PENETRATION OF ELECTROMAGNETIC WAVES INTO A PLASMA, TAKING NONLINEARITY INTO ACCOUNT

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We obtain the structure of the electromagnetic field in a weakly ionized plasma under conditions when the electromagnetic wave affects the ionization balance. We consider the incidence of an electromagnetic wave with a frequency larger than the effective frequency for collisions between electrons and atoms and lower than the plasma frequency. We show that when the electromagnetic wave affects the ionization balance in the plasma the penetration depth of the field into the plasma is strongly decreased but nevertheless stays appreciably larger than the quantity c/ω_p, where ω_p is the plasma frequency established at the boundary of the plasma under the influence of the electromagnetic wave.

NON-LINEAR effects arising when electromagnetic waves propagate in a plasma are basically caused by the heating of the electrons in the field of the wave by striction effects from the exclusion of the plasma because of the inhomogeneity of the electrical field of the wave. It is noteworthy that the non-linearity begins to manifest itself very strongly already at fields which are rather weak compared with the electrical field characteristic for the plasma.

In the present paper we consider the structure of a variable electromagnetic field with a frequency ω in weakly ionized plasma which is not in thermal equilibrium. In the present paper we confine ourselves to the case when the frequency of the electromagnetic field is much larger than the frequency of electron-atom and electron-molecule collisions and less than the plasma frequency.

An important part is played by the non-linear effects connected with the heating of the electrons by the alternating electrical field, as a result of which the electrical field changes the electron concentration. Up to the present only the case of weak fields (much weaker than the characteristic plasma field) has been discussed in the literature[1] and the influence of the electromagnetic field on the ionization-recombination balance in the plasma[2] has not been taken into account.

In this paper we show that allowance for the influence of the electromagnetic wave on the local ionization-recombination balance in the plasma leads to an appreciable change in the penetration depth of the field into the plasma as compared to the quantity c/ω_p, where ω_p is the electron plasma frequency established at the boundary of the plasma under the influence of the electromagnetic wave. The physics of the change in the penetration depth of the field into the plasma as compared with c/ω_p consists in the fact that when the electromagnetic wave penetrates deep into the plasma its intensity decreases and as a result the plasma frequency of the electrons produced by the alternating electrical field of the wave decreases and the deeper it penetrates into the plasma the smaller and smaller is the barrier that it produces for its own penetration.

1. STATEMENT OF THE PROBLEM

We consider the propagation of an electromagnetic wave with frequency ω in a weakly-ionized plasma which is not in thermal equilibrium. Let the non-equilibrium, i.e., the initial deviation of the electron temperature, be caused by some effective constant electrical field E_{eff}. The frequency of the electromagnetic wave ω is much larger than the electron-atom collision frequency $\nu(T_e)$, where T_e is the electron temperature (the degree of ionization in the plasma is small so that we can neglect Coulomb collisions).

According to an elementary consideration[1] the equation of motion of an electron in the electrical field of the wave $\mathbf{E}\cos\omega t$ is

$$\ddot{\mathbf{r}} = \frac{e}{m}\mathbf{E}_{\text{eff}} + \frac{e}{m}\mathbf{E}\cos\omega t - \nu(T_e)\dot{\mathbf{r}}. \tag{1}$$

The energy balance equation is

$$n_e\dot{T}_e = {}^2/_3 n_e e\dot{\mathbf{r}}(\mathbf{E}_{\text{eff}} + \mathbf{E}\cos\omega t) - \delta n_e\nu(T_e)T_e + \nabla(\varkappa(T_e)\nabla T_e), \tag{2}$$

where e, m, n_e, and T_e are the electron charge, mass, concentration, and temperature, $\varkappa(T_e)$ the electron heat conductivity coefficient, and δ the fraction of energy which is transferred when the electrons collide with neutral particles.

We shall assume that the inhomogeneity occurring in the $\mathbf{E}(\mathbf{r})$ and $\mathbf{T}(\mathbf{r})$ distributions is sufficiently small, viz.,

$$l\delta^{-1/2}|\operatorname{grad} E| \ll E, \tag{3}$$

where l is the electron mean free path.

