ULTRASONIC ABSORPTION BY SMALL SIZE CONDUCTORS IN A MAGNETIC FIELD

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Ultrasonic absorption by thin conductors whose thickness d is less than the electron mean free path l but much greater than the sound wavelength is studied theoretically. It is shown that the sound energy absorption coefficient Γ is sensitive to the character of the electron reflection from the sample boundary. If this reflection is almost specular, then resonance oscillations of Γ will arise in the presence of a weak magnetic field (Larmor radius $r > d$) parallel to the surface of the thin plate. It is also shown that acoustic cyclotron resonance should be possible in a weak magnetic field provided that the sound frequency exceeds the intra-volume collision frequency. In strong magnetic fields ($r \ll d$), oscillations of Γ are studied that are similar to Sondheimer oscillations of the resistance of thin conductors. The electron specular reflection parameter and the radius of curvature of the Fermi surface can be determined experimentally on the basis of the effects mentioned.

IN metals, the energy of sound waves is chiefly absorbed by the conduction electrons, and in an external magnetic field the sound absorption coefficient Γ turns out to be very sensitive to the characteristics of the electron energy spectrum. In bulk samples whose dimensions d greatly exceed the sound wavelength $\lambda\!\!\!^-$ and the electronic mean free path l, a whole series of resonance and oscillation magneto-acoustic effects take place[1] in strong magnetic fields (Larmor radius $r \ll l$), which have been used with success for establishing the Fermi surface.[2]

Recently, in connection with newly developed possibilities of obtaining pure materials, interest has increased in investigations of the kinetic and thermodynamic characteristics of thin samples ($d \ll l$), where the presence of the metal boundaries significantly changes the character of the motion of the conduction electrons, and leads at low temperatures to the appearance of a discrete spectrum of surface states[3] if the electrons are almost specularly reflected from the surface of the sample. Allowance for the magnetic surface levels leads in a number of cases to some interesting features in the dependence of the sound absorption on the magnetic field H.[4] However, we shall not consider the effect of the sample boundaries on the electron energy spectrum, and shall show that a whole series of classical magneto-acoustic effects take place in thin conductors. These are connected only with the change in the dynamics of the conduction electrons because of their reflection from the sample boundaries.

In the case of a different character of scattering of electrons by the metallic surface, the dependence of Γ on the value of the magnetic field is seen to be essentially different, which allows us to establish the degree of specularity of the reflection experimentally. Magneto-acoustic measurements in thin samples can also serve as a source of additional information of the Fermi surface. Here, in contrast with the case of bulk samples, a weak magnetic field is already sufficiently effective. In a magnetic field parallel to the surface of the thin plate, the electrons that are specularly reflected from the surface of the conductor move along open periodic orbits (Fig. 1) and the period of the motion is much less than the time of free flight if $r \ll l^2/d$. In this case the sound absorption coefficient $\Gamma(H)$ oscillates with the magnetic field if the wave vector of the sound **k** does not coincide with the normal to the surface of the plate. These oscillations are connected with the fact that for changes in the electron displacement along k by a sound wavelength within one period the conditions of effective interaction of the electron with the sound wave are again repeated.

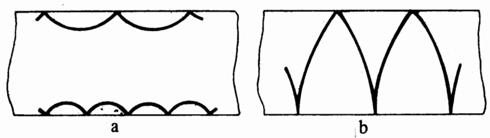

FIG. 1. Types of electron orbits in a magnetic field parallel to the surface of the plate for $q \approx 1$ and $2r > d$.

For sound propagation normal to the plate, the smooth dependence of Γ on the value of the magnetic field contains sufficiently detailed information on the character of the reflection of the electrons by the surface of the sample. The reason is that all the electrons in specular reflection interact in resonant fashion with the sound field and $\Gamma(H)$ increases materially in comparison with the sound absorption coefficient in the absence of a magnetic field, Γ_0, while in diffuse scattering of the electrons by the surface of the plate, the periodicity of the motion of the electrons is absent and Γ even falls off with increase of the weak magnetic field. In a previous work of the authors,[5] a method was shown for the determination of the specularity parameters q from the $\Gamma(H)$ curve.

The magneto-acoustic effects will be considered below for arbitrary orientations of the vectors **k** and **H** under the assumption that the wavelength of the sound is much less than the thickness of the conductor. This enables us to ignore the Rayleigh waves, i.e., for sound waves, the conductors under study are almost bulk conductors and the boundary conditions are important only for the conduction electrons.

