KINETIC RESONANCE IN A BINARY MIXTURE OF MOLECULAR GASES

V. D. BORMAN, B. I. NIKOLAEV, and V. I. TROYAN

Moscow Engineering Physics Institute

Submitted January 27, 1971

Zh. Eksp. Teor. Fiz. 61, 208–221 (July, 1971)

The thermal conductivity of binary mixtures of molecular non-paramagnetic gases in mutually perpendicular constant and alternating magnetic fields is considered. An expression describing the resonance spectrum, each line of which corresponds to thermal conductivity resonance with various mixture components, is obtained by solving the set of kinetic equations and taking into account the pseudo-proper collision operators. Examples of fine structure of the resonance lines are considered. The dependence of the resonant variation of the thermal conductivity of molecular gas mixtures on the concentration of each component is obtained. Cases when the concentration of one of the components is small or when the values of the thermal conductivity co-efficients, molecular weights and nonsphericity scattering parameters of the components are close to each other are considered in greater detail.

1. INTRODUCTION

GAS-kinetic magnetic resonance (GMR) was observed experimentally with the thermal conductivity of O_2 as an example[1]. The theory of the effect is given in[2]. This effect is observed in mutually perpendicular constant and alternating magnetic fields, is exhibited by all molecular gases, and is due to the additional change, compared with the case of a constant field, of the scattering cross section of nonspherical molecules, which has a maximum when the frequency of the alternating field is equal to the frequency of molecule precession in the constant field.

Gorelik, Rukavishnikov, and Sinitsyn[3] observed resonance of the thermal conductivity of the polar gas NF_3 in mutually perpendicular constant magnetic and alternating electric fields. The resonant behavior of the angular momentum of the NF_3 molecules and consequently the resonant increase of the scattering cross section were due to the interaction of the (rotational) magnetic moment with the constant magnetic field and of the dipole moment with the alternating electric field.

As shown earlier[2], the resonant change of the thermal-conductivity coefficient should have several maxima (fine structure of GMR), the presence of which is connected with the symmetry of the perturbed collision operator and with the angular dependence of the nonspherical potential of molecule interaction.

In the present paper we analyze theoretically the phenomenon of kinetic resonance in binary mixtures of molecular gases. By solving the system of kinetic equations we obtain expressions describing the behavior of the thermal-conductivity coefficients of such a mixture in mutually perpendicular constant and alternating fields. We consider the fine structure of the resonance lines corresponding to different mixture components, and the concentration dependences of the effect.

2. SYSTEM OF KINETIC EQUATIONS

The system of linearized kinetic equations describing the binary mixture of gases with rotating molecules in a spherical coordinate system, in which the z axis coincides with the direction of the constant magnetic field H_0, is of the form (we use throughout the indices $\alpha, \beta = 1, 2$)

$$\frac{\partial\chi_\alpha}{\partial t}+N_\alpha+\gamma_\alpha[\mathbf{M}_\alpha\mathbf{H}]\frac{\partial\chi_\alpha}{\partial\mathbf{M}_\alpha}=-n_\alpha\hat{I}_\alpha\chi_\alpha-n_\beta\hat{I}_{\alpha\beta}(\chi_\alpha+\chi_\beta), \qquad (2.1)^*$$

where $f_\alpha = n_\alpha f_{0\alpha}(1+\chi_\alpha)$; $f_{0\alpha}$ is the equilibrium distribution function of the molecules of sort α; $\gamma_\alpha = \mu_{\alpha\mathrm{rot}}h$; $\mu_{\alpha\mathrm{rot}}$ are the rotational magnetic moments of the molecules;

$$\mathbf{H}=-\mathbf{i}H_1\sin\omega t+\mathbf{j}H_1\cos\omega t+\mathbf{k}H_0$$

(H_0 is the intensity of the constant field; H_1 and ω are the amplitude and the frequency of the rotating field); n_α is the density of the molecules.

In the case of thermal conductivity, the quantity N_α has the following value:

$$N_\alpha=(u_\alpha^2+M_\alpha^2-c_{p_\alpha})\left(\frac{2kT}{m_\alpha}\right)^{1/2}(u_\alpha)_i\nabla_i\ln T+\frac{n}{n_\alpha}\left(\frac{2kT}{m_\alpha}\right)^{1/2}(u_\alpha)_i d_i^{\alpha\beta}, \qquad (2.2)$$

where

$$d_i^{\alpha\beta}=\frac{1}{n}\frac{\partial n_\alpha}{\partial r_i}+\frac{n_\alpha n_\beta(m_\alpha-m_\beta)}{n\rho}\frac{\partial\ln p}{\partial r_i}=-d_i^{\beta\alpha},$$

$$\rho=n_\alpha m_\alpha,\ p=nkT,\ n=n_1+n_2,$$

$\mathbf{u}$ and $\mathbf{M}$ are the dimensionless velocity and moment of the molecule, c_{p_α} is the specific heat of the gas at constant pressure. The collision operators in (2.1) are given by

$$\hat{I}_\alpha\chi_\alpha=\int f_{0\alpha}[(\chi_\alpha+\chi_{1\alpha})w_\alpha-(\chi_\alpha'+\chi_{1\alpha}')w_\alpha']d\Gamma_\alpha'\,d\Gamma_{1\alpha}'\,d\Gamma_{1\alpha},$$

$$\hat{I}_{\alpha\beta}(\chi_\alpha+\chi_\beta)=\int f_{0\beta}[(\chi_\alpha+\chi_\beta)w_{\alpha\beta} \qquad (2.3)$$

$$-(\chi_\alpha'+\chi_\beta')w_{\alpha\beta}']d\Gamma_\alpha'\,d\Gamma_\beta'\,d\Gamma_\beta, \qquad \alpha\neq\beta.$$

Here w is the probability of molecule collision and $d\Gamma$ is the phase volume of the molecules.

According to Kagan and Maksimov[4], the homogeneous parts of (2.2) can be represented in the form

$$N_\alpha=n_\alpha\sum_{lm}(a_\alpha)_{lm}{}^*(A_\alpha)_{lm}, \qquad (2.4)$$

where

$$(A_\alpha)_{1m}{}^T=Y_{1m}(\mathbf{u}_\alpha)(u_\alpha^2+M_\alpha^2-c_{p_\alpha}),\quad (A_\alpha)_{1m}{}^D=Y_{1m}(\mathbf{u}_\alpha),$$

---

*$[\mathbf{M}_\alpha\mathbf{H}]\equiv\mathbf{M}_\alpha\times\mathbf{H}$.

$$(a_\alpha)_{1m}{}^T = \frac{1}{n_\alpha}\left(\frac{2kT}{m_\alpha}\right)^{1/2} t_k{}^{1m}\nabla_k \ln T,$$

$$(a_\alpha)_{1m}{}^D = \frac{n}{n_\alpha^2}\left(\frac{2kT}{m_\alpha}\right)^{1/2} d_k{}^{\alpha\beta}\, t_k{}^{1m},$$

$$Y_{lm}(\mathbf{u}_\alpha) = (u_\alpha)^l Y_{lm}(\varphi_{u_\alpha}, \vartheta_{u_\alpha}). \tag{2.5}$$

The indices T and D denote the terms corresponding respectively to the heat transfer and to the number of molecules. The quantities t_k^{1m} are given in[4].

