NONLINEAR RADIATION OF SPIN WAVES IN FERRITES

G. A. MELKOV

T. G. Shevchenko Kiev State University

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The existence of singularities in the dependence of nonlinear ferrite susceptibility in parallel high-frequency and stationary magnetic fields on stationary magnetic field strength is observed experimentally. These singularities are manifest in a sharp increase or decrease of susceptibility for certain values of the stationary field, which may be different for different orientations of magnetization relative to the ferrite crystallographic axes. In contrast to the usual opinion, it should be recognized that a decisive role in nonlinear relaxation of parametrically excited spin waves is played by magnon splitting processes and by interaction between magnons and spin waves moving along the magnetization direction. Three-magnon coalescence processes exist only in a restricted stationary magnetic field range (of the order of 20 Oe). The nature of nonlinear relaxation depends essentially on the orientation of ferrite magnetization with respect to the ferrite axes.

INTRODUCTION

A powerful microwave signal produces a threshold effect of parametric spin-wave excitation in ferrites, with a frequency that is a multiple of half the pump frequency[1-3]. The theory of the threshold of parametric excitation of spin waves has by now been sufficiently fully developed and makes it possible to obtain experimentally very valuable information on ferromagnetic crystals. For example, experiments on parallel pumping of spin-wave instability[2] make it possible to determine the line widths of spin waves with different wave vectors ΔH_k, their acoustic Q, etc.

Great progress has also been made recently in the development of the theory describing the state of the ferrite beyond the threshold of parametric excitation of spin waves[4-6]. This theory, however, encounters difficulties due to the large number of degrees of freedom of the spin system of the parametrically regenerated ferrite. Under these conditions, it is very important to choose the correct initial premises of the theory, something that can be done by organizing special experiments.

This paper reports an experimental investigation of the dependence of the imaginary part of the nonlinear susceptibility on the constant magnetic field H_0, for parallel pumping of the spin wave instability[2]. The present experiments differ from those described in the lierature in that $\chi''(H_0)$ was measured at $P/P_{H_0} = \text{const}$, and not at $P/P_{H_c} = \text{const}$ as earlier[7,8]. Here P is the microwave power acting on the sample, P_{H_0} the threshold power of the spin-wave instability at the constant magnetic field H_0 at which the measurement of χ'' is performed, and P_{H_c} the threshold at the constant magnetic field H_c at which the spin-wave instability threshold is minimal.

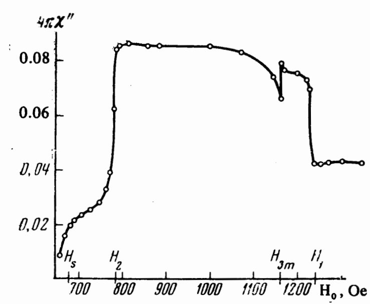

FIG. 1. Dependence of the imaginary part of the nonlinear longitudinal susceptibility of a single-crystal ferrite sphere of 2.11 mm diameter on the external constant magnetic field directed along the difficult magnetization axis. The pump exceeds the threshold of the spinwave instability in the ratio $P/P_{H_0} = 1$ dB.

EXPERIMENT

1. **Experimental setup.** The investigations were performed on 10 single-crystal spheres of yttrium iron garnet (YIG) with diameters from 1.5 to 3.5 mm and with spin-wave resonance line width (at $H_0 = H_c$) $\Delta H_k \approx 0.3$ Oe. The samples were placed in the center of a rectangular reflecting resonator operating in the H_{012} mode. The intrinsic resonator Q was 4000. The loaded Q at a microwave power below the threshold spin-wave instability power was 1500, i.e., the coupling of the resonator with the waveguide exceeded the critical value. The nonlinear susceptibility χ'' was determined by measuring the loaded Q of the resonator at powers above threshold, using the standing-wave-coefficient method[9]. The error in the determination of the absolute value of χ'' did not exceed 10%, and the relative error was less than 2%. Such a large difference between the absolute and relative errors is due to the inaccuracy of the absolute measurements of the microwave power and of the ferrite and resonator parameters (magnetization, volume, Q), which greatly increases the error of the absolute measurements.

The resonator was placed in a permanent magnet that could be adjusted both mechanically and electrically. The magnetic field was measured with a nuclear magnetometer accurate to 1 Oe. The measurements of χ'' were carried out using a pulsed microwave generator operating at 9370 MHz, with a pulse duration 200 μsec and a pulse repetition frequency 1–50 Hz.

