DYNAMIC ELASTIC MODULI AND DISPERSION AND ABSORPTION OF TRANSVERSE SOUND IN A STRONG MAGNETIC FIELD

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A compact representation of the dynamic elastic moduli of a metal is presented in terms of the electroacoustic kinetic coefficients. Asymptotic formulas are given which permit the explicit form of the kinetic coefficients in strong (non-quantized) magnetic fields to be obtained directly. The asymptotic behavior of the absorption and dispersion of transverse sound propagating perpendicular to the magnetic field is found as a function of the field direction relative to the crystallographic axes (for closed cross sections of the Fermi surface). It is shown that, besides the symmetry of the magnetic field direction[4], anomalously small absorption of the transverse sound requires simple-connectedness of the Fermi surface cross sections. For low-symmetry field directions, the absorption of the transverse sound depends significantly on the polarization and differs from the case of high symmetry. Considerable renormalization of the transverse sound velocity is possible here. Sound velocity dispersion near those frequencies for which the sound wavelength and skin depth are comparable is also discussed. For sound polarized perpendicular to the magnetic field, the order of magnitude of dispersion and absorption are about the same in this resonance region, thanks to the contribution of transverse electric fields. The equations of elasticity theory for metals are employed in the dispersion and absorption investigations.[1]

1. INTRODUCTION

IN the present paper, the equations of elasticity theory for metals, which were obtained earlier,[1] are employed for the investigation of the dispersion and absorption of transverse sound in a strong magnetic field. The propagation of transverse sound perpendicular to the magnetic field is discussed for closed Fermi-surface cross sections, when collision-free absorption, which was considered for the isotropic metal in[2], is lacking. Dispersion is also considered along with absorption.[1)]

The interaction of transverse sound with electrons differs in a number of features from that of longitudinal sound. These include the important contribution of the transverse electric fields, which are generated upon propagation of the (transverse) sound wave in the metal (see[2,3]) and the references cited in[2]), and the sensitivity to the anisotropy of the electronic spectrum, which is much greater than for longitudinal sound. The latter is associated with the fact that the shear moduli of the electron gas (and not the compressional modulus as for longitudinal sound, which disappear in the transition to the isotropic spectrum, play a role in the deformation interaction of the transverse sound with the electrons. This is especially evident in the presence of a strong magnetic field; the absorption of transverse sound in the crystal is strongly dependent on the direction of this field, as was first pointed out by Pippard.[4]

The difference in the deformation interaction of the electrons with longitudinal and transverse sound lies in the fact that in the former and latter cases it is determined respectively by the diagonal and nondiagonal components of the deformation potential tensor $\lambda_{ik}(\mathbf{p})$ (one of the indices denotes the direction of propagation and the other the polarization of the wave). This follows from the definition of $\lambda_{ik}(\mathbf{p})$,[5] which describes the change in the dispersion law $\epsilon(\mathbf{p})$ for the deformation $u(r, t)$:[2)]

$$\delta\varepsilon' = \lambda_{ik}(\mathbf{p}')u_{ik} - m\mathbf{v}'\dot{\mathbf{u}}. \tag{1.1}$$

For a constant tensor (independent of the quasimomentum), as a consequence of the point symmetry of the crystal, a number of the nondiagonal components (along the axes of symmetry of the crystal) are identically equal to zero, as is well known. By virtue of the dependence of $\lambda_{ik}(\mathbf{p})$ on the quasimomentum, similar components of the deformation potential, which satisfy more complicated symmetry conditions,[6] are of course different from zero. But their values averaged over the period of revolution of $\bar{\lambda}_{ik}$ (which are responsible for the interaction with the transverse sound in a strong field) can be zero for a symmetric direction of the field, and this should appreciably change the frequency and field dependence of the absorption (and dispersion) of

[1)] Dispersion of sound velocity in metals, like absorption, is due to the interaction of the lattice with the conduction electrons. Here the characteristic lengths and frequencies with which the sound wavelength $\lambda = 2\pi/k$ and the sound frequency ω should be compared are the parameters of the electron gas in the metal, such as the free path of the electron l, the Larmor radius, r, the skin depth (which is proportional to $\delta \equiv c/\omega_p$), the collision frequency ν, the cyclotron frequency Ω, and so on. These lengths are significantly greater than the interatomic distance (l, r, $\delta \gg a$) and the frequencies are much smaller than the atomic and plasma frequencies ($\nu, \Omega \gg \omega_{at}, \omega_p$).

[2)] The prime refers to the local co-moving set of coordinates, where the classification of the states of the periodic lattice is preserved, and the perturbed dispersion law is defined (see, for example, [1]). In the laboratory system $\delta\epsilon = (\lambda_{ik}(\mathbf{p}) + p_i v_k)\partial u_i/\partial x_k + (\mathbf{p}-m\mathbf{v})\dot{\mathbf{u}}$. In [4] this quantity divided by hv_F is used (with neglect of the last component).

the transverse sound[4,7] in comparison with the longitudinal one.[7-9]

In the present paper we obtain the conditions under which the equations $\bar{\lambda}_{ik} = 0$ are satisfied as a consequence of the symmetry properties (Sec. 3) and determine the dispersion and absorption of transverse sound (Sec. 4) as functions of the orientation of the magnetic field relative to the crystallographic axes.

It has turned out that one must add the essential condition of single connectedness of the Fermi-surface cross sections to the requirement of high symmetry[4] (which latter is formulated quantitatively below). The asymptotes found for the absorption in the low-symmetry directions of the field are essentially different from those considered earlier for the high-symmetry directions.

It was shown earlier[1] for H = 0 that in the range of frequencies where the sound wavelength is comparable with the skin depth, i.e., with the damping distance of the electromagnetic wave in the metal, a unique "resonance" sets in, as a result of which the dispersion of the sound velocity increases materially. In the resonance region, the dispersion becomes of the same order as the absorption. Such a behavior of the dispersion is due to the contribution of the transverse electric fields which arise upon propagation of a (transverse) sound wave in the metal.

In a strong field, the analog of a similar "resonance" takes place in compensated metals for sound that is polarized perpendicularly to the external magnetic field. The location of the resonance depends on the value of the field and for excellent metals it lies in the region of frequencies which is much lower than the collision frequency (Sec. 5).

The asymptotic expansions of the electroacoustic kinetic coefficients in strong fields are also given in the work (see Sec. 3 and the Appendices). The representation of the dynamic elastic moduli of the metal in terms of the electroacoustic kinetic coefficients, obtained by the author in[1], is given in closed and more compact form (Sec. 2).

2. THE DISPERSION EQUATION

We start from the equations of elasticity theory in metals:[1,10]

$$\rho \ddot{u}_i = \lambda_{iklm}\frac{\partial u_{lm}}{\partial x_k} + D_i, \qquad \mathbf{D} = \frac{1}{c}[\mathbf{jH}] - \frac{m}{e}\frac{\partial \mathbf{j}}{\partial t} + \mathbf{f}^{d}. \quad (2.1)^*$$

Here e < 0 and m are the charge and mass of the electron, λ_{iklm} is the lattice tensor of the elastic moduli, which takes into account the equilibrium contribution from the conduction electrons, j is the current density, and f^d is the deformation force, equal to the gradient of the mean value of the deformation potential. The force D describes the contribution from the nonequilibrium electrons and the electromagnetic fields. We shall make its physical meaning clear below (see (2.1′)).

In an unbounded metal, the Fourier components of the current density j and the deformation force f^d are expressed in terms of the ω, $\mathbf{k}$ Fourier components of the field and displacement, according to[1], by[3)]

$$j_i = \sigma_{ik}\tilde{E}_k + \partial_{ik}u_k, \qquad f_i^{d} = c_{ik}\tilde{E}_k + b_{ik}u_k, \quad (2.2)$$

where $\tilde{\mathbf{E}} = \mathbf{E} + c^{-1}\mathbf{u}\times\mathbf{H} - me^{-1}\ddot{\mathbf{u}}$ is the effective field, and the field E, which satisfies the Maxwell equations, includes the gradient of the chemical potential.