When inequality (3) is satisfied the stationary value of the temperature which establishes itself in the plasma is determined by the local electron energy balance. It then follows from (1) and (2) that the stationary value of the temperature is up to small terms of order δ and $\delta\nu/\omega$ is given by

$$T_e = \frac{2}{3}\frac{e^2E^2_{\text{eff}}}{m\delta\nu^2(T_e)} + \frac{e^2E^2}{3m\delta(\omega^2+\nu^2(T_e))} \tag{4}$$

Since we are interested in the case $\omega \gg \nu(T_e)$, we can rewrite (4) in the form

$$T_e = \frac{2}{3}\frac{e^2E^2_{\text{eff}}}{m\delta\nu^2} + \frac{e^2E^2}{3m\delta\omega^2}. \tag{5}$$

We assume also that the plasma is quasi-linear, i.e., we assume that the amplitude of the electrical field changes little over a distance equal to the Debye radius r_D

$$r_D|\text{grad}\, E| \ll E.$$

We assume further that the characteristic dimension of the inhomogeneity in $T_e(\mathbf{r})$ and $E(\mathbf{r})$ which establishes itself in the plasma is such that everywhere up to electron energies of the order of the ionization potential of the neutral particles the electron distribution function is determined by the local value of the temperature. (We shall discuss below the criterion for the validity of such an assumption.) The electron distribution function then has the form[1]

$$f_0(v) \sim \exp\left\{-\int_0^v \left[\frac{e^2E^2}{3m\delta(\omega^2+\nu^2(v))} + \frac{2}{3}\frac{e^2E^2_{\text{eff}}}{m\delta\nu^2(v)}\right]^{-1} mv\,dv\right\}. \tag{6}$$

Hence it follows that in the electron energy range of the order of the ionization potential the electron distribution function has the form

$$f_0(v) \sim \exp\left\{-\frac{I\omega^2\beta}{T_{e0}\nu_0^2(1+\beta)^2} - \frac{I^2}{3T_{e0}^2(1+\beta)} + \frac{3}{2}\left(\frac{\omega}{\nu_0}\right)^4 \frac{\beta}{(1+\beta)^3}\ln\left[1+\frac{2I\nu_0^2}{3I_{e0}\omega^2}(1+\beta)\right]\right\}, \tag{7}$$

where $\beta = E^2/E^2_{\text{eff}}$.

We assume here that the cross section for the electron-neutrals collisions is independent of the electron energy, and we choose therefore the electron-neutrals collision frequency in the form $\nu(T_e) = \nu_0\sqrt{(T_e/T_{e0})}$, where ν_0 is the number of collisions at an electron temperature T_{e0} caused by the effective field E_{eff}.

We shall assume that the main role in the ionization-recombination balance in the plasma is played by processes of impact ionization of the neutral particles through an electron collision and by electron-ion recombination processes. The heating of the electrons in the variable electrical field due to the exponential dependence of the coefficient for the ionization of the neutral particles by electron impact on the electron temperature T_e is basically expressed through the increase in the rate of ionization of the neutral particles. The electron concentration established in a weakly ionized plasma is thus determined by the equation

$$\alpha(T_e)n_en_0 - \beta_p n_e^2 + \text{div}(D_a\,\text{grad}\, n_e) = 0. \tag{8}$$

Here n_0 is the gas density (the neutral particle density), $\alpha(T_e)$ the coefficient for ionization of neutral particles by electron impact, β_p the electron-ion recombination coefficient, and D_a the ambipolar diffusion coefficient.

We shall assume everywhere in what follows that the electron balance is local, i.e., that we can neglect the last term in Eq. (8). This assumption is valid if the following inequality holds:

$$(D_a/\beta_p n_e)^{1/2}|\text{grad}\, n_e| \ll n_e. \tag{9}$$

The electron concentration is in that case determined by Eq. (8):

$$n_e(T_e) = \alpha(T_e)n_0/\beta_p = \varphi(T_e)f_0(I), \tag{10}$$

where $\varphi(T_e)$ is some power-law function, and $f_0(I)$ the distribution function (7) in the range of electron energies equal to the ionization potential I. We have assumed here that $I/T_e \gg 1$.