The operator $\hat{I}_\alpha$ describes collisions of molecules of one sort. The properties of these operators are considered in[4-6]. The collision operators of molecules of different sorts have $I_{\alpha\beta}$ can be represented, in accordance with[7] in the form

$$\hat{I}_{\alpha\beta}(\chi_\alpha + \chi_\beta) = \hat{I}_{\alpha\beta}(\chi_\alpha) + \hat{J}_{\alpha\beta}(\chi_\beta),$$

$$\hat{I}_{\alpha\beta}(\chi_\alpha) = \int f_{0\beta}(\chi_\alpha - \chi_\alpha')\, w_{\alpha\beta}\, d\Gamma_\alpha'\, d\Gamma_\beta\, d\Gamma_\beta',$$

$$\hat{J}_{\alpha\beta}(\chi_\beta) = \int f_{0\beta}(\chi_\beta - \chi_\beta')\, w_{\alpha\beta}\, d\Gamma_\alpha'\, d\Gamma_\beta\, d\Gamma_\beta', \quad \alpha \neq \beta \tag{2.6}$$

(it is assumed in these relations that the equality $w_{\alpha\beta} = w'_{\alpha\beta}$ is satisfied in the collisions of these molecules).

Taking (2.6) into account, the system (2.1) takes the form ($\alpha \neq \beta$)

$$\frac{\partial \chi_\alpha}{\partial t} + N_\alpha + \hat{L}_\alpha \chi_\alpha = -n_\alpha \hat{I}_\alpha \chi_\alpha - n_\beta \hat{I}_{\alpha\beta}\chi_\alpha - n_\beta \hat{J}_{\alpha\beta}\chi_\beta, \tag{2.7}$$

$$\hat{L}_\alpha = i[\gamma_{\alpha 0}\hat{l}_{\alpha z} + \gamma_{\alpha_1}\hat{l}_{\alpha +}e^{-i\omega t} + \gamma_{\alpha_1}\hat{l}_{\alpha -}e^{i\omega t}]. \tag{2.8}$$

The operator $\hat{L}_\alpha$ describes the interaction of the molecules with the field[2]. Here $\gamma_{\alpha 0} = \gamma_\alpha H_0$, $\gamma_{\alpha 1} = (1/2)\gamma_\alpha H_1$, and $\hat{l}_\alpha$ is the molecule angular momentum operator.

Following[4], we break up the collision operators (2.3) and (2.6) into two parts:

$$\hat{I}_\alpha = \hat{I}_\alpha^{(0)} + \varepsilon \hat{I}_\alpha^{(1)}, \quad \hat{I}_{\alpha\beta} = \hat{I}_{\alpha\beta}^{(0)} + \varepsilon \hat{I}_{\alpha\beta}^{(1)}, \quad \alpha \neq \beta, \tag{2.9}$$

where ϵ is a small parameter. Here $\hat{I}_\alpha^{(0)}$ and $\hat{I}_{\alpha\beta}^{(0)}$ are operators that are diagonal in the space $\psi_{\alpha n}$,

$$\psi_{\alpha n} = \sum_{m_1+m_2=m} C_{l_1m_1l_2m_2}^{lm} Y_{l_1m_1}(\mathbf{u}_\alpha) Y_{l_2m_2}(\mathbf{M}_\alpha) L_{r_1}{}^{l_1}(u_\alpha^2) L_{r_2}^{l_2+\gamma}(M_\alpha^2), \tag{2.10}$$

where $n = (lm, l_1l_2, r_1r_2)$, $C_{l_1m_1l_2m_2}^{lm}$ are Clebsch-Gordan coefficients and L_r^l are Laguerre polynomials. For linear molecules $\gamma = 0$. For molecules of the symmetrical-top type $\gamma = 1/2$.

The cross-collision operators $\hat{I}_{\alpha\beta}^{(0)}$ and $\hat{J}_{\alpha\beta}^{(0)}$ satisfy relations analogous to those given in[7]:

$$\hat{I}_{\alpha\beta}^{(0)}\psi_{\alpha n} = \lambda_n{}^{\alpha\beta}\psi_{\alpha n}, \tag{2.11a}$$

$$\hat{J}_{\alpha\beta}^{(0)}\psi_{\beta n} = \Lambda_n{}^{\alpha\beta}\psi_{\alpha n}, \quad \alpha \neq \beta; \tag{2.11b}$$

here $\psi_{\alpha n}$ are the eigenfunctions of (2.10), and $\lambda_n^{\alpha\beta}$ are the eigenvalues of the operators $\hat{I}_{\alpha\beta}^{(0)}$. The functions $\psi_{\alpha n}$ describe the anisotropy of the non-equilibrium distribution function in the angular-momentum and velocity space. The expression (2.11b) is a reflection of the pseudo-eigenfunction properties of $\psi_{\alpha n}$. These properties of the functions are very useful, since they make it possible to separate only one type of eigenfunctions for each kinetic equation.

3. SOLUTION OF SYSTEM OF KINETIC EQUATIONS

We represent the solution of the system (2.7) in the form

$$\chi_\alpha = -\sum_{lm} (a_\alpha)_{lm}{}^* \chi_{\alpha lm}. \tag{3.1}$$

Substituting these expressions into the initial equations (2.7) and taking (2.9) into account, we obtain equations for $\chi_{\alpha lm}$:

$$\hat{K}_\alpha{}^{-1}\chi_{\alpha lm} + n_\beta \hat{F}_{\alpha\beta}^{(0)}\chi_{\beta lm} = n_\alpha (A_\alpha)_{lm} - \varepsilon(n_\alpha \hat{I}_\alpha^{(1)} + n_\beta \hat{I}_{\alpha\beta}^{(1)})\chi_{\alpha lm} - \varepsilon n_\beta \hat{F}_{\alpha\beta}^{(1)}\chi_{\beta lm}, \tag{3.2}$$

where

$$\hat{K}_\alpha{}^{-1} = \frac{\partial}{\partial t} + \hat{L}_\alpha + n_\alpha \hat{I}_\alpha^{(0)} + n_\beta \hat{I}_{\alpha\beta}^{(0)}$$

$$\hat{F}_{\alpha\beta}^{(0)} = \frac{n_\alpha}{n_\beta}\left(\frac{m_\alpha}{m_\beta}\right)^{1/2}\hat{J}_{\alpha\beta}^{(0)}, \quad \hat{F}_{\alpha\beta}^{(1)} = \frac{n_\alpha}{n_\beta}\left(\frac{m_\alpha}{m_\beta}\right)^{1/2}\hat{J}_{\alpha\beta}^{(1)}, \quad \alpha \neq \beta. \tag{3.3}$$

We seek $\chi_{\alpha lm}$ in the form of a series in the small parameter ϵ:

$$\chi_{\alpha lm} = \chi_{\alpha lm}^{(0)} + \varepsilon \chi_{\alpha lm}^{(1)} + \varepsilon^2 \chi_{\alpha lm}^{(2)} + \ldots \tag{3.4}$$