The change in the free energy, due to the work per unit time done on the charges by the effective electric field and the deformation field, is equal to[10]

$$-\frac{\partial F}{\partial t} = \int dV(\mathbf{j}\tilde{\mathbf{E}} - \mathbf{f}^{d}\dot{\mathbf{u}}). \quad (2.3)$$

The kinetic coefficients satisfy the symmetry relations of[10], as a consequence of which it is convenient to introduce the quantity

$$\bar{\partial}_{ik}(\mathbf{k}, \mathbf{H}) \equiv \partial_{ik}(-\mathbf{k}, -\mathbf{H}) = i\omega c_{ki}(\mathbf{k}, \mathbf{H}) \quad (2.4)$$

in place of c. With the help of the tensor

$$\Delta_{kl} = me^{-1}(i\omega\delta_{kl} - \varepsilon_{klm}\omega_m^{c}), \qquad \boldsymbol{\omega}^{c} \equiv e\mathbf{H}/mc$$

the Fourier components of the force D and of the effective field E are rewritten in the following fashion:

$$D_i = \Delta_{ik}j_k + f_i^{d}, \qquad \tilde{E}_k = E_k - i\omega\Delta_{kl}u_l.$$

Eliminating the electric field by means of the Maxwell equations (see, for example,[1]) and rewriting the expression for the force D in the form $D_i = D_{ik}u_k$, we obtain the equations for the displacement in the form

$$\{(\eta_p - \zeta)\delta_{pq} + d_{pq}\}u_q = 0, \qquad \eta_p\delta_{pq} \equiv \lambda_{p\kappa q\kappa}/\rho s^2, \quad \zeta \equiv (\omega/ks)^2, \quad d_{il} = -D_{il}/\rho s^2k^2. \quad (2.5)$$

Here p and q are the principal axes and η_p is the principal value of the tensor of the elastic moduli of the lattice, $\lambda_{p\kappa q\kappa} \equiv \lambda_{pjqj}k_jk_i/k^2$, ζ is the dimensionless square of the phase velocity of the wave, and s is the characteristic speed of sound. The terms $d_{ik}u_k$ are the (dimensionless) Fourier components of the force exerted on the lattice by the conduction electrons and the electromagnetic field.

We write the expression obtained in[1] for d_{pq} in a different, more symmetric form:

$$d_{pq} = \sum_{J=I}^{VI} d_{pq}^{(J)},$$

$$d_{pq}^{I} = -\frac{b_{pq}}{\rho s^2k^2}, \quad d_{pq}^{II} = \frac{\bar{\partial}_{p\kappa}\partial_{\kappa q}}{i\omega\sigma_{\kappa\kappa}\rho s^2k^2}, \quad d_{pq}^{III} = -\frac{k^2c^2}{4\pi}\frac{\Delta_{p\beta}\Delta_{\beta q}}{\rho s^2k^2},$$

$$d_{pq}^{IV} = \frac{\bar{\partial}_{p\beta}^{*}\rho_{\beta\gamma}^{*}\partial_{\gamma q}^{*}}{i\omega\rho s^2k^2}, \quad d_{pq}^{V} = \frac{i\omega}{\rho s^2k^2}\left(\frac{k^2c^2}{4\pi\omega}\right)^2\Delta_{p\beta}\rho_{\beta\gamma}\Delta_{\gamma q},$$

$$d_{pq}^{VI} = -\frac{ic^2}{4\pi\omega\rho s^2}\{\Delta_{p\beta}\rho_{\beta\gamma}\partial_{\gamma q}^{*} - \bar{\partial}_{p\beta}^{*}\rho_{\beta\gamma}\Delta_{\gamma q}\}. \quad (2.6)$$

The indices $\alpha, \beta, \gamma, \ldots$ denote the projections on the axes orthogonal to directions of the wave vector k, κ denotes the projection in the direction of the wave vector, the asterisk denotes renormalization which appears upon elimination of the longitudinal electric field (by means of the condition of electric neutrality $j_\kappa = 0$), $\rho_{\alpha\beta}$ are the components of the tensor of the effective transverse resistivity, $\hat{I}$ is the unit operator, and the definition of $\bar{\partial}$ is (2.4):

*[jH] ≡ j × H.

3)We note that the corresponding formula (4.5) of [10] contains errors.

$$\hat{\rho} \equiv \left(\hat{\sigma}^{\cdot} + i\frac{k^2c^2}{4\pi\omega}\hat{I}\right)^{-1}, \quad \sigma_{\alpha\beta}^{\cdot} = \sigma_{\alpha\beta} - \frac{\sigma_{\alpha\varkappa}\sigma_{\varkappa\beta}}{\sigma_{\varkappa\varkappa}},$$
$$\partial_{\alpha l}^{\cdot} = \partial_{\alpha l} - \frac{\sigma_{\alpha\varkappa}\partial_{\varkappa l}}{\sigma_{\varkappa\varkappa}}, \quad \bar{\partial}_{l\alpha}^{\cdot} = \bar{\partial}_{l\alpha} - \frac{\bar{\partial}_{l\varkappa}\sigma_{\varkappa\alpha}}{\sigma_{\varkappa\varkappa}}. \tag{2.7}$$

The components d^{III} and d^{V} are sometimes conveniently joined by using the definition (2.7) of $\hat{\rho}$:

$$d_{pq}^{III} + d_{pq}^{V} = -\frac{c^2}{4\pi\rho s^2}\Delta_{p\beta}\rho_{\beta\gamma}\sigma_{\gamma\varepsilon}^{\cdot}\Delta_{\delta q}. \tag{2.6'}$$

The dispersion equation of the system is

formula $\det\|(\eta_p - \zeta)\delta_{pq} + d_{pq}\| = 0.$

If $|d| \ll 1$, then in the nondegenerate case

$$\zeta \approx \eta_p + d_{pp}, \quad \eta_1 \neq \eta_2 \neq \eta_3, \quad |d| \ll 1. \tag{2.8}$$

Here the relative change in the sound velocity $\Delta s/s$ and the relative absorption γ/ω are equal to

$$\Delta s/s\sqrt{\eta_p} \approx \mathrm{Re}(d_{pp}/2\eta_p), \quad \gamma/\omega = -\mathrm{Im}(d_{pp}/2\eta_p). \tag{2.8'}$$

In the case of double degeneracy (this corresponds, for example, to transverse sound in propagation along an axis of symmetry of order higher than two)

$$\zeta_{1,2} = \eta + {}^1/_2(d_{11} + d_{22} \pm \sqrt{(d_{11} - d_{22})^2 + 4d_{12}d_{21}}), \quad \eta = \eta_1 = \eta_2 \neq \eta_3. \tag{2.9}$$

The expressions (2.5) and (2.6) are the consequence only of the phenomenological relations (2.1) and (2.2) and the Maxwell equations. In the quasiclassical case, when the electron gas is described by the kinetic equation, according to[1,10]

$$j_i = -e\langle v_i\chi\rangle, \quad f_i^{d} = -\frac{\partial}{\partial x_k}\langle\Lambda_{ik}\chi\rangle, \tag{2.10}$$

where $\chi\partial f_0/\partial\epsilon$ is the deviation of the distribution function from instantaneous equilibrium and satisfies the equation

$$\frac{\partial\chi}{\partial t} + (\mathbf{v}\Delta)\chi + \Omega\frac{\partial\chi}{\partial\tau} + \hat{\nu}\chi = e\mathbf{E}\mathbf{v} - \Lambda_{ik}\dot{u}_{ik}, \quad \Lambda_{ik} = \lambda_{ik} - \frac{\langle\lambda_{ik}\rangle}{\langle 1\rangle}. \tag{2.11}$$

The brackets denote integration over the Fermi surface, and $\Lambda_{ik}(\mathbf{p})$ is the value of the deformation potential $\lambda_{ik}(\mathbf{p})$ renormalized by virtue of the electrical neutrality. The deformation potential itself can be represented in the form of two components

$$\lambda_{ik}(\mathbf{p}) = -mv_iv_k + L_{ik}(\mathbf{p}), \tag{2.12}$$

where the first component describes the momentum flux when the electron moves freely and $L_{ik}(\mathbf{p})u_{ik}$ is the work done on an electron with quasimomentum $\mathbf{p}$ upon deformation of the lattice.[2,11]

