

THEORY OF THE SENFTLEBEN EFFECT IN OXYGEN

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The change in the viscosity of oxygen in strong magnetic fields is studied in the case when the level splitting in the magnetic field is comparable with the distance between the levels of the fine structure of the rotational levels. A detailed comparison is carried out between the theory and experiments on the angular dependence of the Senftleben effect in oxygen. In passing, a general form has been found for the change of kinetic coefficients in external fields without the use of any assumption on "weak non-sphericity" of the interaction.

1. In the research of I. K. Kikoin et al.,^[1] the change in the viscosity in a magnetic field was studied in detail for nitrogen and oxygen. The most astonishing result for the latter was the violation of the "B/P law." According to this law, all the kinetic coefficients are functions of the ratio B/P (B = magnetic field, P = pressure). The B/P law is a reflection of the fact that a dependence on the magnetic field **B** enters into the kinetic coefficients only through the ratio of the frequency of precession of the molecule in the magnetic field

$$\Omega_0 = \mu_{\text{eff}} B / \hbar \tag{1.1}$$

to the frequency of collisions between the molecules, Ω , which is proportional to the pressure P .^[2-4]

The departure from the B/P law observed in oxygen is due principally to the complicated nature of the effective magnetic moment of oxygen μ_{eff} . The direct carrier of the magnetic moment of the molecule is the electron spin of the molecule and the latter is connected with the rotational moment of the molecule as a result of the "spin-axis" interaction. In a strong magnetic field, this coupling is weakened, which leads in turn to a reduction of μ_{eff} .

In Sec. 2, we obtain an expression for μ_{eff} by means of elementary quantum mechanics. In Sec. 3, we give a more rigorous consideration, based on the quantum kinetic equation (see also^[5,6]). In Sec. 4, we construct a formal solution of the quasiclassical kinetic equation in a magnetic field. In Sec. 5, a model is formulated, within the framework of which expressions are obtained for the five independent viscosity coefficients, and a comparison is made with experiment. The end of the paper is devoted to the Senftleben effect in superhigh fields.

Along with the departure from the B/P law at low pressures, we also have an anomalous dependence of the collision frequency on the pressure when the free path length is comparable with the minimal characteristic dimension of the apparatus. For systematic calculation of this effect it is necessary to solve the kinetic equation with boundary conditions. In the present research, the role of the apparatus factors is taken into account by the introduction of a simple empirical factor (see Sec. 5).

Taking as the rotational quantum number its mean value at room temperature, $K = 10$, we write out the order of magnitude of the quantities entering into the problem:

- a) the distance between the rotational levels: $\hbar K / I \approx 10$ deg;
- b) the distance between the levels of the fine structure of the rotational levels: $E_0 - E_{\pm 1} \approx 1$ deg, $E_{-1} - E_1 \approx 10^{-1}$ deg;
- c) the distance between the levels of the magnetic structure for $B = 10^3$ G: $\mu_0 B K^{-1} \approx 10^{-2}$ deg for $\sigma = \pm 1$; $\mu_0 B K^{-2} \approx 10^{-3}$ deg for $|\sigma| = 0$;
- d) the collision frequency Ω in energy units for normal temperature and pressures (NTP): $\hbar \Omega \hbar \approx 10^{-2}$ deg.

2. As is known,^[7] the fine structure E_σ of the rotational levels of oxygen with a large value of the rotational moment **K** is determined by the expressions

$$E_0 - E_{\pm 1} \approx \lambda, \quad E_{-1} - E_1 \approx \lambda K^{-1}, \tag{2.1}$$

$$\lambda = \hbar \omega_\lambda = 1.985 \text{ cm}^{-1} = 4 \cdot 10^4 \mu_0 \text{ G}.$$