Substituting these expressions into (3.2) and retaining terms of zeroth order in ϵ, we obtain a system of equations for $\chi_{\alpha lm}^{(0)}$, solutions of which take the form

$$\chi_{\alpha lm}^{(0)} = \sum_{n_0} b_{\alpha lm}^{n_0}\psi_{\alpha n_0}, \tag{3.5}$$

where

$$b_{\alpha lm}^{n_0} = (\mu_{\beta n_0} n_\alpha \langle (A_\alpha)_{lm}, \psi_{\alpha n_0}\rangle - n_\beta{}^2 \Omega_{n_0}^{\alpha\beta}\langle (A_\beta)_{lm}, \psi_{\beta n_0}\rangle)\, \nu_{n_0}^{-1},$$

$$\mu_{\beta n_0} = n_\beta \lambda_{n_0}{}^\beta + n_\alpha \lambda_{n_0}{}^{\beta\alpha},$$

$$\nu_{n_0} = \mu_{\alpha n_0}\mu_{\beta n_0} - n_\alpha n_\beta \Lambda_{n_0}{}^{\beta\alpha}\Lambda_{n_0}{}^{\alpha\beta},$$

$$\Omega_{n_0}^{\alpha\beta} = \frac{n_\alpha}{n_\beta}\left(\frac{m_\alpha}{m_\beta}\right)^{1/2}\Lambda_{n_0}^{\alpha\beta}, \quad \alpha \neq \beta, \tag{3.6}$$

$$\langle q_1, q_2\rangle = \int f_0 q_1{}^* q_2\, d\Gamma, \quad n_0 = (lm, l_10, r_1r_2).$$

The functions $\chi_{\alpha lm}^{(1)}$ satisfy the equations

$$\hat{K}_\alpha^{-1}\chi_{\alpha lm}^{(1)} + n_\beta \hat{F}_{\alpha\beta}^{(0)}\chi_{\beta lm}^{(1)} = -\sum_n B_{\alpha n}^{lm}\psi_{\alpha n}, \tag{3.7}$$

where

$$B_{\alpha n}^{lm} = \sum_{n_0}[b_{\alpha lm}^{n_0}(E_\alpha)_{n_0 n} + b_{\beta lm}^{n_0} n_\beta (F_{\alpha\beta}^{(1)})_{n_0 n}], \tag{3.8}$$

$$(E_\alpha)_{n_0 n} = n_\alpha (I_\alpha^{(1)})_{n_0 n} + n_\beta (I_{\alpha\beta}^{(1)})_{n_0 n}, \quad \alpha \neq \beta.$$

Here $(I_\alpha^{(1)})_{n_0 n}$ are the matrix elements of the operators $\hat{I}_\alpha^{(1)}$. For the operators $F_{\alpha\beta}^{(1)}$ they take the form

$$(F_{\alpha\beta}^{(1)})_{n n_0} = \langle \psi_{\alpha n}, \hat{F}_{\alpha\beta}^{(1)}\psi_{\beta n_0}\rangle. \tag{3.9}$$

The symmetry properties of $\hat{I}_\alpha^{(1)}$, $\hat{I}_{\alpha\beta}^{(1)}$, and $\hat{F}_{\alpha\beta}^{(1)}$ give rise to selection rules for the matrix elements of these operators, analogous to those obtained in[4,6].

We seek the solution of the system (3.7) in the form

$$\chi_{\alpha lm}^{(1)} = -\sum_n \sum_k B_{\alpha n}^{lm}(a_\alpha)_{qm_2k}\varphi_{\alpha n}e^{i\omega t(m_2-k)}. \tag{3.10}$$

Here

$$\varphi_{\alpha n} = C_{l_1m_1l_2m_2}^{lm} Y_{l_1m_1}(\mathbf{u}_\alpha) L_{r_1}^{l_1}(u_\alpha) Y_{l_2k}(\mathbf{M}_\alpha) L_{r_2}^{l_2}(M_\alpha^2). \tag{3.11}$$

Substituting (3.10) in (3.7) and taking (3.3) and (2.8) into account, we obtain equations for the unknown coefficients

$$\sum_k e^{i\omega(m_2-k)t}\{B_{\alpha n}^{lm}[(ik\tilde{\gamma}_{\alpha 0} + \tilde{\lambda}_{\alpha n})(a_\alpha)_{qm_2k} + i\gamma_{\alpha 1}(l_{k,k+1}^{-}(a_\alpha)_{qm_2k+1} + l_{k,k-1}^{+}(a_\alpha)_{qm_2k-1})] + B_{\beta n}^{lm} n_\beta \Omega_n^{\alpha\beta}(a_\beta)_{qm_2k}\} Y_{l_2k}(\mathbf{M}_\alpha) = B_{\alpha n}^{lm}\overline{Y}_{l_2m_2}(\mathbf{M}_\alpha), \tag{3.12}$$

where

$$\tilde{\gamma}_{\alpha 0} = \gamma_{\alpha 0} - \omega, \quad \tilde{\lambda}_{\alpha n} = \mu_{\alpha n} + i\omega m^2.$$

Transforming (3.12) back to operator form, we obtain equations for the determined coefficients:

$$B_{\alpha n}^{lm}[\hat{I}_\alpha + i(\tilde{\gamma}_{\alpha 0}\hat{l}_z + \gamma_{\alpha 1}\hat{l}_+ + \gamma_{\alpha 1}\hat{l}_-)]\chi_\alpha(M_\alpha) + B_{\beta n}^{lm} n_\beta \hat{F}_{\alpha\beta}^{(0)}\chi_\beta(\mathbf{M}_\alpha) = B_{\alpha n} Y_{l_2 m_2}(\mathbf{M}_\alpha), \tag{3.13}$$

where

$$\hat{I}_\alpha = n_\alpha \hat{I}_\alpha^{(0)} + n_\beta \hat{I}_{\alpha\beta}^{(0)} + i\omega m_2, \qquad \chi_\alpha(\mathbf{M}_\beta) = \sum_k (a_\alpha)_{qm_2k} Y_{l_2k}(\mathbf{M}_\beta). \tag{3.14}$$

Solving (3.13) by the method used in[2], we obtain

$$(a_\alpha)_{qm_2k} = \sum_p \{T_{\alpha m_2 p}^{l_2}(T_\alpha^{-1})_{pk}^{l_2}[B_{\alpha n}^{lm}(\mu_{\beta n} + im_2\omega + ip\gamma_{\alpha э\phi\phi}) - B_{\beta n}^{lm} n_\beta \Omega_n^{\alpha\beta}]\}\{B_{\alpha n}^{lm}[(\mu_{\alpha n} + im_2\omega + ip\gamma_{\alpha э\phi\phi})(\mu_{\beta n} + im_2\omega + ip\gamma_{\alpha э\phi\phi}) - n_\alpha n_\beta \Lambda_n^{\alpha\beta}\Lambda_n^{\beta\alpha}]\}^{-1}, \tag{3.15}$$

where $T_{\alpha m_2 p}^{l_2} = i^{m_2-p} P_{m_2p}^{l_2}(\cos\vartheta_\alpha)$ ($P_{m_2p}^{l_2}(\cos\vartheta_\alpha)$ are functions defined in[8]),

$$\gamma_{\alpha э\phi\phi} = \sqrt{\tilde{\gamma}_{\alpha 0}^2 - \gamma_{\alpha 1}^2}, \qquad \cos\vartheta_\alpha = \tilde{\gamma}_{\alpha 0}\gamma_{\alpha э\phi\phi}^{-1}. \tag{3.16}$$

Substituting this solution in (3.10) and using (3.11), we can obtain $\chi_{\alpha lm}^{(1)}$.