2. **Experimental results.** Figure 1 shows the dependence of χ'' on the constant magnetic field, $\chi'' = f(H_0)$,

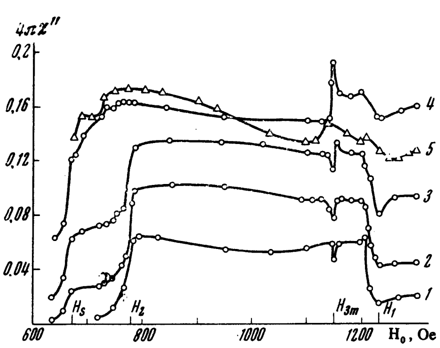

FIG. 2. Dependence of the imaginary part of the nonlinear longitudinal susceptibility of a single-crystal ferrite sphere of 2.2 mm diameter on the constant magnetic field directed along the difficult magnetization axis. The numbers at the curves correspond to the following excesses of the pump over the threshold of the spin-wave instability P/P_{H_0}: 1–0.5; 2–1; 3–2; 4–5; 5–10 dB.

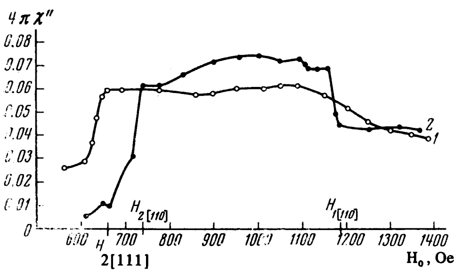

FIG. 3. Dependence of the imaginary part of the nonlinear longitudinal susceptibility of a single-crystal YIG sphere of 2.11 mm diameter on the external constant magnetic field directed along the easy axis (curve 1) or the intermediate axis of crystal magnetization (curve 2). $P/P_{H_0} = 1$ dB.

at $P/P_{H_0} = 1$ dB for a YIG sphere of 2.11 mm diameter. The sample was magnetized along the [100] axis, for in this case the threshold of the low-frequency automodulation of the magnetization is maximal[5], and its amplitude is minimal, thereby increasing the accuracy of the measurements of χ''. It is seen from Fig. 1 that a strong change of χ'' takes place at constant magnetic fields 1238, 1160, and 785 Oe (designated in the figure H_1, H_{3m}, and H_2, respectively). Plots similar to that of Fig. 1 were obtained for all the investigated samples. The following characteristic regions can be separated on the $\chi'' = f(H_0)$ curve:

1) $H_1 < H_0 < H_c$. χ'' is approximately constant in the entire region.

2) When the field decreases below H_1 (which lies in the range 1225 Oe $< H_1 <$ 1238 Oe for all samples), a sharp increase takes place in the nonlinear susceptibility in the field interval 20–30 Oe.

3) Near the field H_{3m} (for different samples 1148 Oe $< H_{3m} <$ 1160 Oe, with 75 Oe $< H_1 - H_{3m} <$ 80 Oe), in the field range 15–25 Oe, a resonant decrease of χ'' takes place, with the minimum at H_{3m}.

4) At fields below H_2 (775 Oe $< H_2 <$ 758 Oe), χ'' decreases by more than a factor of 2 when the field changes by 10 Oe.

5) In fields weaker than the saturation field H_s, further decrease of χ'' takes place.

The evolution of the $\chi'' = f(H_0)$ curves as a function of the excess of pump over threshold power P/P_{H_0} for a sphere of 2.2 mm diameter is shown in Fig. 2. At $H_1 < H_0 < H_c$, χ'' is constant only if $P/P_{H_0} \leq 2$ dB. At large excesses, χ'' increases with increasing field. The relative change of susceptibility near H_1 decreases with increasing P/P_{H_0}. Near H_{3m}, at $P/P_{H_0} \geq 5$ dB, there is a sharp maximum in place of the minimum. The change of susceptibility at H_2 also occurs up to definite values of P/P_{H_0} (ranging from 5 to 10 dB for different samples). With increasing P/P_{H_0}, the value of H_2 shifts towards weaker fields. The decrease of susceptibility at H_s takes place for all values of P/P_{H_0}. It is also seen from Fig. 2 that saturation of χ'' occurs as a function of H_0 for different values of P/P_{H_0}. Whereas at $H_0 = 1200$ Oe the saturation occurs at $P/P_{H_0} \approx 5$ dB, at $H_0 = 750$ Oe χ'' is far from saturation even at $P/P_{H_0} = 10$ dB. The $\chi'' = f(H_0)$ curves for different samples are approximately similar to those of Fig. 2, but the absolute value of the susceptibility changes quite appreciably. For example, the maximum value of the susceptibility χ''_{max} lies in the range $0.18 < \chi''_{max} < 0.24$, and the field at which χ''_{max} is obtained differs for different samples and depends on their diameter. Starting with a diameter 2.8 mm and above, χ''_{max} is located near H_c, whereas for smaller samples χ''_{max} lies near H_{3m}.

