightarrow N + \overline{N}$; therefore there is no reason for a cancellation of the fixed singularities of the type being discussed to appear in a natural way in perturbation theory.[8] An intuitive explanation of a mechanism for the appearance of such singularities in perturbation theory is given in the Appendix.

2. This singularity must be present in $\varphi_j^{+\gamma N}$ because of the existence of the third spectral function $\rho(s, u)$ in the Mandelstam representation.[6,7]

It is natural to expect that as a consequence of the presence of the strong interactions, the partial wave amplitude will in fact be more singular at j = 1. For example, for values of t close to zero

$$\varphi_j^{+\gamma N} = e^2 r(t)/[j-\beta(t)]\sqrt{j-1}. \qquad (1)$$

Here $\beta(t)$ denotes the trajectory of the vacuum pole for which $\beta(0) = 1$, and r(t) specifies the residue of the vacuum pole. (We have neglected vacuum branch points only in order to simplify the discussion. Their contribution to the asymptotic form of the radiative corrections will be discussed in detail below.) For the specific formula (1), the following additional justifications also exist:

1) A partial wave amplitude of this type appears in a natural way in the simple models discussed in[9].

2) Such a form for $\varphi_j^{+\gamma N}$ is necessary in order that the total cross section for the scattering of a real photon by a hadron is constant at high energies where, due to the masslessness of the photon this cross section is expressed only in terms of the nonsense amplitude.[10] In fact, the question of the presence of a fixed singularity in the residue of the vacuum pole is not related to the absence of photon rest mass, since if r(0) = 0 for a virtual quantum then $\sigma\gamma N$ must tend to zero with increasing energy, where $\sigma\gamma N$ is the total cross section for the scattering of a virtual photon, measured in experiments involving electroproduction on a nucleon.

3) If the vacuum pole does not give any contribution to $\sigma\gamma N$ for t = 0, then at the same time the contribution from the two-reggeon vacuum branch cut must also tend to zero since the latter gives a negative contribution to $\sigma\gamma N$.[11]

Iteration of the nonsense partial wave amplitude $\varphi_j^{+\gamma N}$ with the aid of the two-particle unitarity condition in the t-channel and dispersion relations (or the direct investigation of the Feynman graph shown in Fig. 1) leads to the following result for the electromagnetic correction to the partial wave amplitude for NN scattering:

$$f_j^{+NN}(t) \approx e^4\varkappa(t)/(j-1)[j-\beta(t)]^2. \qquad (2)$$

Here $\kappa(t)$ specifies the residue of the vacuum pole. If the photon were a reggeon, then the fixed singularity which we have found would be cancelled by the contribution from the many-particle states.[12] (Here we shall not discuss the hypothesis of Blankenbecler et al.[13]

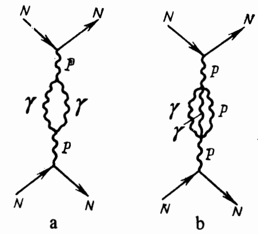

FIG. 2

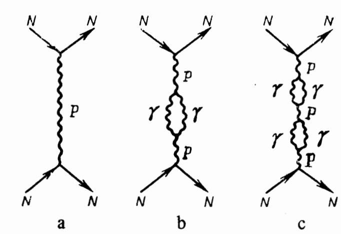

FIG. 3

that the photon can be Reggeized to the lowest order in e^2, since this hypothesis leads to a change in the electrodynamics of leptons.)

