PROPAGATION OF A HIGH-PRESSURE MICROWAVE DISCHARGE

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The propagation of microwave discharges in gases at a pressure of the order of the atmospheric pressure is discussed. Thermal conduction is the main mechanism responsible for the propagation of the discharge toward the incident electromagnetic wave. Heat from the plasma is transferred to the cold gas which becomes ionized and begins to absorb microwave energy. The problem of the steady-state propagation of the discharge wave is formulated and solved approximately. The temperature of the heated plasma, the velocity of the discharge front as a function of the energy flux in the electromagnetic wave, and the threshold for this state are determined. It is shown that the process has much in common with flame propagation in the case of combustion. The phenomenon is frequently observed in microwave devices operating under continuous conditions when a discharge appears in the waveguide and the plasma travels toward the microwave source. However, so far, no satisfactory physical interpretation has been available. Reasonable agreement between the results of our calculations and the experimental data has been obtained.

1. INTRODUCTION

BEUST and Ford[1] have described a phenomenon occasionally observed in microwave devices operating under continuous conditions. A discharge appears from time to time in the waveguide, and the resulting plasma formation travels against the direction of propagation of the microwaves. The discharge is usually initiated by some impurity or foreign body, for example, a metal shaving which has been left behind inside the waveguide. This is heated to a high temperature in the microwave field and produces the initial amount of ionized vapor. Buest and Ford have investigated this phenomenon in a rectangular waveguide using microwaves in the X band. The waveguide dimensions were 2.29 × 1.02 cm and the wavelengths employed were in the range $\lambda_0 = 2.5-5.8$ cm (11.9–5.2 GHz). The discharge was initiated by a small steel screw introduced into the waveguide, which immediately became red hot in the microwave field. The effect was found to have a power threshold of about 0.25 kW which is lower by three orders of magnitude than the power necessary for ordinary breakdown in air at atmospheric pressure in the waveguide.

Insofar as one can judge from the photographs, the plasma formation has the appearance of a column at the center of the waveguide, which is at right angles to its axis and is parallel to the shorter wall, i.e., it lies along the electric vector (the H_{01} mode was employed). The radius of the bright region appears to amount to a few millimeters. The velocity of the plasma moving against the incident wave was found to increase monotonically from about 25 cm/sec up to 6 m/sec as the power was increased from the threshold value up to 2.5 kW. Measurements showed that the energy balance depended on the number of specific conditions and, in a typical case, the plasma absorbed about 75% of the incident power, the remainder being reflected. In order to prevent this phenomenon from occurring it is recommended in[1] that the waveguide be carefully cleaned so that the foreign bodies which initiated the discharge are removed. An increase in air pressure leads to only a slight increase in the threshold power.

The above phenomenon is well known to anyone who has to work with high-power microwave devices operating under continuous conditions. However, no physical interpretation of the process has been given either in[1] or, as far as we know, anywhere else. At the same time, there seems to be no doubt that we are dealing here with a well-defined example of the propagation of discharge waves similar to flame propagation in the case of combustion. The deep analogy between discharge propagation and combustion has facilitated very substantially the analysis of high-frequency discharges in gas flows on which the electrodeless plasmatron is based.[2] The analogy has also been used in the theoretical analysis of the slow propagation of laser sparks, i.e., discharges at optical frequencies[3,4] (the experiments are described in[3,5,6]).

The above process is important not only because of its practical significance but also because it is an interesting physical effect, especially since, in one form or another, it may appear under very different conditions. For example, a related phenomenon of discharge propagation is produced in microwave plasmatrons with transverse gas flow[7] in which the gas passes through the waveguide at right-angles to its axis and the discharge remains stationary. The effect which elsewhere is undesirable is used here for a positive purpose.

In this paper we shall discuss the propagation of high-pressure discharge waves maintained by microwave radiation.[1)] This case has a number of features which distinguish it from discharges at high and optical frequencies. We shall review the main regularities of

[1)]It is important to note that the propagation of the plasma front in the waveguide has also been observed at low pressures of the order of 1 torr. [8] This is a special case because the plasma is under highly nonequilibrium conditions and the propagation mechanism is not thermal conduction, as in the case of high pressures, but the diffusion of resonance radiation. The theory of this phenomenon is given in [18].

the phenomenon, the temperature of the resulting plasma, the velocity of its propagation, and the threshold for the effect. One would hope that this would also be useful for the theory of microwave plasmatrons.