This allows us to give a simple interpretation of the separate components in the force. We rewrite (2.1) in the form

$$\rho_p\ddot{u}_i = \lambda_{iklm}\frac{\partial u_{lm}}{\partial x_k} + D_i',$$
$$D_i' = \frac{1}{c}[\mathbf{jH}]_i - \frac{m}{e}\frac{\partial j_i^{el}}{\partial t} + \frac{\partial}{\partial x_k}\langle mv_iv_k\chi\rangle - \frac{\partial}{\partial x_k}\langle L_{ik}\chi\rangle. \tag{2.1'}$$

We have used the normalization condition $\langle\chi\rangle = 0$, thanks to which $f_i^{d} = -\partial\langle\lambda_{ik}\chi\rangle/\partial x_k$, and also the obvious relations $\rho = \rho_p + nm$ and $\mathbf{j} = \mathbf{j}^{el} - en\dot{\mathbf{u}}$, where n is the concentration of electrons and ρ_p is the lattice density. The first term in D′ is the momentum obtained by the electrons and the lattice from the external field (with account of the fact that the system as a whole is electrically neutral), the remaining terms (with opposite sign) describe the momentum carried away by the electrons, so that the difference here is the momentum transferred to the lattice (per unit time and per unit volume). Actually, $(m/e)\partial\mathbf{j}^{el}/\partial t$ is the change in the momentum of the electrons contained in a given element of volume, the term $-\partial\langle mv_iv_k\chi\rangle/\partial x_k$ describes the momentum carried away in the free motion of the electrons into another element of volume, and the term $\partial\langle L_{ik}\chi\rangle/\partial x_k$, according to the already mentioned results of Gurevich, Lang and Pavlov,[11,2] is nothing but the momentum transferred to the lattice when work is performed to deform it.

We shall now briefly discuss the structure of the matrix $\hat{d}$. The individual components in the expression for $\hat{d}$ contain different kinetic coefficients and have different physical meanings (see[7,8]). The first three terms $\hat{d}^{I}$, $\hat{d}^{II}$ and $\hat{d}^{III}$ do not contain the transverse resistivity, while $\hat{d}^{IV}$, $\hat{d}^{V}$, and $\hat{d}^{VI}$ are proportional to $\rho_{\alpha\beta}$ and are due to the transverse electric fields which are generated upon propagation of sound. The component $\hat{d}^{I}$ describes pure deformation effects. Neither longitudinal nor the transverse fields make any contribution to it. It corresponds to the work done by that part of the force f^{d} which is proportional to deformation only. The corresponding power is $\dot{u}f^{d}$, where by f^{d} we mean only the second term in (2.2). The component $\hat{d}^{II}$ represents the renormalization of the deformation term due to the longitudinal electric field and is unimportant as a rule. The component $\hat{d}^{III}$ is hermitian; it describes the change produced in the sound velocity by the induction effects in an infinitely conducting medium, and was first measured by Galkin and Korolyuk.[12] Not included in it are the terms associated with the Stewart-Tolman effect. This component is universal; it does not contain any kinetic coefficients. In terms of the axes x, y, and z, with the field directed along the z axis, $\hat{d}^{III}$ is equal to

$$\hat{d}^{III} = -\frac{H^2}{4\pi\rho s^2}\begin{pmatrix} 1 & i\omega/\omega_c & 0 \\ -i\omega/\omega_c & 1 & 0 \\ 0 & 0 & (\omega/\omega_c)^2 \end{pmatrix}. \tag{2.13}$$

The component $\hat{d}^{IV}$ describes the contribution of the transverse deformation currents and is the deformation-interaction renormalization due to the transverse electric fields. It is especially important for transverse sound.

Correspondingly, $\hat{d}^{V}$ describes the effect of finite conductivity on the induction term, i.e., renormalization, due to the transverse fields, of the component $\hat{d}^{III}$. In some cases, the separation of these components becomes purely arbitrary. And, finally, the last term $\hat{d}^{VI}$ describes crossing effects, when either the work $\dot{\mathbf{u}}\cdot\mathbf{f}^{d}$ (per unit time) is performed by that part of the deformation force which is proportional to the electric field, or the current in the expression for the work $\mathbf{j}\cdot\mathbf{E}$ done by the field has a deformation origin.

The dynamic elastic moduli of the metal were also obtained and studied (principally as applied to the rotation of the plane of polarization of sound propagated along the magnetic field) by Vlasov and Filippov.[13]

3. KINETIC COEFFICIENTS IN A STRONG MAGNETIC FIELD

The tensor d_{ik}, which determines the absorption, dispersion, and polarization of the sound in the metal, is expressed according to (2.6) in terms of the kinetic coefficients (2.2). The microscopic expressions for the kinetic coefficients are written conveniently by introducing the Green's operator R of the Fourier component of the kinetic equation (2.11):

$$\sigma_{ik} = e^2\langle v_i R^s v_k\rangle, \quad \partial_{il} = e\omega k\langle v_i R^a \Lambda_{lv}\rangle, \qquad b_{il} = i\omega k^2\langle \Lambda_{ix} R^s \Lambda_{lx}\rangle, \tag{3.1}$$

where R^S and R^a denote the symmetric and antisymmetric parts (about p) of the Green's operator. In the calculation of the integrals of $\langle\varphi\rangle$ over the Fermi surface, the principal difficulty encountered is the calculation of the mean $\overline{\varphi}$ over the angle of rotation in the magnetic field:[4)]

$$\langle\varphi\rangle \equiv \frac{2}{h^3}\int dp_H \cdot 2\pi m^*\varphi, \quad \varphi \equiv \frac{1}{2\pi}\int_0^{2\pi}\varphi(\tau)\,d\tau \quad \langle\varphi\rangle \equiv \langle\overline{\varphi}\rangle. \tag{3.2}$$

In the following, we introduce the relaxation time $\nu^{-1}(p)$. For an arbitrary periodic function $g(\tau)$, the quantity Rg satisfies identically the relation

$$\frac{\partial}{\partial\tau}Rg + \frac{\alpha}{\Omega}Rg = \frac{g}{\Omega}, \quad \alpha \equiv i(kv-\omega)+\nu, \tag{3.3}$$

and the explicit expression for Rg has the form

$$Rg = \frac{1}{\Omega(1-e^{-2\pi\alpha/\Omega})}\int_{\tau-2\pi}^{\tau} d\tau_1 g(\tau_1)\exp\frac{1}{\Omega}\int_{\tau}^{\tau_1} d\tau_\parallel \alpha(\tau_\parallel). \tag{3.3'}$$

We confine ourselves to the case of closed Fermi-surface cross sections. In a strong magnetic field, in the calculation of the principal terms of the expansion in powers of 1/H, it is convenient to use identities that follow from (3.3) and (3.3′) for periodic r and g:

$$\overline{\frac{\partial r}{\partial\tau}Rg} = \frac{1}{\Omega}(-\overline{rg} + \overline{r\alpha Rg}), \quad \overline{gR\frac{\partial r}{\partial\tau}} = \frac{1}{\Omega}(\overline{gr} - \overline{gR r\alpha}), \tag{3.4}$$

and also the relation

$$\overline{rRg} \approx \bar{r}\bar{g}/\bar{\alpha} + O(1/\Omega). \tag{3.5}$$

By using the representation of the periodic quantities in the form $r = \bar{r} + \partial s/\partial\tau$ at $\bar{s} = 0$, it is easy to obtain with the aid of the relations (3.4) and (3.5) (see also the Appendix) expansions for the kinetic coefficients in a strong magnetic field.