Let us calculate the asymptotic behavior of A_{NN}—the electromagnetic correction to elastic NN-scattering corresponding to the t-channel partial wave amplitude (formula (2)):

$$A_{NN} \approx e^4\varkappa(0)\{i\ln^2(s/s_0) + \pi\ln(s/s_0)\}s/s_0, \quad t = 0, \qquad (3)$$

$$A_{NN} \approx ie^4\varkappa(t)s/s_0, \quad -\beta' t\ln(s/s_0) \gg 1; \qquad (4)$$

here β' denotes the slope of the trajectory associated with the vacuum pole. Let us note the characteristic properties of formulas (3) and (4). In the first place, the correction to the total cross section increases like $e^4\ln^2(s/s_0)$.[3] (The condition $\kappa(0) > 0$ follows from the positiveness of the imaginary part of the graph shown in Fig. 1). In the second place, in (4) there is no shrinkage of the diffraction cone with energy, that is, formula (4) corresponds to the contribution of finite impact parameters which are not increasing with energy. But the contribution of the strong interactions at such distances is small owing to the shrinkage of the diffraction cone with energy. (For example, the s-channel partial wave amplitude, corresponding to the exchange of the vacuum pole, is of the order of $1/\ln(s/s_0)$.[14]) It is precisely this property of the strong interactions which does not allow them to cancel the contribution from the graph shown in Fig. 1.

The latter qualitative discussion can be confirmed by an analysis of the contribution from the vacuum branch points. The rules for the description of the Reggeon diagrams are the same as in article[11], except for a single exception: The exchange of two photons must be associated with a fixed pole for j = 1 at points of positive signature, with the residue equal to $e^4\lambda(t)$ (here $\lambda(t) = \kappa(t)/r_{NN}^2(t)$, and $r_{NN}(t)$ denotes the nucleon vertex of the vacuum pole). Let us consider typical Reggeon graphs for f_j^{NN} at t = 0. Formula (2) corresponds to the Reggeon graph a in Fig. 2, where P denotes the vacuum pole. It is obvious that the graph shown in Fig. 2b gives a smaller contribution to f_j^{NN}; a factor $\ln(j-1)$ arises from the closed loop[11] instead of the pole $1/(j-1)$ coming from two-photon exchange. This additional smallness appears upon integration over the momentum transfer as a result of the Reggeization of P. As a consequence of the quasistability of the Pomeranchuk pole (the three- and four-Reggeon vertices vanish[15] at zero momentum transfer), an increase of the order of the vacuum branch point leads to an even smaller contribution. The proof of the latter assertion does not differ from the arguments given in article[15], and therefore they are not given here. Since the contribution of the vacuum branch points associated with the quasistability of the Pomeranchuk pole is always smaller than the contribution from the vacuum pole,[15] then to each order of perturbation theory in e^4, the Reggeon graphs shown in Fig. 3 give the principal singularity in j. As is well known from perturbation theory in quantum electrodynamics,[16] the multiphoton intermediate states in the t-channel give, in any specific order of perturbation theory, a less singular contribution than the one coming from the graphs shown in Fig. 3.

In the approximation $e^2/(j-1) \sim 1$, $e^4/(j-1) \ll 1$, one can calculate the effective singularity by summing the dominant Reggeon graphs shown in Fig. 3. The calculation reduces to the summation of the geometrical progression

$$f_j \approx \frac{1}{j-\beta}\sum_{n=0}^{\infty}\left[\frac{e^4\lambda(0)}{(j-1)(j-\beta)}\right]^n = \frac{j-1}{(j-1)(j-\beta)-e^4\lambda(0)} \quad (5)$$

As a result of the summation, two poles appear, given by

$$j_\pm = {}^1/_2[1+\beta\pm\sqrt{(1-\beta)^2+4e^4\lambda(0)}]. \quad (6)$$

One of these poles, j_+, is found to the right of the point $j = 1$ independently of the value of $\beta(0)$. If $\beta(0)$ differs from unity by an amount of the order of e^2:[1,4]

$$\beta(0) = 1 - 2e^2\Delta,$$

then the residue at the pole for $j = j_+$ does not contain e^2:

$$\lim_{j\to j_+} f_j(j-j_+) = [\sqrt{\Delta^2+\lambda}-\Delta]/2\sqrt{\Delta^2+\lambda}, \quad (7)$$

that is, because of two-photon exchange the cross section must increase with energy.