2. ONE-DIMENSIONAL FORMULATION OF THE PROBLEM

The real process in a waveguide is complicated by many details, and to exhibit its main features we shall consider a somewhat simplified problem. Consider a plane electromagnetic wave of frequency ω, propagating through cold gas in the direction of the x axis, and suppose that it encounters a plane front which bounds a region of plasma. The dissipation of electromagnetic energy in the ionized gas leads to its heating but, as a result of thermal conduction, there is heat transfer from the plasma to the cold gas so that further layers are heated, become ionized, and begin to absorb electromagnetic energy. The plasma front, i.e., the boundary of the discharge, is thus found to propagate against the incident electromagnetic wave, leaving behind heated plasma at some temperature T_p. The problem is to determine this temperature and the velocity of the discharge in the cold gas.

We shall suppose that the pressure is of the order of atmospheric pressure, so that the plasma may be approximately regarded as being in thermodynamic equilibrium. This is permissible, at least for nitrogen and air. The thermal conduction rates are very low, and the process proceeds at constant pressure p. The electrodynamic properties of the medium are fully defined by the temperature T (and so is the thermal conductivity λ). The permittivity ϵ and the electrical conductivity σ [9] are given by

$$\varepsilon = 1 - \frac{4\pi e^2 N_e}{m(\omega^2 + \nu_m^2)}, \quad \sigma = \frac{e^2 N_e \nu_m}{m(\omega^2 + \nu_m^2)}, \tag{1}$$

where N_e is the number of electrons per cm^3 and ν_m is their effective collision frequency.

Consider a time-independent state in a coordinate system in which the discharge front is at rest (Fig. 1). Cold gas of initial density ρ_0 enters the discharge with velocity u.

In the time-independent planar process, $dT/dt = v_x dT/dx$, where v_x is the velocity of the gas, $\rho v_x = \text{const} = \rho_0 u$, and hence the energy balance is

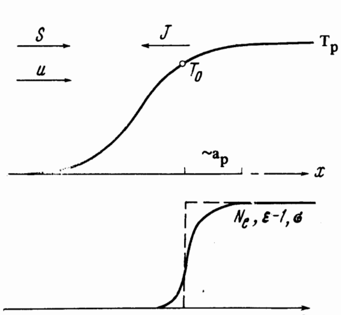

FIG. 1. Schematic distribution of temperature (upper curve) and electron density, permittivity, and conductivity in the wave.

$$\rho_0 u c_p \frac{dT}{dx} = -\frac{dJ}{dx} + \overline{\sigma E^2},$$
$$J = -\lambda \frac{dT}{dx}. \tag{2}$$

In these expressions c_p is the specific heat, J is the heat flux, and the bars over the symbols represent averages per period of the field (E, H $\sim e^{-i\omega t}$). We shall neglect energy losses by radiation because the resulting temperatures are relatively low ($\approx 5000°$ in air), so that they are small in comparison with heat release and do not affect the parameters of the problem. There are no other losses that need be taken into account. The region to which the fields in the plasma are confined are bounded, and this leads to thermal-conduction (and hydrodynamic) heat losses in transverse directions. This can be ignored only if the transverse size r of the discharge is much greater than the width of the discharge front, i.e., it is much greater than the depth a of penetration of the field into the plasma. We shall suppose for the moment that this condition is satisfied. However, it will be shown below that it is precisely the condition for high losses, r $\sim$ a, which determines the threshold for the state.

The fields are given by the Maxwell equations. In the case of monochromatic fields and planar geometry, the complex field amplitudes $E_a \equiv E_y$, $H_a \equiv H_z$ are given by

$$\frac{dE_a}{dx} = \frac{i\omega H_a}{c}, \quad -\frac{dH_a}{dx} = \left(\frac{4\pi\sigma}{c} - i\varepsilon\frac{\omega}{c}\right)E_a. \tag{3}$$

We must now formulate the boundary conditions. The field does not penetrate deep into the plasma where the gas is heated to some constant temperature, i.e. for $x = \infty$ we have $E_a = 0$, $J = 0$. In front of the discharge for $x = -\infty$ the gas is cold (T = 0), the heat flux is fully absorbed (J = 0), and the electromagnetic flux density S_0 is given. The problem is overdefined mathematically. This enables us to determine the unknown velocity of propagation, u, which is an eigenvalue of the system just as in the case of combustion.[10]

Equations (2) and (3) have a first integral which expresses the conservation of the total energy flux:

$$\rho_0 u w + J + S = S_1, \quad S = \frac{c}{4\pi}\overline{EH}, \tag{4}$$

where $w = \int_0^T c_p dT$ is the specific enthalpy and S_1 is the electromagnetic energy flux in front of the discharge for $x = -\infty$, which is equal to the energy flux entering the plasma. It is given by $S_1 = S_0(1 - R)$, where R is the reflection coefficient of the plasma for the incident wave, which must be determined in the course of the solution of the problem. From Eq. (4) we have the general energy-balance equation for the discharge wave without losses:

$$\rho_0 u w_p = S_1, \quad w_p = w(T_p). \tag{5}$$

3. APPROXIMATE SOLUTION

Let us solve Eqs. (2) and (3) by a very approximate method, since an accurate solution can only be obtained by numerical procedures. The analogy with combustion[10] suggests a method of avoiding the difficulties connected with the presence of the unknown parameter u.

