ELECTROPRODUCTION OF PAIRS OF PARTICLES AT HIGH ENERGIES

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The pair production of particles with spins 0 and ½ in the collision of two charged particles at high energies is investigated. Terms containing the third, second, and in a number of important cases even the first power of the logarithm, are found in the cross section for this process, which increases logarithmically with the energy. Together with the total cross section, a number of differential cross sections are derived, and with the aid of these results an analysis of the angular and energy distributions of the final particles is carried out.

1. The cross section for pair production associated with the collision of charged particles (electroproduction) increases logarithmically with energy, and therefore this process turns out to be extremely important at high energies. In particular, the electroproduction of e^+e^- pairs at large angles was recently observed in experiments using electron-positron colliding beams with energies ϵ = 500 MeV at Novosibirsk.[1,2] At high energies the leading terms in the cross section for this process—that is, terms of the type $\ln^2(\epsilon/m)$ ("triple-logarithmic")—can be derived by using the method of equivalent photons, which was first done in the well-known work by Landau and Lifshitz.[3] In order to derive expressions for the cross sections which are valid at relatively small energies, it is also necessary to calculate the terms containing smaller powers of the logarithms. In the present article the cross section for electroproduction is found to the approximation involving "doubly-logarithmic" terms (i.e., terms proportional to the second power of the logarithm), and in a number of important cases the cross section is obtained correct to terms of the order of a single power of the logarithm. As will be clear from what follows, over the entire range of practically accessible energies it is impossible to use the cross section obtained by taking account of only the leading logarithmic terms.

To the lowest order of perturbation theory for nonidentical particles the process is represented by the three "block" diagrams of the type shown in Fig. 1, where the momenta, masses, and spins of the particles are indicated. The meaning of the double circle is shown in Fig. 2. Here we shall only consider processes involving unpolarized particles. After carrying out the operations of averaging and summing over the spin states, the contribution from diagram I takes the form

$$d\sigma_1 = \frac{e^8(-1)^{2s_f}}{(2\pi)^8} J^{(s_f)}_{\rho\sigma}(\mu, -p_6, p_5) J^{(s_1)}_{\mu\nu}(m_1, p_1, p_3) \times \frac{K^{(s_2)\mu\nu\rho\sigma}(m_2, p_2, p_4, q_1, \Delta)}{4(2s_1+1)(2s_2+1)(\nu^2 - m_1^2 m_2^2)^{1/2}\Delta_1^4\Delta^4} d\rho, \tag{1}$$

where s_1, s_2 denote the spins of the initial particles and s_f denotes the spin of the created particles ($s_{1,2,f}$ = 0 or ½),

$$d\rho = \delta(p_1 + p_2 - p_3 - p_4 - p_5 - p_6)\frac{d^3p_3}{2\varepsilon_3}\frac{d^3p_4}{2\varepsilon_4}\frac{d^3p_5}{2\varepsilon_5}\frac{d^3p_6}{2\varepsilon_6}, \tag{2}$$

$$\Delta_{1,2}^2 = -q_{1,2}^2, \quad q_{1,2} = p_{1,2} - p_{3,4}, \quad \Delta = p_5 + p_6, \quad \nu \equiv (p_1 p_2).$$

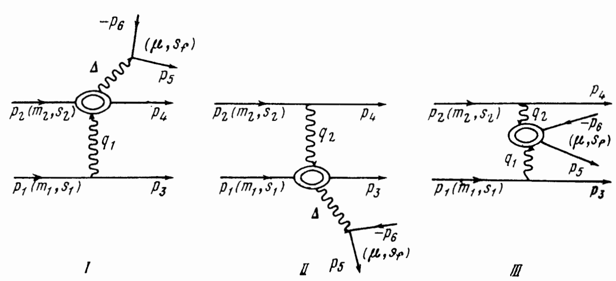

FIG. 1

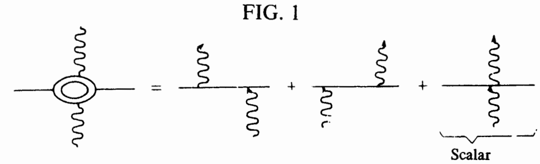

FIG. 2

The current tensors have the form

$$J^{(0)}_{\mu\nu}(m_1, p_1, p_3) = (p_1 + p_3)_\mu (p_1 + p_3)_\nu, \tag{3}$$

$$J^{(1/2)}_{\mu\nu}(m_1, p_1, p_3) = \mathrm{Sp}[(m_1 + \hat{p}_1)\gamma_\mu(m_1 + \hat{p}_3)\gamma_\nu],$$

and the Compton tensors have the form

$$K^{(0)}_{\mu\nu\rho\sigma}(m_2, p_2, p_4, q_1, \Delta) = L^{(0)}_{\mu\rho} L^{(0)}_{\nu\sigma}, \tag{4}$$

where

$$K^{(1/2)}_{\mu\nu\rho\sigma}(m_2, p_2, p_4, q_1, \Delta) = \mathrm{Sp}[(m_2 + \hat{p}_4) L^{(1/2)}_{\mu\rho}(m_2 + \hat{p}_2)\bar{L}^{(1/2)}_{\nu\sigma}],$$

$$L^{(0)}_{\mu\rho} = \frac{(2p_2 + q_1)_\mu (2p_4 + \Delta)_\rho}{(p_2 + q_1)^2 - m_2^2} + \frac{(2p_4 - q_1)_\mu (2p_2 - \Delta)_\rho}{(p_2 - \Delta)^2 - m_2^2} - 2g_{\mu\rho},$$

$$L^{(1/2)}_{\mu\rho} = \gamma_\mu \frac{p_2 - \Delta + m_2}{(p_2 - \Delta)^2 - m_2^2}\gamma_\rho + \gamma_\rho \frac{\hat{p}_2 + \hat{q}_1 + m_2}{(p_2 + q_1)^2 - m_2^2}\gamma_\mu,$$

$$\bar{L}^{1/2}_{\mu\nu} = \gamma^0 L^{(1/2)+}_{\mu\nu}\gamma^0.$$

The contribution from diagram II is obtained from Eq. (1) by making the substitutions $m_1 \longleftrightarrow m_2$, $s_1 \longleftrightarrow s_2$, $p_{1,3} \longleftrightarrow p_{2,4}$. In terms of this notation the contribution from diagram III is given by

$$d\sigma_{\mathrm{III}} = \frac{(-1)^{2s_f}e^8}{(2\pi)^8} J^{(s_1)}_{\mu\nu}(m_1, p_1, p_3) J^{(s_2)}_{\rho\sigma}(m_2, p_2, p_4) \times \frac{K^{(s_f)\mu\nu\rho\sigma}(\mu, -p_6, p_5, q_1, -q_2)}{4(2s_1+1)(2s_2+1)(\nu^2 - m_1^2 m_2^2)^{1/2}\Delta_1^4\Delta_2^4} d\rho. \tag{5}$$

We note that in diagram III the pair is created in the state with positive C-parity, but in diagrams I and II it is created in the state with negative C-parity.

Therefore, the interference between the contributions from diagrams I or II and from diagram III changes sign upon making the substitution $p_5 \longleftrightarrow p_6$. Because of this the contribution of this interference to the total cross section σ and to the cross section $d\sigma/d\Delta^2$ vanishes.

