MANY-PARTICLE STATES IN THE THEORY OF THE KONDO EFFECT

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The role of many-particle states in the unitarity conditions for the amplitude of elastic scattering by a paramagnetic impurity at zero temperature is investigated. An analysis of the spin structure of the inelastic-process amplitudes occurring in these conditions makes it possible to conclude that these conditions have different forms for electron and hole scattering. Namely, the expression for the imaginary part of the exchange amplitude B contains a term that changes sign on passage through the Fermi surface. On the basis of the requirement of analyticity of the amplitude and the unitarity conditions for its non-exchange part, it is concluded that on the Fermi surface this term is exactly equal to zero. Since it has the form of a product of the sign function with a sum of positive terms, each of these is, consequently, equal to zero. But one of these terms is $|B|^2$, i.e., $B(E_F) = 0$. Furthermore, an analysis of the unitarity conditions for inelastic processes in the spirit of scaling theory makes it possible to conclude that, near the Fermi surface, the amplitudes behave in a more complicated manner than simple powers of the energy, and, consequently, the asymptotic forms of the amplitudes in the Kondo effect are not the same as in the scaling theories.

A theory of the Kondo effect based on unitarity conditions and the analytic properties of the scattering amplitude was developed by Suhl and Wong[1] and by the author[2,3]. The basic predictions of this theory[4] agree qualitatively with the experimental data at present available. At the same time, from a theoretical point of view the position is not so satisfactory, since this theory does not take many-particle states into account[1)].

In papers by the author[5,6], an iteration procedure was developed, making it possible to take systematic account of the contribution of many-particle states to the unitarity conditions. The basic result of these papers is the following: if $\ln(\epsilon_0/|\zeta|) \gg 1$, where ζ is the energy reckoned from the Fermi surface and ϵ_0 is the Kondo energy, the many-particle states do not change the amplitude's asymptotic form obtained in the single-particle approximation:

$$F = A + BS\sigma,$$

$$A(\zeta) \approx \frac{i}{k_F}\left(1 - \frac{\pi^2 S(S+1)}{\ln^2(\varepsilon_0/\zeta)}\right), \quad B(\zeta) \approx -\frac{\pi}{2k_F \ln(\varepsilon_0/\zeta)}. \qquad (1)$$

Their role reduces simply to renormalization of the Kondo energy. For $\zeta \lesssim \epsilon_0$, the contribution of three-particle terms is of order $(2S+1)^{-1}$ relative to the one-particle terms and is therefore small for large spins (even for $S = 1/2$ it is about a third as large as the one-particle contribution). It was further shown that the five-particle terms lead to corrections of order $(2S+1)^{-2}$, and so on. Thus, for large spins the single-particle approximation is validated. Moreover, for $S = 1/2$ too it gives the correct qualitative behavior of the scattering amplitude and the correct asymptotic formulas (1).

In the present paper the role of many-particle states at zero temperature is investigated from a different point of view. On the basis of an analysis of the spin structure of the inelastic amplitudes, the general structure of the unitarity conditions for elastic scattering is established. Starting from these conditions, we find a number of constraints that the different amplitudes must satisfy. In particular, we show, in a general form, that the exchange part B of the scattering amplitude is equal to zero at the Fermi surface, and find an upper bound for this quantity valid for all energies. We then give an analysis of the amplitude of inelastic processes and show that we cannot define the asymptotic forms of all the amplitudes as simple powers of the energies, as we could have done if the scaling law proposed recently in[7-9] were valid.