The system of equations for $\chi_{\alpha lm}^{(2)}$ is

$$\hat{K}_\alpha^{-1}\chi_{\alpha lm}^{(2)} + n_\beta \hat{F}_{\alpha\beta}^{(0)}\chi_{\beta\alpha m}^{(2)} = -\sum_{n_0} A_{\alpha n_0}^{lm}\psi_{\alpha n_0}e^{i\omega(m_2-k)t}. \tag{3.17}$$

Here

$$A_{\alpha n_0}^{lm} = -\sum_n \sum_k \sum_{l'=|l_1-l_2|}^{l_1+l_2} C_{l_1m_1l_2m_2}^{lm} C_{l_1m_1l_2k}^{l'm'}[B_{\alpha n}^{lm}(E_\alpha)_{n_0n'}(a_\alpha)_{qm_2k} + B_{\beta n}^{lm}(a_\beta)_{qm_2k}n_\beta(F_{\alpha\beta}^{(1)})_{n_0n'}], \quad \alpha \neq \beta. \qquad m = m_1 + m_2 \tag{3.18}$$

We seek the solution of (3.17) in the form

$$\chi_{\alpha lm}^{(2)} = -\sum_{n_0} D_{\alpha n_0}^{lm}(t)\psi_{\alpha n_0}. \tag{3.19}$$

Substituting (3.19) into the system (3.17) and using (3.3), we obtain equations for the unknown coefficients $D_{\alpha n_0}^{lm}(t)$:

$$\frac{\partial D_{\alpha n_0}^{lm}(t)}{\partial t} + \mu_{\alpha n_0}D_{\alpha n_0}^{lm}(t) + n_\beta \Omega_{n_0}^{\alpha\beta}D_{\beta n_0}^{lm}(t, = A_{\alpha n_0}^{lm}e^{i\omega(m_2-k)t}. \tag{3.20}$$

The periodic solution of this system of inhomogeneous equations of first order is presented in the form

$$D_{\alpha n_0}^{lm}(t) = -\{[\mu_{\beta n_0} + i\omega(m_2-k)]A_{\alpha n_0}^{lm} - n_\beta\Omega_{n_0}^{\alpha\beta}A_{\beta n_0}^{lm}\}\{[\mu_{\beta n_0} + i\omega(m_2-k)][\mu_{\alpha n_0} + i\omega(m_2-k)] - n_\alpha n_\beta \Lambda_{n_0}^{\alpha\beta}\Lambda_{n_0}^{\beta\alpha}\}^{-1}e^{i\omega t(m_2-k)}. \tag{3.21}$$

Knowledge of these coefficients makes it possible to write in explicit form expressions for $\chi_{\alpha lm}^{(2)}$.

4. THERMAL CONDUCTIVITY

The heat flux in a binary gas mixture can be written in the following form:

$$q_i = kT\sum_\alpha n_\alpha\left(\frac{2kT}{m_\alpha}\right)^{1/2}\left[\int f_{0\alpha}\chi_\alpha(u_\alpha^2 + M_\alpha^2 - c_{p\alpha})(u_\alpha)_i\,d\Gamma_\alpha + c_{p_\alpha}(\bar{u}_\alpha)_i\right], \quad (\bar{u}_\alpha)_i = \langle\chi_\alpha, (u_\alpha)_i\rangle. \tag{4.1}$$

Substituting χ_α in (4.1) we obtain, in accord with (3.1),

$$q_i = -k\sum_\alpha\left\{\sum_{mm'} t_i^{1m}(t_k^{1m'})^*\left[\left(\frac{2kT}{m_\alpha}\right)(c_\alpha^T)_{1m1m'}(\nabla T)_k - Tnn_\alpha^{-1}\left(\frac{2kT}{m_\alpha}\right)(c_\alpha^{T-D})_{1m1m'}d_k^{\alpha\beta}\right] + Tc_{p_\alpha}\left(\frac{2kT}{m_\alpha}\right)^{1/2}n_\alpha(\bar{u}_\alpha)_i\right\}, \tag{4.2}$$

where k is Boltzmann's constant,

$$(c_\alpha^T)_{1m1m'} = \langle(A_\alpha^T)_{1m}, (\chi_\alpha^T)_{1m'}\rangle,$$

$$(c_\alpha^{T-D})_{1m1m'} = \langle(A_\alpha^T)_{1m}, (\chi_\alpha^D)_{1m'}\rangle, \quad \alpha \neq \beta.$$

Expression (4.2) shows that the heat flux consists of three parts: 1) the heat flux due to the inhomogeneity of the temperature, 2) the heat flux due to diffusion—the so-called diffusion thermal effect, 3) the flux due to mass transport.

We use furthermore the equation for the diffusion flux

$$(\bar{v}_1)_i - (\bar{v}_2)_i = -\frac{n^2}{n_1n_2}\left[D_{ik}^T\frac{1}{T}(\nabla T)_k + D_{ik}(d_{12})_k\right],$$

$$D_{ik} = n^{-1}\sum_\alpha\sum_{mm'} t_i^{1m}(t_k^{1m'})^*\frac{n_\beta}{n_\alpha}\left(\frac{2kT}{m_\alpha}\right)(c_\alpha^D)_{1m1m'},$$

$$D_{ik}^T = \frac{n_1n_2}{n^2}\sum_{mm'} t_i^{1m}(t_k^{1m'})^*\left[\frac{1}{n_1}\left(\frac{2kT}{m_1}\right)(c_1^{D-T})_{1m1m'} - \frac{1}{n_2}\left(\frac{2kT}{m_2}\right)(c_2^{D-T})_{1m1m'}\right] \tag{4.3}$$

$$(c_\alpha^{D-T})_{1m1m'} = \langle(A_\alpha^D)_{1m}, (\chi_\alpha^T)_{1m'}\rangle, \qquad (c_\alpha^D)_{1m1m'} = \langle(A_\alpha^D)_{1m}, (\chi_\alpha^D)_{1m'}\rangle$$

(D_{ik}, D_{ik}^T are the coefficients of diffusion and thermodiffusion); then the expression for the heat flux in the case of a stationary state at $(\bar{v}_1)_i = (\bar{v}_2)_i = 0$ can be written in the absence of the field in the form

$$q_i = -\varkappa_0(\nabla T)_i, \tag{4.4}$$

$$\varkappa_0 = \varkappa_0' - \frac{n^3}{n_1n_2}kk_T^2D_0. \tag{4.5}$$

Here k_T is the thermodiffusion ratio, κ_0 is the experimentally measured coefficient of thermal conductivity of the mixture, and κ_0' determines the heat transport in the gas mixture without allowance for the diffusion thermoeffect.