In the cross section $d\sigma_I$ given by Eq. (1) it is convenient to isolate the factor equal to the cross section for bremsstrahlung of a virtual photon with momentum Δ during the collision of charged particles:

$$d\sigma_{\rho\sigma}^{(\gamma)} = \frac{e^6}{(2\pi)^5} \frac{J_{\mu\nu}^{(s_1)}(m_1, p_1, p_3)\, K^{(s_2)\mu\nu\rho\sigma}(m_2, p_2, p_4, q_1, \Delta)}{4(2s_1+1)(2s_2+1)(\nu^2 - m_1^2 m_2^2)^{1/2}\Delta_1^4} \times \delta(p_1 + p_2 - p_3 - p_4 - \Delta)\frac{d^3p_3}{2\varepsilon_3}\frac{d^3p_4}{2\varepsilon_4}\frac{d^3\Delta}{2\Delta_0}. \tag{6}$$

It is convenient to carry out the integration over the momenta of the created pair for a given 4-momentum Δ_μ in tensor form (compare with[4]) with gauge invariance taken into consideration:

$$\frac{(-1)^{2s_f}e^2}{(2\pi)^3}\int J_{\mu\nu}^{(s_f)}(\mu, -p_6, p_5)\,\delta(\Delta - p_5 - p_6)\frac{d^3p_5}{2\varepsilon_5}\frac{d^3p_6}{2\varepsilon_6} = -f^{(s_f)}(\Delta^2)\left(g_{\mu\nu} - \frac{\Delta_\mu\Delta_\nu}{\Delta^2}\right); \tag{7}$$

here

$$f^{(0)}(\Delta^2) = \frac{\alpha}{12\pi}\Delta^2\beta^3, \qquad f^{(1/2)}(\Delta^2) = \frac{\alpha}{3\pi}(\Delta^2 + 2\mu^2)\beta \tag{8}$$

and $\beta = \sqrt{(\Delta^2 - 4\mu^2)/\Delta^2}$. As a result the cross section $d\sigma_I$ takes the form

$$d\sigma_I = -f^{(s_f)}(\Delta^2)\,\Delta^{-4}\,d\Delta^2\,d\sigma_{\rho\sigma}^{(\gamma)}g^{\rho\sigma}. \tag{9}$$

In connection with the calculation of the cross section $d\sigma_{III}$ one can perform the integration over the final states of the created pair just like in Sec. 4 of[5], using a different grouping of the terms such that upon integration of the cross section $d\sigma_{III}$ terms do not appear which increase anomalously with $\nu = (p_1p_2)$. As a result we obtain

$$\begin{aligned} d\sigma_{III} = {} & \frac{e^8}{(2\pi)^8}\frac{1}{(\nu^2 - m_1^2m_2^2)^{1/2}\Delta_1^4\Delta_2^4 D}\Big\{c_1^{(s_f)}(\Delta_1^2 b_1 - 2m_1^2\chi)(\Delta_2^2\tilde{b}_1 - 2m_2^2\chi) \\ & + (c_2^{(s_f)} + c_3^{(s_f)})\,D\left[4\left(\nu - \frac{\eta\eta'}{\chi}\right)^2 + s_1s_2\Delta_1^2\Delta_2^2\left(2 + \frac{\Delta_1^2\Delta_2^2}{\chi^2}\right)\right. \\ & \left. - 2s_1\Delta_1^2\left(m_2^2 + \frac{\Delta_2^2\eta'}{\chi} - \frac{\Delta_2^2\eta'^2}{\chi^2}\right) - 2s_2\Delta_2^2\left(m_1^2 + \frac{\Delta_1^2\eta}{\chi} - \frac{\Delta_1^2\eta^2}{\chi^2}\right)\right] \\ & + c_4^{(s_f)}\tilde{b}_2(\Delta_1^2 b_1 - 2m_1^2\chi) + c_5^{(s_f)}b_2(\Delta_2^2\tilde{b}_1 - 2m_2^2\chi) + \\ & + (c_6^{(s_f)} + c_7^{(s_f)})\,\Delta_1^2\Delta_2^2\left[\left(\nu - \frac{\eta\eta'}{\chi}\right)\left(2(\eta + \eta') - \chi - 4\frac{\eta\eta'}{\chi}\right)\right. \\ & - s_1s_2\Delta_1^2\Delta_2^2\left(1 - \frac{\Delta_1^2\Delta_2^2}{\chi^2}\right) + s_1\Delta_1^2\Delta_2^2\left(\frac{1}{2} - \frac{2\eta'}{\chi} + \frac{2\eta'^2}{\chi^2}\right) \\ & \left. + s_2\Delta_1^2\Delta_2^2\left(\frac{1}{2} - \frac{2\eta}{\chi} + \frac{2\eta^2}{\chi^2}\right)\right] + c_8^{(s_f)}b_2\tilde{b}_2\Big\}\frac{d^3p_3}{2\varepsilon_3}\frac{d^3p_4}{2\varepsilon_4}, \end{aligned} \tag{10}$$

where

$$\eta \equiv (p_1q_2), \quad \eta' \equiv (p_2q_1), \quad \chi \equiv (q_1q_2), \quad D = \chi^2 - \Delta_1^2\Delta_2^2,$$

$$b_1 = 2\eta\left(\frac{\eta}{\chi} - 1\right) + s_1\chi\left(2 + \frac{\Delta_1^2\Delta_2^2}{\chi^2}\right), \quad \tilde{b}_1 = b_1(\eta \to \eta', s_1 \to s_2),$$

$$b_2 = \Delta_1^2[2m_1^2 - (3s_1 - 1/2)\Delta_1^2], \quad \tilde{b}_2 = b_2(\Delta_1^2 \to \Delta_2^2,\ m_1^2 \to m_2^2,\ s_1 \to s_2)$$

and the coefficients have the form

$$\begin{aligned} c_1 = {} & {}^1/_8 D^{-1}[f_1(3\chi^2 - 2\Delta_1^2\Delta_2^2) - (f_2 + f_3)(\chi^2 - 2\Delta_1^2\Delta_2^2) - (f_5\Delta_1^2 + f_6\Delta_2^2) \\ & \times (7\chi^2 - 2\Delta_1^2\Delta_2^2) + 2(f_6 + f_7)\chi(\chi^2 - 6\Delta_1^2\Delta_2^2) + \\ & + f_8(39\Delta_1^2\Delta_2^2\chi^2 - 2\chi^4 - 2\Delta_1^4\Delta_2^4)], \\ c_2 = {} & {}^1/_8[-f_1 + 3f_2 - f_3 + f_4\Delta_2^2 + f_5\Delta_1^2 - 6f_6\chi + 2f_7\chi + f_8(2\chi^2 - \Delta_1^2\Delta_2^2)], \\ c_3 = {} & {}^1/_8[-f_1 - f_2 + 3f_3 + f_4\Delta_2^2 + f_5\Delta_1^2 + 2f_6\chi - 6f_7\chi + f_8(2\chi^2 - \Delta_1^2\Delta_2^2)], \\ c_4 = {} & {}^1/_8 D^{-1}\{(f_1 + f_2 + f_3)\chi\Delta_1^2 - \chi[f_4(4\chi^2 + \Delta_1^2\Delta_2^2) \\ & + 5f_5\Delta_1^4 + 10(f_6 + f_7)\chi\Delta_1^2 - f_8\Delta_1^2(30\chi^2 + 5\Delta_1^2\Delta_2^2)]\}, \\ c_5 = {} & \frac{\Delta_2^2}{\Delta_1^2}\left(c_4 + \frac{\chi}{2}f_4\right) - \frac{\chi}{2}f_5, \qquad c_6 = 2c_2 - f_6\chi + f_8\chi^2, \\ c_7 = {} & 2c_3 - f_7\chi + f_8\chi^2, \quad c_8 = c_1 + {}^3/_2c_6 + c_7 - {}^1/_2(f_2 - 3\chi f_8). \end{aligned} \tag{11}$$

Here the values of the superscripts s_f (appearing on the c_n and f_n, that is, $c_n^{(Sf)}$, $f_n^{(Sf)}$) are not explicitly written down since they are the same in the left and right-hand parts;

$$f_n^{(s_f)} = R_{\mu\nu\rho\sigma}^{(n)}(-1)^{2s_f}\int K^{(s_f)\mu\nu\rho\sigma}(\mu, -p_6, p_5, q_1, -q_2) \times \delta(\Delta - p_5 - p_6)\frac{d^3p_5}{2\varepsilon_5}\frac{d^3p_6}{2\varepsilon_6},$$

$$R_{\mu\nu\rho\sigma}^{(n)} = \left(g_{\mu\nu}g_{\rho\sigma},\quad g_{\mu\rho}g_{\nu\sigma},\quad g_{\mu\sigma}g_{\nu\rho},\quad \frac{g_{\mu\nu}q_{1\rho}q_{1\sigma}}{D},\right. \left. \frac{g_{\rho\sigma}q_{2\mu}q_{2\nu}}{D}, \frac{g_{\mu\rho}q_{1\sigma}q_{2\nu}}{D}, \frac{g_{\mu\sigma}q_{1\rho}q_{2\nu}}{D}, \frac{q_{1\rho}q_{1\sigma}q_{2\mu}q_{2\nu}}{D^2}\right). \tag{12}$$

The contractions of $f_n^{(Sf)}$ given by Eq. (12) can be calculated immediately; we shall not present them here due to their cumbersome nature.