We now make one remark, important for what follows, concerning the characteristics of the problem under consideration. The impurity occupies a completely determined position in space. Therefore, the amplitudes of all the processes caused by the interaction with the impurity are amplitudes of transitions between states of free particles, i.e., particles not interacting with the impurity. The unitarity conditions for such amplitudes must contain free particles in the intermediate states, exactly as in the case of the ordinary theory of elastic and inelastic scatterings by a nucleus or atom. In this respect our problem is radically different from, e.g., the problem of second-order phase transitions, where it is completely meaningless to consider free particles[2)]. In other words, we shall in fact analyze below the properties of the matrix elements of the S-matrix between asymptotic states consisting of a product of the function for the

[1)]We shall not concern ourselves here with the question of the so-called CDD ambiguity, which arises when one seeks a solution satisfying the single-particle unitarity conditions. In a paper by the author[5], it was shown on the basis of the equations of motion that the correct choice of solution was made in [1-3].

[2)]Strictly speaking, the unitarity conditions for a many-body system should always be written using free particles or some other complete set of states. The connection between the quantities appearing in these unitarity conditions and quantities that can be determined by Feynman diagrams is discussed in a paper by Ginzburg and the author[10].

ground state (characterized by the total moment J and its projection M) of the "electrons plus impurity" system and a finite number of plane waves describing electrons and holes situated far from the impurity, where the latter's influence can be neglected. Such a description is, of course, possible only if the system of electrons interacting with the paramagnetic impurity has a ground state, and this is essentially the only assumption upon which the results obtained below are based.

As shown in[2,3], the expression for the retarded amplitude for scattering by a pointlike impurity has the form[3])

$$F^{M'M}_{\alpha'\alpha}(E)=\frac{i}{4\pi}\int_0^\infty dt\, e^{iEt}\langle M'|\{j_{\alpha'}(t),j_\alpha^+(0)\}|M\rangle,$$

$$j_\alpha(t)=\int d\mathbf{x}\left(i\frac{\partial}{\partial t}-H_0\right)\psi_\alpha(\mathbf{x},t). \qquad (2)$$

Here α and α' are the spin projections of an electron $(E > E_F)$ before and after the scattering, or of a hole $(E < E_F)$ after and before the scattering, and M and M′ are the spin projections of the impurity. From this formula, the unitarity conditions follow immediately (cf.[2,3,6]):

$$\operatorname{Im} A=k[|A|^2+|B|^2S(S+1)]+\Delta_A,$$

$$\operatorname{Im} B=k[AB^*+A^*B-|B|^2\varepsilon(\zeta)]+\Delta_B,$$

$$\Delta^{M'M}_{\alpha'\alpha}=\Delta_A\delta_{\alpha'\alpha}\delta_{M'M}+\Delta_B\,(\mathbf{S}\sigma)^{M'M}_{\alpha'\alpha}=\begin{cases}{}^1\!/_4\sum\limits_n\int d\rho_n j_{M'\alpha'n}j^+_{n\alpha M}, & E>E_F\\ {}^1\!/_4\sum\limits_n\int d\rho_n j^+_{M'\alpha n}j_{n\alpha' M}, & E<E_F\end{cases}, \qquad (3)$$

where $\epsilon(\zeta) = 1$ for $\zeta > 0$ and $\epsilon(\zeta) = -1$ for $\zeta < 0$; $d\rho_n$ is an element of phase volume of an n-particle state $(n > 1)$.

For $E > E_F$, the quantity $j_{\alpha'n}$ has the meaning of the amplitude of the coalescence of $m + 1$ electrons and m holes into one electron $(n = 2m + 1)$, while for $E < E_F$, the quantity $j_{n\alpha'}$ is the amplitude of the decay of a hole into m electrons and $m + 1$ holes; $j^+_{n\alpha}$ and $j^+_{\alpha n}$ are the Hermitean-conjugate amplitudes of the reverse processes[4]). The simplest three-particle term in (3) is depicted graphically in Fig. 1, where the lines cut by a vertical dashed line represent free particles; these lines correspond to δ-functions expressing the fact that the energy of the particles is $k^2/2m$.