Let us discuss the radiative correction to the hadron amplitude of negative signature coming from one-photon exchange. As a consequence of the fact that the photon's spin is equal to one, the pole graph (see Fig. 4) has the asymptotic form $M^- \sim e^2 s$. Using the methods of the Reggeon diagram technique,[11] one can show that taking the strong interactions into account cannot lead to a decrease with increasing energy of the contribution from one-photon exchange. For example, the sum of the graphs shown in Fig. 5 is asymptotically real:[4]

$$M \sim e^2 s\left\{\frac{1}{\ln(s/s_0)} + \frac{i\pi}{2\ln^2(s/s_0)}\right\}. \quad (8)$$

The increase of the order of the vacuum branch point leads to a smaller contribution due to the fact that the three- and four-Reggeon vertices vanish at zero momentum transfer. (Here we shall not consider the graphs which reduce to the Coulomb phase,[4] due to the vanishing of the photon's rest mass.)

As a consequence of the vector nature of the weak interaction, the considerations developed above are also basically applicable to the weak corrections to the scat-

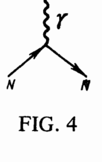

FIG. 4

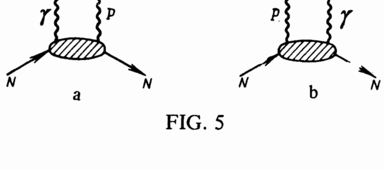

FIG. 5

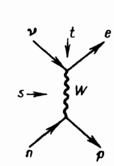

FIG. 6

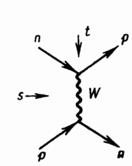

FIG. 7

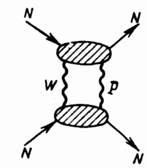

FIG. 8

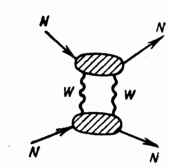

FIG. 9

tering of hadrons. First let us consider the weak correction of first order in G (G is the coupling constant of the Fermi interaction). It is well known that the process of weak scattering of leptons by nucleons is described by graphs of the type shown in Fig. 6, which corresponds to the process $\nu + n \to e + p$. (In order to simplify the discussion we assume that the weak interaction is due to the exchange of a W-meson.) For $s \to \infty$ and fixed t, the matrix element for this process has the form GsF(t)—that is, the same form as for electromagnetic scattering. Within the framework of the usual theory of the universal weak interaction (upon switching the strong interaction off), the scattering of hadrons would be described by a graph of the type shown in Fig. 7, which represents the weak scattering process $p + n \to n + p$. Graphs of the type shown in Fig. 7, but now with form factors in t which take the finite dimensions of the hadron into account, dominate at high energy. The proof of the latter assertion is based on the same reasoning as the proof that single-photon exchange is the dominant process. It does not depend on whether we describe the weak interaction with the aid of the Fermi theory or as a result of the exchange of a W meson. (In connection with the construction of the Reggeon diagram technique, the fact that the integrals over the momentum transfer in Reggeon graphs of the type shown in Fig. 8 are automatically cut off, due to the shrinkage of the diffraction cone with energy, plays an important role.)

As to the correction of second order in the weak interaction (see Fig. 9), here the integral over the momentum transfer may diverge.[17] The divergent part corresponds to the subtraction constant in a dispersion relation; it does not increase with energy. We will only be interested in the contribution from the convergent part of the Feynman diagrams, which is increasing with energy. (For example, in the theory with W mesons we will only be interested in the contribution from the transverse part of the W-meson propagator.)