As is seen from (3.5), the principal term of the expansion in Ω^{-1} (which does not depend on the field) is expressed in the form of the product of values averaged over the period of rotation. We limit ourselves in the following to the case $\mathbf{k} \perp \mathbf{H}$:

$$\mathbf{H} = (0,0,H), \quad \mathbf{k} = (-k,0,0), \quad v_x = m_*^{-1}\frac{\partial p_y}{\partial\tau}, \qquad v_y = -m_*^{-1}\frac{\partial p_x}{\partial\tau}, \quad \bar{\alpha} = \nu^*, \quad \nu^* \equiv \nu - i\omega. \tag{3.6}$$

For the estimates, we shall assume that we are dealing with a good metal, in which the number of electrons per atom is of the order of unity: $n \sim n_0$ and, correspondingly, the only characteristic energy is the atomic energy:

$$\Lambda \sim \varepsilon \sim mv^2 \sim Ms^2. \tag{3.7}$$

Here v and m are the Fermi velocity and the electron mass, and M is the mass of the ion.

We consider the principal term of the expansion of the coefficients b_{il}, which determine the deformation component d^I_{il} in (2.6):

$$b_{il} = i\omega k^2\langle\overline{\Lambda_{ix}R^s\Lambda_{lx}}\rangle, \qquad \overline{\Lambda_{ix}R^s\Lambda_{lx}} \approx \bar{\Lambda}_{ix}\bar{\Lambda}_{lx}/\bar{\nu}^* + O(1/\Omega). \tag{3.8}$$

We denote by g_H those crystal point symmetry transformations for which the plane p_H = const remains unchanged. These transformations form the group G_H which is a subgroup of the point symmetry group G and which depends on the direction of the field.

We first limit ourselves to the case of singly connected Fermi surface cross sections. Then the quantities ϵ and p_H are obviously not changed by the transformations g_H; only τ is transformed. Therefore the quantities $\bar{\Lambda}_{ik}(\epsilon, p_H)$ averaged over the period of rotation transform like components of a constant tensor, whence there directly follow "selection rules" for the components $\bar{\Lambda}_{ix}$, which we shall discuss for axial symmetry (the general results are summarized in the Table). Thus, if the z axis, along which the magnetic field is directed, is not a symmetry axis of the crystal, then $\bar{\Lambda}_{ix} \neq 0$ and, in accord with (3.8), the principal term of the expansion is different from zero and determines the following estimate of the matrix b_{il}:

$$b_{il} \sim i\omega k^2 n\varepsilon_F/\nu^* \quad (n=1). \tag{3.9}$$

(the order of the z axis is shown in parentheses). If H is directed along a twofold axis, then $\bar{\Lambda}_{\mu x} \neq 0$ (μ, ν = x, y) and $\Lambda_{zx} = 0$, in view of which one can set $\Lambda_{zx} = \partial\psi_{zx}/\partial\tau$, where the transformation properties of the quantities ψ_{zx} are such that $\overline{\psi_{zx}v_x} \neq 0$ and the principal terms of the expansion lead to an estimate of the matrix elements b_{il}:

$$b_{\mu\nu} \sim i\omega k^2\frac{n\varepsilon}{\nu^*}, \quad b_{zz} \sim \frac{i\omega k^2 n\varepsilon}{\nu^*}\left[\left(\frac{r}{l}\right)^2 + (kr)^2\right], \quad n=2. \tag{3.9'}$$

Here $r = v/\Omega$ is the twist radius, $l = v/\nu^*$ is the (complex) free path.

In the case in which H is directed along an axis of higher order ($n > 3$) and there is a plane of symmetry, the result is qualitatively the same as in the isotropic case, which we shall therefore consider in more detail. In an isotropic metal,

$$\lambda_{ik}(\mathbf{p}) = \lambda_1\delta_{ik} + \lambda_2\frac{p_ip_k}{p^2}, \quad \Lambda_{ik}(\mathbf{p}) = \lambda_2\left(\frac{p_ip_k}{p^2} - \frac{1}{3}\delta_{ik}\right), \tag{3.10}$$

where λ_1 and λ_2 are functions of the energy.

The components Λ_{ix} and ψ_{ix} of interest to us are equal to

$$\Lambda_{xx} = \lambda_2\left[\left(\frac{p_\perp}{p}\right)^2\cos^2\tau - \frac{1}{3}\right], \quad \Lambda_{yx} = \frac{1}{2}\left(\frac{p_\perp}{p}\right)^2\sin 2\tau,$$
$$\Lambda_{zx} = \frac{p_zp_\perp}{p^2}\cos\tau, \quad \psi_{xx} = \lambda_2\left(\frac{p_\perp}{p}\right)^2\frac{\sin 2\tau}{4},$$
$$\psi_{yx} = -\left(\frac{p_\perp}{p}\right)^2\frac{\cos 2\tau}{4}, \quad \psi_{zx} = \frac{p_zp_\perp}{p^2}\sin\tau,$$

[4)] The brackets also include summation over the zones (index omitted) and all the singly-connected cross sections of a given zone (index a):

$$\langle\varphi\rangle = \frac{2}{h^3}\sum\int dp_H\sum_a 2\pi m_a^*\overline{\varphi}^a.$$

We use the variables of Lifshitz, Azbel' and Kaganov. [14]

G_H	C_1	C_i	C_s			C_{2v}		C_s	C_{3v}		C_4	C_{4v}	C_s	C_{6v}	$C_{\infty v}$
			m_x	m_y	m^*_ζ	$2m_xm_y$	$2m_\zeta m^*_\eta$		$3m_y$	$3m^*_\zeta$					
$\bar\Lambda_{xy}$	$\neq0$	$\neq0$	0	0	$\neq0$	0	$\neq0$	0	0	0	0	0	0	0	0
$p_y\Lambda_{xy}$	—	—	$\neq0$	0	—	0	—	$\neq0$	0	$\neq0$	0	0	0	0	0
d^I_{yy}/d^I_{xx}	1	1	$(kr)^2$	$(r/l)^2$	1	$(r/l)^2$	1	$(kr)^2$	$(r/l)^2$	$(kr)^2$	$(r/l)^2$	$(r/l)^2$	$(r/l)^2$	$(r/l)^2$	$(r/l)^2$
$\bar\Lambda_{xz}$	$\neq0$	0	$\neq0$	0	$\neq0$	0	$\neq0$	0	0	0	0	0	0	0	0
$p_y\Lambda_{xz}$	—	$\neq0$	—	0	—	0	—	$\neq0$	0	0	$\neq0$	0	$\neq0$	0	0
d^I_{zz}/d^I_{xx}	1	$(kr)^2$	1	$(r/l)^2$	1	$(r/l)^2$	1	$(kr)^2$	$(r/l)^2$	$(r/l)^2$	$(kr)^2$	$(r/l)^2$	$(kr)^2$	$(r/l)^2$	$(r/l)^2$

$m_\zeta(m_\eta)$ is the mirror plane which does not pass through the axes Ox and Oy.

In particular, $\overline{\psi_{zx}v_x} = 0$. In this notation,

$$\overline{\Lambda_{ix}R^s\Lambda_{lx}}\approx\begin{pmatrix} \bar\Lambda^2_{xx}/\bar\nu^* & \frac{1}{\Omega}\overline{\frac{\partial\psi_{xx}}{\partial\tau}}\psi_{yx} & 0 \\ -\frac{1}{\Omega}\overline{\frac{\partial\psi_{xx}}{\partial\tau}}\psi_{yx} & \frac{1}{\Omega^2}\overline{\psi^2_{xy}\nu^*} & 0 \\ 0 & 0 & \frac{1}{\Omega^2}\overline{\psi^2_{zx}\nu^*} \end{pmatrix}, \quad n>3.$$

Finally, the estimate for b_{il} has the following form in the isotropic case:

$$b_{il}\sim\frac{i\omega}{\nu^*}k^2n\varepsilon\begin{pmatrix} 1 & r/l & 0 \\ -r/l & (r/l)^2 & 0 \\ 0 & 0 & (r/l)^2 \end{pmatrix}, \quad n>3. \qquad (3.9'')$$

A completely different result is obtained in the case of multiply-connected cross sections, when the individual singly-connected parts transform into one another under the transformations g_H and the group G no longer determines the symmetry properties of each of these regions. The cross section of each of the regions $\bar\Lambda^a_{x\alpha}$ are generally different from zero even for high symmetry of the field direction; the asymptote of b_{il} has the form

$$b_{\alpha\alpha}\approx i\omega k^2\int\frac{2dp_H}{h^3}2\pi\sum_a m_a^*\frac{(\bar\Lambda^a_{\alpha x})^2}{\bar\nu^{*a}} \qquad (3.11)$$

and leads to the estimate (3.9).