1. For the determination of the sound absorption

coefficient, we compute the electron dissipation function

$$T\dot{S} = \overline{\langle \psi \hat{W} \psi^* \rangle} \equiv \frac{2}{h^3} \overline{\int\limits_{\varepsilon(\mathbf{p})=\varepsilon_F} \psi \hat{W} \psi^* \frac{dS_p}{v}}, \tag{1}$$

where T is the temperature, $\dot{S}$ the change in the entropy density of the electrons per unit time, dS_p the element of area of the Fermi surface $\epsilon(p) = \epsilon_F$, and h Planck's constant. The superior bar indicates averaging over the volume of the conductor.

The function ψ characterizes the departure of the electron distribution function from the equilibrium Fermi function $f_0(\epsilon)$

$$f(t, \mathbf{r}, \mathbf{p}) = f_0(\varepsilon) - \frac{\partial f_0}{\partial \varepsilon} \psi(t, \mathbf{r}, \mathbf{p}) \tag{2}$$

and is determined by means of the kinetic equation of Boltzmann, linearized in the small deformation tensor u_{ik}[1]

$$\mathbf{v}\frac{\partial \psi}{\partial \mathbf{r}} - i\omega\psi + \frac{e}{c}[\mathbf{vH}]\frac{\partial \psi}{\partial \mathbf{p}} + \hat{W}\psi = \left(\lambda_{ik} - \frac{\langle \lambda_{ik} \rangle}{\langle 1 \rangle}\right)\dot{u}_{ik} \equiv g. \tag{3*}$$

Here ω is the sound frequency, e the electronic charge, and c the speed of light. The collision integral W takes into account the scattering of the electrons inside the conductor; in what follows, we shall assume W to be the operator of multiplication of the function ψ by the frequency of the intra-volume collisions ν, entirely for the sake of convenience. The collisions of the electrons with the surface of the sample will be taken into account by means of the boundary condition for the function[6] which we shall write down in the form

$$\psi(t, \mathbf{r}_S, p_\eta, p_\zeta, p_\xi')|_{v_\xi > 0} = q(\mathbf{p})\, \psi(t, \mathbf{r}_S, p_\eta, p_\zeta, p_\xi)|_{v_\xi < 0} + \chi(t, \mathbf{r}_S, \varepsilon_F). \tag{4}$$

Using the condition of absence of current through the surface of the sample

$$j_n{}^s = \langle e v_\xi \psi^s \rangle = 0, \tag{5}$$

we can express the function χ in terms of the distribution function of the electrons incident on the surface of the conductor at the point $\mathbf{r}_S$

$$\chi = \frac{\langle (1-q) v_\xi \psi^s \rangle}{\langle v_\xi \rangle}\bigg|_{v_\xi < 0}. \tag{6}$$

The ξ axis coincides with the internal normal to the surface of the sample at the point $\mathbf{r}_S$, and the η and ξ axes are located in the adjoining planes.

We only take into account here the deformation mechanism of sound absorption,[7] which is connected with the fact that in a metal deformed by a sound wave, the energy of the electron acquires a time dependent contribution that is proportional to the deformation tensor $\delta\epsilon = \lambda_{ik}(p) u_{ik}$ in first approximation; here the coefficients λ_{ik} are identical in order of magnitude with the chemical potential of the electrons in the undeformed metal, μ_0. This mechanism is practically always fundamental. In certain cases induction absorption,[8] which is due to electric fields that appear in the conductor because of the inhomogeneity of the deformations in the field of the sound waves, is also important. However, account of this mechanism of dissipation of the sound energy does not lead to any significant change in the dependence of Γ on the value of the magnetic field. The role of the remaining mechanisms of sound absorption (absorption due to thermal conductivity, entrapment of the electrons by the phonons[9], and so forth) at temperatures below the Debye temperature are not large.

For small sound wavelengths, the dependence of g on the coordinates, which is connected with the presence of sample boundaries and with the spatial dispersion of the sound, can be separated out:

$$g(t, \mathbf{r}, \mathbf{p}) = g_0(\mathbf{r}, \mathbf{p})\, e^{i(\mathbf{kr} - \omega t)}.$$

The solution of the kinetic equation (3) must be sought in such a form:

$$\psi(t, \mathbf{r}, \mathbf{p}) = \psi_0(\mathbf{r}, \mathbf{p})\, e^{i(\mathbf{kr} - \omega t)}, \tag{7}$$

where the function ψ_0 satisfies the equation

$$\left\{ i(\mathbf{kv} - \omega) + \nu + \frac{\partial}{\partial \tau} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} \right\} \psi_0(\mathbf{r}, p_z, \tau) = g_0(\mathbf{r}, p_z, \tau). \tag{3a}$$