The weak correction to elastic NN-scattering, coming from the simplest diagram which is shown in Fig. 9, rapidly increases with energy: $\mathrm{Im}\, A_{NN} \approx (Gs)^2$. For the proof it is sufficient in the calculation of $\mathrm{Im}\, A_{NN}$ to substitute the Compton amplitude, which is known from electroproduction,[18] in place of the amplitude for the scattering of a W meson by a nucleon. Such a strong increase with energy follows[3] from Bjorken's scaling hypothesis for the Compton amplitude.[2] (We note that for $s \lesssim m_W^2$ a difference appears between the Fermi theory and the W-meson theory. If the W meson exists, then for $s \geq m_W^2$ the quadratic increase with energy must stop due to the presence of the W propagators.) If the scaling hypothesis is valid for any arbitrary quasielastic amplitudes for the scattering of the W meson by a hadron (the recent experimental data concerning electroproduction on protons and on deuterons[18] indicates this), then the weak corrections to any quasielastic hadron processes increase quadratically with increasing energy in analogy to the weak corrections to lepton-hadron scattering.[19] One can show that the corrections to the graphs of the type shown in Fig. 9 due to vacuum branch points are asymptotically small on account of the shrinkage of the diffraction cone with energy.

The results obtained in this section are valid at ultrahigh energies. In the range of energies which are accessible to experimental verification, the contribution of the corrections from the vacuum branch points no longer contains the parametric small quantity. (The logarithmic decrease with increasing energy is not present.) If it is assumed that the vacuum branch points represent a numerical correction (and this is precisely what is assumed in discussions of the experimental data[20]), then for estimates of the weak and electromagnetic corrections one can use the graphs shown in Figs. 1, 4, 7, and 9. We note that the basic assertion about the increase with energy of the electromagnetic and weak corrections to the scattering of hadrons is also valid in this range of energies.

2. COMPARISON WITH EXPERIMENT

First let us discuss the electromagnetic corrections to the scattering of hadrons. The absence of Reggeization of the photon leads to an increase of the relative contribution due to the electromagnetic corrections. Parametrically the ratio of the electromagnetic amplitude (due to photon exchange) to the strong-interaction amplitude is $e^2(s/t)/(s/s_0)^{\beta(t)}$, where $\beta(t)$ is the trajectory of the Reggeon which dominates in the reaction.

As an example let us consider the ratio of the contribution from single-photon exchange to the cross section for the reaction $\gamma + p \to \pi^0 + p$ compared to the contribution from the strong interactions:

$$H = \frac{d}{dt}\sigma^{\mathrm{em}}(\gamma + p \to \pi^0 + p) \Big/ \frac{d}{dt}\sigma(\gamma + p \to \pi^0 + p).$$

In order to estimate the photon's contribution it is assumed that the vertex $\gamma \to \pi^0\gamma$ depends on the mass of the quantum like the electromagnetic form factor of the nucleon. For E_γ = 6 GeV we have $H(t = -0.1) = 6.5 \times 10^{-3}$ and $H(t = -0.4) = 4.5 \times 10^{-4}$[21] (t is expressed in units of GeV^2). If the experimental dependence from[21] is used for the energy extrapolation

$$\frac{d}{dt}\sigma(\gamma + p \to \pi^0 + p) \sim s^{2\alpha(t)-2}, \qquad \alpha(t) = 0.18 + 0.26t,$$

then at E_γ = 200 GeV we obtain $H(t = -0.1) \approx 2$ and $H(t = -0.4) \approx 0.4$. It is probably somewhat easier to observe the violation of the isotopic-spin selection rules in reactions of the type $N + N \to N + \Delta^+$, $\pi + N \to \pi + \Delta^+$, and so forth due to the exchange of a photon or as a consequence of the correction to the residue of the vacuum pole.

If the electromagnetic correction to the position of the vacuum pole does not vanish,[4] then there exists a correction to the hadron total cross section of the order of $e^2\ln(s/s_0)$, which is impossible to estimate at the present time.