The quantities $\epsilon - 1$ and σ determine the effect of the medium on the field, and are proportional to the electron density N_e. Relatively low temperatures and low degrees of ionization are found in plasma produced by microwaves. The temperature dependence of the electron density is then of the form $e^{-I/2kT}$, where I is the ionization potential, i.e. this dependence is very rapid because $I/2kT \gg 1$. This means that the density N_e which is appreciable in comparison with the finite density $N_e(T_p)$ is reached only at temperatures very close to T_p (Fig. 1). The main fraction of the dissipating field energy is liberated in the gas at temperatures lying within a very narrow temperature interval $T_0 < T < T_p$, approximately given by $T_p - T_0 \approx T_p(2kT_p/I) \ll T_p$ (the density N_e changes by a factor of e within this range). The temperature T_0, which is naturally referred to as the ionization temperature, corresponds to the ignition temperature in the case of combustion. Most of the change in the electromagnetic energy flux occurs in this temperature range, and the reflected wave is generated there as well.

In the region where most of the heat is released the gas itself is not highly heated because most of the liberated heat is lost by thermal conduction to the cold gas, and is used up to heat it to the ionization (ignition) temperature, as in the case of combustion. This means that, when we consider the region in which there is an appreciable change in S, we can approximately replace w(T) by $w(T_p)$ in Eq. (4), and then use Eq. (5) to write down the flux integral in the form

$$J + S = 0. \tag{6}$$

This equation rigorously describes the flux balance in the time-independent static discharge which is obtained if we place a cold wall in the path of the incident electromagnetic waves and in front of the discharge, provided the wall is transparent to the wave and completely removes the heat released in the discharge. However, the fate of the heat flux in the temperature region below T_0, where there are practically no heat sources and $S \approx \text{const} = S_1$, cannot have a substantial effect on the temperature to which the plasma is heated because this temperature is, of course, governed by the conditions in the heat release zone. Therefore, to calculate the temperature T_p (and the field distribution) we can start with the simplified set of equations given by Eqs. (3) and (6), and extend Eq. (6) to the entire temperature range. This means that we separate the determination of T_p from the determination of the velocity u[2)]. The velocity u can be found from Eq. (5). We note that this equation can be written in the form characteristic for the thermal conduction mechanism of propagation.[2]

Let us now consider the electromagnetic wave. In a plane monochromatic wave propagating through a homogeneous medium $E \sim H \sim \exp(-i\omega t + in\omega x/c - \kappa\omega x/c)$,[9] where

$$n = \{{}^1/_2[\varepsilon + \sqrt{\varepsilon^2 + (4\pi\sigma/\omega)^2}]\}^{1/2}, \quad \varkappa = \{{}^1/_2[-\varepsilon + \sqrt{\varepsilon^2 + (4\pi\sigma/\omega)^2}]\}^{1/2}. \tag{7}$$

The energy flux is $S \sim |H_a|^2$ and its attenuation is described by

$$\frac{dS}{dx} = -\mu S, \quad \mu = \frac{2\varkappa\omega}{c} = \frac{4\pi\varkappa}{\lambda_0}. \tag{8}$$

For normal incidence of the waves from a medium with $\epsilon = 1$, $\sigma = 0$ on a sharp boundary with a medium ϵ, σ, the reflection coefficient is given by

$$R_0 = \frac{(n-1)^2 + \varkappa^2}{(n+1)^2 + \varkappa^2}. \tag{9}$$

If the medium is not homogeneous and the frequency is arbitrary, the solution of the Maxwell equations as given by Eq. (3) becomes very difficult. However, the problem is substantially simplified in two limiting cases.

In the low-frequency limit, when $|4\pi\sigma/\omega\epsilon| \gg 1$ ($n \approx \kappa \approx \sqrt{2\pi\sigma/\omega} \gg 1$, reflection almost complete, skin depth much less than λ_0), we can neglect displacement currents in the Maxwell equations. We then have $S = -(c^2/64\pi^2\sigma)d|H_a|^2/dx$, and if we substitute this and Eq. (2) in Eq. (6) the resulting equation is immediately integrable. If we then determine the integration constant with the aid of the boundary conditions we find the relation between the plasma temperature T_p and the magnetic field amplitude in front of the discharge $|H_{a1}|$:

$$\int_0^{T_p} \lambda(T)\sigma(T)\,dT = c^2|H_{a1}|^2/64\pi^2. \tag{10}$$

This formula was obtained in[11] for the case of the static high-frequency discharge (high frequencies of the order of 1 MHz are "low" in the above sense).