For simplicity, we shall assume that the impurity spin $S = 1/2$. Then the matrix element $j_{\alpha'n}$ has the form

$$j_{\alpha'n}=g_{\alpha'n}+\mathbf{S}\mathbf{h}_{\alpha'n} \qquad (4)$$

and for the n-particle contribution to Δ for $E > E_F$ we obtain

$$\Delta_n=a_n+\mathbf{S}\sigma(\mathbf{b}_n+\mathbf{b}_n^*)+S_iS_k(c_n\delta_{ik}+d_n\sigma_i\sigma_k), \qquad (5a)$$

$$a_n=(g_n,\,g_n^+),\qquad \sigma_ib_n=(h_{ni},\,g_n^+),$$
$$c_n\delta_{ik}+d_n\sigma_i\sigma_k=(h_{ni},\,h_{nk}^+), \qquad (5b)$$

where the symbol $(\ldots,\ldots)$ denotes integration over the phase volume of the intermediate particles and summation over their spin projections. Clearly, by virtue of the definition, $a_n > 0$ and $c_n + d_n > 0$ and, therefore, $\Delta_{nA} > 0$. We shall now show that $d_n \geq 0$. This quantity can be written thus:

$$d_n=\frac{1}{4i}\operatorname{Sp}\{\sigma_z[(h_{nx},h_{ny}^+)-(h_{ny},h_{nx}^+)]\}. \qquad (6)$$

To construct the vectors $h^i_{\alpha'n}$ we have at our disposal one vector σ_i and one tensor ϵ_{ikl}. By means of these quantities the most general expression for $h^i_{\alpha'n}$ can be represented in the following form

$$h^i_{\alpha'n}=\sigma^i_{\alpha'\mu}H^{(1)}_{\tilde n}+\sigma^i_{\mu\nu}H^{(2)}_{\alpha',\tilde n}+\varepsilon_{ikl}\sigma^k_{\alpha'\mu}\sigma^l_{\nu\tau}H^{(3)}_{\tilde n}+\varepsilon_{ikl}\sigma^k_{\mu\nu}\sigma^l_{\tau\rho}H^{(4)}_{\alpha',\tilde n}, \qquad (7)$$

where each of the symbols $\tilde{n}$ comprises all the spin indices of the intermediate particles with the exception of those explicitly written out alongside the Pauli matrices. From this formula we obtain

$$d_n=\sum_{p,q}d^{(n)}_{p,q},\qquad d^{(n)}_{pq}=(H^{(p)}_n,H^{(q)+}_n),$$

$$d^{(n)}_{pq}=d^{(n)*}_{qp},\qquad d^{(n)}_{11}>0,\qquad d^{(n)}_{22}=d^{(n)}_{33}=d^{(n)}_{44}=0, \qquad (8)$$

whence it follows immediately that d_n is non-negative.

We now consider the scattering of a hole $(E < E_F)$. For $j_{n\alpha'}$ we have

$$j_{n\alpha'}=g'_{n\alpha'}+\mathbf{S}\mathbf{h}'_{n\alpha'}. \qquad (9)$$

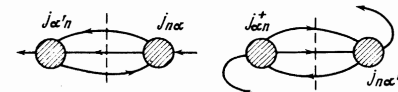
The matrix structure of the quantities $g'_{n\alpha'}$ and $h'_{n\alpha'}$ differs from the matrix structure of $g_{\alpha'n}$ and $h_{\alpha'n}$ only in the order of the sequence of all the spin indices. That this statement is true is seen most simply by examining Fig. 1. In fact, diagram 1(b) differs from 1(a) only in the direction of all the electron lines and, in particular, in the order of the sequence of all the spin indices. As a result, for $E < E_F$ the expression for Δ_n will differ from (5a) only in the ordering of the vector indices in the product of Pauli matrices in the last term ($\sigma_k\sigma_i$ instead of $\sigma_i\sigma_k$), while the coefficients a_n and $c_n + d_n$, expressed now in terms of the primed quantities, will be positive as before, with d_n non-negative.