The spatial distribution of the amplitude of the electrical field and hence also of the electron temperature is determined under the assumptions made above by the solution of the Maxwell's equations, in which we neglect the absorption of the electromagnetic wave and the displacement currents, assuming that the following inequality is always satisfied:

$$\omega_{p0} \gg \omega \gg \nu(T_e), \tag{11}$$

where ω_{p0} is the plasma frequency when there is no field E. The wave equation then has the form

$$\nabla^2\mathbf{E}(\mathbf{r}) = \frac{4\pi e^2}{mc^2}n_e(E)\mathbf{E}, \tag{12}$$

where $n_e(E)$ is the electron concentration determined by Eq. (10).

We consider separately the structure of the field for the case of strong ($E \gg E_{\text{eff}}$) and weak ($E \ll E_{\text{eff}}$) fields.

2. SOLUTION FOR $E/E_{\text{eff}} \gg 1$

We consider the propagation of a strong electromagnetic wave ($E \gg E_{\text{eff}}$) in the plasma. We find in that case from Eq. (7) that the electron distribution function is Maxwellian with a temperature

$$T_e = e^2E^2/3m\delta\omega^2 \tag{13}$$

under the condition that $\omega \gg \nu$ everywhere up to electron energies of the order of the ionization potential.

If, however, the inequality

$$\nu(I) > \omega > \nu(T_e), \tag{14}$$

is satisfied it follows from Eq. (7) that for $E \gg E_{\text{eff}}$ the electron distribution function has the Druyvesteyn form and for an electron energy $mv^2/2 = I$ we get

$$f_0(I) \sim \exp\{-I^2\nu^2(T_e)/3T_e^2\omega^2\}, \tag{15}$$

where T_e is the electron temperature determined by Eq. (13). In the case of a strong field the wave equation therefore becomes

$$\nabla^2\mathbf{E}(\mathbf{r}) = \frac{4\pi e^2}{mc^2}\varphi(E)\mathbf{E}\exp\left(-\frac{\gamma}{E^2(r)}\right), \tag{16}$$

where

$$\gamma = \gamma_1 \equiv 3Im\delta\omega^2/e^2, \tag{17a}$$

if $\omega > \nu(I)$ and

$$\gamma = \gamma_1 I\nu^2(T_e)/3T_e\omega^2, \tag{17b}$$

if inequality (14) holds.

We consider normal incidence of an electromagnetic wave linearly polarized in the z-direction onto a plane plasma boundary. We choose the x-axis at right angles to the boundary and the boundary condition $E(x=0) = E_0$. In the region $E \gg E_{\text{eff}}$ the equation for E will be (16):

$$\frac{d^2E}{dx^2} = \frac{4\pi e^2}{mc^2}\varphi(E)E\exp\left(-\frac{\gamma}{E^2(x)}\right). \tag{18}$$

To solve Eq. (18) we introduce a new function

$$\Theta(x) = \gamma / E^2(x). \quad (19)$$

Equation (18) now becomes

$$\frac{d^2\Theta}{dx^2} - \frac{3}{2\Theta}\left(\frac{d\Theta}{dx}\right)^2 + B(\Theta)e^{-\Theta} = 0, \quad (20)$$

where $B(\Theta) = (8\pi e^2/mc^2)\Theta\varphi(\Theta)$. Solving Eq. (20) close to the boundary we find, when $\Theta \gg 1$, an approximate expression for $E(x)$ in the form

$$E(x) = E_0\left\{1 + \frac{2}{\Theta_0}\ln\left[1 + \frac{x}{c/\omega_p}\Theta_0^{1/2}\right]\right\}^{-1/2}, \quad (21)$$

where $\Theta_0 = \gamma/E_0^2$.

