TRANSITION RADIATION OF γ RAYS IN A CRYSTAL

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An analysis has been made of the production of transition γ radiation in a crystal. It is shown that transition γ rays with a wave vector satisfying the Bragg condition have in the crystal an anomalous penetration similar to the anomalous penetration through a crystal of beams of γ rays and x rays. It is shown also that this leads to a sharp increase in the intensity of transition radiation under the conditions of diffraction.

IT is known (see, for example, refs. 1 and 2) that the main difficulties in the experimental study of transition radiation in the x-ray region are due to the low intensity of the transition radiation and to the fact that the γ rays are emitted at practically the same angle as the electrons which produced them. As will be shown below, these difficulties can apparently be avoided if the transition radiation is obtained by passing electrons through a crystalline target in such a way that the γ rays arising undergo diffraction in the crystal.

It is well known that transition radiation from a system of plates is enhanced in comparison with radiation from a single plate.[1] As a consequence of the periodic location of the atoms (nuclei) in a crystal, it can be considered as a system of a very large number of atomic planes (microplates) and one can hope that under appropriate conditions the intensity of the transition radiation will increase. It is true that, at first glance, it appears that the strong absorption of resonance (Mössbauer) γ rays and x rays will prevent a substantial enhancement. However, as we have pointed out recently,[3] transition γ rays with a wave vector satisfying the Bragg condition have an anomalous penetrating power in the crystal similar to that of beams of γ rays and x rays,[4] which leads to a sharp increase in the intensity of transition radiation under diffraction conditions.

Let us consider this phenomenon in more detail. Maxwell's equations in momentum space for the electric field strength in a crystal are similar to those given by Garibyan[5] with replacement of the usual relation $\mathbf{D}(\mathbf{k}, \omega) = \epsilon(\omega)\mathbf{E}(\mathbf{k}, \omega)$ by the corresponding expression for the crystal

$$\mathbf{D}(\mathbf{k}, \omega) = \sum_{\tau} \varepsilon(\omega, \tau)\mathbf{E}(\mathbf{k}+\tau, \omega), \tag{1}$$

where $\tau/2\pi$ is the reciprocal-lattice vector. For wave vectors $\mathbf{k}$ satisfying the Bragg diffraction conditions

$$\big|\,|\mathbf{k}+\tau| - |\mathbf{k}|\,\big| \lesssim |\mathbf{k}|\,|\varepsilon - 1|, \tag{2}$$

only two waves are important in Eq. (1): $\mathbf{E}(\mathbf{k})$ and $\mathbf{E}(\mathbf{k} + \tau)$, which satisfy a coupled system of equations.

The solution of Maxwell's equations presents no fundamental difficulties, but in the general form is rather tedious. Therefore we will calculate only the radiation field component E'_y perpendicular to the vectors $\mathbf{v}$ and τ. It is well known that for ultrarelativistic particles ($1 - \beta^2 \approx |\epsilon - 1|$, $\beta = v/c$) in the absence of diffraction, the transition radiation is concentrated along the direction of $\mathbf{v}$. This leads to the fact that under diffraction conditions the radiation is concentrated in the diffraction plane, i.e., in the plane of the vectors $\mathbf{v}$ and τ, and as a consequence the equations for E'_y can be written in the form

$$k^2a - \frac{\omega^2}{c^2}(\varepsilon a + \varepsilon_1 b) = 0, \quad k_\tau^2 b - \frac{\omega^2}{c^2}(\varepsilon b + \varepsilon_2 a) = 0, \tag{3}$$

where for convenience we have introduced the designations

$$E_y'(\mathbf{k}, \omega) \equiv a, \quad E_y'(\mathbf{k}_\tau, \omega) \equiv b, \quad \mathbf{k}_\tau = \mathbf{k} + \tau,$$
$$\varepsilon = \varepsilon(\omega, 0), \quad \varepsilon_1 = \varepsilon(\omega, \tau), \quad \varepsilon_2 = \varepsilon(\omega, -\tau).$$

Taking the determinant of the system (3) as zero, we obtain the following condition for existence of nonzero solutions:

$$k_{1,2}^2 = \frac{\omega^2}{c^2}n_{1,2}^2, \quad n_{1,2}^2 = \varepsilon - \alpha \pm \sqrt{\alpha^2 + \varepsilon_1\varepsilon_2}, \quad \alpha = \frac{2\mathbf{k}\tau + \tau^2}{2k^2}. \tag{4}$$

In addition, the amplitudes a and b are related by the equations

$$\frac{n_1 - \varepsilon}{\varepsilon_1}a_1 = b_1, \quad a_2 = \frac{\varepsilon_1}{n_2 - \varepsilon}b_2.$$