As a result we can write down the unitarity conditions for the scattering amplitude for all E in the form

$$\operatorname{Im} A=k[|A|^2+|B|^2S(S+1)]+\sum_{n>1}[a_n+S(S+1)(c_n+d_n)], \qquad (10a)$$

$$\operatorname{Im} B=k[AB^*+A^*B]+\sum_{n>1}(b_n+b_n^*)-\varepsilon(\zeta)\left[k|B|^2+\sum_{n>1}d_n\right]. \qquad (10b)$$

The reason for the appearance of the factor $\epsilon(\zeta)$ in (10b) can be elucidated as follows. The transition from electrons to holes for the spin operators is analogous



a b

FIG. 1

[3])The assumption of a pointlike impurity is not essential for what follows. Just as in [4], we could examine all the processes for a fixed value of the orbital angular momentum.

[4])Other variants of this interpretation are also possible. For example, $j^+_{n\alpha}$ and $j_{n\alpha'}$ are decay amplitudes of an electron and a hole, and so forth. This is connected with the fact that the unitarity conditions can be written in two ways: $SS^+ = S^+S = 1$. Formally, the different variants are obtained by means of the relation, used in[6], between the matrix elements of S and j or j⁺ when the different times $t = +\infty$ and $t = -\infty$ pertain to the intermediate states.

to "time reversal"[11]. Therefore, in such a transition the ordering of the electron-spin operators should change. But this "time reversal" does not, of course, refer to the impurity spin, i.e., the ordering of the operators S_i is not changed. This also gives a change of sign in the last term in Im B.

As is well known, a knowledge of the functions Im A and Im B enables us to determine the whole scattering amplitude by means of the dispersion integral:

$$F(\zeta)=F_0(\zeta)+\frac{1}{\pi}\int_{-E_F}^{\infty}\frac{d\zeta' \,\mathrm{Im}\, F(\zeta')}{\zeta'-\zeta-i\delta} \tag{11}$$

where $F_0(\zeta)$ is the rational part of F, consisting of a constant and pole terms corresponding to the bound states for $\zeta < -E_F$. The latter lie far from the Fermi surface and bear no relation to the Kondo effect. The functions Im A and Im B must be continuous for all $\zeta > -E_F$, and at the point $\zeta = 0$ in particular. Indeed, if for one of these functions this is not so, the corresponding dispersion integral must behave as $\ln \zeta$. Substitution of this logarithm into the expression for Im A immediately leads to a contradiction: quantities of different order appear on the right and the left (the same contradiction arises if we assume the existence of a pole in the amplitude at $\zeta > -E_F$; hence follows, in particular, the impossibility of genuine bound states near the Fermi surface).

But in (10b) there is a term proportional to $\epsilon(\zeta)$. Therefore we must have one of two possibilities: either the coefficient of $\epsilon(\zeta)$ goes to zero for $\zeta = 0$, or else the second term in (10b) behaves in such a way as to compensate the term with $\epsilon(\zeta)$.

In Appendix I it is shown that such a compensation is impossible, and therefore only the first possibility remains, by virtue of which, taking into account that d_n is non-negative we obtain

$$B(0)=0, \tag{12a}$$

$$d_n(0)=0, \tag{12b}$$

$$\sum_{n>1}[b_n(0)+b_n^*(0)]=0. \tag{12c}$$

No approximations have been made in the derivation of these formulas. They are the consequences of the exact unitarity and analyticity conditions. We note also that in[5,6] these conditions were fulfilled. The physical meaning of the condition (12a) is as follows. The entire problem of scattering by an impurity with spin has arisen because of the fact that an electron and a hole are scattered in different ways by such an impurity. This is due to the fact that the expressions for their spins (σ and $-\sigma^T$ [11]) are different. But on the Fermi surface the difference between an electron and a hole disappears and consequently they must be scattered in the same way; this, evidently, is possible only if the scattering amplitude does not depend on the spin, and this is just condition (12a).