If the magnetic field is not too large, then the operator of the energy of the electron spin moment in the magnetic field

$$\hat{V}_B = \hbar \omega_B \hat{S}_z; \tag{2.2}$$

$$\hbar \omega_B = 2 \mu_0 B, \quad \omega_B \ll \omega_\lambda$$

(μ_0 is the Bohr magneton) can be considered as the excitation of the states $|\sigma\rangle = |K, j, j_z\rangle$ with definite values of the rotational angular momentum **K**, the total angular momentum $j = K + \sigma$ and the projection of the total angular momentum in the direction of the magnetic field j_z . Strictly speaking, it is not possible to use perturbation theory in the narrow range close to the intersection of levels with the same j_z and different $\sigma = \pm 1$. But the kinetic phenomena are determined by characteristics averaged over both the magnitude and the direction of the rotational angular momentum. Therefore, the behavior of the levels in the vicinity of the intersection points is of little importance, and the effective magnetic moment of the oxygen can be obtained by differentiating the Zeeman splitting of the levels, computed with accuracy $(\omega_B / \omega_\lambda)^2$ with respect to the magnetic field:

$$\mu_{\text{eff}} = -\partial \Delta E / \partial B = \gamma M, \tag{2.3}$$

$$\gamma = \frac{2\mu_0}{M} \left\{ \sigma + \frac{\hbar}{M} \left(1 - \frac{1}{2} \sigma^2 \right) + \frac{2\mu_0}{M\lambda} (2 - 3\sigma^2) M B \right\}.$$

Here the transition to the quasiclassical limit $\hbar j \rightarrow |M|$, $\hbar j_z \rightarrow M_z$ has been carried out. The expression (2.3) should be substituted in the magnetic terms of the quasiclassical kinetic equation:^[2,3]

$$\hat{\Omega}_\chi + \hat{\Omega}_B \chi + N = 0, \quad f = f_0(1 + \chi),$$

$$N = \left\{ \frac{\partial}{\partial t} + (\mathbf{v}\nabla) \right\} \ln f^{(0)}, \quad \hat{\Omega}_B \chi = \gamma[\mathbf{MB}] \frac{\partial \chi}{\partial M}, \quad (2.4)^*$$

where $f^{(0)}$ is the equilibrium distribution function, and $f^{(0)} \hat{\Omega}_\chi$ is the linearized collision integral.

The last term in γ of (2.3) leads to the violation of the B/P law. For two thirds of the gas ($\sigma = \pm 1$), this term is small in comparison with the first term. For one third of the gas ($\sigma = 0$) the first term of (2.3) disappears and the second and third terms of this expression are comparable for fields of the order of 10^3 G. Inasmuch as the effective precession (1.1) for the component $\sigma = 0$ is an order of magnitude smaller than for the components $\sigma = \pm 1$, we can sketch the following picture of the Senftleben effect in oxygen with increase in B/P. For comparatively small values of B/P ($\omega_B \approx \Omega \bar{K}$) with maintenance of the B/P law, the odd effect appears, which is due to the dependence of the collision probability on the number σ and the principal part of the even effect reaches saturation.^[2,3] In the next interval of B/P ($\omega_B \approx \Omega \bar{K}^2$), the principal part of the odd effect develops^[3] and the even effect approaches its asymptotic value. These processes take place with violation of the B/P law: in the transition from one fixed value of the pressure to another, the curves of the effects (as functions of B/P) approach the ordinate axis. This is connected with the fact that the degree of violation of the spin-axis coupling depends only on the value of the magnetic field, but not on the pressure, and the point on the B/P axis for given B is shifted to the left with increase in P.

3. We shall now show that Eqs. (2.3) and (2.4) can be obtained more rigorously, directly from the quantum kinetic equation

$$\partial \hat{\rho} / \partial t + (\mathbf{v}\nabla) \hat{\rho} + (i/\hbar)[H, \hat{\rho}] = -\hat{I}_{\text{coll}} \quad (3.1)$$

Here $\hat{\rho}$ is the single particle density matrix, so normalized that its local equilibrium part is identical in form with the local equilibrium Maxwell function $f^{(0)}$; \hat{I}_{coll} is the collision integral, \hat{H} the internal energy of the molecule:

$$H = \hbar^2 \bar{K}^2 / 2I + \hat{V}_{ks} + \hat{V}_B. \quad (3.2)$$

The first term describes the rotational energy of the molecule ($\hbar^2/2I = 2$ deg), the second, the spin-axis interaction energy, which is responsible for the fine structure (2.1), the third, the magnetic energy (2.2).