The shape of the $\chi'' = f(H_0)$ curve depends essentially on the orientation of $\mathbf{H}_0$ relative to the crystallographic axes of the ferrite sphere. Typical plots for a ferrite sphere of 2.11 mm diameter magnetized along the [110] and [111] axes are shown in Fig. 3. For magnetization along the intermediate axes [110], the shape of the $\chi'' = f(H_0)$ curve resembles analogous plots for magnetization along the [100] axis. The field $H_{1[110]}$ (see Fig. 3) is 55–60 Oe lower than H_1, and the field $H_{2[110]}$ is 38–42 Oe lower than H_2 for different ferrites. The rise of the susceptibility at the field $H_{1[110]}$ is always smaller than for H_1 with magnetization along the difficult axis. In fields $H_2 < H_0 < H_1$, the $\chi'' = f(H_0)$ plot has a nonuniform character and there is no minimum analogous to that appearing at $H_0 = H_{3m}$ in the case of magnetization along [100]. For magnetization along the easy axis [111], there is no analog of the field H_1 at all. The field $H_{2[111]}$ ranges from 645 to 660 Oe for different samples.

When the sample is rotated along the [110] axis, a smooth transition takes place between the curves shown in Figs. 1 and 3.

To investigate the causes of the susceptibility anomalies in fields H_1, H_2, and H_{3m}, we plotted the dependence of the threshold excitation power of the spin-wave instability on the constant magnetic field. This plot has made it possible in accordance with the well-known formulas[2] to construct Fig. 4, which shows ΔH_k as a function of the constant magnetic field H_0 for three orientations of the single-crystal sphere of 2.11 mm diameter. From Figs. 1, 3, and 4 it is seen that the kink of the $\chi'' = f(H_0)$ curves coincides, within ±2 Oe, with the kink in the plot of ΔH_k against H_0, and lies approximately 100 Oe above the saturation field H_s. It should be noted here that plots similar to those shown in Fig. 4 were obtained earlier, but it was assumed, probably because of the low accuracy with which the constant magnetic fields were measured, that the rapid increase of

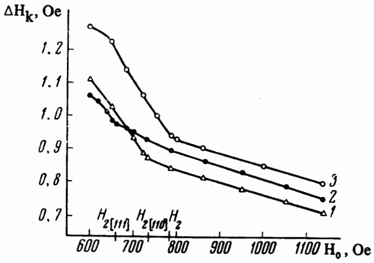

FIG. 4. Spin-wave resonance line width, measured by the parallel-pumping method, vs. the external constant magnetic field directed along the intermediate (1), easy (2) or difficult (3) magnetization axis of the ferrite.

ΔH_k occurs in fields weaker than the saturation field.

During the course of the experiments we also investigated the low-frequency automodulation of the magnetization. The maximum of the automodulation excitation threshold following orientation of the magnetization along the difficult axis takes place at $H_0 \approx H_1$. This result is in numerical agreement with[5], where, as can be determined from the plot, the maximum lies at $H_0 \approx 1250$ Oe. As indicated earlier, the accuracy of the measurements of χ'' is higher in the absence of magnetization automodulation. No other effect of automodulation on the $\chi'' = f(H_0)$ dependence was observed. For example, curves 1 and 2 of Fig. 2 were obtained in the absence of automodulation. Automodulation of the magnetization was observed on curve 3 at fields $H_0 < 850$ Oe, but there were no anomalies on the $\chi'' = f(H_0)$ curve. In fields $H_0 \approx H_1$, automodulation arises at $P/P_{H_0} \gtrsim 4.5$ dB, but again no essential changes were observed in the $\chi'' = f(H_0)$ plot with increasing power.