The results obtained in[5,6] were essentially based on an estimate of $|B|^2$ obtained in the single-particle approximation:

$$|B|^2 \leqslant 1/k^2(2S+1)^2. \tag{13}$$

We shall now show that the rigorous upper bound for $|B|^2$ differs little from this expression. For this we eliminate A from (10a) with the aid of the formula $2ikA = Bu - 1$. As a result we obtain

$$|B|^2=\frac{1-4\Delta_A}{|u|^2+4k^2S(S+1)} \tag{14}$$

where Δ_A represents the many-particle terms in (10a). It follows from (14) that

$$|B|^2 \leqslant \frac{1}{4k^2S(S+1)}=\frac{1}{k^2[(2S+1)^2-1]}, \quad \Delta_A \leqslant \frac{1}{4k}. \tag{15}$$

These formulas are also exact. They are, obviously, valid for any values of the spin S.

Recently, in a number of papers[7-9], attempts have been made to construct a theory of the Kondo effect in the spirit of the scaling theories widely used to describe second-order phase transitions (cf., e.g., the papers of Migdal and Polyakov[12,13]). We shall now concern ourselves with elucidating whether the basic ideas of these theories can be reconciled with the results obtained above. In the case we are considering $(T = 0)$, the scaling laws reduce to the following two statements: 1) all amplitudes of inelastic processes have the form $\Gamma_{n_1,n_2} = \zeta^{-z} f_{n_1,n_2}(\zeta_1, \ldots, \zeta_n)$ where ζ_i are the energies of the particles participating in the process; 2) many-point vertices with the same number of points are quantities of the same order (e.g., $\Gamma_{13} \sim \Gamma_{22}$, where Γ_{22} is the scattering amplitude for two particles); the discontinuities of the many-point functions with respect to any of the energies are also of the same order.

The contribution of the n-particle term to the unitarity conditions (10) is, in order of magnitude, equal to $\zeta^{2m}|\Gamma_{n1}|^2$ where Γ_{n1} is the inelastic amplitude, which is expressed in terms of the matrix elements of j and j^+. It follows from this estimate that $z \leq m$. By virtue of (4) and (9), $\Gamma_{n1} = \Gamma_{n1}^{(1)} + \Gamma_{n1}^{(2)} \cdot \mathbf{S}$, and, generally speaking, $\Gamma_{n1}^{(1)}$ and $\Gamma_{n1}^{(2)}$ can have different indices $z_1 = m - x$ and $z_2 = m - y$ (we recall that, in perturbation theory, $x = y = 0$[5,6]). We consider the unitarity condition for scattering of a particle and a hole (see Fig. 2). Schematically it has the form

$$\begin{gathered}\Delta\Gamma_{22}^{(1)}=\Gamma_{22}^{(1)+}\zeta\Gamma_{22}^{(1)}+\Gamma_{22}^{(2)+}\zeta\Gamma_{22}^{(2)}+\Gamma_{24}^{(1)+}\zeta^3\Gamma_{42}^{(1)}+\ldots,\\ \Delta\Gamma_{22}^{(2)}=\Gamma_{22}^{(1)+}\zeta\Gamma_{22}^{(2)}+\Gamma_{22}^{(2)+}\zeta\Gamma_{22}^{(1)}+\ldots\end{gathered} \tag{16}$$

Here Δ denotes the discontinuity with respect to the energy $\zeta_1 - \zeta_2$. It follows immediately from these conditions that $x = 0$, while y remains undetermined. The unitarity conditions for other processes also do not allow us to determine y.

In Appendix II, it is shown with the aid of invariance with respect to time reversal that y is either a whole number or zero. This means in fact that $y = 0$, since otherwise the principal term in $\Gamma_{22}^{(2)}$ will be the constant term, and this is sufficient to violate all the scaling theory concepts.