In the high-frequency limit when $4\pi\sigma/\omega\epsilon \ll 1$, $\epsilon > 0$ ($n \approx \sqrt{\epsilon} \sim 1$; $\kappa \approx 2\pi\sigma/\omega\sqrt{\epsilon} \ll 1$; absorption over a wavelength and reflection are small) we may assume that Eq. (8) remains valid even for an inhomogeneous medium because the inhomogeneity is "weak." Equation (6) can then again be integrated and yields

$$S(T) = S_1 - \int_0^T \lambda\mu\,dT. \tag{11}$$

Here the integration constant has been determined from the boundary condition $S = S_1$ for $T = 0$. Referring the integral given by Eq. (11) to the point $T = T_p$, where $S = 0$, we find the relation between the temperature T_p and the energy flux S_1 entering the plasma by analogy with Eq. (10):

$$\int_0^{T_p} \lambda(T)\mu(T)\,dT = S_1. \tag{12}$$

The high-frequency limit corresponds to, for example, optical frequencies, but the microwave frequencies in which we are interested occupy the intermediate position between the two limiting cases because $|4\pi\sigma/\omega\epsilon| \sim 1$ (Table I). Equation (6) cannot then be

Table I

T, °K	N_e, cm^{-3}	$\nu_m \cdot 10^{-10}$, sec^{-1}	$\sigma \cdot 10^{-10}$, sec^{-1}	ε	$\frac{4\pi\sigma}{\omega\,\|\varepsilon\|}$	n	$\varkappa$	a, cm	R_0
3500	$6.6 \cdot 10^{11}$	7.5	0.13	0.78	0.33	0.89	0.14	1.7	0.0089
4000	$4.4 \cdot 10^{12}$	7.1	0.88	−0.53	3.3	0.81	1.1	0.22	0.28
4500	$1.6 \cdot 10^{13}$	6.6	3.3	−5.1	1.3	1.3	2.6	0.091	0.57
5000	$4.8 \cdot 10^{13}$	6.4	9.9	−18	1.1	2.1	4.7	0.050	0.71
5500	$9.3 \cdot 10^{13}$	6.0	19.0	−39	1.0	2.8	7.3	0.032	0.83
6000	$2.1 \cdot 10^{14}$	5.8	41.0	−88	1.0	4.3	11	0.022	0.88

2) We note that in the case of high-frequency discharges for which $T_0 \approx T_p$ as well, the plasma temperature calculated with allowance for flow is found to be very close to the static value. [2]

integrated exactly, but Eqs. (11) and (12) remain approximately valid. They can be looked upon as a result of a first approximation when Eqs. (3) and (6) are solved by the method of successive approximations. In fact, one can start the iteration process by using step functions as the zero-order approximations to $T(x)$, $\epsilon(x)$, and $\sigma(x)$ (dashed curve in Fig. 1). By solving the field equations in the plasma we obtain Eq. (8). If we now substitute $\mu = \mu(T)$ in Eq. (8) and integrate Eqs. (6) and (8), we obtain Eqs. (11) and (12) as the next approximation.

It is readily seen that Eq. (12), which is rigorously valid in the limit of high frequencies, gives satisfactory results even in the limit of low frequencies. In fact, the energy flux received by a homogeneous skin layer is $S_1 = (c|H_{a1}|^2/16\pi)(\omega/2\pi\sigma_p)^{1/2}$, where $\sigma_p = \sigma(T_p)$. Substituting this equation and $\mu = 2\sqrt{2\pi\sigma\omega}/c$ in Eq. (12), we obtain, instead of Eq. (10), an approximate relation in which, instead of $\sigma(T)$ in the integrand, we now have $\sqrt{\sigma(T)\sigma_p}$. Integrals of this kind can be evaluated approximately by expanding $1/T$ in the exponential around its upper value by the Frank-Kamenetskiĭ method[10]: $1/T \approx 1/T_p + (T_p - T)/T_p^2$. This procedure shows that the exact and the approximate integrals differ by a factor of only two, and this characterizes the error in the square of the field amplitude at a given temperature. The error in T_p for a given field will be quite small because the temperature is only a logarithmic function of all the factors apart from the ionization potential. It follows that we can use Eq. (12) to determine the microwave discharge temperature.

It is interesting that Eq. (12) is practically equivalent to the condition that the optical thickness of the heating zone is unity. In fact,

$$\tau = \int_0^{x(T_0)} \mu\, dx = \int_0^{T_0} \frac{\mu\lambda\, dT}{\lambda\, dT/dx} \approx \frac{1}{(\lambda\, dT/dx)\, T_p} \int_0^{T_p} \lambda\mu\, dT \approx \frac{1}{S_1} \int_0^{T_p} \lambda\mu\, dT \approx 1,$$

i.e., we have Eq. (12).