It follows from (4.5) that the change of the thermal conductivity coefficient of the mixture in the magnetic field ($\Delta\kappa$) can consist of the change of the coefficients κ', D, and k_T:

$$\frac{\Delta\varkappa}{\varkappa_0} = \frac{\Delta\varkappa'}{\varkappa_0} - \frac{nkDk_T^2}{x_1x_2\varkappa_0}\left(\frac{\Delta D}{D_0} + \frac{2\Delta k_T}{k_T}\right), \quad x_\alpha = \frac{n_\alpha}{n}. \tag{4.6}$$

Let us estimate the contribution of the second term in (4.5), which describes the change of the heat flux connected with the diffusion thermoeffect in a magnetic field. As shown in[9] and noted in[10], the change of the thermodiffusion ratio of the mixtures O_2-Kr and O_2-He in a magnetic field amounts to less than 0.4%. The diffusion coefficient of the mixture $O^{16}O^{18}$-O_2^{16} in a magnetic field changes by an amount $\sim 1.5 \times 10^{-4}$[11]. Unfortunately, we do not know the changes of D and k_T of other mixtures in a magnetic field, but it can be assumed that the changes of D and k_T will be ~1% (as is the case, for example, for the change in the thermal conductivity).

Under conditions typical of measurements of the thermal-conductivity coefficient of gases in a magnetic field, p = 1 mm Hg, T = 300°K, the second term in (4.6) for the O_2-Kr mixture is of the order of 10^{-6} (D = 115 cm²/sec, $k_T = 0.045$, $x_1 = x_2 = 0.5$, $\kappa_0 = 22.5$

× 10^3 erg/cm-sec-deg). It follows from the work of Senftleben[12] that the relative change of the thermal-conductivity coefficients of the mixtures investigated by him depends on the concentration and amounts, say for O_2-Kr, to 0.1 – 1%. We shall therefore consider only the coefficient κ' in the analysis of the behavior of the thermal conductivity of a gas mixture in magnetic fields.

Let us examine the dependence of κ' on the intensity of the constant field and on the amplitude and frequency of the alternating field. We are interested only in the real part of the time-averaged thermal-conductivity tensor, which is even in the external field[4]

$$\varkappa_{ik} = \sum_{\alpha} \frac{2k^2 {}_1}{3m} \{\delta_{ik} \operatorname{Re}(c_\alpha{}^T)_{11,11} + h_i h_k [(c_\alpha{}^T)_{10,10} - \operatorname{Re}(c_\alpha{}^T)_{11,11}]\}, \quad (4.7)$$

$h_i = H_i/H$. It is seen from (4.7) that, just as in the case of a molecular gas[2], the dependence of κ_{ik} on H_0, H_1, and ω is determined by the coefficients

$$(c_\alpha)_{l'mlm} = \langle (A_\alpha)_{l'm}, (\chi_\alpha^{(2)})_{lm} \rangle, \quad (4.8)$$

where $(\chi_\alpha^{(2)})_{lm}$ are non-equilibrium distribution functions, obtained in the preceding section in the second approximation in the small parameter ϵ (here and throughout the index T has been left out from the quantities $(c_\alpha)_{l'mlm}$, $(A_\alpha)_{l'm}$, $(\chi_\alpha^{(2)})_{lm}$). Using expressions (3.18), (3.19), and (3.21), we can express the coefficients $(c_\alpha)_{l'mlm}$ in the form

$$(c_1)_{l'mlm} = \sum_{\alpha} \sum_{nn'} B_{\alpha n}^{lm} H_{\alpha n'}^{l'm} (G_\alpha)_{nn'},$$

$$(c_2)_{l'mlm} = \sum_{\alpha} \sum_{nn'} B_{\alpha n}^{lm} Q_{\alpha n'}^{l'm} (G_\alpha)_{nn'}, \quad (4.9)$$

where

$$H_{1n'}^{l'm} = \sum_{n_0} \langle (A_1)_{l'm}, \psi_{1n_0} \rangle [\mu_{2n_0} (E_1)_{n_0 n'} - n_1 n_2 \Lambda_{n_0}^{12} (J_{21}^{(1)})_{n_0 n'}] \nu_{n_0}^{-1}, \quad (4.10)$$

$$H_{2n'}^{l'm} = \sum_{n_0} n_2 \langle (A_1)_{l'm}, \psi_{1n_0} \rangle [\mu_{2n_0} (F_{12}^{(1)})_{n_0 n'} - \Omega_{n_0}^{12} (E_2)_{n_0 n'}] \nu_{n_0}^{-1},$$

$$Q_{1n'}^{l'm} = \sum_{n_0} \langle (A_2)_{l'm}, \psi_{2n_0} \rangle [\mu_{1n_0} (E_1)_{n_0 n'} - n_1 n_2 \Lambda_{n_0}^{21} (J_{12}^{(1)})_{n_0 n'}] \nu_{n_0}^{-1},$$

$$Q_{2n'}^{l'm} = \sum_{n_0} n_1 \langle (A_2)_{l'm}, \psi_{2n_0} \rangle [\mu_{1n_0} (F_{21}^{(1)})_{n_0 n'} - \Omega_{n_0}^{21} (E_1)_{n_0 n'}] \nu_{n_0}^{-1}$$

and

$$(G_\alpha)_{nn'} = \sum_{m_1 + m_2 = m} C_{l_1 m_1 l_2 m_2}^{l'm} C_{l_1 m_1 l_2 m_2}^{lm} \delta_{n_0 n_0'} (a_\alpha)_{q m_2 m_2}. \quad (4.11)$$

The quantities B_{1n}^{lm} and B_{2n}^{lm} were defined earlier (see (3.8)).

Using the equality

$$\sum_p |P_{mp}{}^l (\cos\vartheta)|^2 = 1,$$

we separate in the quantities $(G_\alpha)_{nn'}$ the parts $(\Delta G_\alpha)_{nn'}$ that vanish in the absence of magnetic fields:

$$(G_\alpha)_{nn'} = (B_{\alpha n}{}^{lm} \mu_{\beta n} - B_{\beta n}{}^{lm} n_\beta \Omega_n{}^{\alpha\beta}) (B_{\alpha n}{}^{lm} \nu_n)^{-1} \delta_{nn'} + (\Delta G_\alpha)_{nn'},$$

$$(\Delta G_\alpha)_{nn'} = -(B_{\alpha n}^{lm} \mu_{\beta n} - B_{\beta n}^{lm} n_\beta \Omega_n^{\alpha\beta}) (B_{\alpha n}^{lm} \nu_n)^{-1} \cdot$$

$$\sum_{m_1 + m_2 = m} C_{l_1 m_1 l_2 m_2}^{lm} C_{l_1 m_1 l_2 m_2}^{l'm} \delta_{n_0 n_0'} (\Delta a_\alpha)_{q m_2 m_2}. \quad (4.12)$$