Measurements with the sample magnetization oriented along the [111] and [110] axes were almost always made in the presence of strong automodulation, the amplitude and frequency of which were sometimes strongly dependent on the field. The $\chi'' = f(H_0)$ curve had no noticeable anomalies at such sharp-dependence points.

DISCUSSION OF RESULTS

Let us discuss first the data pertaining to the orientation of H_0 along the [100] axis of the ferrite.

From an examination of the plots in Figs. 1, 2, and 4 it is perfectly obvious that significant changes in the system of parametrically-excited spin waves occur in fields H_1, H_{3m}, and H_2.

1) The minimum of the susceptibility at the field H_{3m} can obviously be attributed to the three-magnon coalescence process considered by Gottlieb and Suhl[4]. Indeed, it turned out that for all the measured samples, with allowance for the scatter of the crystallographic-anisotropy field and of the magnetization, the field H_{3m} coincides within ±3 Oe with the three magnon-coalescence field calculated in accordance with the exact formula[7] (but not in accordance with the approximate formula of[4]). However, unlike in the Gottlieb and Suhl model, it is necessary to assume that the parametric spin waves having a polar angle $\theta_k = \pi/2$ have a perfectly defined azimuthal angle φ_k, and are not distributed with equal probability in all directions in the (100) plane. In such a case, the vector condition of the three-magnon-coalescence process[4] will be satisfied only near the field H_{3m}, and the width of the peak of χ'' at $H_0 = H_{3m}$ is determined by the scatter of the angles θ_k and φ_k for the different parametric spin waves. If θ_k is strictly equal to $\pi/2$, then the probability of the three-magnon-coalescence process is equal to zero, since the corresponding term of the Hamiltonian vanishes[5,10], and consequently the depth of the dip at χ'' at H_{3m} indicates to some degree the magnitude of the deviation of θ_k from $\pi/2$. The fact that the parametrically excited spin waves have a definite angle φ_k can be attributed to the anisotropy of the spin-wave parameters in the (100) plane. It is known, for example, that the spin waves propagate mainly along the intermediate axis[11].

2) The drop of the susceptibility at $H_0 = H_2$ is explained with the aid of Fig. 4, which shows clearly the acceleration of the growth of ΔH_k in magnetic fields $H_0 \le H_2$. According to Sparks[10], the relaxation processes of spin waves with $\theta_k = \pi/2$ at a frequency $\omega_k = 2\pi \cdot 4685$ MHz are determined mainly by three-magnon splitting and coalescence processes.

Processes of coalescence (of a parametric magnon with a nonparametric one) exist in the entire range of magnetic fields. The region of existence of three-magnon splitting processes was determined in[8] — these processes exist only in fields $H_0 \lesssim H_S + 100$ Oe. Consequently, it is perfectly possible that the drop of χ'' at $H_0 < H_2$ is due to the occurrence of additional relaxation of the spin waves, namely three-magnon splitting. It is known that the probability of the splitting process increases strongly with increasing spin-wave amplitude.

3) To explain the causes of the jump of the susceptibility of H_1, we calculated the regions of existence of all possible three- and four-magnon processes for different spin waves. It turned out that the only relaxation process having an existence-region boundary close to H_1 is three-magnon splitting for spin waves with $\theta_k = 0$ and frequency $\omega_k = 2\pi \cdot 4685$ MHz. Indeed, the three-magnon splitting process takes place under the conditio $\omega_k \ge 2\omega_k/2$, from which we get for spin waves with $\theta_k = 0$ an existence region for the splitting process

$$0 < \gamma(H_0 - H_s) \leqslant {}^1/_3\omega_k. \qquad (1)$$

For the frequency $\omega_k = 2\pi \cdot 4685$ MHz we have $H_0 - H_S \le 557$ Oe. The upper limit of this region coincides, within ±5 Oe, with the experimental values of H_1 for all the investigated samples.

Condition (1) indicates that for our experiment, at $H_0 - H_S = 557$ Oe, the line width of the spin-wave resonance of waves with $\theta_k = 0$, henceforth designated $\Delta H_k'$, increases, since in these fields the process of three-magnon splitting is added to the existing relaxation processes. Consequently, for $\Delta H_k'$, in principle, the curves obtained are analogous to those shown in Fig. 4, but the point of inflection for a sample magnetized along the difficult axis lies near H_1.