We shall consider a range of problems in which the density matrix is close to its local equilibrium value, and rewrite (3.1) in the form

$$N f^{(0)} + (i/\hbar)[H, \hat{\rho}^{(0)}] + \hat{I}_{\text{coll}} = 0, \quad \hat{\rho}^{(0)} = \hat{\rho} - f^{(0)}. \quad (3.3)$$

The ratio of the value of the nondiagonal elements of the density matrix to the diagonal elements is determined by the ratio of the transition frequency to the transition energy. Therefore, the density matrix of oxygen is known beforehand to be diagonal relative to the rotational quantum numbers K , while the terms that are nondiagonal in σ are small in comparison with the terms that are diagonal in σ , which we shall denote

by $f(\sigma)$. Expressing the nondiagonal elements of the density matrix in terms of the diagonal in the usual fashion, we get

$$N f^{(0)} + i\omega_B [S, f(\sigma)] + I_{cr} = 0,$$

$$S = S_{\sigma\sigma} - \hbar\omega_B \sum_{\sigma' \neq \sigma} S_{\sigma\sigma'} S_{\sigma'\sigma} / (E_{\sigma'} - E_{\sigma}) =$$

$$= (j_z/K) [\sigma + (1 - 1/2\sigma^2)/K] - (\omega_B j_z^2 / 2\omega_K K^2) (2 - 3\sigma^2). \quad (3.4)$$

Equation (3.4) is the matrix equation in the space of the projection j_z of the total angular momentum. The transition to the quasiclassical limit is given by the relations

$$i[j_z, f] \rightarrow [\mathbf{bM}] \partial f / \partial M, \quad \mathbf{b} = \mathbf{B} / |\mathbf{B}|,$$

$$i[j_z^2, f] \rightarrow (2/\hbar) \mathbf{bM} [\mathbf{bM}] \partial f / \partial M, \quad (3.5)$$

which transform Eq. (3.4) to Eq. (2.4), which was required. The excitation of the states of the fine structure through the magnetic field has the order of magnitude ω_B/ω_K while the excitation by collisions is much less: Ω/ω_K . But the magnetic transitions are completely taken into account in the elementary consideration of Sec. 2. The almost complete coincidence of Eqs. (3.4) and (2.4) is explained in this way (strictly speaking, the collision integral I_{coll} in (3.4) contains a small term which depends explicitly on the magnetic field).

4. We now formulate the formal solution of the kinetic Eq. (2.4) in the presence of an external field.

The space of states $\{\psi_n\}$ in which the nonequilibrium part of the distribution function χ is expanded can be divided into two subspaces \mathfrak{N} and \mathfrak{M} . We refer to the subspace \mathfrak{N} those states which do not depend on the direction of the rotational moment. Introducing the projection operators on these subspaces $\hat{P}_{\mathfrak{N}}$ and $\hat{P}_{\mathfrak{M}}$, we can show that the change in the kinetic coefficient

$$c = -\langle N, \chi \rangle = - \int d\Gamma f^{(0)} N^* \chi$$

in the magnetic field is determined by the formula¹⁾

$$\Delta c = - \langle N, (\hat{\Omega}_{\mathfrak{M}\mathfrak{M}})^{-1} \hat{\Omega}_{\mathfrak{M}\mathfrak{M}} (\hat{\Omega}_{\mathfrak{M}\mathfrak{M}} + \hat{\Omega}_B)^{-1} \hat{\Omega}_B$$

$$(\hat{\Omega}_{\mathfrak{M}\mathfrak{M}})^{-1} \hat{\Omega}_{\mathfrak{M}\mathfrak{M}} (\hat{\Omega}_{\mathfrak{M}\mathfrak{M}})^{-1} N \rangle. \quad (4.1)$$

Here we have introduced notation of the type

$$\hat{\Omega}_{\mathfrak{M}\mathfrak{M}} = \hat{P}_{\mathfrak{M}} \hat{\Omega} \hat{P}_{\mathfrak{M}}, \quad \hat{\Omega}_{\mathfrak{M}\mathfrak{M}} = \hat{\Omega}_{\mathfrak{M}\mathfrak{M}} - \hat{\Omega}_{\mathfrak{M}\mathfrak{M}} (\hat{\Omega}_{\mathfrak{M}\mathfrak{M}})^{-1} \hat{\Omega}_{\mathfrak{M}\mathfrak{M}}. \quad (4.2)$$