Here

$$(\Delta a_\alpha)_{q m_2 m_2} = \sum_p |P_{m_2 p}^{l_2} (\cos\vartheta_\alpha)|^2 (m_2 \omega + p\gamma_{\alpha \text{eff}})^2 \{[(m_2 \omega + p\gamma_{\alpha \text{eff}})^2$$

$$+ \mu_{\beta n}{}^2 + n_\alpha n_\beta \Lambda_n{}^{\alpha\beta} \Lambda_n{}^{\beta\alpha}] (B_{\alpha n}^{lm} \mu_{\beta n} - n_\beta B_{\beta n}^{lm} \Omega_n{}^{\alpha\beta})$$

$$+ (B_{\alpha n}^{lm} n_\alpha n_\beta \Lambda_n{}^{\alpha\beta} \Lambda_n{}^{\beta\alpha} - \mu_{1n} n_\beta B_{\beta n}^{lm} \Omega_n{}^{\alpha\beta}) (\mu_{1n} + \mu_{2n})\} \quad (4.13)$$

$$\times [(B_{\alpha n}^{lm} \mu_{\beta n} - n_\beta B_{\beta n}^{lm} \Omega_n{}^{\alpha\beta}) \Delta_{\alpha n}]^{-1},$$

$$\Delta_{\alpha n} = [(m_2 \omega + p\gamma_{\alpha \text{ eff}})^2 + \mu_{\alpha n}{}^2] [(m_2 \omega + p\gamma_{\alpha \text{ eff}})^2 + \mu_{\beta n}{}^2]$$
$$-2[\mu_{\alpha n} \mu_{\beta n} - (m_2 \omega + p\gamma_{\alpha \text{ eff}})^2] n_\alpha n_\beta \Lambda_n{}^{\alpha\beta} \Lambda_n{}^{\beta\alpha} +$$
$$+ (n_\alpha n_\beta \Lambda_n{}^{\alpha\beta} \Lambda_n{}^{\beta\alpha})^2, \qquad \alpha \neq \beta.$$

Taking these expressions into account, the change of the coefficients $(c_\alpha)_{l'mlm}$ in mutually-perpendicular fields will be

$$(\Delta c_1)_{l'mlm} = \sum_{\alpha} \sum_{nn'} B_{\alpha n}^{lm} H_{\alpha n}^{l'm} (\Delta G_\alpha)_{nn'},$$

$$(\Delta c_2)_{l'mlm} = \sum_{\alpha} \sum_{nn'} B_{\alpha n}^{lm} Q_{\alpha n}^{l'm} (\Delta G_\alpha)_{nn'}. \quad (4.14)$$

Expressions (4.14) together with (4.12) and (4.13) determine completely the dependence of the time-averaged changes of the thermal-conductivity coefficient of a gas mixture on the value of the constant field, on the amplitude and frequency of the alternating field, and on the pressure.

Let us consider the case of strong fields, when

$$\gamma_{\alpha 0}, \; \gamma_{\alpha 1} \gg n_\alpha \lambda_{\alpha n}. \quad (4.15)$$

Then expression (4.12), with allowance for (4.13), takes the form $(\alpha \neq \beta)$

$$(\Delta G_\alpha)_{nn'} = -(B_{\alpha n}^{lm} \mu_{\beta n} - n_\beta B_{\beta n}^{lm} \Omega_n{}^{\alpha\beta}) (B_{\alpha n}^{lm} \nu_n)^{-1} \delta_{nn'} (G_\alpha)_{l_1 l_2}^{lml'm},$$

$$(G_\alpha)_{l_1 l_2}^{lml'm} = \sum_{\substack{m_1 + m_2 = m \\ p}} C_{l_1 m_1 l_2 m_2}^{lm} C_{l_1 m_1 l_2 m_2}^{l'm} |P_{m_2 p}^{l_2} (\cos\vartheta_\alpha)|^2 \quad (4.16)$$

$$= \sum_{\substack{m_1 + m_2 = m \\ m_2 \neq 0}} C_{l_1 m_1 l_2 m_2}^{lm} C_{l_1 m_1 l_2 m_2}^{l'm} + C_{l_1 m_1 l_2 0}^{lm} C_{l_1 m_1 l_2 0}^{l'm} [1 - P_{l_2}{}^2 (\cos\vartheta_\alpha)]. \quad (4.17)$$

It follows from (4.17) that in this case the dependence of the thermal-conductivity coefficient of a gas mixture on H_0 ($\gamma_{\alpha 0} = \mu_{\alpha \text{rot}} H_0/\hbar$), H_1, and ω ($\gamma_{\alpha 1} = \mu_{\alpha \text{rot}} H_1/\hbar$) is determined by the function P_{l_2}, and consequently by the selection rules with respect to the index l_2 for the matrix elements of the operators $\hat{I}_\alpha^{(1)}$, $\hat{I}_{\alpha\beta}^{(1)}$, and $\hat{J}_{\alpha\beta}^{(1)}$, which describe non-spherical scattering of molecules. In the case when the probabilities of the direct and inverse transitions in collisions are equal, the simplest operator model is the one in which the matrix elements differ from zero only for "transitions between states" $n_0 = (1m, 10, r_1 r_2)$ and $n = (1m, 12, 00)$. If the interaction of the mixture molecules is described by this model, then the resonant change of the thermal conductivity of the mixture in accordance with (4.7), (4.9), (4.12), (4.14), and (4.17), in fields satisfying the condition (4.15), is described by the expressions

$$\Delta c_{1m,1m} = -F_1 (G_1)_{1,2}^{1m,1m} - F_2 (G_2)_{1,2}^{1m,1m}, \quad (4.18)$$

$$(G_\alpha)_{1,2}^{1m,1m} = (C_{1m,20}^{1m})^2 \frac{4(\gamma_{\alpha 0} - \omega)^2 + \gamma_{\alpha 1}{}^2}{[(\gamma_{\alpha 0} - \omega)^2 + \gamma_{\alpha 1}{}^2]^2}. \quad (4.19)$$

Here $F_1 (>0)$ and $F_2 (>0)$ are complicated coefficients that depend on the non-zero matrix elements of the operators $\hat{I}_1^{(1)}$, $\hat{I}_2^{(1)}$, $\hat{I}_{12}^{(1)}$, and $\hat{J}_{12}^{(1)}$ (see (4.14) and (4.9)) and determine the concentration dependence of the resonant change in the thermal conductivity. At low concentrations of the first gas we have $F_1 \sim n_1$, and at small concentrations of the second gas $F_2 \sim n_2$. (The concentration dependences will be discussed below.)

It follows from (4.18) and (4.19) that at a fixed frequency and amplitude of the alternating field the quantity $\Delta c_{1m,1m}$ and consequently also the change of the thermal-conductivity coefficient, has two maxima at constant-field values

$$H_0 = \frac{\hbar\omega}{\mu_{1\mathrm{rot}}} \pm \frac{1}{2\sqrt{2}} H_1, \tag{4.20}$$

corresponding to resonance with the molecules of the first gas, and two maxima at constant-field values

$$H_0 = \frac{\hbar\omega}{\mu_{2\mathrm{rot}}} \pm \frac{1}{2\sqrt{2}} H_1, \tag{4.21}$$

corresponding to resonance with the second gas.

Thus, a resonant spectrum, each line of which has a fine structure, should be observed in a gas mixture.

As seen from (4.18) and (4.19), the amplitudes of the resonance lines also change with changing concentration, but the form of the fine structure of the lines does not depend on the concentration. The latter circumstance can be attributed to the fact that, in the nonspherical scattering model considered above, the collision-operator matrix elements differ from zero for both identical and different molecules only in "transitions into states" with identical $l_2 = 2$. The anisotropy of the non-equilibrium distribution functions (χ_1 and χ_2) in (v, M) space will then be the same and, as can be seen from (3.8), (3.10), and (3.11), it is described in the first approximation in ϵ by the functions φ_n (3.11) with $l_1 = 1$ and $l_2 = 2$. This form of the anisotropy of χ_1 and χ_2 is likewise independent of the concentration.