Here τ is the time of motion of the electron in the magnetic field, i.e., the phase in the orbit ϵ = const, p_z = const (p_z is the projection of the momentum in the direction of the magnetic field). The boundary condition (4) can be satisfied by an appropriate choice of the solution of Eq. (3a):

$$\psi_0(\mathbf{r}, p_z, \tau) = f(\mathbf{r} - \mathbf{r}(\tau))\, \mathscr{E}_\lambda^\tau + \int\limits_\lambda^\tau d\tau' g_0(\tau', p_z, \mathbf{r} + \mathbf{r}(\tau') - \mathbf{r}(\tau))\, \mathscr{E}_{\tau'}^\tau, \tag{8}$$

where f is an arbitrary function of its argument, $\mathscr{E}_a^b = \exp\{\int_a^b \alpha(\tau) d\tau\}$ $\alpha = -i(\mathbf{k}\cdot\mathbf{v} - \omega) - \nu$; $\mathbf{r}(\tau) = \int^\tau \mathbf{v}(\tau') d\tau'$, and λ is the moment of reflection of the electron from the surface of the sample, that is, the root of the equation

$$\mathbf{r} - \mathbf{r}(\tau) = \mathbf{r}_S - \mathbf{r}(\lambda); \quad \lambda \leqslant \tau \tag{9}$$

that is closest to τ. For electrons, which do not generally collide with the sample boundaries, one should set $\lambda = -\infty$.

On the surface of the conductor, the function f coincides with the distribution function of the reflected electrons and, by force of Eq. (9), we have

$$f(\mathbf{r} - \mathbf{r}(\tau)) = f(\mathbf{r}_S - \mathbf{r}(\lambda)) = \psi_0(\mathbf{r}_S, p_z, \lambda)|_{v_\xi > 0}.$$

In the case of diffuse scattering of the electrons (q = 0), the function f depends only on the energy of the electrons and is easily determined from Eq. (5). In scattering that is almost specular ($q_1 \equiv 1 - q \ll 1$), difficulties also arise in the determination of the function f, since $f(\mathbf{r} - \mathbf{r}(\tau))$ is constant along the trajectory of the electron between two collisions of the electron with the surface of the sample.

In a thin conducting plate ($d \ll l, r$) placed in a magnetic field parallel to its surfaces $\xi = 0$ and d, it is possible to obtain the exact solution of the kinetic equation (3a) for any character of reflection of the electrons by the sample boundaries. For example, for electrons colliding only with one of the surfaces of the plate, the function $f(\xi - \xi(\tau))$ is constant along the entire open orbit of the electron and is equal to

$$f(-\xi(\lambda_1)) = \frac{1}{1 - q\mathscr{E}_{\lambda_2}^{\lambda_1}} \left[\chi_0 + q \int\limits_{\lambda_2}^{\lambda_1} d\tau'\, g_0(\tau', p_z, \xi(\tau') - \xi(\lambda_1))\, \mathscr{E}_{\tau'}^{\lambda_1} \right]. \tag{10}$$

*$[\mathbf{vH}] \equiv \mathbf{v} \times \mathbf{H}$.

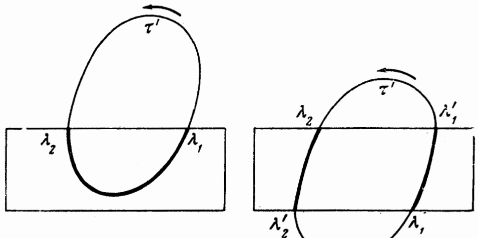

FIG. 2. Portions of electronic orbits in an unbounded metal formed by the trajectories of the electrons in a thin plate ($2r > d$) of type a and b.

As before λ_1 is the root of Eq. (9) closest to τ, and the constant χ_0 on the Fermi surface is determined easily from condition (6). For electrons colliding with both surfaces of the plate, the function f has only two different values along the entire trajectory, because of the periodicity of the trajectory of the electron:

$$f_1 = \frac{1}{1 - q^2 \mathscr{E}_{\lambda_1}^{\lambda_1'} \mathscr{E}_{\lambda_2}^{\lambda_2'}} \left[\chi_0 + q \int_{\lambda_2}^{\lambda_2'} d\tau' g_0 \mathscr{E}_{\tau'}^{\lambda_2'} + q \mathscr{E}_{\lambda_2}^{\lambda_2'} \left\{\chi_0 + q \int_{\lambda_1}^{\lambda_1'} d\tau' g_0 \mathscr{E}_{\tau'}^{\lambda_1'}\right\}\right],$$
$$(v_\xi(\lambda_1) < 0);$$
$$f_2 = \frac{1}{1 - q^2 \mathscr{E}_{\lambda_1}^{\lambda_1'} \mathscr{E}_{\lambda_2}^{\lambda_2'}} \left[\chi_0 + q \int_{\lambda_1}^{\lambda_1'} d\tau' g_0 \mathscr{E}_{\tau'}^{\lambda_1'} + q \mathscr{E}_{\lambda_1}^{\lambda_1'} \left\{\chi_0 + q \int_{\lambda_2}^{\lambda_2'} d\tau' g_0 \mathscr{E}_{\tau'}^{\lambda_2'}\right\}\right],$$
$$(v_\xi(\lambda_2) > 0), \tag{11}$$

where

$$\chi_0 = \frac{\langle (1-q) v_\xi \psi_0 \rangle_-}{\langle v_\xi \rangle_-}\bigg|_{\xi=0} = \frac{\langle (1-q) v_\xi \psi_0 \rangle_+}{\langle v_\xi \rangle_+}\bigg|_{\xi=d},$$

and the signs + and − denote integration over these portions of the Fermi surface where $v_\xi > 0$ and $v_\xi < 0$, respectively. In Eqs. (10) and (11), τ' denotes the phase on the orbit of the electron in unbounded space,[5] and λ_2' and λ_1' are the roots of the equations $\xi(\lambda_1') - \xi(\lambda_1) = -d$ and $\xi(\lambda_2') - \xi(\lambda_2) = d$ (see Fig. 2).

Using the relations (8), (10), and (11), there is no difficulty in finding the sound energy absorption coefficient

$$\Gamma = \overline{2\langle \nu \psi_0 \psi_0^* \rangle} / \rho \dot{u}^2, \tag{12}$$

where ρ is the density of the crystal and $\dot{u}$ the rate of migration of the atoms under the action of the sound wave.

2. We first consider the case in which the sound wave vector k is perpendicular to the direction of the magnetic field and makes an angle θ with the ξ axis, while the scattering of the electrons by the surface of the plate is almost specular ($q_1 \equiv 1 - q \ll 1$). For not too small values of θ ($\theta \gg \sqrt{d/r}$), the sound absorption is due chiefly to electrons colliding with both surfaces of the plate (Fig. 1b), since there are points of stationary phase $\mathbf{k} \cdot \mathbf{v}(\varphi) = 0$ only on orbits of such a type. Assuming the Fermi surface to be symmetric relative to the plane (p_ξ, p_η)[1)] we write down the sound absorption coefficient in the form

$$\Gamma \approx -\frac{8eHv}{\rho \dot{u}^2 ch^3 d} \int dp_z \int_{\varphi-\sigma(\varphi)}^{\varphi} d\lambda\, v_\xi(\lambda) \left\{ \int_{\lambda}^{\lambda'} |f_2|^2 d\tau + \int_{-\lambda'}^{-\lambda} |f_1|^2 d\tau \right\}, \tag{13}$$

[1)]This condition allows us to separate the contribution of the electrons incident on the surface of the plate at a particular angle β, which is important for the experimental investigation of the dependence $q(\beta)$. The calculation can be carried out for any electron dispersion law.

where $\sigma(\varphi)$ is determined from the condition $\xi(\varphi) - \xi(\varphi - \sigma) = -d$, and we omit from the functions f_1 and f_2 of (11) the terms proportional to χ_0, which are small when $q_1 \ll 1$. When $kd^2/r \gg 1$, the integrals with respect to τ' in Eqs. (11) are computed by the stationary phase method which, after uncomplicated transformations, gives

$$\Gamma \approx -\frac{8\pi eHv}{\rho \dot{u}^2 ch^3 d}\left\{I_0 + 2\sum_{n=1}^{\infty} I_n\right\}, \tag{14}$$

$$I_n = \int dp_z \frac{|g_0(\varphi)|^2}{|\mathbf{k}\mathbf{v}_\varphi'|} \int_{\varphi-\sigma}^{\varphi} d\lambda \frac{v_\xi(\lambda)\sigma(\lambda)}{\delta(\lambda)} e^{-n\delta(\lambda)} \cos[nA(\lambda, p_z)]. \tag{15}$$