There remains the very important problem of the reflection coefficient, since it is not clear in advance to what extent it will be reduced by a diffuse plasma boundary. Since in the zero-order approximation, corresponding to a sharp boundary, the reflection coefficient r_0 is given by Eq. (9), let us consider the next approximation. Since the function $S(T)$ is now known [it is given by Eqs. (11) and (12)], the distribution $T(x)$ can be found from

$$x = \int \lambda(T)\, dT/S(T), \tag{13}$$

which follows from Eq. (6).

For the approximate evaluation of the integrals given by Eqs. (11) and (12) and then Eq. (13), we note that in the limit as $\omega \to \infty$ we have $\mu \sim \sigma$, and as $\omega \to 0$ we have $\mu \sim \sigma^{1/2}$. Calculations show that in the intermediate case of microwave frequencies we can approximately substitute $\mu \sim \sigma^{1/\beta}$ with constant β $(1 < \beta < 2)$. Therefore, $\mu \sim \exp(-I/2\beta kT)$, and if we evaluate the integrals given by Eqs. (11) and (12) by the above method, we find that

$$S(T) \approx S_1\left\{1 - \exp\left[-\frac{I}{2\beta k T_p^2}(T_p - T)\right]\right\}, \tag{14}$$

$$S_1 \approx \lambda(T_p)\, \mu(T_p)\, 2\beta k T_p^2 / I. \tag{15}$$

Substituting Eqs. (14) and (15) in Eq. (13) and integrating again on the assumption that $\lambda(T)$ is a slowly varying function, we finally obtain the distribution $T(x)$. Let us write down the explicit expression for the electron density: $N_e \sim \exp(-I/2kT) \approx \exp[-I(T_p - T)/2kT_p^2]$. The quantities $\epsilon - 1$ and σ are proportional to this density. We have

$$\frac{N_e}{N_{ep}} \approx \left(\frac{e^{x/a_p}}{1 + e^{x/a_p}}\right)^{\beta}, \quad a_p = \frac{1}{\mu_p} = a(T_p). \tag{16}$$

To estimate the reflection coefficient for a layer with this distribution of ϵ and σ we can use the well-known solution of the wave equation[12] for a transparent transition layer with the distribution

$$\varepsilon(x) = 1 - (1 - \varepsilon_\infty)\frac{e^{x/\Delta}}{1 + e^{x/\Delta}}, \quad \varepsilon_\infty = \varepsilon(\infty). \tag{17}$$

Generalization to an absorbing medium can also be made quite easily if we assume that ϵ is complex. After some simple rearrangements we find that the reflection coefficient for the transition layer defined by Eq. (17) is

$$R = \frac{1}{R_0(n, \varkappa)}\left|\Gamma\left[\frac{\Delta}{2a}\left(1 - i\frac{n+1}{\varkappa}\right)\right] \Big/ \Gamma\left[\frac{\Delta}{2a}\left(1 - i\frac{n-1}{\varkappa}\right)\right]\right|^4, \tag{18}$$

where Γ is the gamma function and the quantities n, κ, and $a = 1/\mu = \lambda_0/4\pi\kappa$ are referred to the point $x = \infty$. When $\Delta \to 0$ we have $R \to R_0$, and when $\Delta \to \infty$ we have $R \approx R_0^{-1}\exp(-4n\Delta/\kappa^2 a)$. The dependence of R on the relative layer thickness is illustrated in Fig. 2. Comparison of Eqs. (16) and (17) shows that to estimate R we can substitute in our case $\Delta = a/\beta$. Since $1 < \beta < 2$, the values of R are not much less than R_0, especially at higher temperatures (Fig. 2).

4. THE REGIME WITH LOSSES AND THE LIMIT FOR ITS EXISTENCE

In the absence of energy losses the discharge wave can exist at very low electromagnetic energy fluxes, which is clear from Eq. (12). In practice, the state always has a threshold which is connected with heat losses from the discharge through heat conduction in transverse directions out of the region in which strong fields operate (the hydrodynamic energy loss is estimated to be less than the thermal conduction loss).

Let us now include the losses, at least approximately. Suppose that the incident wave and the discharge have a radius r in the transverse directions. The mean energy loss per unit volume of the gas at a point x per second is $(2\pi r/\pi r^2)(-\lambda\, \partial T/\partial r')_{r'=r}$. This can be written in the form $A\Theta/r^2$, where the heat flux potential is given by

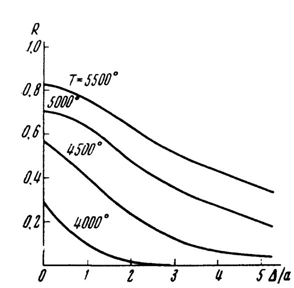

FIG. 2. Reflection coefficient of a plane transition layer as a function of the relative thickness in air (at a pressure of 1 atm and wavelength λ_0 = 3 cm).