For actual computation of Eq. (4.1), we need to solve three integral equations which determine the functions $X_{\mathfrak{M}}$, $X_{\mathfrak{N}}$ and \tilde{N} :

$$\hat{\Omega}_{\mathfrak{M}\mathfrak{M}} X_{\mathfrak{M}} = -N, \quad \hat{\Omega}_{\mathfrak{M}\mathfrak{M}} X_{\mathfrak{M}} = -\hat{\Omega}_{\mathfrak{M}\mathfrak{M}} X_{\mathfrak{M}},$$

$$\tilde{N} = -[(\hat{\Omega}_{\mathfrak{M}\mathfrak{M}})^{-1} \hat{\Omega}_{\mathfrak{M}\mathfrak{M}}]^+ N \quad (4.3)$$

and one integro-differential equation which yields the function $Y_{\mathfrak{M}}$:

$$(\hat{\Omega}_{\mathfrak{M}\mathfrak{M}} + \hat{\Omega}_B) Y_{\mathfrak{M}} = -\hat{\Omega}_B X_{\mathfrak{M}}. \quad (4.4)$$

If the solutions of Eqs. (4.3) and (4.4) are known, then

$$\Delta c = -\langle \tilde{N}, Y_{\mathfrak{M}} \rangle. \quad (4.5)$$

¹⁾ It is easy to obtain this formula by representing the solution of Eq. (2.4) in the form $\chi = X_{\mathfrak{M}} + Y_{\mathfrak{M}} + Y_{\mathfrak{N}} + X_{\mathfrak{N}}$, where X is the solution for $B = 0$ and Y the change in the solution in the presence of the field.

*[MB] $\equiv \mathbf{M} \times \mathbf{B}$.

It is convenient to solve Eq. (4.4) in a spherical system of coordinates, in which the external field operator is diagonal in the projection of the rotational moment on the direction of the magnetic field:

$$\begin{aligned}\hat{\Omega}_B Y_{l_2 m_2}(\mathbf{M}) &= im_2 \gamma B Y_{l_2 m_2}(\mathbf{M}), \\ Y_{l_2 m_2}(\mathbf{M}) &= M^l Y_{l_2 m_2}(\theta_M, \varphi_M),\end{aligned}\quad (4.6)$$

and (see (2.3))

$$\gamma = \gamma(\sigma, M^2, \theta_M) = \begin{cases} 2\mu_0 \sigma / M, & \sigma = \pm 1 \\ (2\mu_0 \hbar / M^2) \{1 + (4\mu_0 / \lambda \hbar) B M \cos \theta_M\}, & \sigma = 0. \end{cases} \quad (4.7)$$

In this set of coordinates, the inhomogeneous term of the kinetic equation (2.4) is equal (for the viscosity) to

$$N = Y_{2m}(\nu). \quad (4.8)$$

We introduce the complete set of orthonormalized states:^[3]

$$\begin{aligned}\psi_n &= \sum_{m_1, m_2} C_{l_1 m_1, l_2 m_2}^{l m} Y_{l_1 m_1}(\nu) Y_{l_2 m_2}(\mathbf{M}) T_{r_1}^{l_1}(\nu^2) L_{r_2}^{l_2}(M^2) \varphi_n(\sigma), \\ n &= (lm, l_1 l_2, r_1 r_2, s).\end{aligned}\quad (4.9)$$

In correspondence with the definition of the subspaces \mathfrak{R} and \mathfrak{M} , \mathfrak{N} states with $l_2 = 0$ enter into the expansion of the functions $X_{\mathfrak{R}}$ and \tilde{N} . States with $l_2 \neq 0$ enter into $X_{\mathfrak{M}}$ and \tilde{N} :

$$X_{\mathfrak{R}} = -\sum_{\nu} P_{\nu} \psi_{\nu}, \quad X_{\mathfrak{M}} = \sum_{\mu} Q_{\mu} \psi_{\mu}, \quad \tilde{N} = \sum_{\mu} R_{\mu} \psi_{\mu}. \quad (4.10)$$

The specific form of the coefficients P , Q , and R depend on the model of the interaction between molecules. It is clear from symmetry considerations that for all terms of the expansion (4.10), $l = 2$, while l_1 takes on only even values.