In connection with the calculation of the total cross section σ_{III} one can change to covariant variables

$$\frac{d^3p_3}{2\varepsilon_3}\frac{d^3p_4}{2\varepsilon_4} = \frac{\pi}{2}\frac{d\eta\, d\Delta_2^2}{(\nu^2 - m_1^2m_2^2)^{1/2}}\frac{d\Delta^2\, d\Delta_1^2\, d\varphi}{8(\eta^2 + m_1^2\Delta_2^2)^{1/2}} \tag{13}$$

where φ denotes the angle between the planes $(\mathbf{p}_1, \mathbf{p}_2)$ and $(\mathbf{p}_1, \mathbf{q}_1)$ in the system where the vectors $\mathbf{p}_1$ and $\mathbf{q}_2$ are colinear. The invariant variables appearing in Eq. (13) vary within the following limits:

$$\begin{aligned} & \Delta_{1-}^2 \leqslant \Delta_1^2 \leqslant \Delta_{1+}^2, \quad \Delta_{2-}^2 \leqslant \Delta_2 \leqslant \Delta_{2+}^2, \\ & (m_1 + \sqrt{\Delta^2})^2 \leqslant w^2 \leqslant (\sqrt{(p_1 + p_2)^2} - m_2)^2, \\ & 4\mu^2 \leqslant \Delta^2 \leqslant (\sqrt{(p_1 + p_2)^2} - m_1 - m_2)^2, \end{aligned} \tag{14}$$

where

$$w^2 = (p_1 + q_2)^2 = 2\eta + m_1^2 - \Delta_2^2, \tag{15}$$

$$\Delta_{1\pm}^2 = -2m_1^2 + {}^1/_2 w^{-2}\{(w^2 + m_1^2 + \Delta_2^2)(w^2 + m_1^2 - \Delta^2) \pm ([(w^2 - m_1^2 + \Delta_2^2)^2 + 4m_1^2\Delta_2^2][(w^2 - m_1^2 - \Delta^2)^2 - 4m_1^2\Delta^2])^{1/2}\},$$

and $\Delta_{2\pm}^2$ is obtained from $\Delta_{1\pm}^2$ by making the following substitutions:

$$w^2 \to (p_1 + p_2)^2, \quad \Delta_2^2 \to -m_1^2, \quad m_1 \to m_2, \quad \Delta^2 \to w^2.$$

The cited expressions (9) and (10) for the cross sections $d\sigma_I$ and $d\sigma_{III}$ are exact. Now let us go on to an investigation of the cross sections at larger energies when $\nu \gg \mu^2$, m_1m_2, $m_1\mu$, $m_2\mu$.

2. Let us start with a calculation of the cross section $d\sigma_{III}$. Let us discuss the important range of variation of the variables (14):

1) Since the cross section for the photoprocess (the conversion of two photons into a pair) falls off according to a power law with increasing energy, then $\Delta^2 \sim 4\mu^2$.

2) The cross section for the virtual photoprocess decreases according to a power law for $\Delta_{1,2}^2 \gg \Delta^2$; therefore $\Delta_{1,2}^2 \lesssim \Delta^2$.

3) The variable η is the energy of the photon q_2 in the system where $\mathbf{p}_1 = 0$. The cross section for photoproduction of a pair on a particle does not have a power-law dependence on the energy of the photon. Therefore, the entire range of variation of η gives a contribution, except for the case when $m_2 \gg \mu$ since then for $\eta \sim \mu\nu/m_2$ the quantity Δ_{2-}^2 is comparable

with μ^2, which effectively cuts off the integral over η.

We shall seek the cross section σ_{III} with "doubly-logarithmic" accuracy, that is, in the cross section we retain terms containing the products of two or three large logarithms (both the logarithms of the ratio of the energy to the mass as well as the logarithm of the mass ratio can be large), where we shall perform the calculation in such a way that the last integration is carried out with respect to Δ^2. The following properties should be taken into consideration.

A. The desired logarithmic terms arise upon integration over Δ_1^2, Δ_2^2, and η. Therefore, one can set $D = \chi^2$ since two logarithms immediately occur in connection with the expansion in powers of $\Delta_1^2\Delta_2^2/\chi^2$. Because of this reason one can omit the terms $\Delta_1^4\Delta_2^4$ inside the curly brackets in expression (10).

B. One can set $\eta' = \nu\chi/\eta$ everywhere except in the combination $(\nu - \eta\eta'/\chi)^2$ in which cancellation occurs, where to the required degree of accuracy one has

$$\int d\varphi\left(\nu - \frac{\eta\eta'}{\chi}\right)^2 = \frac{\pi}{\chi^2\eta^2}[\nu(\nu-\eta)\Delta_2^2 - m_2^2\eta^2][\eta(\eta-\chi)\Delta_1^2 - m_1^2\chi^2]. \tag{16}$$

With this taken into account, it is seen that the cross section (10) does not contain terms which increase anomalously with ν, since the coefficients c_n given by Eqs. (11) are of the order of unity.

C. From formulas (10) and (13) it follows that inside the curly brackets in Eq. (10) one should keep the terms proportional to ν^2. In the terms where the quantity ν^2 does not enter explicitly, it may arise upon integration with respect to $\eta(\Delta_1^2)$ (the contribution from the upper (lower) limit); in this connection, however, the corresponding logarithm drops out. It is also necessary to take into account that $\int d\Delta_2^2/\Delta_2^2$ ($\int d\Delta_1^2/\Delta_1^2$) gives a "large" logarithm in the region of large (small) values of η only in the case $m_2(m_1) \ll \mu$.

On the basis of what has been said above, one can omit the terms $c_n (n \geq 4)$ in expression (10) for the cross section and one can also substantially simplify the remaining expression. As a result we obtain

$$d\sigma_{III} = \frac{\alpha^4}{8\pi^2}d\Delta^2\frac{d\eta}{\eta}\frac{d\Delta_2^2}{\Delta_2^4}\frac{d\Delta_1^2}{\Delta_1^4}\frac{1}{\chi^2}\left\{f_1^{(s_f)}\left[\Delta_1^2\Delta_2^2\left(1 - \frac{\eta}{\nu} - \frac{\chi}{\eta} + s_1\frac{\chi^2}{\eta^2} + s_2\frac{\eta^2}{\nu^2}\right) - m_2^2\Delta_1^2\frac{\eta^2}{\nu^2} - m_1^2\Delta_2^2\frac{\Delta^4}{4\eta^2}\right] - 3\left(f_4^{(s_f)}\Delta_2^2 + f_5^{(s_f)}\Delta_1^2\right)\Delta_1^2\Delta_2^2\right\}, \tag{17}$$

where with the assumed degree of accuracy

$$f_1^{(0)} = \pi\beta\left\{4 + \frac{\Delta^2}{\chi^2}(4\mu^2 + \Delta_1^2 + \Delta_2^2) - \left[2(4\mu^2 + \Delta_1^2 + \Delta_2^2) - \frac{1}{\chi}(8\mu^4 + 6\mu^2(\Delta_1^2 + \Delta_2^2) + \Delta_1^4 + \Delta_2^4)\right]\frac{l_0}{\chi}\right\},$$

$$f_1^{(1/2)} = -8\pi\beta\left\{1 + \frac{\Delta^2}{2\chi^2}(2\mu^2 - \Delta_1^2 - \Delta_2^2) + \left[2\mu^2(\mu^2 - \chi) - \chi^2 + \frac{\Delta^2}{2}(\Delta_1^2 + \Delta_2^2)\right]\frac{l_0}{\chi^2}\right\}, \tag{18}$$

$$f_4^{(0)} = f_5^{(0)} = \frac{\beta\pi}{\chi^2}[3\Delta^2 - (2\mu^2 + \Delta^2)l_0],$$

$$f_4^{(1/2)} = f_5^{(1/2)} = \frac{4\pi\beta}{\chi^2}(-\Delta^2 + 2\mu^2 l_0).$$