If we take into account that $|\epsilon - 1| \lesssim 10^{-4}$ in the x-ray region of the spectrum which is of interest here, we can neglect waves which undergo mirror reflection at the vacuum-crystal boundaries. Then, using the boundary conditions, we obtain for the Fourier component of the transition radiation arising in passage of a particle through a crystal of thickness d and propagated at an acute angle to $\mathbf{v}$ the following expression:

$$E_y'(\mathbf{k}, \omega) = \frac{ei}{2\pi^2}\varkappa_y e^{-i\lambda_0 d}\left[(\lambda_0 + \lambda_1)\psi_2 - \frac{n_1 - \varepsilon}{n_2 - \varepsilon}(\lambda_0 + \lambda_2)\psi_1\right]^{-1}$$
$$\times \left\{\left[\psi_2(1 + R)(F_1 - \varphi_1) - \frac{\varepsilon_1}{n_2 - \varepsilon}(\lambda_0 + \lambda_2)(1 + Re^{-i\tau_z d})F_1'\right.\right.$$
$$\left. - \frac{\varepsilon_1}{n_2 - \varepsilon}(\lambda_0 + \lambda_2)\varepsilon_2\frac{\omega^2}{c^2}\frac{\lambda_0^\tau\chi + \omega v/c^2}{\delta}\right]e^{i\lambda_1 d}$$
$$- \frac{\varepsilon_1}{n_2 - \varepsilon}\left[\frac{n_1 - \varepsilon}{\varepsilon_1}\psi_1(F_1 - \varphi_1) - (\lambda_0 + \lambda_1)F_1'\right.$$
$$\left.\left. + (\lambda_0 + \lambda_1)\varepsilon_2\frac{\omega^2}{c^2}\frac{\lambda_0^\tau\chi + \omega v/c^2}{\delta}\right]e^{i\lambda_2 d}\right\}$$
$$- \frac{ei}{2\pi^2}\frac{\varkappa_y}{\lambda_2 + \lambda_0}e^{i(\omega/v - \lambda_0)d}(F_2 - \varphi_2), \tag{5}$$

where $\omega = \mathbf{k} \cdot \mathbf{v}$,

$$\lambda_{1,2} = \pm\left[\frac{\omega^2}{c^2}n_{1,2} - \varkappa^2\right]^{1/2}, \quad \lambda_0^\tau = \left[\frac{\omega^2}{c^2} - (\varkappa + \tau_\perp)^2\right]^{1/2}, \quad \lambda_0 = \lambda_0^0,$$

$$\delta = \left(k^2 - \frac{\omega^2}{c^2}\varepsilon\right)\left(k_\tau^2 - \frac{\omega^2}{c^2}\varepsilon\right) - \varepsilon_1\varepsilon_2\frac{\omega^4}{c^4}.$$

$$\delta_1 = \left(k^2 - \frac{\omega^2}{c^2}\varepsilon\right)\left(k_\tau^2 - \frac{\omega^2}{c^2}\varepsilon\right) + \varepsilon_1\varepsilon_2\left(\mathbf{k}\boldsymbol{\tau} + \frac{\omega^2}{c^2}\varepsilon\right)\left(\boldsymbol{\tau}\mathbf{k}_\tau - \frac{\omega^2}{c^2}\varepsilon\right),$$

$$\chi = \frac{1}{\varepsilon} + \frac{\varepsilon_1\varepsilon_2}{\delta_1}\left(\boldsymbol{\tau}\mathbf{k}_\tau - \frac{\omega^2}{c^2}\varepsilon\right)\left(\frac{\omega}{c^2}\boldsymbol{\tau}\mathbf{v} - \frac{1}{\varepsilon}\boldsymbol{\tau}\mathbf{k}\right),$$

$$\psi_{1,2} = \lambda_0^\tau + \lambda_{1,2} + \tau_z,$$

$$\varphi_{1,2} = \frac{\varepsilon_1\varepsilon_2}{\delta}\left[\left(\chi - \frac{1}{\varepsilon}\right)\left(k_\tau^2 - \frac{\omega^2}{c^2}\varepsilon\right)\lambda_{0,2} - \frac{\omega^4}{c^4}\varepsilon\frac{\lambda_{0,2}\chi}{k^2 - \omega^2\varepsilon/c^2} - \frac{\omega^4}{c^4}\varepsilon\frac{\omega v/c^2}{k^2 - \omega^2\varepsilon/c^2}\right],$$

$$F_{1,2} = \frac{\lambda_{0,2}/\varepsilon + \omega v/c^2}{k^2 - \omega^2\varepsilon/c^2} - \frac{\lambda_{0,2} + \omega v/c^2}{k^2 - \omega^2/c^2},$$

$$F_1' = \frac{\lambda_0/\varepsilon + \omega v/c^2}{k_1^2 - \omega^2\varepsilon/c^2} - \frac{\lambda_0 + \omega v/c^2}{k_1^2 - \omega^2/c^2};$$

$$R = \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_0}e^{i(\omega/v - \lambda_1)d}, \quad k_1^2 = (\varkappa + \tau_\perp)^2 + \frac{\omega^2}{v^2}, \quad k^2 = \varkappa^2 + \frac{\omega^2}{v^2}$$

$\boldsymbol{\kappa}$ and $\boldsymbol{\tau}_\perp$ are the respective projections of the vectors **k** and $\boldsymbol{\tau}$ on the plane of the plate, and the z axis coincides with the direction of the velocity **v**.