We shall assume that the operator $\hat{\Omega}_{\mathfrak{M}\mathfrak{M}}$ in the representation (4.9) is diagonal:

$$\hat{\Omega}_{\mathfrak{M}\mathfrak{M}} \psi_{\mu} = \Omega_{\mu} \psi_{\mu}. \quad (4.11)$$

This relation can be understood as the definition of the orthogonal functions T , L and φ in (4.9). But our information on the interaction between the molecules is too weak to allow us to pretend an accurate solution of (4.11). It would be more accurate to consider (4.11) as the result of the solution of the problem of the eigenvalues in a single-moment approximation, choosing normalized orthogonal polynomials as the functions T , L and φ .

Strictly speaking, the relations (4.6) and (4.11) are insufficient to obtain a simple exact solution of Eq. (4.4). The fact is that the effective gyromagnetic ratio (4.7) depends on the internal variables of the molecules and the operator $\hat{\Omega}_B$ entangles \mathfrak{M} states with different values of the numbers l , l_2 , r_2 and s (without affecting the numbers l_1 , r_1). But if we neglect the dependence of the eigenvalues of $\hat{\Omega}_{\mu}$ on these numbers, then

$$Y_{\mathfrak{M}} = -\sum_{\mu} Q_{\mu} \psi_{\mu} \{im_2 x_{\mu} (1 + im_2 x_{\mu})\}, \quad x_{\mu} = \gamma B / \Omega_{\mu} \quad (4.12)$$

(the expression $\{\dots\}$ is under the summation over m_2 , which is implied in ψ_{μ}). Substituting (4.12) in (4.5), we find

$$\begin{aligned}\Delta c_m &= -\sum_{\mu, \mu', m_1, m_2} C_{l_1 m_1, l_2 m_2}^{2m} C_{l_1 m_1, l_2 m_2}^{2m} \delta_{l_1 l_1'} \delta_{r_1 r_1'} R_{\mu} Q_{\mu'} \langle Y_{l_2 m_2} L_{r_2} \varphi_{\mu}, im_2 x_{\mu} (1 \\ &+ im_2 x_{\mu})^{-1} Y_{l_2 m_2} L_{r_2} \varphi_{\mu'} \rangle.\end{aligned}\quad (4.13)$$

The expression (4.13) essentially coincides with the corresponding expression $(\Delta c_{2m, 2m})$ which is found

in^[3] by perturbation theory, although the meaning of some of the quantities is different. This shows that the results of the theory of the Senftleben effect are not sensitive to any assumption on the "small non-sphericity" of the interaction, or, more accurately, to any assumption as to the smallness of the nondiagonal elements of the collision operator $\hat{\Omega}$ in comparison with its diagonal elements. The advantage of (4.13) in comparison with the corresponding formulas in^[3] can be noted only in the case in which we seek to obtain information about the quantities R_{μ} , Q_{μ} and $\hat{\Omega}_{\mu}$ from direct solution of Eqs. (4.3) and (4.11) by making use of the specific form of the collision probability.

At the present stage of the theory, these quantities are chosen directly from comparison with experiment (see below) and unique establishment of the operator $\hat{\Omega}$ is not possible. The situation is similar to that which prevails in the quantum theory of collisions—the scattering cross section in a strong potential can sometimes be described as the cross section in the Born approximation for a weak pseudopotential.