One of the possible mechanisms of nonlinear relaxation with participation of spin waves with $\theta_k = 0$ is as follows: first, primary spin waves with $\theta_k \approx \pi/2$ and frequency ω_k equal to half the pump frequency are produced. This is followed by a process perfectly analogous to a phenomenon described by Suhl[12], namely saturation of the main resonance. The primary spin waves are coupled via two and four-magnon interactions with the secondary spin waves having $\theta_k = 0$, as a result of which the amplitude of the primary spin waves becomes limited at the expense of amplification of the secondary wave. It is then perfectly obvious that the amplitude of the primary waves (and the quantity χ'' associated with it) will increase with increasing losses of the secondary waves, since this leads, in accordance with[12], to a decrease of the coupling between the primary and secondary spin waves. Thus, within the framework of the given mechanism, the increase of χ'' in fields $H_0 < H_1$ is perfectly natural. The validity of the proposed mechanism of nonlinear relaxation is confirmed, furthermore, by the following two circumstances:

a) The influence of two-magnon scattering on the surface inhomogeneities for spin waves with $k \gg 0$ is quite small, since it was not observed in special experiments. However, two-magnon scattering of spin waves by volume defects (microinhomogeneities, dislocations, etc.) apparently does exist. Thus, for example, we measured ΔH_k of samples subjected to a quasihydrostatic pressure 3×10^8 g/cm^2 (at which the density of the dislocations and microdefects in the ferrite was altered). It turned out that by varying the time of action of the pressure it is possible to vary ΔH_k on the average by 30%.

b) Using the method described in[12] and the results of[13], it is possible to obtain approximately the maximum value χ''_{max}. It is quite difficult to find the exact value of χ'', and this problem is not considered here. If we denote by a_k the amplitude of the primary spin wave and by ψ_k its time phase measured relative to the pump phase, then the maximum amplitude of the primary spin waves is determined from the condition

$$\left| \sum_k a_k a_k{}^* e^{2i\psi_k} \right| = \frac{\gamma \Delta H_k'}{T_{kk'}}, \tag{2}$$

where $T_{kk'}$ is a numerical coefficient in the Hamiltonian that describes the parametric interaction of a pair of waves with wave vectors $\mathbf{k}$ and $-\mathbf{k}$ with a pair of waves with vectors $\mathbf{k}'$ and $-\mathbf{k}'$. An exact expression for $T_{kk'}$ can be found, for example, in[14]. For our purposes, it can be assumed that $T_{kk'} \approx \omega_M = 4\pi\gamma M_0$. The phase ψ_k of the parametrically-excited spin waves is strictly connected with the pump phase, with ψ_k dependent on the spin-wave frequency, on the spin-wave losses, etc.

The determination of ψ_k as a function of the wave vector $\mathbf{k}$ is the most laborious task of the theory. If it is assumed that the distribution of the spin waves with respect to the different ψ_k is Lorentzian (since we are dealing with a stimulated process, such a distribution is preferable to all others), we can rewrite (2) with the aid of the statistical methods in the form

$$A \sum a_k a_k{}^* = \frac{\gamma \Delta H_k'}{\omega_M}. \tag{3}$$

Here A is a constant that depends on the distribution parameter. For example, if it is assumed that the half-width of the Lorentzian is $\pi/4$ (this means that more spin waves have phases in the range $\psi_0 - \pi/4 < \psi < \psi_0 + \pi/4$, where ψ_0 is the phase of the waves most strongly coupled with the pump), then, as shown by numerical calculations, A = 0.7. If the phases of all the spin waves are equal to ψ_0, then A = 1. The real situation obviously lies between the first case and the second.

Physically, the presence of A in (3) is due to the fact that different primary waves act on a secondary wave with different phases, as a result of which the integral effect of excitation of the secondary wave will be the smaller the larger the phase difference between the primary waves.

With the aid of (3) and the results of[13], we can obtain the following expression for χ''_{max}:

$$\chi''_{max} = \Delta H_k \Delta H_k' / A 8\pi h^2, \tag{4}$$

where h is the amplitude of the magnetic microwave field at which χ'' is maximal. It is known from experiment that $h \approx 1$ Oe; in such a case it is easily seen that $\chi''_{max} \sim 10^{-2}$, which corresponds to the experimental results.