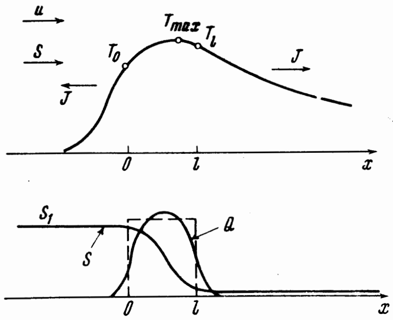

FIG. 3. Schematic distribution of temperature, radiation flux, and heat release in the presence of energy losses. Broken curves shows the heat-release approximation adopted for the linearization of the equations.

$$\Theta = \int_0^T \lambda(T)dT \qquad (d\Theta = \lambda dT, J = -d\Theta/dx)$$

and corresponds to the mean temperature at the point x, A depends on the radial profile $T(r')$, and when $T(r) = 0$ we have $A = A_{max} \approx 5.8$.[4] In practice, the temperature is still high on the discharge boundary and A is a few times lower. Let us apply Eq. (8) to radiation, and introduce a loss term in the energy-balance equation of Eq. (2) by rewriting it in the form

$$\frac{\rho_0 u c_p}{\lambda}\frac{d\Theta}{dx} = \frac{d^2\Theta}{dx^2} + Q - \frac{A\Theta}{r^2}, \quad Q = S\mu = -\frac{dS}{dx}. \tag{19}$$

In front of the discharge, where $x = -\infty$, we have $T = 0$, $J = 0$, and $S = S_1$ as before, and behind the wave, when $x = \infty$, the gas is now fully cooled because of the losses, so that $T = 0$, $J = 0$ (Fig. 3). The radiation flux is absorbed by the wave quite strongly, but not completely, since the "optical thickness" is now finite. The phrase "plasma temperature" must now be interpreted as the maximum temperature $T_{max} \equiv T_m$ reached in the wave. This quantity replaces T_p. The threshold for the existence of this state corresponds to zero propagation velocity ($u = 0$), when heat release is just sufficient to compensate the losses but not as yet sufficient to "move" the wave.

The approximate solution can be found by linearizing Eq. (19) by analogy with the theory of combustion.[13] We shall suppose that $Q = 0$ for $T < T_0$, $x < 0$ so long as the absorption coefficient is still low, and also for $x > l$ where flux S is highly absorbed.[3)] In the region $0 \leq x \leq l$ the heat release Q will be assumed constant and we shall substitute $d = \lambda/\rho_0 u c_p = \text{const} = d(T_m)$. The general solution of the linearized equation (19) which satisfies the boundary conditions for $x = \pm\infty$ is

$$\Theta = \Theta_0 e^{k_1 x}, \quad \Theta_0 = \Theta(T_0) \quad \text{for } x \leqslant 0,$$
$$\Theta = Qr^2/A + C_1 e^{k_1 x} + C_2 e^{k_2 x} \quad \text{for } 0 \leqslant x \leqslant l,$$
$$\Theta = C_3 e^{k_2(x-l)} \quad \text{for } x \geqslant l,$$

where

$$k_{1,2} = \frac{1}{d}\left(\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{4Ad^2}{r^2}}\right); \quad k_1 > 0,\ k_2 < 0. \tag{20}$$

3)This is the essential difference from the case of optical frequencies where the absorption of the flux can be completely neglected. [4]

By matching the values of Θ and of the derivatives on the boundaries, we obtain four equations relating the integration constants C_1, C_2, C_3 and the unknown parameter d (the wave velocity). After some rearrangement we have

$$C_1 = -\Theta_0 \frac{q-z}{z}, \quad C_2 = -\Theta_0 \frac{q}{z}, \quad C_3 = \Theta_0 \frac{q}{z}(1 - e^{-y/\sqrt{z-1}}), \tag{21}$$
$$q = z/(1 - e^{-y\sqrt{z-1}}),$$

where we have used the dimensionless parameters

$$q = \frac{Qr^2}{A\Theta_0}, \quad y = \sqrt{A}\frac{l}{r}, \quad z = 1 - \frac{k_1}{k_2}. \tag{22}$$

Let us now use the condition which gives the ionization temperature T_0 introduced at the beginning of Sec. 3:

$$\frac{\Theta_m - \Theta_0}{\Theta_0} \approx \frac{1}{\Theta_0}\left(\frac{d\Theta}{dT}\right)_m (T_m - T_0) \approx \frac{\lambda_m T_m}{\Theta_m}\frac{2\beta k T_m}{I} \equiv \delta. \tag{23}$$