It is clear from Eq. (21) that close to the boundary everywhere as long as Eq. (18) is valid, i.e., $E \gg E_{\text{eff}}$, the amplitude of the electrical field decreases logarithmically. It is then clear that a very steep—linear—decrease in the field is observed near the boundary when $x \ll (c/\omega_p)\Theta_0^{-1/2}$; we have then from (21)

$$E(x) = E_0\left(1 - \frac{x}{\Theta_0^{1/2}c/\omega_p}\right). \quad (22)$$

It follows from Eq. (22) that the surface impedance is

$$\zeta = \Theta_0^{1/2}\omega/\omega_p. \quad (23)$$

The quantity Θ_0 has the value I/T_e in the case when the electron distribution function everywhere up to electron energies of the order I is Maxwellian with a temperature determined from Eq. (13). If, however, in the electron energy range of the order of I the tail of the distribution function is the Druyvesteyn one given by (15), we have

$$\Theta_0 = I^2\nu^2(T_e) / T_e^2\omega^2.$$

The solution (21) is valid for x-values such that $E/E_{\text{eff}} \gg 1$. One verifies easily that then

$$x \ll c\Theta_0^{-1/2}/\omega_{p0}(\varepsilon). \quad (24)$$

These results are also valid when a strong wave is incident upon a plasma which is initially at equilibrium.

3. SOLUTION FOR $E \ll E_{\text{eff}}$

We consider now the incidence of a weak ($E \ll E_{\text{eff}}$) electromagnetic wave onto a plasma, which is not in thermal equilibrium, with an electron temperature T_{e0} which is much larger than the neutral-particles temperature. Under the same assumptions as before the distribution function will have the form (7).

We simplify Eq. (7) expanding the index of the exponent in the small parameter E^2/E_{eff}^2:

$$f_0(I) \sim \exp\{-I^2/3T_{e0}^2 + \tilde{\gamma}2\beta\}, \quad (25)$$

where

$$\tilde{\gamma} = \frac{I^2}{6T_{e0}^2} - \frac{I\omega^2}{2T_{e0}\nu^2(T_{e0})} + \frac{3\omega^4}{4\nu^4(T_{e0})}\ln\left[1 + \frac{2I\nu^2(T_{e0})}{3T_{e0}\omega^2}\right]. \quad (26)$$

The condition for the validity of the expansion (25) has the form

$$\tilde{\gamma}\beta^2 \ll 1. \quad (27)$$

Since $\tilde{\gamma} \gg 1$, inequality (27) allows us to take into account the influence of the field of the electromagnetic wave only in the exponent in $n_e(E)$:

$$n_e(E) = n_{e0}\exp(2\tilde{\gamma}\beta). \quad (28)$$

Here n_{e0} is the electron concentration in the plasma which is not perturbed by the field of the electromagnetic wave. Therefore the distribution of the amplitude of the electrical field of the wave well inside the plasma is determined by the Maxwell equation analogous to Eq. (18):

$$\frac{d^2E}{dx^2} = \frac{\omega_{p0}^2}{c^2}E\exp\left(\tilde{\gamma}\frac{E^2}{E_{\text{eff}}^2}\right). \quad (29)$$

Introducing the dimensionless variables

$$\Theta = \tilde{\gamma}\frac{E^2}{E_{\text{eff}}^2}, \quad \xi = \frac{x}{c/\omega_{p0}}, \quad (30)$$

we write Eq. (29) in the form

$$\frac{d}{d\xi}\left(\frac{1}{\Theta^{1/2}}\frac{d\Theta}{d\xi}\right) = 2\Theta^{1/2}e^{\Theta}. \quad (31)$$

Making the substitution

$$d\Theta / d\xi = P(\Theta)\Theta^{1/2},$$

we can lower the order of Eq. (31):

$$\frac{1}{\Theta}\left(\frac{d\Theta}{d\xi}\right)^2 = 4e^{\Theta} + C_1. \quad (32)$$

We find the arbitrary constant C_1 from the following considerations. Far from the plasma boundary for $x \to \infty$ the solution (32) must change to the solution of Eq. (29) with $E \to 0$, i.e., in the usual solution of the linear theory

$$E(x \to \infty) \sim \exp\left(-\frac{x}{c/\omega_{p0}}\right). \quad (33)$$

Equation (32) gives this asymptotic solution for $E \to 0$, if $C_1 = -4$.