Here

$$l_0 = \frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}, \quad \beta = \sqrt{\frac{\Delta^2 - 4\mu^2}{\Delta^2}}. \tag{19}$$

With the assumed degree of accuracy the limits in (14) can be set equal to the following:

$$m_1^2\Delta^4/4\eta^2 \leqslant \Delta_1^2 \leqslant \infty, \quad m_2^2\eta^2/\nu^2 \leqslant \Delta_2^2 < \infty,$$
$$\sqrt{\Delta^2}(m_1 + \sqrt{\Delta^2}/2) \leqslant \eta \leqslant \nu. \tag{20}$$

After these simplifications are made, the integrals with respect to Δ_1^2 and Δ_2^2 are elementary to carry out. As a result we obtain

$$d\sigma_{III} = \frac{4\alpha^4}{\pi}\frac{d\Delta^2}{\Delta^4}\frac{d\eta}{\eta}\beta\left[d_1^{(s_f)}\left(1 - \frac{\eta}{\nu} - \frac{\Delta^2}{2\eta} + s_1\frac{\Delta^4}{4\eta^2} + s_2\frac{\eta^2}{\nu^2}\right)l_1 l_2 + (d_2^{(s_f)} - d_1^{(s_f)})\left(l_2 + \frac{l_1}{1 + m_2^2\eta^2/\Delta^2\nu^2}\right)\right], \tag{21}$$

where

$$d_1^{(0)} = \frac{1}{2}\left[1 + \frac{4\mu^2}{\Delta^2} - \frac{4\mu^2}{\Delta^2}\left(1 - \frac{2\mu^2}{\Delta^2}\right)l_0\right],$$

$$d_1^{(1/2)} = -1 - \frac{4\mu^2}{\Delta^2} + \left(1 + \frac{4\mu^2}{\Delta^2} - \frac{8\mu^4}{\Delta^4}\right)l_0,$$

$$d_2^{(0)} = \frac{1}{2}\left[-\frac{11}{3} - \frac{22}{3}\frac{\mu^2}{\Delta^2} + \left(\frac{2}{3} + \frac{10\mu^2}{\Delta^2} - \frac{44}{3}\frac{\mu^4}{\Delta^4}\right)l_0\right], \tag{22}$$

$$d_2^{(1/2)} = \frac{11}{3} + \frac{22}{3}\frac{\mu^2}{\Delta^2} - \left(\frac{5}{3} + \frac{10\mu^2}{\Delta^2} - \frac{44}{3}\frac{\mu^4}{\Delta^4}\right)l_0,$$

$$l_1 = \ln\frac{4\eta^2}{m_1^2\Delta^2}, \quad l_2 = \ln\left(\frac{m_2^2\eta^2 + \Delta^2\nu^2}{m_2^2\eta^2}\right).$$

We note that the quantity $4\alpha^2\pi\Delta^{-2}\beta d_1^{(Sf)} = \sigma_{\gamma\gamma}^{(Sf)}$ is the cross section for the conversion of two photons into a pair of particles, and the term containing $l_1 l_2$ in Eq. (21) for the cross section can be obtained with the aid of the pole approximation (the method of equivalent photons).

Carrying out the integration over η within the limits given by Eq. (20), we obtain

$$d\sigma_{III} = \frac{8\alpha^4}{\pi}\frac{d\Delta^2}{\Delta^4}\beta\left\{d_1^{(s_f)}\left[\frac{1}{3}\ln^3\frac{2\nu}{m_1m_2} - \ln\frac{2\nu}{m_1m_2}\left(\ln^2\left(1 + \frac{\Delta}{m_1}\right) + \ln^2\left(1 + \frac{\Delta}{m_2}\right)\right) + \frac{2}{3}\left(\ln^3\left(1 + \frac{\Delta}{m_1}\right) + \ln^3\left(1 + \frac{\Delta}{m_2}\right)\right) - \ln\frac{2\nu}{m_1m_2}\ln\frac{\nu}{(m_1+\Delta)(m_2+\Delta)} - (2 - s_1)\ln\frac{\nu}{m_2\Delta}\ln\left(1 + \frac{\Delta}{m_1}\right) - (2 - s_2)\ln\frac{\nu}{m_1\Delta}\ln\left(1 + \frac{\Delta}{m_2}\right)\right] + d_2^{(s_f)}\ln\frac{2\nu}{m_1m_2}\ln\frac{\nu}{(m_1+\Delta)(m_2+\Delta)}\right\}, \tag{23}$$

where $\Delta = \sqrt{\Delta^2}$ and $d_{1,2}^{Sf}$ are given by formula (22). The obtained cross section $d\sigma_{III}/d\Delta^2$ is valid for any arbitrary ratio between the masses of the participating particles. The integrated cross section σ_{III} has a different form depending on the ratio between the masses.

In the case when $\mu \gg m_{1,2}$ we have the following result for the electroproduction cross section, correct to "doubly-logarithmic" terms (the logarithm of the ratio $\mu/m_{1,2}$ is assumed to be "large"):

$$\sigma_{III}^{(s_f)}(s_1, s_2) = \frac{\alpha^4}{27\pi\mu^2}\{a_1^{(s_f)}[L^3 - 3L(L_1^2 + L_2^2) + 2(L_1^3 + L_2^3)] - a_2^{(s_f)}L^2 - a_3^{(s_f)}L(L_1 + L_2) + a_4^{(s_f)}(L_1^2 + L_2^2) + a_5^{(s_f)}(s_1 - 2)L_1(L - L_1) + a_5^{(s_f)}(s_2 - 2)L_2(L - L_2)\}, \tag{24}$$

where

$$L = \ln\frac{2\nu}{m_1m_2}, \quad L_1 = \ln\frac{\mu}{m_1}, \quad L_2 = \ln\frac{\mu}{m_2}$$

and

$$a_1^{(0)} = 4, \quad a_1^{(1/2)} = 28; \quad a_2^{(0)} = 19, \quad a_2^{(1/2)} = 178; \quad a_3^{(0)} = 14, \quad a_3^{(1/2)} = 80;$$

$$a_4^{(0)} = 33, \quad a_4^{(1/2)} = 258; \quad a_5^{(0)} = 12, \quad a_5^{(1/2)} = 84. \tag{25}$$

In the case when $m_1 \gtrsim \mu$, $m_2 \gtrsim \mu$, one should set $L_1 = L_2 = 0$ in Eq. (24); then we obtain the following result which is independent of the spins s_1 and s_2:

$$\sigma_{III}^{(s_f)} = \frac{\alpha^4}{27\pi\mu^2}(a_1^{(s_f)}L^3 - a_2^{(s_f)}L^2). \tag{26}$$

In the case when $m_{1(2)} \gtrsim \mu$, $\mu \gg m_{2(1)}$, one should set $L_{1(2)} = 0$ in Eq. (24).