As a consequence of the smallness of e^2, it is helpful to estimate the two-photon correction only to the total cross section, because of its rapid increase with energy. The large (logarithm squared) contribution to the cross section arises only from the three graphs shown in Fig. 10. As an example, let us estimate the two-photon contribution to the total cross section for proton-proton scattering. (In Fig. 10 the hatched blocks represent the amplitude for Compton scattering, not containing the proton pole. Such a separation of the Feynman graphs is useful for a discussion of the complications associated with the masslessness of the photon.) The asymptotic behavior of the graph shown in Fig. 10a was discussed above, see formulas (3) and (4). (The vanishing of the photon's rest mass because of gauge invariance does not affect the asymptotic behavior of this diagram.) The doubly logarithmic contribution to the cross) section, coming from diagrams b and c, arises from two different causes: from the constancy of $\sigma_T^{\gamma p}$ at high energies and from the vanishing of the photon's rest mass. Let us present the answer for the contribution to the total cross section coming from graph a in Fig. 10 and from the summation of graphs b and c:

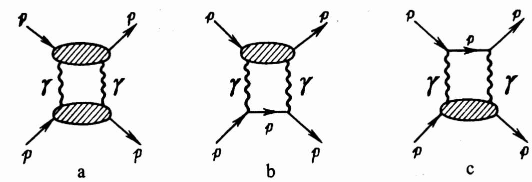

FIG. 10

$$\delta\sigma_{pp}^{(a)} = \frac{a[\sigma_T^{\gamma p}]^2}{(2\pi)^3}\ln^2\frac{s}{m^2}, \tag{9}$$

$$\delta\sigma_{pp}^{(b)} + \delta\sigma_{pp}^{(c)} = \frac{e^2}{2\pi^2}\sigma_T^{\gamma p}\ln^2\frac{s}{m^2}. \tag{10}$$

In calculating the asymptotic behavior we neglected $\sigma_L^{\gamma p}$ and carried out the parametrization $\sigma_T^{\gamma p}(s, k^2) = \sigma_T^{\gamma p}(s)/(1 - k^2/a)$ where $a = (1/2)$ GeV^2 in agreement with the observations concerning the smallness of $\sigma_L^{\gamma p}$ and the decrease of $\sigma_T^{\gamma p}$ with increasing values of k^2.[18] Here $\sigma_L(\sigma_T)$ denotes the total cross section for the scattering of a longitudinal (transverse) virtual photon, which is known from electroproduction.[18]

Let us calculate the ratio $\delta\sigma_{pp}/\sigma_{pp}$:

$$\frac{\delta\sigma_{pp}}{\sigma_{pp}} = \frac{\delta\sigma_{pp}^{(a)} + \delta\sigma_{pp}^{(b)} + \delta\sigma_{pp}^{(c)}}{\sigma_{pp}} \approx 1.3\cdot 10^{-5}\ln^2\frac{s}{m^2} \tag{11}$$

In order to obtain the last estimate, it was assumed that $\sigma_T^{\gamma p}(s \to \infty) \approx 0.1$ mb,[22] and $\sigma_{pp}(s \to \infty) \approx 40$ mb. In the colliding beams accelerator, where $s = (56)^2$ GeV^2 will be reached, the value of $\delta\sigma_{pp}/\sigma_{pp} \approx 10^{-3}$.

For scattering by heavy nuclei the effect is larger as a result of the coherent scattering of light by the protons in the nucleus (graph c shown in Fig. 10):

$$\delta\sigma_{pA} = \frac{e^2}{4\pi^2}Z^2\sigma_T^{\gamma p}\ln^2\frac{E}{M_p}. \tag{12}$$

Here E denotes the energy of the incident proton in the laboratory system, where the nucleus is at rest, and Ze is the charge of the nucleus. Let us calculate the ratio

$$\frac{\delta\sigma_{pA}}{\sigma_{pA}} = \frac{e^2}{4\pi^2}\frac{Z^2}{A^{2/3}}\frac{\sigma_T^{\gamma p}}{\sigma_{pp}}\ln^2\frac{E}{M} \approx 6\cdot 10^{-6}\frac{Z^2}{A^{2/3}}\ln^2\frac{E}{M}. \tag{13}$$