Moreover, there exist simple physical considerations from which it follows that $y = 0$. In fact, for $y > 0$ we have $\Gamma_{n1}^{(2)} \ll \Gamma_{n1}^{(1)}$, i.e., inelastic scattering

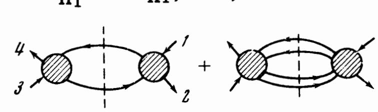


FIG. 2

processes accompanied by a spin-flip are small compared with non-spin-flip processes. For $\zeta \to 0$ the total probability of the former is proportional to ζ^{2y} and of the latter is constant. In other words, an electron that has collided with an impurity can generate a certain number of pairs, but the impurity spin cannot be flipped in this process. Moreover, it follows from (10b) that in the case under consideration $B \sim \zeta^y$ and, consequently, the spin does not flip in elastic scattering either.

Thus, a very strange situation arises; the inelastic scattering processes have a finite probability, whereas the only impurity degree of freedom responsible for the interaction between the electrons—its spin—turns out to be frozen. And only when $y = 0$ do the inelastic processes with a spin-flip become of the same order as the non-spin-flip processes.

Neither, however, can the scaling laws be fulfilled for $x = y = 0$. In fact, if these laws hold, then for $\zeta \to 0$ the many-particle contribution to (10) cannot depend on ζ, i.e., it must be constant[5)], and it is impossible to reconcile this with the conditions (12). This means that the inelastic amplitudes Γ_{n1} must have, in addition to a power factor of order ζ^{-m}, an additional energy dependence of a non-power type, the latter being responsible for the fulfillment of the conditions (12). This conclusion is in complete agreement with the results of[5,6], where it is shown that

$$\Gamma_{n_1} \sim \zeta^{-m}(\ln\zeta)^{-m-1}, \quad B \sim (\ln\zeta)^{-1}.$$

In the present article, unlike in[7-9], all the amplitudes are considered "on the mass-shell," i.e., under the condition that the impurity energy is the same before and after the scattering.

Experimentally observed quantities such as the resistance, thermopower, etc., are directly expressed in terms of precisely these amplitudes. Because of this fact, our results mean that the asymptotic behavior of the observed quantities in the limit cannot be a power behavior, and this is in accordance with the results of paper[4]. At the same time, the results obtained above cannot, generally speaking, be used for the analysis of quantities whose definitions contain amplitudes (many-point functions) "off the mass-shell," e.g., for the calculation of the impurity Green's function or the magnetic susceptibility.

In conclusion, the author expresses his gratitude to S. L. Ginzburg and E. F. Shender for discussions, to A. A. Abrikosov and A. A. Migdal for constant criticism and for the opportunity to become acquainted with the results of[9] before it was published, and also to Fowler and Zawadowski for sending their preprint.

APPENDIX I

We now show that the second term in (10b) cannot compensate the term proportional to $\epsilon(\zeta)$.

First of all, we note that the phase volume of an n-particle state is ζ^{2m} in order of magnitude, and the problem arises only when the inelastic amplitudes can compensate this small quantity, i.e., when for these amplitudes the estimate $\zeta^{-m}f(\zeta)$, where $f(\zeta)$ is bounded, holds; in particular, as $\zeta \to 0$ this estimate can decrease more slowly than any power of ζ. A strong singularity is impossible, since the contribution of Im A becomes infinite. Furthermore, each inelastic amplitude is an analytic function of the energies of all the particles participating in the process and of a number of linear combinations of these energies (cf.[14]), while in each of its arguments it has a cut along the real axis. The choice of branches of such an analytic function is fixed by the usual rules for causal amplitudes: 1) the imaginary parts of the fermion energies (combinations of an odd number of energies of electrons and holes) have the same sign as the energies themselves; 2) as a function of each of the boson energies (combinations of an even number of energies of fermions), the amplitude consists of two terms, in the first of which the energy has a positive, and in the second a negative imaginary part. For the Green's functions, these rules follow from the formulas available in[15]; in more complicated cases, they are easily derived by means of an expansion, analogous to that used in[14], in the intermediate states.