Here $\delta(\lambda) = 2\,(q_1 + \nu\sigma)$ and

$$\frac{1}{k} A(\lambda, p_z) = 2 \sin\theta \int_{\lambda}^{\lambda'} v_\eta\, d\tau$$

is the displacement of the electron in the direction of propagation of the sound over a period $T_\lambda = 2\,(\lambda' - \lambda)$. For $n > 1$, the integrand in (15) oscillates rapidly with change in λ and p_z and, inasmuch as $\partial A/\partial\lambda \neq 0$ for $\varphi - \sigma \leq \lambda \leq \varphi$, the principal contribution to the integral with respect to λ is made by the ends of the integration interval:

$$I_n \approx \int dp_z \frac{|g_0(\varphi)|^2}{|\mathbf{k}\mathbf{v}_\varphi'|}\left\{\left(\frac{v_\xi \sigma e^{-n\delta}}{n\delta A'}\right)_\varphi \sin[nA(\varphi)] - \left(\frac{v_\xi \sigma e^{-n\delta}}{n\delta A'}\right)_{\varphi-\sigma} \sin[nA(\varphi-\sigma)]\right\} \approx -2\int dp_z \frac{|g_0(\varphi)|^2}{|\mathbf{k}\mathbf{v}_\varphi'|}\left(\frac{v_\xi\sigma}{n\delta A'}\right)_\varphi$$
$$\times e^{-n\delta(\varphi)} \cos(2nkd\cos\theta) \sin\left[\frac{nkd^2}{r_\varphi \sin^2\theta}\right],$$

where r_φ is the radius of curvature of the electron trajectory at the point $\tau = \varphi$. The integral over p_z is computed by the saddle-point method. Assuming for simplicity that there is only one point $p_z = p_e$ such that $dr_\varphi/dp_e = 0$, we get

$$I_n = 2\left\{\frac{|g_0(\varphi)|^2}{|\mathbf{k}\mathbf{v}_\varphi'|}\left(\frac{v_\xi \sigma}{A'\delta}\right)_\varphi \left|\frac{kd^2}{\pi r_\varphi \sin^2\theta}\frac{d^2 r_\varphi}{dp_z^2}\right|^{-1/2} \frac{e^{-n\delta(\varphi)}}{n^{3/2}}\right\}_{p_z=p_e}$$
$$\times \cos(2nkd\cos\theta) \sin[nkd^2/r_{\varphi e}\sin^2\theta + \pi/4]. \tag{16}$$

The sound absorption coefficient consists of a component that depends weakly on the magnetic field and a resonant part that oscillates rapidly with change in H:

$$\Gamma(H) \approx \frac{2\Gamma_0 d}{l\delta(\theta)\sin\theta}\left\{h_1 + h_2\left(\frac{kd^2}{r_{\varphi e}\sin^2\theta}\right)^{-1/2} \sum_{n=1}^{\infty} \frac{e^{-n\delta(\theta)}}{n^{3/2}} \cos(2nkd\cos\theta) \times \sin\left[\frac{nkd^2}{r_{\varphi e}\sin^2\theta} + \frac{\pi}{4}\right]\right\}. \tag{17}$$

Here

$$\delta(\theta) \equiv 2(q_1 + d/l \sin\theta), \quad \Gamma_0 = \frac{8\pi^2}{\rho \dot{u}^2 h^3 k}(m^* |g_0(\varphi)|)_e^2 \sim \frac{n_0 \mu_0 \omega}{\rho s v},$$

m^* is the cyclotron mass, n_0 the electron density, s the speed of sound and h_1 and h_2 numerical factors of the order of unity.

As is seen from the formulas written out above, the resonant part of $\Gamma(H)$ is determined by the characteristics of a small group of electrons that are incident on the surface of the sample at angles close to θ. If $\cos(2nkd\cos\theta) = 1$, Eq. (17) is easily symmetrized at the points $H = H_m$, where $kd^2/r_{\varphi e}\sin^2\theta = 2m$. By computing $\partial\Gamma/\partial H_m$ also, we can easily obtain the relation

$$q_1 + \frac{d}{l\sin\theta} \approx \frac{\pi}{2}\left[\frac{\Gamma_{\text{res}}(H_m)}{\partial\Gamma/\partial H_m}\frac{1}{\Delta(H)\zeta_R(3/2)}\right]^2, \tag{18}$$

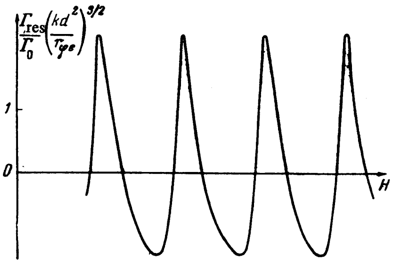

FIG. 3. Resonance oscillations of the ultrasonic absorption coefficient in a weak magnetic field in the case of almost specular scattering of the electrons by the surface of the sample.

which allows us to determine experimentally the dependence of the specularity coefficient of the reflection of electrons from the surface of the plate on the angle of incidence ($\xi_R(3/2) \approx 2.612$ is the Riemann zeta function).