The structure of the variable electric field in the plasma is thus determined by the equation

$$d\Theta / d\xi = -2\Theta^{1/2}(e^{\Theta} - 1)^{1/2}. \quad (34)$$

The solution of Eq. (34) and thus also of (29) can easily be written in the form of quadratures, but we are only interested in the solution near the plasma boundary when $\Theta \gg 1$. When $\Theta \gg 1$, we get from Eq. (34)

$$\Theta(\xi) = \Theta_0 - 2\ln(1 + \xi\Theta_0^{1/2}e^{\Theta_0/2}),$$

where $\Theta_0 = \tilde{\gamma}E_0^2/E_{\text{eff}}^2$. Hence, returning to the variables E and x we find a logarithmic law for the decrease of the field of the electromagnetic wave down to such values that the process of the self-action of the electromagnetic wave can be neglected:

$$E(x) = E_0\left\{1 - \frac{2}{\Theta_0}\ln\left[1 + \frac{x}{c/\omega_{p0}}\Theta_0^{1/2}e^{\Theta_0/2}\right]\right\}^{1/2}. \quad (35)$$

Here E_0 is the amplitude of the variable electrical field at the plasma boundary.

We get from Eq. (35) near the surface a formula analogous to (22):

$$E(x) = E_0\left[1 - \frac{x}{\Theta_0^{1/2}c/\omega_p(E_0)}\right], \quad (36)$$

where $\omega_p(E_0)$ is the plasma frequency caused by the electrical field of the wave on the plasma boundary.

Using Eq. (35) we can estimate the penetration depth of the electrical field as follows:

$$L_E \lesssim \frac{c}{\omega_{p0}}\frac{1}{\tilde{\gamma}^{1/2}}\frac{E_{\text{eff}}}{E_0}.$$

From this it is clear that when the intensity of the incident electromagnetic wave increases, its penetration depth decreases remaining, however, by virtue of condition (27)

$$L_E > c / \omega_{p0} \tilde{\gamma}^{1/4}.$$

Using Eq. (26) for $\tilde{\gamma}$ this inequality can be written in a translucent form:

$$L_E > (c / \omega_{p0}) (T_{e0} / I)^{1/2}. \tag{37}$$

We can thus conclude that, under the assumptions made above, taking the self-action into account leads to the result that the penetration depth of electromagnetic waves with a frequency $\omega_{p0} > \omega > \nu(T_{e0})$ is decreased as compared to the value from the linear theory but is appreciably larger than the penetration depth calculated in the linear theory with the boundary plasma frequency:

$$\frac{c}{\omega_p} \ll \frac{c}{\omega_{p0}} \left(\frac{T_{e0}}{I}\right)^{1/2} \ll \frac{c}{\omega_{p0}}. \tag{38}$$

4. DISCUSSION

We now discuss the conditions under which the effect considered by us must occur.

In the derivation of the expressions obtained above we assumed that the tail of the electron distribution function up to electron energies of the order of the ionization potential of the neutral particles is determined by Eq. (7) which follows from the kinetic equation in the Fokker-Planck approximation neglecting spatial inhomogeneity. It is clear that this approximation must give a lower limit to the size of the inhomogeneity of E(r) occurring in our problem. To determine this limitation we analyze the conditions under which the solution of the kinetic equation is the function $f_0(v)$ given by Eq. (7).

We write down the kinetic equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\mathrm{grad}_r f + \frac{e}{m}\mathbf{E}\nabla_v f + S = 0.$$

We write, as usual,[1] the electron distribution function in the form

$$f = f_0(t, \mathbf{r}, v) + \frac{\mathbf{v}\mathbf{f}_1(t, \mathbf{r}, v)}{v} + \ldots \tag{39}$$

and write down the first two kinetic equations from the chain, assuming the subsequent moments of the distribution function to be small:[3]