In passing from $E > E_F$ to $E < E_F$, apart from the rearrangement of the spin indices already taken into account, the signs of all the energies change (we neglect the change, small for $|\zeta| \ll E_F$, in the density of states). With the choice of branches indicated above, such a change in the signs is equivalent to a rotation of each of the arguments through an angle π in the positive direction. As a result of such a rotation, the imaginary parts of all the energies turn out to have signs opposite to the signs they must have in a causal amplitude, i.e., with the rotation we go over from a causal amplitude to a quantity that is the Hermitean conjugate of another causal amplitude. But on rotation through an angle π, the factor ζ^{-m} goes to $\zeta^{-m}(-1)^m$ and, consequently, cannot lead to a change of sign in the corresponding contribution to Im B. It remains to discuss the role of the non-power factor $f(\zeta)$. It is also an analytic function of its arguments and for each of them can be represented in the form of a Cauchy integral along the real axis. Elementary estimates show that such an integral, for small values of its argument x, has the form

$$C_0 + C_1\rho_1(|x|)\ln|x| + C_2\rho_2(|x|)x + i[C_3\rho_1(|x|)\varepsilon(x) + C_4\rho_2(|x|)],$$

where the C_i are functions of all the remaining arguments, and ρ_1 and ρ_2 are the odd and even parts of the discontinuity ρ of the function $f(\zeta)$ with respect to x: $\rho(x) = \rho_1(|x|)\epsilon(x) + \rho_2(|x|)$. It follows from this formula that the odd part in $f(\zeta)$ is small compared with the even, and therefore cannot contribute to the part of Im B of interest to us, namely, the part that does not fall off as $\zeta \to 0$. Generally speaking, it could be the case that the principal contribution to Im B from certain terms diverges and is finite only in the sum; then it would become necessary to take account of the odd corrections. But such divergences would inevitably lead to divergences in Im A, where such a cancellation is impossible.

APPENDIX II

The amplitude $\mathbf{S}\cdot\boldsymbol{\Gamma}_{22}^{(2)}$ is a matrix in spin space; to construct it we have at our disposal the spin vectors of an electron (σ_e) and of a hole (σ_h) and the impurity-

[5)] It is shown in [6] that substitution into (10) of the amplitude calculated by perturbation theory leads to a logarithmic infinity.

spin vector. Obviously, for $S = \frac{1}{2}$ the most general expression for this amplitude has the form

$$S\Gamma_{22}^{(2)} = S\sigma_e\gamma_1 + S\sigma_h\gamma_2 + S[\sigma_1 \times \sigma_2]\gamma_3, \tag{A.1}$$

in which the first two combinations of spins are unchanged on time reversal but the last changes sign. Therefore, the functions γ_1 and γ_2 must by t-even, and γ_3 t-odd. We now change the signs of the energies of all the particles taking part in the scattering. In accordance with what was said in Appendix I, we have, taking into account the assumed uniformity of the functions f_{22},

$$\begin{aligned} S\Gamma_{22}^{(2)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) &\to -e^{i\pi y} S\Gamma_{22}^{(2)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) \\ &= S\Gamma_{22}^{(2)+}(-\zeta_1, -\zeta_2, -\zeta_3, -\zeta_4), \end{aligned} \tag{A.2}$$

where Γ^+ differs from Γ by a change in the signs of the imaginary parts of all the energies. On the right and left of (A.2) are the scattering amplitudes of a particle and a hole. We now choose the energies as follows: $\zeta_1 = -\zeta_3 = \epsilon$, $\zeta_4 = -\zeta_2 = \eta$. Then (A.2) goes over to the equality

$$-e^{i\pi y} S\Gamma_{22}^{(2)}(\varepsilon, -\eta, -\varepsilon, \eta) = S\Gamma_{22}^{(2)+}(-\varepsilon, \eta, \varepsilon - \eta). \tag{A.3}$$