From the period of oscillations of $\Gamma(H)$

$$\Delta(H) = 2\pi c P \sin^2\theta / ekd^2 \tag{19}$$

we can find $P = eHr_{\varphi e}/c$, the radius of curvature of the cross section of the Fermi surface $p_z = p_e$ at the point $\mathbf{k}\cdot\mathbf{v} = 0$.

In the case $\theta = \pi/2$, the expression for the resonant part of $\Gamma(H)$ is simplified and has the form

$$\Gamma_{res} \sim \frac{2\Gamma_0 d}{l\delta(\pi/2)}\left(\frac{kd^2}{r_{\varphi e}}\right)^{-3/2}\sum_{n=1}^{\infty}\frac{e^{-n\delta(\pi/2)}}{n^{3/2}}\sin\left(\frac{nkd^2}{r_{\varphi e}}+\frac{\pi}{4}\right) \tag{20}$$

(see Fig. 3).

At small angles θ $\sqrt{d/r} \gg \theta \gg 1/\sqrt{kr}$), the sound is absorbed chiefly by electrons colliding only with one of the surfaces of the plate. In this case, the absorption coefficient is

$$\Gamma \approx -\frac{8eH\mathrm{v}}{\rho \dot{u}^2 ch^3 d}\int dp_z \int_{\varphi}^{\sqrt{d/v_\perp}\Omega} d\lambda\, v_\xi(\lambda)\int_{-\lambda}^{\lambda}|f|^2\, d\tau, \tag{21}$$

where $v_\perp$ is the projection of the velocity of the electron on the plane $p_z = \mathrm{const}$ at the turning point ($v_\xi = 0$), chosen for the beginning of measurement of λ.

Calculations similar to the above lead to the following result

$$\Gamma(H) \approx \Gamma_0\frac{\sqrt{dr_0}}{l\delta_0}\left\{1 + (k\theta\sqrt{dr_0})^{-3/2}\sum_{n=1}^{\infty}\frac{e^{-n\delta_0}}{n^{3/2}}\sin\left(2nk\theta\sqrt{2dr_0} - \frac{\pi}{4}\right)\right\}, \tag{22}$$

where $\delta_0 = a_1 + 2\sqrt{2}dr_0/l$, r_0 is the radius of curvature of the trajectory of the electron at the point $v_\xi = 0$, $p_z = p_e$, and the numerical factors of order unity are omitted. The electrons responsible for the resonant oscillations of $\Gamma(H)$ at small θ are those colliding with the surface at an angle $\beta \approx \sqrt{2d/r_0}$.

At angles θ less than $1/\sqrt{kr}$, the oscillations of the absorption coefficient are almost entirely absent and for $\theta < 1/kl$, the expression (21) is identical with the result obtained in[5] for $\theta = 0$. It must be noted that for $\theta = 0$ and in the bulk sample ($d \gg l \gg \lambda$), the presence of groups of electrons skipping along each of the surfaces leads to a significant increase in the absorption coefficient in a weak magnetic field:

$$\Gamma(H) \sim \Gamma_0(1 + d/r) \qquad (l^2/\lambda \gg r \gg d). \tag{23}$$

3. The most convenient case experimentally is the one in which the sound wave vector $\mathbf{k}$ and the magnetic field $\mathbf{H}$ are parallel to the surface of the plate. Here the points $\mathbf{k}\cdot\mathbf{v}(\varphi) = 0$, at which the electrons effectively interact with the field of the sound waves, occur only for $p_z \le p_0(\theta')$ (θ' is the angle between $\mathbf{k}$ and $\mathbf{H}$) and, depending on the value of p_z, can be found on orbits of different types. However, the basic contribution to the resonant part of the sound absorption coefficient is made by electrons colliding with both surfaces of the plate. Using Eq. (11) for the distribution function of such electrons, we write down the resonant part of $\Gamma(H)$ in the form

$$\Gamma_{res} \approx -\frac{16\pi eH\mathrm{v}}{\rho \dot{u}^2 ch^3 d}\sum_{n=1}^{\infty}\int_0^{p_0(\theta')}\frac{dp_z}{|\mathbf{k}\mathbf{v}_\varphi'|}\cdot$$

$$\cdot\int_{\varphi-\sigma(\varphi)}^{\varphi} d\lambda\frac{v_\xi\sigma}{\delta}\{|g_0(\varphi)|^2 + |g_0(-\varphi)|^2 + 2g_0(\varphi)g_0^*(-\varphi)\sin B\}e^{-n\delta}\cos(nA), \tag{24}$$