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\,\mathrm{div}_r\,\mathbf{f}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) + S_0 = 0, \tag{40}$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\,\mathrm{grad}_r f_0 + \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \mathbf{S}_1 = 0, \tag{41}$$

where

$$S_0 = -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta\nu(v)\left[\frac{T_0}{m}\frac{\partial f_0}{\partial v} + vf_0\right]\right\}, \tag{42a}$$

$$\mathbf{S}_1 = \nu\mathbf{f}_1. \tag{42b}$$

Expression (6) for the distribution function for $\delta\nu/\omega \ll 1$ follows from Eqs. (40) to (42) if the following inequality holds:

$$v|\partial f_{1x}/\partial x| \ll |S_0|. \tag{43}$$

Using (42b) to get expressions for the functions f_{1x} and f_{1z} from Eq. (41), we get

$$f_{1z} = \frac{eE}{m}\frac{i\omega - \nu}{\omega^2 + \nu^2}\frac{\partial f_0}{\partial v}e^{i\omega t}, \tag{44a}$$

$$f_{1x} = -\frac{v}{\nu(v)}\frac{\partial f_0}{\partial x}. \tag{44b}$$

If $\omega > \nu(I)$ up to electron energies of the order I, the solution of Eqs. (40) to (42) under the condition (43) is a Maxwellian function with a temperature determined from Eq. (13). If, however, $\nu(T_e) < \omega < \nu(I)$ the solution of these equations under the condition (43) is a function which has as its central part a Maxwellian form and as its tail a Druyvesteyn form. In both cases we can use Eqs. (44a) and (44b) to write inequality (43) in the following form:

$$\frac{v^2}{\nu(v)}\left|\frac{\partial^2 f_0}{\partial x^2}\right| \ll \frac{1}{v^2}\left|\frac{\partial}{\partial v}\{v^3\delta\nu(v)f_0\}\right|. \tag{45}$$

We have used here the condition

$$\frac{T_0}{T_e}\frac{I}{T_e}\frac{\nu^2(T_e)}{\omega^2} \ll 1.$$

As the maximum inhomogeneity in the distribution function is the same as the maximum inhomogeneity in the field which occurs for a strong field, $E \gg E_{eff}$, we use the solution (21) which gives the following expression for the Maxwellian function $f_0(x, v)$:

$$f_0(x, v) \sim \left[1 + x\frac{\omega_p}{c}\left(\frac{I}{T_e}\right)^{1/2}\right]^{-mv^2/I}. \tag{46}$$

Using (46) we get from inequality (45) in the electron energy range of order T_e the following condition:

$$l / \sqrt{\delta} \ll (c / \omega_p)(I / T_e)^{1/2}. \tag{47}$$

In the range of electron energies of order I, however, inequality (45) gives

$$l / \sqrt{\delta} \ll c / \omega_p. \tag{48}$$

From a comparison of inequalities (47) and (48) it is clear that when $\omega > \nu(I)$ the electron distribution function can everywhere up to electron energies of order I be assumed to be Maxwellian with a temperature determined by Eq. (13) provided inequality (48) is satisfied.

We consider now the consequences of inequality (45) when the tail of the electron distribution function in the region $mv^2/2 \sim I$ has a Druyvesteyn form, i.e., when $\nu(T_e) < \omega < \nu(I)$.

In that case the distribution function in the region $mv^2/2 = T_e$ is Maxwellian and solution (20) gives the following expression for $f_0(x, v)$ in the region of those electron energies:

$$f_0(x, v) \sim \left[1 + x\frac{\omega_p}{c}\frac{I\nu(T_e)}{3T_e\omega}\right]^{-\eta}, \qquad \eta = \frac{3mv^2T_e\omega^2}{I^2\nu^2(T_e)}. \tag{49}$$

In the region $mv^2/2 = I$ the solution (20) which we obtained gives, when we bear Eq. (15) in mind, the expression

$$f_0(x, v) \sim \left[1 + x\frac{\omega_p}{c}\frac{I\nu(T_e)}{3\omega T_e}\right]^{-m^2v^4/2I^2}. \tag{50}$$

Substituting (49) and (50) into the inequality (45) we get the following inequalities:

$$\frac{l}{\sqrt{\delta}} \ll \frac{c}{\omega_p}\frac{I\nu(T_e)}{T_e\omega}, \tag{51}$$

$$l/\sqrt{\delta} \ll c/\omega_p. \qquad (52)$$

One sees easily that inequality (52) is stronger than (51). Noting now that inequality (52) is identical with inequality (48) we conclude that if condition (48) is satisfied we can neglect in Eqs. (40) and (41) the spatially inhomogeneous terms.