3. We have obtained the total cross section σ_{III} correct to "doubly-logarithmic" terms for an arbitrary ratio between the masses. In a number of cases one is also able to obtain simple formulas for the cross sections to within the accuracy of single powers of the logarithm. In order to do this it is convenient to use the results of Kel'ner,[6] who calculated $d\sigma_{III}/d\epsilon_5 d\epsilon_6$ correct to terms $\sim\mu^2/\epsilon_{5,6}^2$, $m_2^2/(\epsilon-\omega)^2$ for the creation of a pair of relativistic particles with energy ϵ and mass m_2 in a Coulomb field. Carrying out the integration of these cross sections, we obtain the energy spectrum $d\sigma/d\omega$ of the created pair ($\omega = \epsilon_5 + \epsilon_6$). We cite this spectrum for certain cases.

In the case $m_2 = \mu$, $s_2 = s_f$ we have

$$\frac{d\sigma_{III}}{d\xi} = \frac{2\alpha^4}{9\pi\mu^2\xi}\left[g_1^{(s_f)} \ln\frac{2\omega}{\mu} \ln\frac{1}{\xi} + g_2^{(s_f)} \ln\frac{1}{\xi} + g_3^{(s_f)} \ln\frac{2\omega}{\mu} + g_4^{(s_f)} \ln\frac{2\omega}{\mu} \ln(1-\xi) + g_5^{(s_f)}\right], \tag{27}$$

where $\xi = \omega/\epsilon$ and

$$\begin{gathered} g_1^{(1/2)} = 28\tau + {}^{32}/_5\xi^2, \quad g_2^{(1/2)} = -{}^{218}/_3, \\ g_3^{(1/2)} = \frac{8}{5}\left(\frac{11}{\xi^2} - \frac{22}{\xi} + 2 + 9\xi\right), \\ g_4^{(1/2)} = 4\left(\frac{22}{5\xi^3} - \frac{11}{\xi^2} + \frac{3}{2\xi} + \frac{49}{4} - 10\xi + \frac{51\xi^2}{10} - \frac{9}{4}\frac{\xi}{2-\xi}\right), \\ g_5^{(1/2)} = {}^{7}/_2\pi^2 - {}^{811}/_9; \end{gathered} \tag{28}$$

$$\begin{gathered} g_1^{(0)} = 4(\tau + {}^{1}/_5\xi^2), \quad g_2^{(0)} = -{}^{26}/_3, \\ g_3^{(0)} = \frac{4}{3}\left(-\frac{11}{\xi^2} + \frac{22}{\xi} - \frac{63}{4} + \frac{19\xi}{4}\right), \\ g_4^{(0)} = -\frac{44}{5\xi^3} + \frac{22}{\xi^2} - \frac{23}{\xi} + \frac{29}{2} - \frac{11\xi}{2} + \frac{4\xi^2}{5}, \\ g_5^{(0)} = \pi^2/2 - {}^{121}/_9; \\ \tau = 1 - \xi + s_2\xi^2. \end{gathered}$$

In the case $m_2 \ll \mu$ we have

$$\frac{d\sigma_{III}}{d\xi} = \frac{2\alpha^4}{9\pi\mu^2}\frac{1}{\xi}\left[h_1^{(s_f,s_2)} \ln\frac{2\omega}{\mu} \ln\frac{(1-\xi)\mu^2}{\xi^2 m_2^2} + h_2^{(s_f,s_2)} \ln\frac{2\omega}{\mu} + h_3^{(s_f,s_2)} \ln\frac{\mu}{\xi m_2} + g_5^{(s_f)}\right], \tag{29}$$

where

$$\begin{gathered} h_1^{(1/2,s_2)} = 7h_1^{(0,s_2)} = 14\tau, \quad h_2^{(1/2,s_2)} = {}^{40}/_3(1-\xi) + \xi^2(1 + {}^{64}/_3 s_2), \\ h_3^{(1/2,s_2)} = -{}^{218}/_3\tau, \\ h_2^{(0,s_2)} = {}^{7}/_3(1-\xi) + \xi^2({}^{1}/_4 + {}^{17}/_6 s_2), \quad h_3^{(0,s_2)} = -{}^{26}/_3\tau. \end{gathered} \tag{30}$$

In the case $m_2 \gg \mu$ we have

$$\frac{d\sigma_{III}}{d\xi} = \frac{2\alpha^4}{9\pi\mu^2}\frac{1}{\xi}\left[e_1^{(s_f)} \ln z \ln\frac{2\varepsilon}{m_2} + e_2^{(s_f)} \ln\frac{2\varepsilon}{m_2} + e_3^{(s_f)} \ln\frac{2\varepsilon}{m_2} \times \ln\left(\frac{z+\sqrt{4+z^2}}{2}\right)\frac{1}{z\sqrt{z^2+4}} + e_4^{(s_f)}\frac{\ln^2 z}{1+z^2} + e_5^{(s_f)}\frac{\ln z}{1+z^2} + \frac{g_5^{(s_f)}}{1+z^2}\right], \tag{31}$$

where $z = \xi m_2/\mu$ and

$$\begin{gathered} e_1^{(1/2)} = -28 - \frac{32}{5}z^2, \quad e_2^{(1/2)} = \frac{8}{5}\left(-4 + \frac{11}{z^2}\right), \\ e_3^{(1/2)} = \frac{4}{5}\left(-\frac{88}{z^2} + 84 + 51z^2 + 8z^4\right), \quad e_4^{(1/2)} = 7e_4^{(0)} = -28, \\ e_5^{(1/2)} = 86, \quad e_1^{(0)} = 4\left(-1 - \frac{z^2}{5}\right), \quad e_2^{(0)} = -\frac{4}{5}\left(1 + \frac{11}{z^2}\right), \\ e_3^{(0)} = \frac{4}{5}\left(\frac{44}{z^2} + 23 + 7z^2 + z^4\right), \quad e_5^{(0)} = 11. \end{gathered} \tag{32}$$

These cross sections are given correct to terms giving a logarithmic contribution to the total cross sections, and these results are valid for $\omega \gg \mu$ and $\epsilon - \omega \gg m_2$. Therefore, in order to calculate the total cross section it is necessary to consider separately the region $\omega \sim \mu$. This region gives a contribution proportional to a single power of the logarithm, which one can determine with the aid of the method of equivalent photons:

$$d\sigma_{III}(\omega \sim \mu) = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\ln\frac{\varepsilon}{\omega}\sigma_\gamma^{(s_f)}(\omega); \tag{33}$$

$\sigma_\gamma^{(s_f)}(\omega)$ is the cross section for the photoproduction of a pair on a nucleus. Expressions (27)–(32) take on a similar form, to within logarithmic accuracy at $\omega \sim \mu$, but they contain the quantities $\sigma_\gamma(\omega \gg \mu)$, evaluated to within a constant. In order to avoid cross-linking, it is necessary in (33) to subtract $\sigma_\gamma^{(s_f)}(\omega \gg \mu)$ from $\sigma_\gamma^{(s_f)}(\omega)$; then the resulting expression behaves like $1/\omega^2$ for $\omega \gg \mu$. In order to obtain the total cross sections with "single-logarithmic" accuracy, it is necessary to integrate this expression within the limits $2\mu \le \omega < \infty$ and to add to it the integrals of the cross sections (27)–(32) within the limits $2\mu < \omega < \epsilon$ (it is not difficult to see that the region $\epsilon - \omega \sim \epsilon$ gives the major contribution, which makes it possible to use formulas (27)–(32) over the entire interval). Finally we arrive at the cross section for the electroproduction of a pair in a Coulomb field.