The asymptote of the conductivity tensor has been calculated a number of times. In our case, $\mathbf{k}\perp\mathbf{H}$, $r/l \ll kr \ll 1$ and, according to the formulas (B.1)–(B.7) of the Appendix, we obtain in terms of the x, y, z axes, $\sigma = \sigma_0\hat M$ ($\sigma_0 \equiv ne^2/m\nu^*$ and $\delta n \equiv n_- - n_+$),[8] where the matrix $\hat M$ has the form

$$M_{xx}=(r/l)^2,\quad M_{xy}=-\frac{\delta n}{n}\frac{r}{l}+\left(\frac{r}{l}\right)^2C_3-(kr)^2\frac{r}{l},\quad M_{xz}=\frac{r}{l}C_2,$$
$$M_{yx}=\frac{\delta n}{n}\frac{r}{l}+\left(\frac{r}{l}\right)^2C_3+(kr)^2\frac{r}{l},\quad M_{yy}=\left(\frac{r}{l}\right)^2+(kr)^2,$$
$$M_{yz}=\left(\frac{r}{l}+(kr)^2\right)C_2,\quad M_{zx}=\frac{r}{l}C_2,$$
$$M_{zy}=\left(-\frac{r}{l}+(kr)^2\right)C_2,\quad M_{zz}=1. \qquad (3.12)$$

The coefficients C_2 and C_3 are of the order of unity and vanish if **H** is directed along the axes of second (third) order and higher, respectively. We shall write down the estimates of the remaining kinetic coefficients for an isotropic metal. Here

$$\hat\sigma^*=\begin{pmatrix}\sigma_{xx}^* & 0\\ 0 & \sigma_{yy}^*\end{pmatrix}$$
$$\sigma_{xx}^*=\frac{e^2(n_- - n_+)^2}{\langle p_y^2\nu^*\rangle}+\left(\frac{c}{H}\right)^2\left(\langle p_x^2\nu^*\rangle+k^2\left\langle\frac{(p_xv_x)^2}{\nu^*}\right\rangle\right),$$
$$\sigma_{yy}^*=e^2\langle v_x^2/\nu^*\rangle,$$

i.e.,

$$\hat\sigma^*\sim\sigma_0\begin{pmatrix}a & 0\\ 0 & 1\end{pmatrix},\quad a=\left(\frac{\delta n}{n}\right)^2+\left(\frac{r}{l}\right)^2+(kr)^2,$$
$$\hat\rho=\frac{1}{\sigma_0}\begin{pmatrix}b & 0\\ 0 & c\end{pmatrix},$$
$$b=\left[\left(\frac{\delta n}{n}\right)^2+\left(\frac{r}{l}\right)^2+(kr)^2+i(kr)^2\frac{\nu^*}{\omega}\right]^{-1},$$
$$c=\left[1+i(k\delta)^2\frac{\nu^*}{\omega}\right]^{-1}. \qquad (3.13)$$

The high-frequency skin depth for $\mathbf{H}=0$ and $\mathbf{k}=0$ is designated $\delta = c/\omega$. In σ^*_{yy} and so on we have kept only those small terms which do not vanish for $\delta n = 0$.

The coefficients ∂_{il}, which describe the deformation part of the current and which correspond to the renormalization of the deformation interaction with the electric fields, have the following order in the isotropic case:

$$\partial_{xx}\sim i\omega ne(kr)^2,\quad \partial_{xy}\sim i\omega ne(kr)^2r/l,\quad \partial_{xz}=0,$$
$$\partial_{\alpha l}\sim\partial_{\alpha l}^*\sim i\omega ne(kr)(kl)\begin{pmatrix}1 & r/l & 0\\ 0 & 0 & r/l\end{pmatrix},\quad \alpha=y,z,\quad n>3, \qquad (3.14)$$

and for the low symmetry direction the fields are determined by the means

$$\overline{v_xR^s\Lambda}\approx\frac{ik}{2(m_*\Omega)^2\bar\nu^*}\{\overline{\Lambda p_y^2\nu^*}+\overline{\Lambda}\,\overline{p_y^2\nu^*}-\overline{2p_y\nu^*}\,\overline{p_y\Lambda}\},$$

$$\overline{v_yR^s\Lambda}\approx\frac{ik}{m_*\Omega\bar\nu^*}\overline{p_xv_x\Lambda},\quad \overline{v_zR^s\Lambda}\approx-\frac{ik}{m_*\Omega\bar\nu^*}\{\overline{v_z\Lambda p_y}-\overline{\Lambda v_zp_y}\}. \qquad (3.15)$$

Knowing the asymptotic behavior of the kinetic coefficients, we can determine d_{il} and find the absorption, dispersion, and polarization of the sound wave in the metal.

4. ABSORPTION AND DISPERSION OF SOUND IN METALS IN A STRONG MAGNETIC FIELD

Let us first consider purely deformation effects, described by d^I which, in a strong field and for $\delta n \sim n$, play a fundamental role (and at $\delta n = 0$ give the same contribution as the electric fields). The contribution of the remaining mechanisms (at $n_- = n_+$) will be discussed briefly in the next section. Outside of resonance

($\omega \gg \omega_{res}$) the electric fields lead to the same dependence on ω and H as $\hat{d}^{I}$.

We begin with the well studied case of longitudinal sound. For strong spatial dispersion of longitudinal sound, and by using for b_{xx} the asymptote (3.8) and (3.9) and the estimate (3.7),[5)] we obtain

$$d_{xx} = -i\omega \frac{\langle \bar{\Lambda}_{xx}^2/\bar{\nu}^* \rangle}{ps^2} \sim \frac{i\omega}{\nu^*} = \frac{i\omega}{\nu - i\omega}. \qquad (4.1)$$

For $\omega \ll \nu$ it follows therefore that $\gamma \sim \omega \operatorname{Im} d_{xx} \sim \omega^2/\nu$, i.e., the absorption agrees with the absorption in the absence of the magnetic field for weak dispersion[7,8] (for references on the experiment, see the reviews[16]). For an arbitrary relation of ω and ν it follows from (4.1) that

$$\frac{\gamma}{\omega} \sim \frac{\omega\gamma}{\nu^2+\omega^2}, \quad \frac{\Delta s}{s} \sim \frac{\omega^2}{\nu^2+\omega^2}. \qquad (4.1')$$

It is seen from (4.1) that for $\omega \gtrsim \nu$ a strong (of the order of unity) renormalization of the longitudinal sound velocity takes place, which was predicted by Kulik and observed by Bezuglyĭ and Burma.[9] The relative absorption γ/ω reaches a maximum for $\omega \sim \nu$ (also of the order of unity, i.e., sound is generally not propagated in this region of frequencies). Before going on to transverse sound, let us comment on these results. They have a simple physical meaning. The behavior of both absorption and dispersion corresponds to the contribution of second viscosity, thanks to the presence of a slow process (collisions) in the electronic subsystem, and is described by the general theories of Leontovich-Mandel'shtam and Kneser. Actually, the kinematic viscosity of the electron gas is $\eta_{el} \sim nmvl$, and inasmuch as the corresponding viscous force enters into the equation of motion of the lattice the absorption coefficient is $\gamma \sim \eta_{el}k^2/\rho$, where $\rho \sim n_0 M$ is the density of the metal. Recognizing that $Ms^2 \sim \epsilon$, we obtain the estimate (4.1).