5. For comparison of theory with experiment, we reduce a number of parameters in the form (4.13) to a minimum. For this purpose, we assume that the terms with $l_1 = r_1 = r_2 = 0$ differ from zero in the expansions of $x_{\mathfrak{M}}$ and \tilde{N} (4.10). From the viewpoint of the method of momentums in cartesian coordinates, this means that the function $X_{\mathfrak{M}}$ is chosen in the form of a superposition of terms:

$$\{M, M\} \varphi_n(\sigma), \quad \varphi_1 = 1, \quad \varphi_2 = \sqrt{3} \sigma, \quad \varphi_3 = \sqrt{7} (\sigma^2 - 3/5). \quad (5.1)$$

In this model, the odd Senftleben effect is described by the formula

$$\begin{aligned}I_m &= \text{Im}(\Delta c_m) / c = -\Lambda_1 \langle |Y_{2m}|^2 \frac{m x_0}{1 + (m x_0)^2} \rangle \\ &- \Lambda_2 \langle |Y_{2m}|^2 m x_1 (1 + (m x_1)^2)^{-1} \rangle,\end{aligned}\quad (5.2)$$

and the even effect, by the formula

$$\begin{aligned}R_m &= \text{Re}(\Delta c_m) / c = -\Lambda_1 \langle |Y_{2m}|^2 (m x_0)^2 (1 + (m x_0)^2)^{-1} \rangle \\ &- \Lambda_2 \langle |Y_{2m}|^2 (m x_1)^2 (1 + (m x_1)^2)^{-1} \rangle.\end{aligned}\quad (5.3)$$

Here we have used the notation

$$\begin{aligned}x_0 &= (x / K^2) (1 + xy K \cos \theta_M), \\ x_1 &= x / K, \quad x = 2\mu_0 B / \hbar \Omega, \quad y = 2\hbar \Omega / \lambda.\end{aligned}\quad (5.4)$$

The three positive parameters Λ are the quadratic combinations of the coefficients R_S and Q_S corresponding to the functions (5.1).

We note that in our model $R_0 = 0$, so that the measurement of this coefficient can give direct information on the role in (4.13) of states which depend on the direction of the velocity ($l_1 \neq 0$).

In the experiments of I. K. Kikoin et al.,^[1] the coefficients I_1 , I_2 and the combinations $(R_1 - R_2)$ and $(3R_0 - 4R_1 + R_2)$ are measured as functions of the ratio B/P for different fixed pressures. In these experiments, a violation of the B/P law is observed even for small values of the magnetic field. This violation cannot be explained within the framework of the proposed theory. Evidently the anomalous decrease in the Senftleben effect at low pressures is associated with surface phenomena.

In order to eliminate this anomaly from consideration, we multiply the experimental curves by the common factor $(P + P_0)/P$, as is done in the account of

the slippage phenomenon. The parameter P_0 is determined by the geometry of the apparatus and is chosen equal to 5 mm Hg. As a result, curves are obtained that are identical for small values of B/P . These curves are satisfactorily described by the present theories within the framework of the simplest model (5.1) (see the drawing) if we assume that

$$\Lambda_1 : \Lambda_2 : \Lambda_3 = 7 : 3 : 22, \quad \Omega = 10^{10} \text{ sec}^{-1} \text{ at } 760 \text{ mm Hg.} \quad (5.5)$$

We call attention to the fact that the relaxation time of the internal degrees of freedom Ω^{-1} has the same order of magnitude as the general relaxation time, which is obtained from the usual coefficients of viscosity and thermal conductivity.

We shall now make clear on what considerations the choice of the model (5.1) is based. We see from (4.13) that quantities such as Δc are basically determined as sums of contributions from the separate terms of the expansion of the function X_{eq} (4.10). But the moment

of the form $\{M_i M_j\}$ gives a contribution of the same order to the odd effects I_1 and I_2 . At the same time, the moment of the form $\{M_i M_l\} \{v_k v_l\}$ (proposed earlier in^[5]), because of the effect of the Clebsch-Gordan coefficients, leads to the result that the contribution to I_1 turns out to be an order of magnitude smaller than that to I_2 . This means that in the description of the experiment (see the drawing), the latter moment can be regarded only as a small correction to the moments (5.1). The hint of a maximum in I_1 and I_2 in the region of small values of B/P means that the odd effect is built up not only on the component $\sigma = 0$ but also on the components $\sigma = \pm 1$.