(we have taken into account the fact that $T_m - T_0 \ll T_m$). The maximum of Θ lies in the heat release zone and approaches its end as the losses become less important. By calculating Θ_{max} and using Eqs. (21) and (22), we obtain one further equation, namely,

$$\frac{\Theta_m}{\Theta_0} - 1 = q - 1 - q\left(1 - \frac{z}{q}\right)^{1/z} = \delta. \tag{24}$$

Physically, it is clear that the quantities Q and l, given during the linearization, are

$$Q = \alpha S_1 / l, \quad l = \gamma a(T_{max}), \tag{25}$$

where α and γ are of the order of unity. More accurate results can be obtained by going over to the low-loss limit $r \to \infty$ for which a more exact solution was found in Sec. 3. When $r \to \infty$ we have $y \to 0$, $q \to \infty$, $z \to \infty$, and $y\sqrt{z} = \text{const}$, $q/z = \text{const}$. If $\delta \ll 1$, which corresponds to the solution of Sec. 3, both these constants are found to be equal to 2δ and we do, in fact, obtain Eq. (15). The coefficients α and γ must be chosen to be $\alpha = 3/2\beta$, $\gamma = 3/\beta$ (where we recall that $1 < \beta < 2$).

The threshold for the $u = 0$ state corresponds to $d \to \infty$, $z = 2$. Equations (21) and (24) then give $q = (1 + \delta)^2/2\delta$, $y = 2\ln[(1 + \delta)/(1 - \delta)]$. The second of these two relations gives us the plasma temperature T_t under the threshold conditions, and the first gives the minimum flux S_{1l} which must be introduced into the plasma. The temperature T_t is not very dependent on the transverse "radius" r of the wave. For atmospheric air $T_t \approx 4000-4500°$. The main contribution to the ionization of air at such temperatures is due to nitric oxide with $I = 9.4$ eV, and for $T \approx 4000-6000°$ we have $\Theta \approx \lambda T/3$, $\beta \approx 1.5$ (Table II). Moreover, if we suppose that $A = 2$, we obtain the following formulas which define the threshold conditions for air ($\delta \approx 0.35$):

Table II

T, °K	$\lambda\cdot10^5$, kW/cm·deg	$\Theta\cdot10^2$, kW/cm	$c_p\cdot10^3$, kJ/deg	w, kJ/g	S_1, kW/cm²	u, cm/sec	v, m/sec	S_1^*, kW/cm²	u^*, cm/sec
4200	0.92	1.1	2.6	—	0.2	0	0	—	—
4500	095	1.4	2.3	9	0.23	6.2	1.2	0.045	—
5000	1.1	1.9	2.8	10	0.35	22	4.8	0.14	11
5500	1.3	2.5	4.4	12	0.56	26	6.7	0.30	19
6000	1.55	3.3	7.5	15	1.06	30	8.7	0.60	31

$$l = 2a(T_t) \approx 1{,}0\, r, \quad S_{1t} \approx 5.4\Theta_t / r. \qquad (26)$$

We note that similar formulas are obtained when the threshold is estimated simply from the obvious condition that all three terms on the right-hand side of Eq. (19) must be comparable in the heat-release zone. Equations (21) and (24) together with the known function $l(T_m)$, i.e., $y(\delta)$, enable us to calculate T_m and u as functions of S_1.[4)]

5. DISCUSSION OF RESULTS. DISCHARGES IN WAVEGUIDES

The numerical calculations were performed for air at a pressure p = 1 atm for λ_0 = 3 cm (frequency 10 GHz, $\omega = 6.3 \times 10^{10}$ sec^{-1}), i.e., for the conditions of the experiments described in[1]. The electromagnetic parameters are shown in Table I and the electron collision frequencies were calculated for a cross section $\sigma_c = 10^{-15}$ cm^2 which corresponded to nitrogen molecules.[14] Let us first estimate the threshold for the process. Without going into the reasons for the limited transverse size of the discharge, let us suppose that r = 0.3 cm, which corresponds approximately to the experiment in[1]. According to Eq. (26), a ≈ 0.5 cm, which corresponds to a temperature $T_t \approx 4200°$. Using the thermal conductivity data,[5] we find that $\Theta_t = 1.1 \times 10^{-2}$ kW/cm and from Eq. (26) we obtain $S_{1t} \approx 0.2$ kW/cm^2. The reflection coefficient R ≈ 0.28 (see Fig. 2; $\Delta/a = 1/\beta \approx 0.7$) and the threshold flux in the incident electromagnetic wave is $S_{0t} \approx 0.28$ kW/cm^2.

Comparison with experiment can be carried out by using the power introduced into the plasma. Experimentally[1] the threshold power in the incident wave is 0.25 kW. If, as suggested in[1], the power absorbed amounted to 75%, the power actually absorbed by the plasma was 0.19 kW. If we take the surface of the plasma column facing the incident wave to be πrh with r = 0.3 cm, h = 0.8 cm, we obtain S_{1t} = 0.25 kW/cm^2 as compared with the calculated value of 0.2 kW/cm^2. The agreement is so good that, bearing in mind all the approximations, one can hardly ascribe to it any great significance.