The change of the anisotropy of χ_1 and χ_2 with changing concentration, and consequently the change of the fine structure of the resonance lines, can occur in a gas mixture if the non-spherical interaction of molecules of different sorts differs. An example is a mixture of two molecular gases, one of which exhibits the anomalous effect of the change of thermal conductivity in a constant magnetic field[13]. Such an effect can be represented as a superposition of two changes of the thermal-conductivity coefficients, with opposite signs. As shown in[5,6,14], the anomalous effect can be explained by considering collisions for which the probabilities of the direct and inverse transitions are different ($w \neq w'$). Allowance for these collisions leads to the need for breaking up the operator $\hat{I}_1^{(1)}$ into symmetrical and antisymmetrical parts, having the properties[6]

$$\hat{I}_1^{(1)} = \hat{I}_c + \hat{I}_a,$$
$$\langle\psi_{1n}, \hat{I}_c\psi_{1n'}\rangle = \langle\psi_{1n'}, \hat{I}_c\psi_{1n}\rangle, \quad \langle\psi_{1n}, \hat{I}_a\psi_{1n'}\rangle = -\langle\psi_{1n'}, \hat{I}_a\psi_{1n}\rangle. \tag{4.22}$$

When describing the interactions of molecules in a mixture of gases, in one of which the anomalous effect is observed (the first gas), in accordance with the selection rules for the operators $\hat{I}_c$ and $\hat{I}_a$[6], the simplest model will be the one with non-zero matrix elements of $\hat{I}_a$ for "transitions into states" with $l_1 = 1$, $l_2 = 1$ and for "transitions into states" with $l_1 = 1$, $l_2 = 2$ in the case of the operators $\hat{I}_c$, $\hat{I}_{\alpha\beta}$, $\hat{J}_{\alpha\beta}$, and $\hat{I}_2$. Then the anisotropy of χ_1 will be described by a linear combination of the functions φ_n with $l_1 = 1$ and $l_2 = 1$ and 2, while that of χ_2 will be described by the functions φ_n with $l_1 = 1$ and $l_2 = 2$. As shown by the calculations, the expression for $\Delta c_{1m,1m}$ (4.14) will contain in addition to the terms analogous to (4.18) and (4.19) also the supplementary term

$$\Delta c^{+}_{1m1m} = F_3(G_1)_{11}^{1m,1m} = F_3(C^{1m}_{1m10})^2 \frac{\gamma_{11}{}^2}{(\gamma_{01} - \omega)^2 + \gamma_{11}{}^2}, \tag{4.23}$$

which describes the resonant increase of the thermal conductivity connected with the resonance with the molecules of the first gas; $F_3 (>0)$ determines the maximum increase of the thermal-conductivity coefficient and depends on the matrix elements of $\hat{I}_a$.

It follows from (4.23 that $\Delta c^{+}_{1m,1m}$ reaches a maximum at

$$H_0 = \omega\hbar / \mu_{1\mathrm{rot}} \tag{4.24}$$

When this relation is satisfied, in accord with (4.18) and (4.19), the resonant change (decrease) of the thermal-conductivity coefficient should have a minimum.

At low concentrations ($n_1 \to 0$), when the collisions of the molecules of the first gas with one another do not make an appreciable contribution, we have $F_3 \sim n_1^2$ (as follows from (4.9), (4.14), and (4.22)), and therefore the resonant change of the thermal-conductivity coefficient in the approximation linear in the concentration will be described by expressions analogous to (4.18) and (4.19), and the anisotropy of χ_1, which is described by the function φ_n with $l_1 = 1$ and $l_2 = 1$, vanishes.

According to[2,4], the resonant change of the thermal conductivity in the case when H_0 is parallel to ∇T is determined by the coefficient $\Delta c_{10,10}$, and consequently also by the quantities $G^{1010}_{l_1 l_2}$ (4.17). However, $C_{1010} = 0$, and therefore (see (4.23)) in a field $\mathbf{H}_0 \parallel \nabla T$ there should be no resonant increase of the thermal-conductivity coefficient.

In the second simplest model of the interaction of the molecules of the same mixture, the non-zero matrix elements will be those of the operators $\hat{I}^{(1)}_{\alpha\beta}$ and $\hat{J}^{(1)}_{\alpha\beta}$ ($\alpha \neq \beta$) in "transitions to the states" with $l_1 = 1$ and $l_2 = 1$ or 2. It can be shown that in this case a change of the concentration will be accompanied by a change of the anisotropy of the distribution function and of the fine structure of the line of the second gas.

An analogous behavior of χ_1, χ_2, and of the fine structure can take place in a mixture of a diatomic gas with a gas of the SF_6 type. It was shown in[15] that in a gas with molecules of the type SF_6, which have octahedral symmetry, the main term in the expansion of the nonequilibrium distribution function is the one containing the product of irreducible third- and fourth-rank tensors ($[\mathbf{v}]^3[\mathbf{M}]^4$), made up of the components of the velocity and angular momentum vectors. As follows from the performed analysis, the fine structure of the line of such a gas consists of four maxima, the positions of which are determined by the relations

$$(H_{\alpha 0})_{1,2} = \frac{\hbar\omega}{\mu_{\alpha\mathrm{rot}}} \pm \frac{1}{6} H_1, \qquad (H_{\alpha 0})_{3,4} = \frac{\hbar\omega}{\mu_{\alpha\mathrm{rot}}} \pm \frac{3}{4} H_1.$$

In a mixture with SF_6 the fine structure of the line of the first or of the second gas will vary with the concentration, depending on whether the operator for collisions between molecules of different sorts has or does not have non-zero matrix elements for "transitions to the state" with $l_1 = 3$ and $l_2 = 4$.

It should be noted that the perturbed-collision-operator model describes satisfactorily the interaction of molecules in a number of gases[16] when transitions to the state with $l_2 = 2$ are allowed and the anisotropy of χ is described by the functions φ_n with $l_1 = 1$ and $l_2 = 2$. This result follows from measurements of the ratio $(\Delta\kappa_\perp/\Delta\kappa_\parallel)_{\text{sat}}$ of the components of the thermal-conductivity tensor at saturation when the constant magnetic field is perpendicular and parallel to ∇T. However, in the presence of other terms of the expansion of the non-equilibrium distribution function (for example v, M, $[\mathrm{v}]^3[\mathrm{M}]^4$), a deviation of the ratio $(\Delta\kappa_\perp/\Delta\kappa_\parallel)_{\text{sat}}$ from $3/2$ (this value corresponds to an anisotropy of χ described by φ_n, $l_1 = 1$, $l_2 = 2$) should be observed in different directions. In this case measurements of $(\Delta\kappa_\perp/\Delta\kappa_\parallel)_{\text{sat}}$ cannot give reliable information concerning the true contributions of the different terms to the expansion of χ.