For a description of this fact, we have introduced in (5.1) a term that is odd in σ . Finally, we shall consider the moment ψ in (5.1) so that the coefficient Λ_3 , which determines the principal part of the even effect (5.3) can be assumed to be independent of the coefficients Λ_1 and Λ_2 . Account of moments of more complicated form incorrectly increases the number of fitting parameters. We emphasize that the number of parameters in our model is reduced to a minimum: one parameter (Ω) determines the scale of the sixteen curves along the B/P axis and three parameters (Λ_1 , Λ_2 and Λ_3) the amplitudes of these curves.

In conclusion, we consider the Senftleben effect in oxygen in the region of limitingly large magnetic fields, when the spin-axis coupling is destroyed.

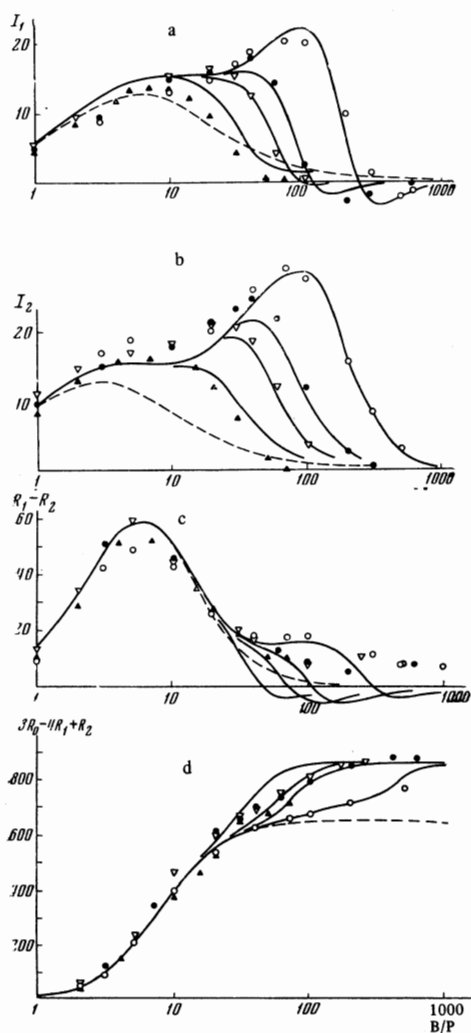
It can be thought that for $B \gg 10^4$ G, when a break occurs between the spin and the axis of the molecule, the rotational moment will not keep up with the precession of the large magnetic moment of the spin, and the paramagnetic Senftleben effect disappears and transforms upon further increase (in B) into the nonparamagnetic Senftleben-Beenakker effect. Actually, the transition from the case $\Omega \ll \omega_B \ll \omega_\lambda$, which describes the saturation region of the Senftleben effect to the case $\Omega \ll \omega_\lambda \ll \omega_B$ does not change the transport phenomena: in both cases, the inequalities $\Omega \ll \omega_\lambda$ and $\Omega \ll \omega_B$ require that the density matrix satisfy the set of equations

$$[\hat{V}_{ks}, \rho] = 0, \quad [\hat{V}_B, \rho] = 0, \quad N^{(0)} + \hat{I}_{\text{coll}} = 0. \quad (5.6)$$

It follows from the vanishing of the commutators in (5.6) that the density matrix is diagonal both in the representation $|\sigma\rangle = |K, j, j_z\rangle$ and in the representation $|S_z\rangle = |K, K_z, S_z\rangle$. The first is characteristic for saturation of the paramagnetic effect, and the second, for saturation of the nonparamagnetic effect. Therefore the transition of molecules from states of the type $|\sigma\rangle$ to states of the type $|S_z\rangle$ is not accompanied by a dip in the curve $\Delta c(B/P)$. Only in the region of high pressures ($\gtrsim 100$ atm), when $\Omega > \omega_\lambda$ can break in the spin-axis coupling materially change the character of the Senftleben effect. But here the break in this coupling takes place primarily from collisions.

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Dependence of the quantities I_1 (a), I_2 (b), $(R_1 - R_2)$ (c) and $(3R_0 - 4R_1 + R_2)$ (d) on the ratio B/P . The experimental points \circ , \bullet , ∇ and \blacktriangle refer respectively to the pressures 6.2, 24, 50 and 150 mm Hg. The continuous curves indicate the results of calculations by the formulas (5.2), (5.3). The dashed curves indicate the contribution of states with $\sigma = \pm 1$.

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