Table II gives the calculated flux S_1 which must be introduced into the plasma to heat it to a temperature T and the corresponding wave propagation velocities u. Losses were taken into account (r = 0.3 cm) as described in the preceding section. The last columns give S_1^* and u* calculated for finite temperatures T but without losses, i.e., using Eqs. (5) and (12). There is a considerable difference between the data near the threshold, but this difference rapidly decreases for fluxes above the threshold value, as expected. We do not reproduce the incident radiation flux S_0 because the reflection coefficients R, calculated for the planar problem, turn out to be extremely high at temperatures in excess of 4500° (see below). As the electromagnetic energy flux increases, the temperature rises relatively slowly and the main increase is in the discharge propagation velocity. The table also gives the velocity of the discharge front relative to the heated gas, i.e., $v = u\rho_0/\rho_m$, where σ_m is the air density at a temperature T and pressure p = 1 atm. This should be equal to the velocity of the plasma front in the laboratory system if the heated gas is at rest, as in the case of a flame in a tube away from the closed end.

The measured velocities increase more or less proportionally to the power, and agree with the calculated values of v but not those of u. This shows that the situation in the waveguide is in some way closer to combustion in a tube with a closed end. This is natural because the heated gas expands in all directions, including the direction of motion of the discharge wave. The discharge front acts as a piston pushing in front of it the cold gas which, consequently, itself moves relative to the waveguide. However, in the absence of the usual combustion in the tube, the "flame" does not occupy the entire tube but is confined to its central region, and this exceedingly complicates any consideration of the hydrodynamic flow in the waveguide.

The transverse size of the discharge in the waveguide and its shape are determined by the distribution of the electromagnetic field near the plasma, and are also connected with the hydrodynamics of the process. It is clear from the photographs given in[1] that the discharge does not propagate in the transverse cross section in the direction of the narrower wall of the waveguide well away from the axis. It appears that this occurs because, owing to the presence of the plasma, the field in its neighborhood falls with distance from the axis more rapidly than the sinusoidal expression corresponding to the H_{01} wave in the empty waveguide. The radius r of the column can be determined theoretically only by solving the field problem for the plasma-waveguide. Unless this is done, one cannot determine the actual amount of energy absorbed from the incident wave. The reflection coefficients of the plasma calculated for the planar problem, i.e., without taking into account the transverse size, were found to be too high at temperatures for which absorption is sufficiently high to ensure the required plasma heating. When the diffuseness of the plasma boundary was taken into account this did not result in any essential reduction in the reflection. Moreover, it is noted in[1] that only 25% of the incident power was reflected, but it was not indicated how reflection varied with power.

A solution for the scattering of H_{01} waves in a waveguide from a very thin conducting rod parallel to the electric vector is given in[16]. According to this solution, one-half, at most, of the power can be dissipated, one-quarter being reflected and the other quarter transmitted, and this is in conflict with experiment.[1] It is clear that the plasma rod cannot be regarded as thin.

Another solution is given in[17], where a thick plasma column in a waveguide is considered for a smooth (parabolic) radial electron density distribution. This again is not in agreement with the experimental data in[1]. The solution results in reflection coefficients which are too high for electron densities which could ensure sufficient absorption of energy. Unfortunately, no data are given in[17], either on the field structure or on the absorption and transmission of waves. This

4)It is more convenient to take T_m as the independent variable in the calculations. By specifying T_m we determine δ and y. If we eliminate q from Eqs. (21) and (24), we can solve the resulting equation for z and then find q from Eq. (21) and S_1, d, and u from Eqs. (20), (22), and (25).

means that it is almost impossible to use the results of that paper for our purposes. It would be necessary to repeat the complicated and extensive calculations on a computer.

In conclusion, we must say a few words about the initiation of the discharge. To start off the "combustion" of the main gas, the initial ionized vapor cloud must be large enough and hot enough so that ionization is produced. The situation is completely analogous to the firing of a combustible mixture.

It is clear that, even for comparable threshold powers necessary to maintain an existing discharge wave, it will be more difficult to initiate the process at a higher ionization (combustion) temperature of the gas. One must assume that the discharge will be most difficult to initiate in helium because this gas has the highest ionization potential of all the gases. Calculations for helium with $p = 1$ atm and $\lambda_0 = 3$ cm, analogous to those performed for air, shows that, for the same discharge radius $r = 0.3$ cm, the threshold temperature is higher by a factor of two than in air (approximately 9000°). The threshold flux at which the "combustion" is possible is also much higher than in air, namely, $S_{1t} \approx 0.9$ kW/cm^2.

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