Let us consider the dependence of the resonant effect on the concentration. We assume that the second gas constitutes an admixture $x = n_2/n \ll 1$. In this case, retaining in (4.7) terms that are linear in x, we can reduce the experimentally measured relative change of the thermal-conductivity coefficient in crossed fields, $\epsilon = (\epsilon_\parallel + \epsilon_\perp)/2$, where $\epsilon_\perp = \Delta\kappa_\perp/\kappa_0$ and $\epsilon_\parallel = \Delta\kappa_\parallel/\kappa_0$ (κ_0 is the thermal-conductivity coefficient of the mixture in the absence of fields), to the form

$$\varepsilon = \sum_\alpha \psi_\alpha^{-1}[\operatorname{Re}(\Delta c_\alpha)_{11,11} + (\Delta c_\alpha)_{10,10}],$$

$$\psi_\alpha = \sum_m \{(1-x)[(A_1)_{12}^{1m1m} + (A_4)_{12}^{1m1m}] + x\eta[(A_2)_{12}^{1m1m} + (A_3)_{12}^{1m1m}]\}, \tag{4.25}$$

where $\eta = 1$ if $\alpha = 1$ and $\eta = m_1/m_2$ if $\alpha = 2$, and

$$(\Delta c_1)_{1m1m} = -[(1-x)(A_1)_{12}^{1m1m}(G_1)_{12}^{1m1m} + x(A_2)_{12}^{1m1m}(G_2)_{12}^{1m1m}],$$

$$(\Delta c_2)_{1m1m} = -[x(A_3)_{12}^{1m1m}(G_2)_{12}^{1m1m} + (1-x)(A_4)_{12}^{1m1m}(G_1)_{12}^{1m1m}].$$

The coefficients $(A_\eta)_{12}^{1m1m}$, where $\eta = 1, 2, 3$, and 4, are given by cumbersome equations that depend on the collision frequencies of the like and unlike molecules and on the matrix elements of the operators $\hat{\mathrm{I}}_{\alpha\beta}^{(1)}$ and $\hat{\mathrm{J}}_{\alpha\beta}^{(1)}$. The expressions for $(G_\alpha)_{12}^{1m1m}$ in the case of large values of the fields ($\gamma_{\alpha 0}, \gamma_{\alpha 1} \gg n_\alpha\lambda_{\alpha n}$) are given in (4.19) above.

The concentration dependences of the GMR in the case of arbitrary concentrations become much simpler if it is assumed that the binary mixture under consideration consists of gases with large thermal conductivities and with close values of the masses and interaction nonsphericities of the molecules. This makes it possible to put

$$\lambda_{n_0}^{\alpha\beta} = \lambda_{n_0}, \qquad (I_{\alpha\beta}^{(1)})_{n_0 n} = (I^{(1)})_{n_0 n}, \qquad m_1 = m_2. \tag{4.26}$$

Using these assumptions, and also the relations (3.6), (3.8), and (4.10), we obtain

$$B_{1n}^{lm} = (1-x)\sum_{n_0} \lambda_{n_0}^{-1}\langle A_{lm}, \psi_{n_0}\rangle (I^{(1)})_{n_0 n},$$

$$B_{2n}^{lm} = x\sum_{n_0} \lambda_{n_0}^{-1}\langle A_{lm}, \psi_{n_0}\rangle (I^{(1)})_{n_0 n},$$

$$H_{1n'}^{l'm} = Q_{2n'}^{l'm} = \sum_{n_0} \lambda_{n_0}^{-1}\langle A_{l'm}, \psi_{n_0}\rangle (I^{(1)})_{n_0 n'},$$

$$Q_{1n'}^{l'm} = H_{2n'}^{l'm} = 0. \tag{4.27}$$

In this case the relative change of the thermal conductivity (ϵ) takes the form

$$\varepsilon = \sum_\alpha \frac{1}{\psi}[\operatorname{Re}(\Delta c_\alpha)_{11,11} + (\Delta c_\alpha)_{10,10}], \tag{4.28}$$

where

$$\psi = \sum_m \Lambda^{1m,1m}(1 - 2x + 2x^2),$$

$$\begin{aligned} (\Delta c_1)_{1m,1m} &= -(1-x)^2\Lambda^{1m,1m}(G_1)_{12}^{1m,1m}, \\ (\Delta c_2)_{1m,1m} &= x^2\Lambda^{1m,1m}(G_2)_{12}^{1m,1m}, \end{aligned} \tag{4.29}$$

$$\Lambda_n^{lm,lm} = (n\lambda_n)^{-1}\sum_{n_0} [\lambda_{n_0}^{-1}\langle A_{lm}, \psi_{n_0}\rangle (I^{(1)})_{n_0 n}]^2, \qquad m = 0, 1.$$

Expressions for $(G_\alpha)^{1m1m}$ are given in (4.17). It should be noted that, according to (4.28) and (4.29), a certain deviation of the effect from linearity with respect to $x = n_2/n$ is observed even for such a simplified case of binary mixtures. For arbitrary gas mixtures, this relation is more complicated.

The authors are grateful to L. L. Gorelik, L. A. Maksimov, and V. V. Sinitsyn for a discussion of the work.

---

[1] V. D. Borman, L. L. Gorelik, B. I. Nikolaev, and V. V. Sinitsyn, ZhETF Pis. Red. 5, 105 (1967) [JETP Lett. 5, 85 (1967)].

[2] V. D. Borman, L. A. Maksimov, Yu. V. Mikhaĭlova, and B. I. Nikolaev, Zh. Eksp. Teor. Fiz. 53, 2143 (1967) [Sov. Phys.-JETP 26, 1210 (1968)].

[3] L. L. Gorelik, V. I. Rukavishnikov, and V. V. Sinitsyn, Zh. Eksp. Teor. Fiz. 55, 1238 (1968) [Sov. Phys.-JETP 28, 647 (1969)].

[4] Yu. M. Kagan and L. A. Maksimov, Zh. Eksp. Teor. Fiz. 51, 1893 (1966) [Sov. Phys.-JETP 24, 1272 (1967)].

[5] A. C. Levi and F. R. McCourt, Physica, 38, 415 (1968).

[6] V. D. Borman, F. G. Gordienko, A. V. Medvedev, B. I. Nikolaev, and V. I. Troyan, Zh. Eksp. Teor. Fiz. 59, 1067 (1970) [Sov. Phys.-JETP 32, 578 (1971)].

[7] L. Sirovich, The Physics of Fluids, 5, 908 (1962).

[8] I. M. Gel'fand, R. A. Minlos, and Z. Ya. Shapiro, Predstavlenie gruppy vrashcheniĭ i gruppy Lorentsa, ikh primenenie (Representation of the Rotation Group and the Lorentz Group and Their Applications), Fizmatgiz, 1958.

[9] A. Tip, A. E. DeVries, and J. Los, Physica, 32, 1429 (1966).

[10] J. J. M. Beenakker, The influence of electric and magnetic fields on the transport properties of polyatomic dilute gases, in Festkörperprobleme VIII, ed. O. Madelung, Vieweg Braunschweig, 1968.

[11] H. F. Vugts, A. Tip, and J. Los, Physica, 38, 579 (1968).

[12] H. Senftleben and J. Pietzner, Ann. der Phys., 30, 541 (1937).

[13] V. D. Borman, B. I. Nikolaev, and V. I. Troyan, ZhETF Pis. Red. 9, 229 (1969) [JETP Lett. 9, 134 (1969)].

[14] A. C. Levi, F. R. McCourt, and A. Tip, Physica, 39, 165 (1968).

[15] S. Hess, Phys. Lett., 28A, 87 (1968).

[16] I. J. F. Hermans, J. M. Koks, H. F. P. Knaap, and J. J. M. Beenakker, Phys. Lett., 30A, 139 (1969).

Translated by J. G. Adashko
22