In the derivation of Eq. (13) it was assumed that

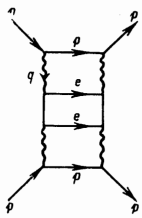

FIG. 11

$\sigma_{pA}(s \to \infty) \to A^{2/3}\sigma_{pp}$.[23] (For large values of Z ($\alpha Z \sim 1$) it is impossible to use formula (13) as a consequence of the large contribution due to rescatterings on the nucleus.) For example, for A = 60, Z = 30, and E_p = 70 GeV we obtain $\delta\sigma_{pA}/\sigma_{pA} \approx 6 \times 10^{-3}$. At E_p = 400 GeV one will find $\delta\sigma_{pA}/\sigma_{pA} \approx 10^{-2}$. In order to compare this prediction with experiment, it is necessary beforehand to experimentally isolate the large effect calculated by L. D. Landau and E. M. Lifshitz[24]—the production of electron-positron pairs in pp-scattering (see Fig. 11). Namely,

$$\frac{\delta\sigma_{pp}}{\sigma_{pp}} = \frac{28}{27}\frac{\alpha^4 \ln^3(E/M_p)}{\pi m_e^2 \sigma_{pp}} \approx 3.6 \cdot 10^{-5} \ln^3 \frac{E}{M_p}, \qquad (14)$$

where $\alpha = e^2/4\pi$. For E_p = 70 GeV the quantity $\delta\sigma_{pp}/\sigma_{pp} \approx 3 \times 10^{-3}$, and for E_p = 400 GeV it is equal to $\delta\sigma_{pp}/\sigma_{pp} \approx 10^{-2}$. For scattering by nuclei the ratio $\delta\sigma_{pA}/\sigma_{pA}$ is larger by a factor of $Z^2/A^{2/3}$ times.

The increase with energy of the hadronic total cross sections is also predicted by the theory of complex angular momenta.[15,25] It can be distinguished from the increase of the cross section associated with the two-photon correction since, in contrast to the contribution from the vacuum branch points, the contribution from the two-photon corrections depends on the charges of the hadrons—graphs b and c in Fig. 10. For example, the difference between the total cross sections for proton-proton and proton-neutron scattering should increase with energy due to the two-photon correction. This effect is easier to observe in scattering by nuclei with different values of Z but containing an identical number of nucleons. (In order for it to be possible to observe the predicted increase of the total cross sections, it is important that the contribution of the nonvacuum Regge poles to σ_{NN} be small.[20]) It is more interesting to observe the increase with energy of the violation of isotopic-spin invariance with regard to the difference between the total cross sections for πN scattering, $\sigma(\pi^+p) \neq \sigma(\pi^-n)$, said difference being due to graphs of the type shown in Fig. 10. The magnitude of the effect is given by formula (10), in which it is necessary to substitute $\sigma_T^{\gamma\pi}$ in place of $\sigma_T^{\gamma p}$.

In the range of energies which are accessible to experimental test, it is difficult to separate the contribution of the two-photon correction to the total hadronic cross section from the contribution due to single-photon exchange (from graphs of the type shown in Fig. 5). One can distinguish the two effects according to signature. For example, single-photon exchange does not give any contribution to the difference $\sigma_{pp} + \sigma_{\bar{p}p} - \sigma_{pn} - \sigma_{\bar{p}n}$.

Now let us go on to the weak corrections to the scattering of hadrons. In this case the graph shown in Fig. 7 dominates. The simplest method of testing this prediction is to search for reactions involving nonconservation of strangeness of the type $\pi^- + p \to K^0 + n$, $p + n \to \Sigma^0(\Lambda) + p$, etc. The cross sections for these reactions are parametrically equal to $G^2M_N^2 \sin^2\theta$ (M_N denotes the nucleon mass). In the estimate, it was assumed that the form factors of all of the hadrons are identical, and the additional smallness due to the Cabibbo angle[26] for processes involving nonconservation of strangeness was also taken into account. Substitution of the graph shown in Fig. 7 as the matrix element for the process $\pi^- + p \to K^0 + n$ leads to the cross section $\sigma \approx 10^{-40}$ cm^2. (The cross section for reactions involving a double violation of the conservation of strangeness, such as $\pi^- + p \to K^0 + \Sigma^0$, $p + n \to \Sigma^0(\Lambda) + \Sigma^+$, etc. is smaller in magnitude by a factor of $\sin^2\theta$.)