4) In the case when $\mathbf{H}_0$ is oriented along the [110] axis of the crystal, the $\chi'' = f(H_0)$ curve corresponds in general outline to the analogous curve with $\mathbf{H}_0$ oriented along [100]. Then $H_2 - H_{2[110]} \approx 40$ Oe, this being due to the shift of the saturation field H_s by an amount equal to half the crystallographic-anisotropy field[15]. However, the difference $H_1 - H_{1[110]} \approx 60$ Oe is smaller by more than a factor of 1.5 than the calculated value (~ 100 Oe), possibly because of the need for taking into account the dependence of ω_k on φ_k when $\mathbf{H}_0$ is oriented along the [110] axis of the ferrite. When $\mathbf{H}_0$ is oriented along the easy axis, as indicated above, there is no analog of the field H_1. This may be due to the absence of anisotropy in the (111) plane, by virtue of which waves with all possible φ_k are parametrically excited. Following the terminology adopted in[6], we have here a stochastic regime of parametric excitation of spin waves—a regime in the form of a continuous distribution of pairs of spin waves over the wave vector, and these pairs have random individual phases. When $\mathbf{H}_0$ is oriented along the difficult axis, it can be assumed that beyond the instability threshold there exist two groups of spin waves—primary with $\theta_k \approx \pi/2$ and a definite azimuthal angle φ_k, and secondary with $\theta_k \approx 0$. Excitation of waves with $0 < \theta_k < \pi/2$ is hindered by the simultaneous action exerted on them by the pump field and by the primary spin waves with $\theta_k \approx \pi/2$, acting in antiphase[6]. Contributing to the production of the group of waves with $\theta_k \approx 0$ is the fact that the pump field does not act on them directly[2], so that the influence of the waves with $\theta_k \approx \pi/2$ is not offset by the pump.

When $\mathbf{H}_0$ is oriented along the [111] axis, the primary spin waves no longer act as a unit, since they have a whole set of φ_k, as a result of which they cannot offset the action of the pump field on the spin waves with $\theta_k \neq \pi/2$. Consequently, in this case, in addition to the waves with $\theta_k \approx \pi/2$ and the waves with $\theta_k \approx 0$ excited by the primary spin waves, there can exist any spin wave having $0 < \theta_k < \pi/2$. Thus, in this case nonlinear dissipation may be connected with any spin wave having $0 \leq \theta_k \leq \pi/2$. Therefore the waves with $\theta_k = 0$ no longer play a special role against the background of the entire set of parametrically-excited spin waves, and an increase in the damping of the waves with $\theta_k = 0$ should no longer noticeably influence the nonlinear susceptibility.

It is seen from Fig. 2 that singularities on the $\chi'' = f(H_0)$ curve at a field H_1 are also absent when $\mathbf{H}_0$ is oriented along the difficult axis, if P/P_{H_0} exceeds a certain value (usually on the order of 5 dB). Just as for the case considered above, it can be assumed here, too, that at such values of P/P_{H_0} there are excited spin waves with $0 \leq \theta_k \leq \pi/2$, since it is perfectly understandable that the primary spin waves can offset the action of the pump only within definite limits.

CONCLUSIONS

The experimental results show that the character of the nonlinear relaxation of the spin waves in ferrites depends essentially on the orientation of $\mathbf{H}_0$ relative to the crystallographic axes. If $\mathbf{H}_0$ is directed along the [100] axis, then there exist in the sample, past the stability threshold, two groups of spin waves—primary with $\theta_k \approx \pi/2$, which are excited directly by the pump, and secondary with $\theta_k \approx 0$.

The presence of waves with $\theta_k \approx 0$ is attributed to the existence of a process analogous to saturation of the main resonance, a phenomenon described by Suhl[12]. The gist of this phenomenon is that the primary spin waves act, via four-magnon interaction, as a pump for the secondary waves with $\theta_k = 0$, and at a certain threshold amplitude of the primary waves this can lead to parametric generation of waves with $\theta_k = 0$.

Even if the amplitude of the primary waves is below the threshold, the amplitude of the secondary waves is appreciable, since parametric amplification of the waves with $\theta_k = 0$ at the expense of the primary waves is possible, the "signal" for such an amplifier being also primary waves transformed via two-magnon scattering into spin waves with $\theta_k = 0$.

If $\mathbf{H}_0$ is directed along the [111] axis, then by virtue of the isotropy of the (111) plane, there exists in the ferrite an entire set of spin waves with $0 \leq \theta_k \leq \pi/2$. When $P/P_{H_0} \gtrsim 5$ dB, spin waves with arbitrary θ_k are excited at all orientations.

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