where $\sigma(\lambda, p_z)$ is the root of the equation $\xi(\lambda) - \xi(\lambda + \sigma) = d$, and $\delta(\lambda, p_z) \equiv 2(q_1 + \nu\sigma)$. The functions

$$A(\lambda, p_z) = 2\int_{\lambda}^{\lambda+\sigma}\mathbf{k}\mathbf{v}\,d\tau, \qquad B(\lambda, p_z) = 2\int_{\lambda}^{\varphi}\mathbf{k}\mathbf{v}\,d\tau \tag{25}$$

describe the displacement of the electron in a single period and the distance between the two nearest saddle points on the given orbit along the wave vector $\mathbf{k}$. Completing the integration in (24) by the saddle-point method, it is not difficult to establish the fact that the sound absorption coefficient undergoes oscillations of a resonant type with change in the value of the magnetic field. These oscillations (Fig. 3) are periodic in the magnetic field:

$$\Gamma_{res} \sim \frac{2\Gamma_0 d}{l\delta(\pi/2)}\left(\frac{kd^2}{r_{\varphi e}}\sin\theta'\right)^{-3/2}\sum_{n=1}^{\infty}\frac{e^{-n\delta(\pi/2)}}{n^{3/2}}\sin\left(\frac{nkd^2}{r_{\varphi e}}\sin\theta' + \frac{\pi}{4}\right). \tag{26}$$

Here as before, $\delta(\pi/2) = /(q_1 + d/l)$ and $r_{\varphi e}$ is the radius of curvature of the electron trajectory at the point $\mathbf{k}\cdot\mathbf{v} = 0$ on the central cross section of the Fermi surface. As $\theta' \to \pi/2$, Eq. (26) becomes identical with the result (20). In the angle range $\theta' < r/kd^2$ the oscillations of $\Gamma(H)$ virtually disappear and

$$\Gamma \sim \Gamma_0/(1 + q_1 l/d). \tag{27}$$

4. Up to this time, by assuming $\omega/\nu \ll 1$, we have assumed the phase distribution of the sound wave in the sample to be static. However, the case $\omega/\nu \gg 1$ is very interesting, although difficult to achieve experimentally.[2)] Here, the phase of the sound wave changes by an amount $\omega T_\lambda \gg 1$ within the period T_λ of motion of the electron along the open periodic orbit. Under the condition $\omega T_\lambda = 2m\pi$ (m is a positive integer), resonance appears, analogous to the acoustic cyclotron resonance in the bulk specimen.[11] For ultrasound propagating normal to the plate, simple calculation according to (21) for $\nu T_\lambda \approx 2\sqrt{2dr}/l \ll 1$ leads to the result

$$\Gamma(H) \sim \Gamma_0\frac{\sqrt{dr_0}}{l\delta_0}\left\{1 + \left(\frac{\omega}{\Omega}\sqrt{\frac{2d}{r_0}}\right)^{-3/2}\sum_{n=1}^{\infty}\frac{e^{-n\delta_0}}{n^{3/2}}\sin\left(\frac{2n\omega}{\Omega}\sqrt{\frac{2d}{r_0}} + \frac{\pi}{4}\right)\right\}, \tag{28}$$

where, as before, $\delta_0 \equiv q_1 + 2\sqrt{2dr_0}/l$, Ω is the Larmor frequency and r_0 the extremal radius of curvature of

[2)]The case $\omega/\nu \gg 1$ can be achieved at least in Ga at low temperatures.[10]

the trajectory of the electron at the turning point $v_\xi = 0$. As in the case of cyclotron resonance in a thin plate,[12] the study of this effect allows us to obtain information on the curvature of the Fermi surface and on the character of the reflection of the electrons from the surface of the sample.

5. In the effects considered above, the periodicity of motion of the electrons reflected specularly from the surface of the plate in a weak ($2r > d$) magnetic field parallel to the surface was very important. If the magnetic field is inclined at an angle γ to the surface of the plate, this periodicity is destroyed: in each collision with the boundaries of the sample, the electron proceeds along the orbit with new values of p_z and τ. However, for small values of γ, the trajectory remains quasiperiodic and, by calculating the change in the electronic displacement along **k** within a "period", produced by the jumps δp_z and $\delta\tau$, it is easy to find the region of angles of inclination of the magnetic field in which the results obtained above remain valid:

$$\gamma \ll (r/d)^{1/4}(kl)^{-1/2} \quad (\theta \ll \sqrt{d/r});$$
$$\gamma \ll (r/kld)^{1/2} \quad (\theta \gg \sqrt{d/r}). \tag{29}$$

For large angles γ, the oscillatory effects become diffuse; for $\gamma \sim 1$ the periodicity of motion of the electrons is entirely absent and the weak magnetic field has practically no effect on the sound absorption coefficient: $\Gamma(H) \approx \Gamma_0$.