However, the results can also be used to obtain the cross sections of electroproduction associated with the collision of two particles. In order to verify this, we consider that the difference between the reaction in a Coulomb field and the reaction on a particle at rest with mass m_1 consists in the fact that the latter acquires a recoil energy equal to $\Delta_1^2/2m_1$, where $\Delta_1^2 \lesssim \mu^2$ in the important region, as mentioned above. Therefore, it is clear that in the case when $\omega \gg \mu^2/m_1$, $\epsilon - \omega \gg \mu^2/m_1$, the obtained formulas describe the process associated with the collision of two particles. If the quantity $\omega = \eta/m_1$ is introduced for $m_1 \ne \infty$, then the cross sections (27)–(32) can be rewritten in covariant form ($\epsilon = \nu/m_1$, see Eq. (11)), and for $\eta \gg \mu^2$, μm_1 and $\nu - \eta \gg \mu^2$, $m_1 m_2$ these cross sections are valid for any arbitrary mass m_1.

Let us discuss the procedure for calculating the total electroproduction cross sections associated with the collision of particles. In the case $m_1 = \mu$ this calculation is carried out in the same way as in a Coulomb field; only as $\sigma_\gamma^{(s_f)}(\omega)$ in Eq. (33) it is necessary to substitute the cross section for photoproduction on a particle with mass m_1 (formula (2.19) of article[5]), and in the cross sections obtained from Eqs. (27)–(32) it is necessary to carry out the integration within the limits $2\mu(\mu + m_1) \le \eta \le \nu$. The corresponding procedure may also be developed for $m_1 \ll \mu$, but one cannot do this if it is taken into consideration that the cross section must be symmetric with respect to the substitutions ($p_1, s_1 \longleftrightarrow p_2, s_2$), which makes it possible to determine the contribution for small values of η by making use of the results for $m_2 \ll \mu$. As a result of rather cumbersome calculations we arrive at the cross sections in the "single-logarithmic" approximation. In order to derive their explicit form it is necessary to add the following expression to the cross sections (24)—(26):

$$\Delta\sigma_{\mathrm{III}}^{(s_f)}(s_1, s_2) = \frac{\alpha^4}{27\pi\mu^2} w^{(s_f)}(s_1, s_2), \tag{34}$$

where for $m_1 \gg \mu$, $m_2 \gg \mu$

$$w^{(1/2)} = (7\pi^2 + 370)L, \quad w^{(0)} = (\pi^2 + 22)L; \tag{35}$$

for $m_1 \gg \mu$, $m_2 = \mu$, and $s_2 = s_f$ we have

$$w^{(1/2)} = (-{}^{75}/_2\pi^2 + 430)L, \quad w^{(0)} = (-{}^{27}/_2\pi^2 + 100)L; \tag{36}$$

for $m_1 \gg \mu$, $m_2 \ll \mu$

$$\begin{gathered} w^{(1/2)} = (-7\pi^2 + {}^{223}/_3 + 43s_2)L + A_2^{(1/2)}L_2, \\ w^{(0)} = (-\pi^2 - {}^{311}/_{12} + {}^{11}/_2 s_2)L + A_2^{(0)}L_2, \\ A_2^{(1/2)} = -7\pi^2 + {}^{3917}/_3 - 303s_2, \\ A_2^{(0)} = -\pi^2 + {}^{2183}/_{12} - {}^{75}/_2 s_2; \end{gathered} \tag{37}$$

for $m_1 = \mu$, $m_2 = \mu$, $s_1 = s_2 = s_f$

$$w^{(1/2)} = (-32\pi^2 + 490)L, \quad w^{(0)} = (-28\pi^2 + 178)L; \tag{38}$$

for $m_1 = \mu$, $m_2 \ll \mu$, $s_1 = s_f$

$$\begin{gathered} w^{(1/2)} = (-{}^{103}/_2\pi^2 + {}^{403}/_3 + 43s_2)L + A_2^{(1/2)}L_2, \\ w^{(0)} = (-{}^{31}/_3\pi^2 + {}^{625}/_{12} + {}^{11}/_2 s_2)L + A_2^{(0)}L_2; \end{gathered} \tag{39}$$

and for $m_1 \ll \mu$, $m_2 \ll \mu$,

$$\begin{gathered} w^{(1/2)} = [-21\pi^2 - {}^{664}/_3 + 43(s_1 + s_2)]L + A_2^{(1/2)}L_2 + A_1^{(1/2)}L_1, \\ w^{(0)} = [-3\pi^2 - {}^{443}/_6 + {}^{11}/_2(s_1 + s_2)]L + A_2^{(0)}L_2 + A_1^{(0)}L_1, \\ A_1^{(s_f)} = A_2^{(s_f)}(s_2 \to s_1). \end{gathered} \tag{40}$$

The expansion used above of the cross sections in powers of logarithms is not valid in a rather wide region near threshold. A similar situation is well known in the problem of photoproduction.[7] The most favorable situation is the case of heavy initial particles, where the correction terms have the form $(\epsilon_t/\epsilon)^2L^5$ (ϵ denotes the energy of the initial particle in the laboratory system, ϵ_t denotes the threshold energy in this same system). However, in the case of the production of particles with equal masses or $\mu \gg m_{1,2}$ the correction terms $\sim(\epsilon_t/\epsilon)L^5$.

4. Let us discuss the angular and energy distributions of the particles in the region where the contribution of diagram III is essential. It is not difficult to see that $\Delta_{1(2)}^2 > p_{3(4)\perp}^2$ where $p_{3(4)\perp}$ are the components of the momentum perpendicular to the initial momentum. In the important region $\Delta_{1,2}^2 \lesssim \Delta^2 \sim 4\mu^2$ (one can easily see this from the approximate expression (17), containing the factor $\chi^{-2} = 4(\Delta^2 + \Delta_1^2 + \Delta_2^2)^{-2}$), from which it follows that $|p_{3(4)\perp}| \lesssim \mu$ and therefore the transverse momentum of the created pair satisfies $|\Delta_\perp| \lesssim \mu$. Since $\Delta^2 \sim 4\mu^2$, we have $|p_{5(6)\perp}| \lesssim \mu$. From Eqs. (27)—(32) it is seen that the spectrum of the created particles in the laboratory system is $d\omega/\omega$, where in connection with the electroproduction of light particles by heavy ones (see Eq. (31)) the spectrum is cut off at $\omega \approx \epsilon\,\mu/m_2$, i.e., when the velocity of the particles in the pair reaches the velocity of the initial particle. Thus, the pair is emitted in the direction of motion of the initial particle at an angle $\vartheta_{5(6)} \lesssim \mu/\omega$. As mentioned $\epsilon_4 \sim \epsilon$, that is, the fast incoming particle is deflected through a small angle $\vartheta_4 \lesssim \mu/\epsilon$, but the energy transfer to the recoiling particles is $\sim\mu^2/m_1$. The smallness of the transverse momentum makes it possible to change to another system, for example, to calculate the spectrum of the created particles for $\omega \gg \mu$ in the center-of-mass system.

5. Now let us go on to an investigation of the contribution due to diagram I. We immediately note that for $\mu \ll m_2$ the contribution of this diagram to the total cross section is proportional to $1/m_2^2$ (the cross section for bremsstrahlung is inversely proportional to the square of the mass of the incident particle) and consequently is negligible in comparison with the contribution from diagram III. Therefore, one should only consider the case $\mu \gtrsim m_2$; then from the condition $\nu \gg \mu^2$ it follows that $\nu \gg m_2^2$.

In order to obtain the cross section σ_{I} let us calculate the cross section for the emission of a virtual photon with mass $\sqrt{\Delta^2}$, that is, $d\sigma_{\rho\sigma}^{(\gamma)} g^{\rho\sigma}$ given by Eq. (6). Let us introduce the covariant variables

$$\begin{gathered} \varkappa = (p_1 + p_2)\Delta - \Delta^2/2, \quad \varkappa_2 = (p_2\Delta) - \Delta^2/2, \\ \varkappa_4 = p_4\Delta + \Delta^2/2, \quad \Delta_1^2 = -(p_1 - p_3)^2 \end{gathered} \tag{41}$$

and we shall seek $g^{\rho\sigma} d\sigma_{\rho\sigma}^{(\gamma)}/d\kappa$ assuming that $\kappa \gg \Delta^2$, $\kappa^2 \gg m_1^2\Delta^2$, $\nu - \kappa \sim \nu$.

