INFLUENCE OF INHOMOGENEITIES ON SUPERCONDUCTOR PROPERTIES

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Inhomogeneities in the electron interaction constant and in the mean free path lead to a smearing-out of the superconductivity transition and to a pinning of the vortices in the mixed state. We find the connection between the width of the smearing-out of the phase transition and the critical pinning current in thin films. We obtain the current density and vortex density distribution across the film.

IN real superconductors there are always small inhomogeneities connected with crystal defects. Such inhomogeneities can be dislocations or crystallites. Superconductors with large inhomogeneities are called hard superconductors. As an example we can give a superconducting matrix in which particles of another phase are included.

All these inhomogeneities lead to the result that the effective parameters which determine the superconductor properties, such as the electron mean free path and the electron interaction constant, turn out to be random functions of the coordinates. Even weak inhomogeneities turn out to be important near the transition temperature and lead to its smearing out. If there is no magnetic field, the fluctuations in the mean free path little affect the superconductor thermodynamics and only the inhomogeneities in the effective electron interaction turn out to be important. The most marked influence of the inhomogeneities turns out to be on the behavior of type II superconductors in the mixed state. They lead to a pinning of the vortex structure, and as a result a nondissipative current can flow along the superconductor.

The temperature spread of the transition and also the critical current can be expressed in terms of the same single parameter which characterizes the distribution and magnitude of the inhomogeneities. In what follows we shall find the correction to the specific heat and to the penetration depth of a static magnetic field near T_c. We find an expression for the critical field for thin superconducting films in a transverse magnetic field. We also find the current distribution across the film.

1. TEMPERATURE DEPENDENCE OF THE ORDER PARAMETER

We use for our description of the inhomogeneities the model considered in ref. 1. We assume that the coefficients A and C in the free energy expression[2]

$$F = F_n + \nu \int d\mathbf{r} \left\{ A|\Delta|^2 + \frac{B}{2}|\Delta|^4 + C\left|\left(\frac{\partial}{\partial \mathbf{r}} - 2ie\mathbf{A}\right)\Delta\right|^2\right\} + \frac{1}{8\pi}\int \{(\operatorname{rot}\mathbf{A})^2 - 2\mathbf{H}\operatorname{rot}\mathbf{A}\}\, d\mathbf{r} \quad (1)$$

are random functions of the coordinates. Here **H** is the external magnetic field, $\nu = mp_0/2\pi^2$ is the density of states at the Fermi surface, $B = 7\zeta(3)/8\pi^2T^2$, where ζ is the Riemann zeta-function. For a very dirty superconductor the coefficient C is connected with the mean free path through the relation[3] $C = \pi\nu l_{tr}/24T$. The deviation of C from the average value is connected with the inhomogeneity in the mean free path. We can write the coefficient A in the form

$$A = -(T_{c0} - T)/T + A_1, \quad (2)$$

where the average $\langle A_1\rangle = 0$.

The coefficient A_1 is connected with the dimensionless interaction constant as follows:

$$A_1 = g^{-1} - \langle g^{-1}\rangle.$$

The random quantities A_1 and C are characterized by correlation functions

$$\langle A_1(\mathbf{r})A_1(\mathbf{r}_1)\rangle = \varphi(\mathbf{r}-\mathbf{r}_1), \quad \langle (C(\mathbf{r}) - C_0)(C(\mathbf{r}_1) - C_0)\rangle = \chi(\mathbf{r}-\mathbf{r}_1), \quad C_0 = \langle C\rangle. \quad (3)$$

The functions φ and χ differ from the corresponding quantities in ref. 1 by a factor ν^{-2}.

The equations for Δ and the vector potential **A** are obtained by minimizing the free energy (1):

$$\left\{\frac{T_{c0}-T}{T} - B|\Delta|^2 + \left(\frac{\partial}{\partial \mathbf{r}} - 2ie\mathbf{A}\right)C\left(\frac{\partial}{\partial \mathbf{r}} - 2ie\mathbf{A}\right)\right\}\Delta = A_1\Delta,$$

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = 4\pi\mathbf{j} = -8\pi eC\nu\left\{4e\mathbf{A}|\Delta|^2 + i\left(\Delta^*\frac{\partial\Delta}{\partial\mathbf{r}} - \Delta\frac{\partial\Delta^*}{\partial\mathbf{r}}\right)\right\}. \quad (4)$$

Near the transition temperature even weak inhomogeneities lead to a large change in Δ and the phase transition turns out to be smeared out. Below we consider the temperature range not too close to T_c where the inhomogeneities in Δ are still small and where we can take them into account using perturbation theory.