In a strong magnetic field ($r \ll d$) inclined to a plate surface that scatters electrons diffusely, the electric conductivity of the metal undergoes the oscillations described by Sondheimer.[13] A similar effect should be observed in the ultrasonic absorption of sound propagating normal to the plate, as a consequence of the periodic dependence of $g_0(\tau)$:

$$g_0(\tau) = \sum_{n=-\infty}^{+\infty} g_n e^{in\Omega\tau}.$$

The ultrasonic absorption coefficient in the case $r \ll \lambda\!\!\!^{-}$[3)] can be written in the form

$$\Gamma \approx \frac{16\pi}{\rho \dot{u}^2 h^3 d} \int m^* dp_z \sum_{n=-\infty}^{+\infty} \int_0^d d\xi \left|\frac{g_n}{\alpha_n}\right|^2 \left|1 - \exp\left(-\frac{\alpha_n \xi}{\bar{v}_\xi}\right)\right|^2, \tag{30}$$

where

$$\alpha_n \equiv ik\bar{v}_\xi + \nu + in\Omega, \qquad \bar{v}_\xi = \frac{\Omega}{2\pi} \int_0^{\Omega/2\pi} v_\xi d\tau.$$

The oscillating part of Γ is determined by the contribution of electrons close to the limiting point (along **H**) of the Fermi surface, which drift between the surfaces of the plate:

$$\Gamma_{osc} \approx \frac{32\pi\nu}{\rho \dot{u}^2 h^3 d} \operatorname{Im} \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{+\infty} \int m^* dp_z \frac{|g_n|^2 \bar{v}_\xi}{n^3 \Omega^3} \exp\left(-\frac{\alpha_n d}{\bar{v}_\xi}\right). \tag{31}$$

By calculating the integral over p_z and keeping only the first harmonic in (31), we get

$$\Gamma \approx \Gamma_0 \left[1 + \frac{kr^5}{ld^3} \cos(kd) \sin\left(\frac{\Omega d}{v_H \sin\gamma}\right)\right], \tag{32}$$

where v_H is the velocity of the electron at the limiting point.

Account of the induction mechanism of damping of the sound also leads to a result of form (32) where, in place of Γ_0, there is a smooth function of the magnetic field, which reaches a maximum and can be comparable with Γ_0 in the region of magnetic fields in which the electromagnetic wavelength λ_{em} generated by the sound field in the metal is identical with s/ω.[8] The relative amplitude of the oscillations of $\Gamma(H)$ remains unchanged here.

Thus, in thin conductors, besides the well-known types of oscillations of $\Gamma(H)$ in a strong magnetic field ($2r < d$), a series of new magneto-acoustic resonance effects take place in a weak field ($2r > d$). These are due to the change in the dynamics of the conduction electrons because of their reflection from the sample boundaries. The experimental study of these effects allows us to measure the curvature of the Fermi surface directly.

However, resonance effects in a weak magnetic field are possible only for sufficiently high degree of specularity ($q_1 \ll 1$) of the reflection of those electrons which are responsible for oscillations of each type. For sound propagated at not too small an angle θ to the normal ($\theta \gg d/r$), these are electrons colliding with both surfaces of the plate at angles close to θ. Specular reflection of such electrons and, consequently, oscillations of the given type can be expected only in semimetals. However, for oscillations of the resonant type at small angle ($\theta < \sqrt{d/r}$), and also for acoustic cyclotron resonance (for $\omega/\nu > 1$), these electrons which skip along each of the surfaces of the plate at small angles ($\beta \sim \sqrt{d/r}$) are important, so that these effects can be observed even in samples of excellent metals with sufficiently smooth surfaces. Oscillations of the Sondheimer type in a strong field ($r \ll d$), which are connected with diffuse scattering of the electrons by the surfaces of the samples, can also be important in metals.

Such a high degree of sensitivity of magnetoacoustic effects to the character of the interaction the electrons with the boundaries of a conductor allow us to establish experimentally the presence of specular reflection of the electrons by the surface of a given sample and also to measure the parameter of specularity and its dependence on the angle of incidence.

3)The oscillations of $\Gamma(H)$ in the opposite limiting case were considered by Kaner and Fal'ke. [14]

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