The solution of this problem is similar to finding the spectrum of the bremsstrahlung in the center-of-mass system[8]. After changing to covariant variables, selecting the major terms and integrating over κ_4, Δ_1^2 (in the important region $\kappa_4 \sim \Delta^2$, $m_1^2\kappa_2^2/\nu^2 \lesssim \Delta_1^2 \lesssim \Delta^2$) we obtain the following result which does not depend on the spin s_1:

$$d\sigma_{\rho\sigma}^{(\gamma)} g^{\rho\sigma} = -\frac{\alpha^3}{\nu^3}\left\{ r_1^{(s_2)} \frac{(l_3 - 1)}{\varkappa_2^2} + r_2^{(s_2)} \frac{\varkappa^2(\nu - \varkappa)}{\nu\varkappa_2^4} \left[\left(m_2^2\Gamma - \frac{2\varkappa_2\nu}{\varkappa} \right)(l_3 - 4) - 2\frac{\varkappa_2^2\nu^2}{m_2^2\varkappa^2\Gamma} \right] \right\} d\varkappa\, d\varkappa_2, \tag{42}$$

where

$$\begin{gathered} r_1^{(1/2)} = \nu^2 + (\nu - \varkappa)^2, \quad r_2^{(1/2)} = m_2^2 + \Delta^2/2, \\ r_1^{(0)} = 2\nu(\nu - \varkappa), \quad r_2^{(0)} = m_2^2 - \Delta^2/4, \\ l_3 = \ln\frac{4\nu^2(\nu - \varkappa)^2}{m_1^2 m_2^2 \varkappa^2 \Gamma}, \quad \Gamma = 1 + \frac{\Delta^2\nu(\nu - \varkappa)}{m_2^2\varkappa^2}. \end{gathered} \tag{43}$$

For $\Delta^2 = 0$ we obtain from here the angular and energy distributions of the bremsstrahlung (for $s_2 = 1/2$, $m_1 = m_2 = 1$—we obtain formula (11) of article[8]). The limits of the integration with respect to κ_2 can be easily determined in the reference frame in which $\mathbf{p}_1 + \mathbf{p}_2 = 0$ (the vector Δ is directed along or opposite to $\mathbf{p}_2$). In terms of covariant variables we have

$$\varkappa_{2\pm} = [2\varkappa(\nu + m_2^2) - \Delta^2(\nu + m_1^2) \pm \{(\nu^2 - m_1^2 m_2^2)[(2\varkappa + \Delta^2)^2 - 4\Delta^2(2\nu + m_1^2 + m_2^2)]\}^{1/2}][2(2\nu + m_1^2 + m_2^2)]^{-1} \tag{44}$$

Under the assumed assumptions, the integration over κ_2 can be carried out within the limits

$$m_2^2\varkappa\Gamma/2\nu \leqslant \varkappa_2 < \infty, \tag{45}$$

since small angles of emission give a contribution (see Eq. (42)). As a result we obtain

$$d\sigma_{\rho\sigma}^{(\gamma)} g^{\rho\sigma} = -\frac{2\alpha^3 d\varkappa}{\nu^2 m_2^2 \varkappa\Gamma}\left[r_1^{(s_2)} - \frac{2}{3} r_2^{(s_2)} \frac{\nu(\nu - \varkappa)}{m_2^2\Gamma} \right](l_3 - 1). \tag{46}$$

For $\Delta^2 = 0$ this cross section goes over into the bremsstrahlung spectrum (compare with[8]). The limits of the integration over κ are determined by the following inequalities

$$(p_1 + p_2 - \Delta)^2 = (p_3 + p_4)^2 \geqslant (m_1 + m_2)^2, \qquad (p_1 + p_2 - \Delta)^2 \leqslant (\sqrt{(p_1 + p_2)^2} - \sqrt{\Delta^2})^2, \tag{47}$$

or

$$[(2\nu + m_1^2 + m_2^2)\Delta^2]^{1/2} - \Delta^2/2 \leqslant \varkappa \leqslant \nu - m_1 m_2. \tag{48}$$

In order to obtain $d\sigma_I/d\Delta^2$ one should, having taken the integral with respect to κ of (46), substitute the result into Eq. (9). It is seen that for $\Delta^2 \ll m_2^2$ the quantity $\sigma_{\rho\sigma}^{(\gamma)} g^{\rho\sigma} \propto 1/m_2^2$, and hence for $\mu \ll m_2$ the cross section $\sigma_I \propto m_2^{-2}$ (this case is not considered in what follows). If $\mu \gtrsim m_2$, then from (46) it is seen that

$$\frac{d\sigma_{\rho\sigma}^{(\gamma)}}{d\varkappa} g^{\rho\sigma} \propto \frac{1}{\Delta^2},$$

and since values $\Delta^2 \sim 4\mu^2$ give the major contribution (see Eq. (9)), then $\sigma_I \propto 1/\mu^2$. Let $\mu \sim m_2$; then from (46) it is seen that the important region corresponds to $\kappa \sim \nu$, $\nu - \kappa \sim \nu$. In the case $\mu \gg m_2$ it follows from (46) (taking into account that $\Delta^2 \sim 4\mu^2$) that in the important region $\nu - \kappa \gtrsim m_2^2\nu/\mu^2 \ll \nu$. However, formula (46) itself is derived under the assumption that $\nu - \kappa \sim \nu$; at the same time by using (44) it is not difficult to verify that this formula is not valid for $\nu - \kappa \sim \mu^2 m_1^2/\nu$ since in this case

$$\varkappa_{2-} = \frac{m_2^2\varkappa}{2\nu}\Gamma_1, \quad \Gamma_1 = 1 + \frac{\Delta^2}{m_2^2\varkappa^2}\left[\nu(\nu - \varkappa) + \frac{\Delta^2 m_1^2}{4}\right]. \tag{49}$$

The detailed analysis shows that expression (46) is applicable for $\nu - \kappa \gg m_1 m_2$, $\nu - \kappa \gg \mu^2 m_1^2/\nu$; consequently for $m_2^2\nu/\mu^2 \lesssim m_1 m_2$ and $m_2^2\nu/\mu^2 \lesssim \mu^2 m_1^2/\nu$, that is, for $\mu^2 \gtrsim m_2\nu/m_1$ the important values of κ turn out to be outside the region of applicability. However, even in this case, by using expression (46), one can obtain the total cross section σ_I, albeit not with power-law accuracy but with logarithmic accuracy, if the integration over κ is carried out up to $\nu - \kappa \sim \mu^2 m_1^2/\nu$. In fact, from (49) it follows that in this case the values $\nu - \kappa \gtrsim \mu^2 m_1^2/\nu$ are essential, where formula (46) is valid for $\nu - \kappa \gg \mu^2 m_1^2/\nu$ but the region $\nu - \kappa \sim \mu^2 m_1^2/\nu$ does not give any logarithmic contributions since in this expression the logarithm l_3 is not "large." One can verify this directly: l_3 appears upon integrating $d\Delta_1^2/\Delta_1^2$, but for $\nu - \kappa \sim \mu^2 m_1^2/\nu$ we have $\Delta_{1+}^2 \sim \Delta_{1-}^2 \sim \kappa_2$.