We now obtain the conditions under which the distribution function can be written in the form (39) and the infinite chain of coupled kinetic equations can be broken off after the first two: (40) and (41). It is well known that this can be done if the following inequalities are satisfied:[3]

$$\frac{e^2E^2}{m^2(\omega^2+\nu^2(v))}\frac{1}{v^2}\left|\frac{\partial}{\partial v}\left(v^2\frac{\partial f_0}{\partial v}\right)\right| \ll f_0, \qquad (53)$$

$$\frac{v}{\sqrt{\omega^2+\nu^2(v)}}\left|\frac{\partial^2 \mathbf{f}_1}{\partial x^2}\right| \ll \left|\frac{\partial f_0}{\partial x}\right|. \qquad (54)$$

Substituting Eqs. (44) into (54) we find that inequality (54) is always satisfied, if inequality (48) is satisfied.

If the electron distribution function f_0 is Maxwellian, inequality (53) is satisfied for all electron energies up to the ionization potential provided

$$\delta I/T_e \ll 1. \qquad (55)$$

If, however, the electron distribution function in the range of electron energies of the order of the ionization potential has the Druyvesteyn form, to satisfy (51) we need the condition

$$\delta\left(\frac{I}{T_e}\right)^2\frac{\nu^2(T_e)}{\omega^2} \ll 1. \qquad (56)$$

Summarizing we can say that for the occurrence of the effect considered by us it is necessary that inequalities (19), (48), and (56) are satisfied. Inequality (3) will then also be satisfied because the characteristic dimension for the change in the amplitude of the electrical field (21) is appreciably larger than the characteristic dimension over which the electron distribution function, and thus the electron concentration, changes.

The minimum size of the inhomogeneities in the electron concentration is determined from the solution of (20) and for Maxwellian and Druyvesteyn tails of the distribution function it is, respectively, equal to

$$L_{1n_e} = \frac{c}{\omega_p}\left(\frac{T_e}{I}\right)^{1/2}, \qquad L_{2n_e} = \frac{c}{\omega_p}\frac{T_e\omega}{I\nu(T_e)}. \qquad (57)$$

When Eqs. (57) are taken into account the condition (9) that the electron balance is local can be written in the form

$$(D_a/\beta_p n_e)^{1/2} \ll L_{n_e}. \qquad (58)$$

In conclusion we estimate the plasma parameters for which one must expect the occurrence of the effect considered by us. The possibility of the simultaneous satisfying of inequalities (11) and (48) leads to the following restriction on the electron mean free path:

$$\delta^{1/2}c/\omega_p > l > v/\omega_{p0} \qquad (59)$$

(here v is the electron thermal velocity).

Satisfying inequality (59) leads to the following condition:

$$\delta^{1/2}c/v > \omega_p/\omega_{p0} > 1. \qquad (60)$$

If $\delta \approx 10^{-2}$ to 10^{-3} which occurs for collisions of electrons with molecules which have low-lying rotational and vibrational excited levels, inequality (60) is satisfied for T_e of the order of 1 eV. Conditions (55) and (56) will be violated when the electron temperature is too low.

We use (60) to estimate the necessary degree of ionization of the plasma:

$$\sqrt{n_e}/n_0 = vm^{1/2}Q_{e0}/e \approx 10^{-11} - 10^{-12}\text{cm}^{3/2}$$

(here Q_{e0} is the electron-neutrals collision cross-section). Therefore, the effect studied by us must occur, for instance, when $n_0 = 10^{18}$ to 10^{19} cm^{-3} and $n_e = 10^{12}$ to 10^{13} cm^{-3}.

Inequality (58) leads to the following requirement for the value of the electron-ion recombination coefficient:

$$\beta_p > \frac{\mu^+ e}{mT_e}\frac{I^2}{c^2}. \qquad (61)$$

Using known values for the mobilities of atomic and molecular ions for $n_0 = 10^{18}$ to 10^{19} cm^{-3} we get $\beta_p > 10^{-10}$ to 10^{-11} cm^3/sec. These values are characteristic for the coefficients of dissociative electron-ion recombination for T_e = 1 eV.

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