We now apply the time-reversal operation to the quantity on the right. Taking (A.1) and the t-parity properties of the functions γ_i into account, we obtain

$$\begin{aligned} -e^{i\pi y}\gamma_{1,2}(\varepsilon, -\eta, -\varepsilon, \eta) &= \gamma_{1,2}{}^{+}(\varepsilon, -\eta, -\varepsilon, \eta), \\ e^{i\pi y}\gamma_3(\varepsilon, -\eta, -\varepsilon, \eta) &= \gamma_3{}^{+}(\varepsilon, -\eta, -\varepsilon, \eta). \end{aligned} \tag{A.4}$$

The functions γ_i can be represented in the form $\gamma_i = \gamma_i^{(1)} + i\gamma_i^{(2)}$, where $\gamma_i^{(1)}$ is the part of the function γ_i defined by the principal values of the Cauchy integrals in all the variables (cf. Appendix I), and $\gamma_i^{(2)}$ are the imaginary parts of the energies in the corresponding denominators; therefore, $\gamma_i^{+} = \gamma_i^{(1)} - i\gamma_i^{(2)}$ and it follows from (A.4) that

$$\begin{aligned} e^{i\pi y}(\gamma_{1,2}^{(1)} + i\gamma_{1,2}^{(2)}) &= -\gamma_{1,2}^{(1)} + i\gamma_{1,2}^{(2)}, \\ e^{i\pi y}(\gamma_3^{(1)} + i\gamma_3^{(2)}) &= \gamma_3^{(1)} - i\gamma_3^{(2)}. \end{aligned} \tag{A.5}$$

These equalities are possible only if $e^{i\pi y} = \pm 1$; in this case, some of the $\gamma_i^{(1,2)}$ must be equal to zero. We note also that the expression for $S \cdot \Gamma_{22}^{(2)}$ obtained in[5] in first order of perturbation theory satisfies the conditions (A.5) for $y = 0$.

[1] H. Suhl and D. Wong, Physics 3, 17 (1967).

[2] S. V. Maleev, Zh. Eksp. Teor. Fiz. 51, 1940 (1966) [Sov. Phys.-JETP 24, 1300 (1967)].

[3] S. V. Maleev, Zh. Eksp. Teor. Fiz. 53, 1038 (1967) [Sov. Phys.-JETP 26, 620 (1968)].

[4] S. L. Ginzburg and S. V. Maleev, Zh. Eksp. Teor. Fiz. 55, 1483 (1968) [Sov. Phys.-JETP 28, 776 (1969)].

[5] S. V. Maleev, Zh. Eksp. Teor. Fiz. 58, 2031 (1970) [Sov. Phys.-JETP 31, 1094 (1970)].

[6] S. V. Maleev, Zh. Eksp. Teor. Fiz. 56, 619 (1969) [Sov. Phys.-JETP 29, 340 (1969)].

[7] M. Fowler and A. Zawadowski, Preprint.

[8] P. W. Anderson, G. Yuval, and D. R. Hamman, Phys. Rev. B1, 4464 (1970).

[9] A. A. Abrikosov and A. A. Migdal, J. Low Temp. Phys. 3, 519 (1970).

[10] S. L. Ginzburg and S. V. Maleev, Teor. Mat. Fiz. 3, 420 (1970) [Theor. Math. Phys. (USSR) 3, 616 (1970)].

[11] S. V. Maleev, Fiz. Metal. Metalloved. 29, 1121 (1970) [English translation to appear in Phys. Metals Metallogr. (USSR)].

[12] A. A. Migdal, Zh. Eksp. Teor. Fiz. 55, 1964 (1968) [Sov. Phys.-JETP 28, 1036 (1969)].

[13] A. M. Polyakov, Zh. Eksp. Teor. Fiz. 57, 2144 (1969) [Sov. Phys.-JETP 30, 1164 (1970)].

[14] S. V. Maleev, Teor. Mat. Fiz. 4, 86 (1970) [Theor. Math. Phys. (USSR) 4, 694 (1970)].

[15] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, M., 1962 (English translation published by Pergamon Press, Oxford, 1965).

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