The nonconservation of spatial parity leads to a large effect thanks to the possibility of interference between the strong and weak amplitudes. The cross section of the inelastic reactions with nonvacuum quantum numbers in the t-channel rapidly falls with energy; therefore, the relative contribution of the weak interactions must increase with increase of energy. Parametrically, the weak effects (involving the nonconservation of spatial and charge parities) at high energies are of the order of the ratio $Gs/(s/s_0)^{\beta(t)}$ of the weak and strong-interaction amplitudes, where $\beta(t)$ is the trajectory of the nonvacuum Reggeon which dominates in this reaction.[3] For observation the charge-exchange reactions $\pi^- + p \to \pi^0 + n$, $K^+ + n \to K^0 + p$, etc. are convenient, and in these reactions at the present time a shrinkage of the diffraction cone with increasing energy is observed.

It is possible to observe the effect of nonconservation of spatial parity in the appearance of a dependence of the cross section on the polarization of one of the colliding particles[27] (it is necessary to sum over the polarizations of the remaining particles). At zero momentum transfer, the differential cross section for scattering on a polarized target has the form

$$\frac{d\sigma}{dt} = \left\langle \frac{d\sigma}{dt} \right\rangle \left(1 + \delta \frac{\boldsymbol{\xi}\mathbf{k}}{|\mathbf{k}|}\right), \qquad (15)$$

where $\langle d\sigma/dt \rangle$ denotes the differential cross section summed over the polarizations of all of the particles participating in the reaction: $\mathbf{k}$ denotes the momentum of the incident particle and $\boldsymbol{\xi}$ is the polarization vector of the target particle. The coefficient δ in formula (16) characterizes the magnitude of the violation of conservation of spatial parity. As an example we have estimated δ for the charge-exchange reaction $\pi^- + p \to \pi^0 + n$. In order to extrapolate the energy dependence, we used the parametrization of the strong-interaction amplitude as the result of exchange of a ρ-Reggeon, given in[28]. As the result of the calculation, it was found that $\delta(\pi^-p \to \pi^0 n) = 4 \times 10^{-6} (E_\pi/E_0)^{0.4}$, where E_0 = 1 GeV and E_π is the energy of the π meson in the laboratory system. For E_π = 100 GeV the quantity $\delta_\pi = 2.5 \times 10^{-5}$.

In order to estimate the effect of nonconservation of spatial parity in the charge-exchange reaction $p + n \to n + p$, let us investigate the ratio of the differential cross section for the weak process described by the graph shown in Fig. 7 to the cross section for the purely hadronic process:

$$R^2 = \frac{d}{dt}\sigma^w \Big/ \frac{d}{dt}\sigma.$$

For E_p = 19 GeV the quantity $R^2(t=0) \approx 10^{-10}$, and $R^2(t=-0.1) \approx 3 \times 10^{-10}$ (t is expressed in units of GeV^2). The enhancement appeared as a consequence of the rapid fall in the differential cross section for the charge-exchange reaction $p + n \to n + p$ with momentum transfer (for small values of t). The experimental data concerning the reaction $p + n \to n + p$ is taken from[29]. The differential cross section of this reaction falls off with energy like s^{-2}, or even faster like $s^{-2.3}$.[29] If this dependence is used for the extrapolation of R to the region of higher energies, then at E_p = 400 GeV we obtain $R(t=0) \approx (2 \text{ to } 3) \times 10^{-4}$, and $R(t=-0.1) \approx (4 \text{ to } 6) \times 10^{-4}$. (The nonconservation of parity in the reaction is characterized by the value of R, which is of the order of the ratio of the amplitudes of the weak and strong interactions.