Performing the integration over κ with what has been said above taken into consideration (in (46) one can make the substitution $\Gamma \to \Gamma_1$ and integrate within the limits $0 \le \kappa \le \nu$), we obtain the following result with "singly-logarithmic" accuracy for $m_2 \le \mu$:

$$d\sigma_I = \alpha^3 \frac{d\Delta^2}{\Delta^6} f^{(s_f)}(\Delta^2) \left\{ z_1^{(s_2)} \ln \frac{4\nu^2}{m_1^2\mu^2} + z_2^{(s_2)}\left[\ln^2\left(\frac{\Delta^2}{m_2^2}\right) - \ln^2\left(1 + \frac{m_1^2\Delta^4}{4m_2^2\nu^2}\right)\right] + z_3^{(s_2)} \ln\left(\frac{\Delta^2}{m_2^2}\right) + z_4^{(s_2)} \ln\left(1 + \frac{m_1^2\Delta^4}{4m_2^2\nu^2}\right)\right\}; \tag{50}$$

here

$$z_1^{(1/2)} = \left(-\frac{4}{3}\frac{\Delta^2}{m_2^2} + 2\frac{\Delta^4}{m_2^4} - \frac{5}{3}\frac{\Delta^6}{m_2^6}\right)\ln\frac{\Delta^2}{m_2^2} + \left(-\frac{4\Delta^2}{\Delta^2 - 4m_2^2} + \frac{2\Delta^2}{m_2^2} - \frac{16}{3}\frac{\Delta^4}{m_2^4} + \frac{5}{3}\frac{\Delta^6}{m_2^6}\right)\sqrt{\frac{\Delta^2}{\Delta^2 - 4m_2^2}} \ln\frac{\sqrt{\Delta^2} + \sqrt{\Delta^2 - 4m_2^2}}{\sqrt{\Delta^2} - \sqrt{\Delta^2 - 4m_2^2}} + \frac{8\Delta^2}{\Delta^2 - 4m_2^2} + \frac{\Delta^2}{m_2^2} + \frac{10}{3}\frac{\Delta^4}{m_2^4},$$

$$z_1^{(0)} = \left(-\frac{4}{3}\frac{\Delta^2}{m_2^2} + \frac{\Delta^4}{2m_2^4} + \frac{\Delta^6}{3m_2^6}\right)\ln\frac{\Delta^2}{m_2^2} + \left(\frac{\Delta^4}{6m_2^4} - \frac{\Delta^6}{3m_2^6} + \frac{3\Delta^2}{m_2^2}\right)\sqrt{\frac{\Delta^2}{\Delta^2 - 4m_2^2}} \ln\frac{\sqrt{\Delta^2} + \sqrt{\Delta^2 - 4m_2^2}}{\sqrt{\Delta^2} - \sqrt{\Delta^2 - 4m_2^2}} - 2\frac{\Delta^2}{m_2^2} - \frac{2}{3}\frac{\Delta^4}{m_2^4}; \tag{51}$$

$$z_2^{(1/2)} = -2/3, \quad z_2^{(0)} = -1/6, \quad z_3^{(1/2)} = -4/3\ln(\Delta^2/\mu^2) - 2,$$

$$z_3^{(0)} = -1/3\ln(\Delta^2/\mu^2), \quad z_4^{(1/2)} = 2, \quad z_4^{(0)} = 0,$$

where for $\Delta^2 \gg 4m_2^2$ we have

$$z_1^{(1/2)} = \frac{4}{3}\ln\frac{\Delta^2}{m_2^2} + \frac{2}{9}, \quad z_1^{(0)} = \frac{1}{3}\ln\frac{\Delta^2}{m_2^2} + \frac{19}{18}. \tag{52}$$

Thus, the region $\Delta^2 \sim 4\mu^2$ is essential and one can integrate with respect to Δ^2 within the limits $4\mu^2 \le \Delta^2 < \infty$.

From (50) and (51) it follows that the cross section σ_I is "doubly-logarithmic" only for $\mu \gg m_2$. In this case, with "singly-logarithmic" accuracy we have

$$\sigma_I = \frac{\alpha^4}{45\pi\mu^2} b_1^{(s_2, s_f)} \left\{\ln\frac{\mu^2}{m_2^2}\ln\frac{4\nu^2 m_2}{m_1^2\mu^3} + \frac{1}{2}\ln^2\left(1 + \frac{m_1^2\mu^4}{4m_2^2\nu^2}\right) + \frac{1}{15} b_2^{(s_2, s_f)} \ln\frac{4\nu^2}{m_1^2\mu^2} + \frac{1}{15} b_3^{(s_2, s_f)}\left[\ln\left(1 + \frac{m_1^2\mu^4}{4m_2^2\nu^2}\right) - \ln\frac{\mu^2}{m_2^2}\right]\right\}, \tag{53}$$

where

$$b_1^{(1/2,1/2)} = 4, \quad b_1^{(1/2,0)} = 1/2, \quad b_1^{(0,1/2)} = 1, \quad b_1^{(0,0)} = 1/8;$$
$$b_2^{(1/2,1/2)} = 41, \quad b_2^{(1/2,0)} = 97/2, \quad b_2^{(0,1/2)} = 86, \quad b_2^{(0,0)} = 187/2; \tag{54}$$
$$b_3^{(1/2,1/2)} = 199/2, \quad b_3^{(1/2,0)} = 229/2, \quad b_3^{(0,1/2)} = 77, \quad b_3^{(0,0)} = 92.$$

For $m_2 = \mu$ the cross section σ_I involves a single power of the logarithm:

$$\sigma_I = \frac{\alpha^4}{81\pi\mu^2} c^{(s_f)} \ln\frac{2\nu}{m_1\mu}, \tag{55}$$

where

$$c^{(1/2)} = 1/2(231\pi^2 - 2198), \quad c^{(0)} = 1/4(425 - 42\pi^2). \tag{56}$$

Let us discuss the angular and energy distributions of the particles in the region where the contribution from diagram I is important. From (42) it follows that values of κ_2 near the lower limit give the contribution, that is, $|\Delta_\perp| \lesssim \mu$, and since $\Delta^2 \sim 4\mu^2$ then $|\mathbf{p}_{5(6)\perp}| \lesssim \mu$. Since $\Delta_1^2 \lesssim 4\mu^2$, then $|\mathbf{p}_{3\perp}| \lesssim \mu$, but this implies that $|\mathbf{p}_{4\perp}| \lesssim \mu$, that is, exactly the same situation occurs as for the contribution from diagram III.

From (46) it follows that the angular and energy distribution of the particles of the created pair in the system $\mathbf{p}_1 = 0$ has the form

$$\frac{1}{\varepsilon_2^3}\left[\vartheta^2 + \frac{m_2^2}{\varepsilon_2^2} + \frac{\Delta^2}{\omega^2}\left(1 - \frac{\omega}{\varepsilon_2} + \frac{\Delta^2}{4\varepsilon_2^2}\right)\right]^{-2} d\omega\, d\vartheta^2, \tag{57}$$

where ω and ϑ denote the energy and angle of flight of the pair as a whole. In this system the recoiling particle obtains an energy $\sim\mu^2/m_1$, but the energy spectrum of the bremsstrahlung particle is given by

$$\frac{d\varepsilon_4}{\varepsilon_4 + m_2^2\varepsilon/\mu^2 + \mu^2/\varepsilon} \tag{58}$$

6. The cross sections $d\sigma_{II}/d\Delta^2$ and σ_{II} are obtained from (50)–(56) by making the substitutions $m_1 \longleftrightarrow m_2$ and $s_1 \longleftrightarrow s_2$. The contributions from diagrams I and II do not interfere since the angular distributions of the final particles essentially do not over-

lap. Therefore the integrated cross section for electroproduction at high energies is given by

$$\sigma = \sigma_{\mathrm{I}} + \sigma_{\mathrm{II}} + \sigma_{\mathrm{III}}. \tag{59}$$

This formula remains valid when the initial particles are identical, since the interference between the direct and exchange diagrams is negligible, the contributions of the direct and exchange diagrams are identical, but it is necessary to divide their sum by two due to the identical nature of the particles. In the case of collisions between particles and anti-particles, it is necessary to add the annihilation diagrams to the diagrams shown in Fig. 1; however, the contribution from the annihilation diagrams is small $(\sim 1/\nu)$ at large energies due to the large magnitude of the momentum transfers (for more details see[5]). In cases when there are particles among the created particles which are identical to the initial particles, formula (59) is valid with "doubly-logarithmic" accuracy, since the interference terms do not contain logarithms to powers higher than the first.

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