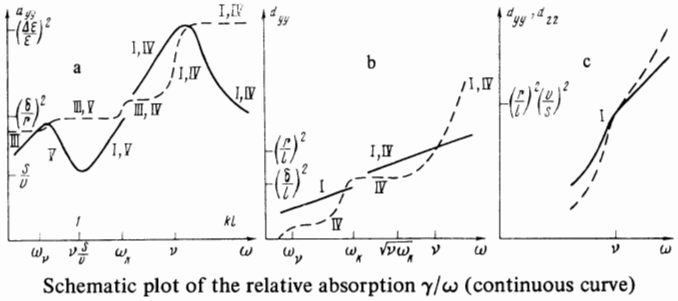
Although the magnetic field does not enter in (4.1), its role is very important. As $\mathbf{H} \to \infty$, we have $r \to 0$ and the electron moves only along the direction $\mathbf{H} \perp \mathbf{k}$. The collision-free absorption connected with Landau damping, is impossible here, no matter how high the frequency. (We note in this connection that the condition $\mathbf{k} \perp \mathbf{H}$ is very critical, since the electrons are seen to be already in resonance with the wave at inclinations through the angle s/v.) The absorption has a collision character even for $kl \gg 1$ (in contrast with the situation for $\mathbf{H} = 0$). Being displaced along the magnetic field, the electron travels relative to the phase fronts of the sound wave, owing to the motion of the latter. Here the work is thus performed in the longitudinal sound wave against the forces of the electron pressure averaged over the magnetic field, which is in turn described by the component $\bar{\Lambda}_{xx} \neq 0$. The collisions lead to a viscous absorption; the average work done by the sound field on the electrons becomes different from zero. For purely deformation interaction in a strong field, the work down by the sound field on the electrons is proportional to $\bar{\Lambda}_{ix}^2$, where i is the direction of the displacement and x the direction of propagation of the sound. Actually, as $H \to \infty$, in accord with (3.5), only the averages of electronic quantities over the period of rotation play a role. One of the factors $\bar{\Lambda}_{ix}$ results from the definition of the deformation force: $\dot{u} \cdot f^{d} = -\dot{u}_i \partial \langle \Lambda_{ix\chi} \rangle / \partial x$, the second factor $\bar{\Lambda}_{ix}$ is due to the fact that the number of electrons contributing to the purely deformation effect is also proportional to Λ_{ix}: $\chi \sim \Lambda_{ix} \partial \dot{u}_i / \partial x$. Thus, in order that the sound field perform work on the electrons (by means of purely deformation interactions) it is necessary that the longitudinal component of the momentum flux in the direction of displacement in the sound field differ from zero. Thus,

$$d_{pq}^{I} = -\frac{i\omega}{\rho s^2} \left\langle \frac{\bar{\Lambda}_{px}\bar{\Lambda}_{qx}}{\bar{\nu}^*} \right\rangle, \quad n = 1. \qquad (4.2)$$

But, while the quantity $\bar{\Lambda}_{xx}$, which plays the role of the partial electronic pressure for the longitudinal sound as $H \to \infty$, is always different from zero, the quantities $\bar{\Lambda}_{xy}$ and $\bar{\Lambda}_{xz}$, which describe the required momentum flux as $\mathbf{H} \to \infty$ for transverse sound, are equal to zero only if **H** is directed along a symmetry axis of order greater than 2 (for more detail, see Sec. 3 and Appendix A). In other words, the electronic shear moduli vanish for an infinitely strong field in the case of a symmetric direction of the field in the crystal. In particular, they vanish for an isotropic electron spectrum. This means that for arbitrary direction of the field **H**, the quantities $\bar{\Lambda}_{yx}$ and $\bar{\Lambda}_{zx} \sim \epsilon(\Delta\epsilon/\epsilon)$, where $\Delta\epsilon/\epsilon$ characterize the relative anisotropy of the spectrum, while the principal term in d^{I} is given by the expression (4.2) and is of the order of

$$d_{\alpha x}^{I} \sim \frac{i\omega}{\nu^*}\frac{\Delta\varepsilon}{\varepsilon}, \quad d_{\alpha\beta}^{I} \sim \frac{i\omega}{\nu^*}\left(\frac{\Delta\varepsilon}{\varepsilon}\right)^2, \quad \alpha, \beta = y, z; \quad n = 1. \qquad (4.2')$$

For a symmetric direction of the magnetic field in the crystal (see the Table) the electronic shear moduli vanish for an infinitely strong field. Therefore, the nonzero electronic effects for transverse sound are connected with the finiteness of **H** and are due to this incomplete averaging, namely, to the finiteness of the period of rotation in the magnetic field compared with



Schematic plot of the relative absorption γ/ω (continuous curve) and dispersion $\Delta s/s$ (dashed curve) for transverse sound with various symmetries of direction of the magnetic field. The Roman numerals show which components give the principal contribution to the dynamic elastic modulus d: a—$G_H = C_1$, $\delta_n \equiv n_- - n_+ = 0$; b—$G_H = C_\infty$, $\delta n = 0$; c—$G_H = C_3$, $\delta_n = 0$.

[5)] We note that the estimates used in the present section are very rough. Actually, even in the approximation of the relaxation time, an appreciable dependence of $\bar{\nu}$ on p_z can lead to a blurring of the region $\omega \sim \bar{\nu}$. Even more significant can be the fact that the introduction of the relaxation time for $\mathbf{k} \perp \mathbf{H}$ is inconsistent for $kl \gg 1$ if $kr \ll 1$ and the sections are closed. Actually, in this case $\chi \sim \bar{g}/\bar{\nu}^*$ and is a smooth function of p_z and does not have the resonance peaks that allow us to introduce the relaxation time for strong spatial dispersion. [15]

the free path time. According to (2.6) and (3.9″) we have, in the isotropic case,

$$\hat{d}^{I} \sim \frac{i\omega}{\nu^*}\begin{pmatrix} 1 & r/l & 0 \\ -r/l & (r/l)^2 & 0 \\ 0 & 0 & (r/l)^2 \end{pmatrix}, \quad n>3. \tag{4.3}$$

The damping and dispersion for both polarizations of the transverse sound are described by the components

$$d_{yy}^{I} \sim d_{zz}^{I} \sim (i\omega\nu - \omega^2)/\Omega^2, \; n>3. \tag{4.3'}$$

For $\omega > \nu$ the dispersion exceeds the absorption (see the drawing). Like the absorption, it is also very small. The absorption under these conditions can actually be determined either by other mechanisms or by an admixture of a longitudinal component in the sound wave, by the inhomogeneity of the field, and by its inclination to the symmetry axes. According to (2.5) a wave polarized along the magnetic field remains purely transverse, and a small (relative to ω/Ω) longitudinal component of displacement appears in the wave of y polarization.[6)]

These same results are preserved qualitatively if the crystal is anisotropic but **H** is directed along axes of symmetry higher than second and have planes of symmetry (see the Table) (and k is along axes which guarantee the existence of transverse linearly-polarized sound).[17] If the same z axis is a twofold axis then, according to (3.9′),

$$\hat{d}^{I} \sim \frac{i\omega}{\nu^*}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & (r/l)^2 + (kr)^2 \end{pmatrix}, \quad n=2. \tag{4.4}$$

In this case the transverse y-polarized sound behaves like longitudinal sound, whereas in absorption and dispersion of transverse z-polarized sound there appear effects due to spatial dispersion, i.e., the finiteness of the twist radius over the wavelength comes into play. For $kl \gg 1$, this leads to asymptotic behavior different from that discussed in[4]. We note that a similar behavior must take place for purely axial symmetry C_n (to the extent that the spectrum is anisotropic) for any n.