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APPENDIX

Let us indicate, following Gribov,[11] how the fixed singularities arise in the partial wave amplitudes. Let us write down the expansion of the amplitudes for the scattering of particles with identical masses but arbitrary spins in terms of partial waves in the t-channel:

$$\langle\lambda_a\lambda_b|T|\lambda_c\lambda_d\rangle = \sum (2j+1)\, d_{\mu\lambda}^{j}(z_t)\, f^{\lambda_c\lambda_d}_{\lambda_a\lambda_b}(t); \qquad (A.1)$$

here $z_t = 1 + 2s/(t - 4\mu^2)$, and $\lambda = \lambda_a - \lambda_b$, $\mu = \lambda_c - \lambda_d$. Let us replace (A.1) by the Sommerfeld-Watson integral, and $d^j_{\mu\lambda}(z)$ by its asymptotic value:

$$d_{\mu\lambda}^{j}(z) \to \frac{(2z)^j}{\sqrt{\pi}} \frac{\Gamma(j+1)\Gamma(j+1/2)}{\{\Gamma(j+\mu+1)\Gamma(j-\mu+1)\Gamma(j+\lambda+1)\Gamma(j-\lambda+1)\}^{1/2}},$$

that is,

$$\langle\lambda_a\lambda_b|T^{\pm}|\lambda_c\lambda_d\rangle = -\frac{1}{i}\int \varphi_j^{\pm} \frac{s^j \pm (-s)^j}{\sin \pi j}\, dj, \qquad (A.2)$$

where

$$\varphi_j^{\pm} = f^{\lambda_c\lambda_d}_{\lambda_a\lambda_b} \frac{1}{4\sqrt{\pi}} \left(\frac{t-4\mu^2}{4}\right)^{-j} \times \frac{(2j+1)\Gamma(j+1)\Gamma(j+1/2)}{[\Gamma(j+\mu+1)\Gamma(j-\mu+1)\Gamma(j+\lambda+1)\Gamma(j-\lambda+1)]^{1/2}}. \qquad (A.3)$$

The ± signs indicate the signature. Let us take the transform of the integral in (A.2) to obtain:

$$\varphi_j^{\pm} = \frac{1}{\pi}\int_{s_0}^{\infty} \mathrm{Im}\,\langle\lambda_a\lambda_b|T^{\pm}|\lambda_c\lambda_d\rangle \frac{ds'}{(s')^{j+1}}.$$

Due to the presence of the Γ functions, near $j = \mu - 1$ we have

$$f_j^{\pm} \sim \frac{1}{\sqrt{j-\mu+1}}\int_{s_0}^{\infty} \mathrm{Im}\,\langle\lambda_a\lambda_b|T^{\pm}|\lambda_c\lambda_d\rangle \frac{ds'}{(s')^{j+1}}. \qquad (A.4)$$

In the rigorous investigation given in[8], the integrand in formula (A.4) has a more complicated form, but this change does not affect the structure of the nonsense singularities. If the integral in (A.4) does not vanish, then the singularity is of the square-root type. If $j \le \mu - 1$, $\lambda - 1$, then the singularity is a pole.[8] This integral does not vanish to lowest order in perturbation theory.[8] In the general case the integral (A.4) does not vanish as a consequence of the existence of the third spectral function $\rho(s, u)$ in the Mandelstam representation.[6,7] The contribution from $\rho(s, u)$ differs from zero only at the wrong signature. From formula (A.1) it follows that a nonsense singularity at points of wrong signature does not give any contribution to the asymptotic behavior of the amplitude. It is important only as a consequence of the nonlinearity of the unitarity condition, since as the result of squaring, a clustering of singularities near the fixed singularity appears[6] in the absence of Reggeization of the particle (in perturbation theory).

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4