The expression for $E_y'(k\omega_\tau)$ is obtained from Eq. (5) by replacement of $\omega = \mathbf{k}\cdot\mathbf{v}$ by $\omega_\tau = \mathbf{k}_\tau\cdot\mathbf{v}$.

We can show that the expressions obtained for transition radiation in a crystal differ from the corresponding expressions for radiation in a uniform plate only for those values of k which satisfy the Bragg condition (2).

On analyzing the solution (5), we may note that it rises sharply for simultaneous fulfillment of conditions which can be written in the form

$$\left|2\frac{\omega}{c}\frac{\tau\rho}{\rho}\sin\theta + 2\frac{\omega}{v}\tau_z + \tau^2\right| \lesssim |\varepsilon - 1|^2\frac{\omega^2}{c^2}, \qquad (6)$$

$$\left|2\frac{\omega}{c}\frac{\tau\rho}{\rho}\sin\theta + 2\frac{\omega}{c}\cos\theta\tau_z + \tau^2\right| \lesssim |\varepsilon - 1|^2\frac{\omega^2}{c^2}$$

where $\boldsymbol{\rho}$ is the projection of the vector **r** on the plane of the plate. It can be shown that the conditions (6) are equivalent to fulfillment of the Bragg conditions for the wave vector in the Fourier expansion of the intrinsic field of the particle and the radiation field.

The conditions (6) are simultaneously satisfied only for ultrarelativistic particles when $v \gtrsim c|\epsilon - 1|$. In this case the two equalities (6) are the same and take the following form:

$$\left|2\frac{\omega}{c}\tau_z + \tau^2\right| \lesssim |\varepsilon - 1|^2\frac{\omega^2}{c^2}.$$

As in the case of a beam of γ rays in a crystal,[4] two refractive indices arise for transition photons under diffraction conditions (see ref. 4), which for $\boldsymbol{\alpha} = 0$ take the form

$$n_1 = (\varepsilon - \sqrt{\varepsilon_1\varepsilon_2})^{1/2}, \quad n_2 = (\varepsilon + \sqrt{\varepsilon_1\varepsilon_2})^{1/2}.$$

The refractive index n_1 contains the difference of the imaginary parts ϵ and $\sqrt{\epsilon_1\epsilon_2}$. Therefore a wave moving with this refractive index is essentially not absorbed in the crystal.[4,6]

Using Eq. (5), we find the energy flux under diffraction conditions, from which we can obtain for the number of γ rays emitted from the crystal at an acute angle to the incident beam direction, for passage of a single electron, the following expression:

$$n_\gamma^{1,2} \approx \frac{e^2}{\hbar c}\left(\frac{E}{mc^2}\right)^4 |\varepsilon - 1| \exp\left\{-2\frac{\omega_{1,2}}{c}\operatorname{Im} n_1 d\right\}. \qquad (7)$$

Equation (7) is valid if the thickness of the crystal satisfies the conditions

$$d \gg \frac{c}{\omega_{1,2}\operatorname{Im} n_2}, \quad \exp\left\{-2\frac{\omega_{1,2}}{c}\operatorname{Im} n_1 d\right\} \gg |\varepsilon - 1|,$$

where E is the electron energy, $\omega_{1,2}$ are the γ-ray frequencies satisfying the condition (6); it is easy to show that $\omega_1 = -\tau^2 c/2\tau_z$, $\omega_2 = \omega_1 + \tau_z$.

According to (7) for E = 10^3 MeV

$$n_\gamma^{1,2} \approx 10^6 \exp\left\{-2\frac{\omega_{1,2}}{c}\operatorname{Im} n_1 d\right\}.$$

As we can see, under diffraction conditions the number of transition photons is substantially greater than without diffraction (~ 10^{-3} photons per electron, according to refs. 2 and 7). Here the transition radiation in the crystal is concentrated in a region of angles $\Delta\theta \sim \sqrt{1 - \beta^2}$ near the directions **v** and **v** + $\boldsymbol{\tau}$. As a result, the diffracted γ rays can be separated from the incident electron beam. A further increase of intensity can be obtained, as usual, by means of a system of crystalline plates.

We note that for $(\omega/c)\operatorname{Im} n_1 d \lesssim 1$ the number of γ rays becomes so large that it is necessary to take into account the effect of the radiation on the nature of the electron motion in the crystal.

We further note that, for thin crystals in which absorption can be neglected, Ter-Mikaelyan[1] and Kudryavtsev and Ryazanov[8] have noted the possibility of enhancement of bremsstrahlung and recoil-electron radiation. It is evident that, as the result of the anomalous penetration, we can also use thick crystals here, which, generally speaking, will enhance the effect.[3]

In conclusion we note that for the γ-ray intensities which apparently can be obtained by the means discussed above, it becomes experimentally possible to observe interference phenomena in independently produced beams of photons.[9] In fact, for a 10 μA current of 10^3 MeV electrons, we will have $n_\gamma \sim 10^{20}$ sec^{-1} and in accordance with the results obtained by Baryshevskiĭ and Podgoretskiĭ[9] the observation time becomes quite realistic: T ~ 10^3 sec.

We note here also that with such γ-ray fluxes it is necessary, generally speaking, to take into account the possibility of induced emission of γ rays by excited nuclei.

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