5. DISPERSION OF ULTRASOUND IN THE RESONANCE REGION. CONTRIBUTION OF TRANSVERSE ELECTRIC FIELDS

It has been shown previously[1] that for **H** = 0, in the range of frequencies where the sound wavelength is comparable with the damping length of the electromagnetic wave (the skin depth), the dispersion increases materially and becomes of the same order as the sound absorption, owing to the contribution of the transverse electric field. We shall show below that a similar effect in sound dispersion should be observed also in a strong magnetic field for **k** ⊥ **H**, but here the position of the "resonance" frequency depends on the magnetic field, which materially simplifies its observation. In most cases, the resonance frequency lies in the range of ultrasonic frequencies ω appreciably lower than the collision frequency ν, while the relative change in the sound velocity can reach several percent. Inasmuch as we are interested in the resonance region, where $\lambda_{ac} \sim \lambda_{em}$, it is then seen from (3.11) that the most interesting case from this viewpoint is the case of a compensated metal with $\delta n = 0$ and with the number of electrons per atom of the order of unity ($n \sim n_0$). Actually, resonance corresponds to equality of the real and imaginary parts of the denominator in the transverse resistivity ρ_{yy} (3.11). If $\delta n/n \sim 1$, then ρ_{yy} is small and the resonance frequency (see below) does not depend on the magnetic field. In view of the smallness of $\hat{\rho}$, the contribution of terms in $\hat{d}$ containing it is very small and resonance does not occur. On the contrary, for $\delta n = 0$, in view of the absence of a Hall current, the transverse resistance of the metal is very large and the contribution of transverse electric fields to the work performed by the sound on the electrons is very substantial. On the other hand, the location of the "resonance" depends here essentially on the magnetic field. Limiting ourselves to the case $\omega \ll \nu$, we can write down the resonance condition in the form $(r/l)^2 + (kr)^2 \sim (k\delta)^2\nu/\omega$, or, in the case of strong spatial dispersion $kl \gg 1$,

$$\omega \sim \omega_k \equiv \nu(\delta/r)^2. \tag{5.1}$$

Inasmuch as $n \sim n_0\delta \sim 10^{-5}$ and consequently $\delta \ll r \gtrsim 10^{-4}$ cm, the frequency $\omega_k \ll \nu$ and increases quadratically with the magnetic field intensity. For weak dispersion, the resonance corresponds to the frequency

$$\omega \sim \omega_v \equiv \nu\left(\frac{s}{v}\frac{r}{\delta}\right)^2, \tag{5.2}$$

which satisfies the condition $\omega_\nu \ll \nu$ if $s/v \ll \delta/r$. It is inversely proportional to the square of the magnetic field intensity. Finally, for ρ_{zz}, and also for ρ_{yy} in the case $\delta n \sim n$, the resonance condition takes the form

$$\omega \sim \omega_s \equiv \frac{\omega_p^2}{\nu}\left(\frac{s}{c}\right)^2 \equiv 4\pi\sigma_0\left(\frac{s}{c}\right)^2. \tag{5.3}$$

We now estimate the amount of dispersion and absorption near the frequency ω_k. For this purpose, we compare the non-resonance terms $\hat{d}^{I}$ and $\hat{d}^{III}$, given above [(2.13), (4.1)], with the contribution from $\hat{d}^{IV} - \hat{d}^{VI}$, which is proportional to $\hat{\rho}$. In the isotropic case, for $\delta n = 0$ and $n \sim n_0$, we have in terms of the axes $\mu, \nu = x, y$

$$\begin{aligned} d_{\mu\nu}^{IV} &\sim \frac{i\omega}{\nu^*}\frac{1}{1+i\omega_k/\omega}\begin{pmatrix} 1 & r/l \\ -r/l & (r/l)^2 \end{pmatrix}, \\ d_{\mu\nu}^{V} &\sim \frac{i\nu^*}{\omega}\frac{(\delta/r)^4}{1+i\omega_k/\omega}\begin{pmatrix} 1 & i\omega/\Omega \\ -i\omega/\Omega & (\omega/\Omega)^2 \end{pmatrix}, \\ d_{\mu\nu}^{VI} &\sim \left(\frac{\delta}{r}\right)^2\frac{1}{1+i\omega_k/\omega}\begin{pmatrix} 0 & r/l \\ r/l & 0 \end{pmatrix}. \end{aligned} \tag{5.4}$$

These components of $\hat{d}$ describe the contribution of the electric fields to sound polarized orthogonally to the external magnetic field. For the zz components of d we have

$$d_{zz}^{IV} \sim \frac{i\omega}{\nu^*}\frac{(kr)^2(r/l)^2}{1+i\omega/\omega_s}, \quad d_{zz}^{V} \sim \frac{i\omega}{\nu^*}\frac{(k\delta)^2(\delta/l)^2}{1+i\omega/\omega_s}, \tag{5.5}$$

and d_{zz}^{VI} is smaller than d_{zz}^{V} by a factor of at least $(\delta/r)^2$.

[6)] Actually, the principal directions of the tensor $\lambda_{p\kappa q\kappa}$ are x, y and z, while $\eta_x = \eta_\parallel$, $\eta_y = \eta_z = \eta_\perp$. From (2.5) and (2.9), with account of $d \ll |\eta_\parallel - \eta_\perp|$ we have ($\eta_\parallel > \eta_\perp$) $\zeta_+ \approx \eta_\parallel + d_{xx} + d_{xy}d_{yx}/(\eta_\parallel - \eta_\perp)$, $u_y^{(+)} \approx u_x^{(+)}d_{yx}/(\eta_\parallel - \eta_\perp)$, $\zeta_- \approx \eta_\perp + d_{yy} - d_{yx}/(\eta_\parallel - \eta_\perp)$, $u_x^{(-)} \approx u_y^{(-)}d_{xy}/(\eta_\parallel - \eta_\perp)$.

It is seen from (5.4) that "resonance" should take place both for transverse (y polarization) and longitudinal sound, which is natural in view of the twisting of the electrons in the magnetic field. Limiting ourselves to the most interesting case of strong spatial dispersion, we obtain the following characteristic dependence of the absorption and dispersion (see the drawing). For y polarization, Δs is reckoned from the level determined by the frequency-independent but field-dependent component $d_{yy}^{III} \sim (\delta/r)^2$. Here the absorption and dispersion have the form

$$d_{xx} \sim \left(\frac{\delta}{r}\right)^2 (1+i), \quad d_{yy} \sim \left(\frac{\delta}{l}\right)^2 (1+i), \quad n > 3,$$

$$d_{xx} \sim \left(\frac{\delta}{r}\right)^2 (1+i), \quad d_{yy} \sim \left(\frac{\delta}{r}\right)^2 \left(\frac{\Delta\varepsilon}{\varepsilon}\right)^2 (1+i), \quad n \leqslant 2. \quad (5.6)$$

For z polarization of transverse sound, when the displacement vector is directed along the magnetic field, the principal term for good metals, by virtue of the condition $\delta \ll r$, is d_{zz}^{I} and resonance does not occur.

As seen from (5.4), for $\omega \gg \omega_k$ ($\delta n = 0$) the transverse deformation electric fields (the component $d_{\mu\nu}^{IV}$) give the same contribution and the same dependence on H and ω as $d_{\mu\nu}^{I}$ (in correspondence with[4]). We shall not write down the explicit formulas, but they are easily obtained by means of the general expressions (B.7).

In conclusion, the author thanks G. Ya. Lyubarskiĭ and N. A. Sapogov for useful discussions.

APPENDIX A

The deformation potential $\hat{\lambda}(p)$ is a tensor function of the quasimomentum and is invariant to transformations g of the point group G of the crystal:[6]

$$g\hat{\lambda}(g^{-1}p)g^{-1} = \hat{\lambda}(p), \quad \hat{\lambda}(p) = \hat{\lambda}(-p); \quad (A.1)$$

$\hat{\lambda}(p)$ is even because of the symmetry relative to time reversal. The following expansion of $\hat{\lambda}(p)$ in independent invariant tensors (similar to (3.10)) can be useful:

$$\lambda_{ik}(p) = \sum \lambda_1 a_{ik} + \sum \lambda_2 P_{ik} + \sum \lambda_3 (a_i P_k + a_k P_i), \quad (A.2)$$

λ_i are functions of the invariants of the group G (λ_1 and λ_2 are even and λ, are odd):

$$\lambda_{(i)}(p) = \lambda_{(i)}(g^{-1}p), \; g\hat{a}g^{-1} = \hat{a}, \; ga = a. \quad (A.3)$$

The quantities P_i are transform like p_i, and P_{ik} like $p_i p_k$ (for example, the quantities P_{ik} can be $p_i p_k$, v_i, v_k, $\partial^2\epsilon/\partial p_i \partial p_k$ and so on).

We now return to the symmetry properties averaged over the period of rotation in the magnetic field. In the variables ϵ, p_H, τ, we have $\hat{\lambda}(p) \equiv \hat{\lambda}(\epsilon, p_H, \tau)$ and $\hat{\lambda} = \hat{\lambda}(\epsilon, p_H)$ if the cross section (in a given zone) is singly connected. We now consider the subgroup of those transformations of symmetry g_H which do not change the projection of the quasimomentum in the direction of the field $g_H p_H = p_H$. The quantities λ_{ik} behave as components of the following constant tensor relative to the group G_H of these transformations:

$$g_H \bar{\hat{\lambda}}(\varepsilon, p_H) g_H^{-1} = \bar{\hat{\lambda}}(\varepsilon, p_H). \quad (A.4)$$

The only components that can differ from zero here are invariants of the group G_H. The latter is determined by the symmetry of direction Oz of the magnetic field in the crystal. In the calculation of the asymptotic form that determines the absorption of transverse sound in strong fields, in accord with the formulas of Sec. 3 and Appendix B, the principal terms are determined by the quantity $\overline{\Lambda}_{x\alpha}$ (α = y, z), and for $\overline{\Lambda}_{x\alpha} = 0$ the following terms contain the mean values $\overline{v_x \psi_{x\alpha}} = -m^{-1}\overline{p_y \Lambda_{x\alpha}}$ and $\overline{\psi_{x\alpha}^2 \nu}$ ($\Lambda_{x\alpha} = \partial\psi_{x\alpha}/\partial\tau$). The transformation properties of the quantities $A_{ik}(\tau)$ which enter into the mean values

$$\bar{A}_{ik} = \frac{1}{2\pi}\int_0^{2\pi} d\tau A_{ik}(\tau),$$

are conveniently determined with the aid of (A.2), assuming the tensor indices to be fixed. Here the P_i transform like components of a vector, P_{ik} is a tensor of second rank, and the quantities a_{ik} and a_i do not change. It is clear that for arbitrary symmetry $\overline{\psi_{x\alpha}^2 \nu} \neq 0$, which leads to a term $\sim d_{xx}^{I}(r/l)^2$ in the asymptote of $d_{\alpha\alpha}^{I}$. The mean values $\overline{v_x \psi_{x\alpha}}$ lead in $d_{\alpha\alpha}^{I}$ to components $\sim d_{xx}^{I}(kr)^2$. Let us consider the conditions under which they can differ from zero. The groups of interest to us (G_H) admit of the existence of the invariant vector p_z: C_1, C_s, C_n, C_{nv}. The quantity $p_y\Lambda_{xz}$ transforms like the xy component of the nonsymmetric tensor (with the exception of the isotropic case, when $\Lambda_{xz} \sim p_x p_z$). Therefore it can be different from zero either because of of its symmetric part (groups C_s and C_{2v}, which do not contain the plane m_y), or because of the antisymmetric part, which transforms like the z component of the momentum (group C_n). The selection rules for $\overline{p_y\Lambda_{xy}}$ are found in similar fashion. The results are given in the Table.

An essentially different picture appears if the cross sections are multiply connected and transform into one another under transformations of the group C_H. Then what the statements above pertain only to the sum $\sum_a \overline{\Lambda_{ik}^a}$. Each of the mean values Λ_{ik}^a is in general different from zero. For this reason, the principal term of the expansion (3.8) does not vanish even for high symmetry of the direction of the magnetic field in the crystal:

$$\sum_a \overline{\Lambda_{\alpha x} R^a \Lambda_{\alpha x}{}^a} \approx \sum_a (\bar{\Lambda}_{\alpha x}{}^a)^2/\bar{\nu}^a. \quad (A.5)$$

We have not considered here the special situation in which the deformation leads to lifting of degeneracy and zone splitting (cf.[18]).

APPENDIX B

We shall write down a number of exact and approximate equations by means of which we can compute the means over the period of rotation in a strong magnetic field.

It follows from (3.3) and (3.4) that

$$\overline{\frac{\partial r_1}{\partial\tau} R \frac{\partial r_2}{\partial\tau}} = -\frac{1}{\Omega}\overline{r_1 \frac{\partial r_2}{\partial\tau}} + \frac{1}{\Omega^2}(\overline{r_1 a r_2} - \overline{r_1 a R r_2 a}). \quad (B.1)$$

With account of the next expansion term in $1/\Omega$, Eq. (3.5) takes the form

$$\overline{rRg} \approx \frac{\bar{r}\bar{g}}{\bar{a}} + \frac{1}{\Omega}\left\{\pi\bar{r}\bar{g} + \frac{\overline{r(g,a)}}{\bar{a}}\right\} + O\left(\frac{1}{\Omega^2}\right), \quad (B.2)$$

where we have introduced the notation

$$(g, r) \equiv \frac{1}{2\pi} \int_{\tau-2\pi}^{\tau} d\tau_1 g(\tau_1) \int_{\tau}^{\tau_1} r(\tau_\parallel) d\tau_\parallel. \quad \text{(B.3)}$$

The means of the brackets (g, r) have the following properties:

$$\overline{r_1 \left(r_2, \frac{\partial r_3}{\partial \tau}\right)} = \overline{\frac{\partial r_3}{\partial \tau}(r_1, r_2)} = \overline{r_2 \left(\frac{\partial r_3}{\partial \tau}, r_1\right)} = \bar{r}_1 \overline{r_2 r_3} - \bar{r}_2 \overline{r_1 r_3}, \quad \text{(B.4)}$$

for $\mathbf{r} = \bar{\mathbf{r}} + \partial \mathbf{s}/\partial t$, where $\bar{\mathbf{s}} = 0$, we get the identity

$$\overline{r_1(r_2 r_3)} = -\pi \bar{r}_1 \bar{r}_2 \bar{r}_3 - \bar{r}_2 \overline{\frac{\partial s_1}{\partial \tau} s_3} + \bar{r}_1 \overline{\frac{\partial s_2}{\partial \tau} s_3} - \bar{r}_3 \overline{\frac{\partial s_2}{\partial \tau} s_1}, \quad \text{(B.5)}$$

which yields

$$\overline{r(g, g)} = \overline{g(r, g)} = \overline{g(g, r)} = -\pi \bar{g}^2 \bar{r}; \quad \overline{(1, 1)} = -\pi \quad \text{(B.6)}$$

in the special case of two identical quantities. Combining (B.1) and (B.2), we obtain expansions for the mean values that enter in the kinetic coefficients:

$$\overline{\frac{\partial r_1}{\partial \tau} R \frac{\partial r_2}{\partial \tau}} \approx \frac{1}{\Omega} \overline{r_1 \frac{\partial r_2}{\partial \tau}} + \frac{1}{\Omega^2} \left(\overline{r_1 a r_2} - \frac{\overline{r_1 a}\, \overline{r_2 a}}{\bar{a}} \right) - \frac{1}{\Omega^3} \left(\pi \overline{r_1 a}\, \overline{r_2 a} + \overline{\frac{r_1 a (r_2 a, a)}{\bar{a}}} \right) + O\left(\frac{1}{\Omega^4}\right), \quad \text{(B.7)}$$

$$\overline{\frac{\partial r}{\partial \tau} R g} \approx -\frac{\overline{rg}}{\Omega} + \frac{1}{\Omega} \frac{\overline{ra}\, \bar{g}}{\bar{a}} + \frac{1}{\Omega^2} \left(\pi \overline{ra}\, \bar{g} + \frac{\overline{ra(g, a)}}{\bar{a}} \right) + O\left(\frac{1}{\Omega^3}\right)$$

$$\overline{gR \frac{\partial r}{\partial \tau}} \approx \frac{\overline{gr}}{\Omega} - \frac{1}{\Omega} \frac{\bar{g}\, \overline{ra}}{\bar{a}} + \frac{1}{\Omega^2} \left(\pi \bar{g}\, \overline{ra} + \frac{\overline{g(ra, a)}}{\bar{a}} \right) + O\left(\frac{1}{\Omega^3}\right)$$

and so on.

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