We find the temperature dependence of the ordering parameter Δ when there is no magnetic field. If the characteristic size of the inhomogeneities L is large compared to the pair dimensions $\xi(T)$, the order parameter Δ follows in the leading approximation the change in $A_1(r)$. Of more interest is the case of temperatures sufficiently close to T_c when $\xi(T) \gg L$. If the inhomogeneities in A_1 are sufficiently small we can find the order parameter Δ as a series in A_1. We must then recognize that there will occur an effective shift in the transition temperature $T_c - T_{c0}$. The magnitude of T_c will be found below from the condition that $\langle\Delta\rangle$ vanishes. The equation for Δ becomes

$$\left\{\tau - B|\Delta|^2 + \frac{\partial}{\partial\mathbf{r}}C\frac{\partial}{\partial\mathbf{r}}\right\}\Delta = \left(A_1 + \frac{T_c - T_{c0}}{T}\right)\Delta, \quad (5)$$

where $\tau = (T_c - T)/T$. We look for a solution of this equation in the form

$$\Delta = \langle\Delta\rangle + \Delta_1. \tag{6}$$

We find Δ_1 in first order in A_1 from the solution of the linearized Eq. (5). As a result we get

$$\Delta_1(\mathbf{r}) = \int \frac{d\mathbf{k}}{(2\pi)^3} \Delta_1(\mathbf{k}) \exp(i\mathbf{kr}), \quad \Delta_1(\mathbf{k}) = -\frac{\langle\Delta\rangle A_1(\mathbf{k})}{C_0 k^2 + 2\tau}. \tag{7}$$

We used here the fact that in zeroth approximation $\Delta_0^2 = \tau/B$.

Substituting Eqs. (6) and (7) into Eq. (5) and averaging, we find

$$\tau - B\langle\Delta\rangle^2 = \frac{3B\langle\Delta\rangle^2}{(2\pi)^3} \int \frac{\varphi(\mathbf{k})\, d\mathbf{k}}{(C_0 k^2 + 2\tau)^2} - \left(\frac{1}{(2\pi)^3} \int \frac{\varphi(\mathbf{k})\, d\mathbf{k}}{C_0 k^2 + 2\tau} - \frac{T_c - T_{c0}}{T} \right) \tag{8}$$

Here $\varphi(\mathbf{k})$ is the Fourier component of the function $\varphi(\mathbf{r})$ defined by Eq. (3). The renormalized temperature T_c is found from the condition that the last term in Eq. (8) vanishes when $\tau = 0$:

$$T_c = T_{c0}\left(1 + \frac{1}{(2\pi)^3} \int \frac{\varphi(\mathbf{k})\, d^3\mathbf{k}}{C_0 k^2}\right). \tag{9}$$

Substituting this expression into Eq. (8) we verify that the region $k^2 \sim \tau/C_0 = \xi^{-2}(T)$ is important in the integrals. In that region $\varphi(\mathbf{k})$ depends weakly on k since, by assumption, $\xi(T) \gg L$. Evaluating the integrals we get

$$\langle\Delta\rangle^2 = \frac{\tau}{B}\left\{1 - \frac{7}{8\pi} \frac{\varphi_0}{C_0^{3/2}\sqrt{2\tau}}\right\}, \quad \varphi_0 = \int \varphi(\mathbf{r})\, d^3\mathbf{r}. \tag{10}$$

Similarly we have

$$\langle\Delta^2\rangle = \langle\Delta\rangle^2 + \langle\Delta_1^2\rangle = \frac{\tau}{B}\left\{1 - \frac{3}{4\pi} \frac{\varphi_0}{C_0^{3/2}\sqrt{2\tau}}\right\}. \tag{11}$$

The relative magnitude of the corrections increases thus proportional to $\tau^{-1/2}$, when we approach T_c.

2. SPECIFIC HEAT AND PENETRATION DEPTH

The entropy is found by differentiating Eq. (1) for the free energy with respect to the temperature:

$$S - S_N = -\frac{\nu}{T}\langle\Delta^2\rangle. \tag{12}$$

Substituting $\langle\Delta^2\rangle$ from Eq. (11) into (12) and differentiating with respect to T we get an expression for the specific heat

$$C_S - C_N = \frac{4mp_0 T}{7\zeta(3)}\left\{1 - \frac{3}{8\pi} \frac{\varphi_0}{C_0^{3/2}\sqrt{2\tau}}\right\}. \tag{13}$$

The presence of inhomogeneities leads thus to a spreading of the phase transition to the region

$$\tau \sim \varphi_0^2 T^3 / v^3 l_{tr}^3. \tag{14}$$

The correction to the specific heat arising from the inhomogeneities has the same temperature dependence as the correction due to thermodynamic fluctuations. However, these corrections have opposite signs and for not too small $\varphi_0 > (\nu T)^{-1}$ the contribution arising from taking the inhomogeneities into account turns out to be dominant.

As in the case of the thermodynamic fluctuations the width of the smearing-out region increases in thin films. If the thickness of the film is less than the pair size $\xi(T)$ the width of the smearing-out of the phase transition is determined by the relation

$$\tau \sim \varphi_0 T / v l_{tr} d, \tag{14a}$$

where, as before, $\varphi_0 = \int \varphi(\mathbf{r}) d^3\mathbf{r}$. If the size of the inhomogeneities is less than the film thickness d, φ_0 is the same as its value in a bulk sample. In the opposite limiting case, $L \gg d$, the magnitude of φ_0 is proportional to d.

Similar corrections occur for the penetration depth of a weak magnetic field. The inhomogeneities led to the result that even in the approximation which is linear in the field there appears a correction $\Delta^{(1)}$ to the ordering parameter. In the approximation linear in the inhomogeneities we get from Eq. (4)

$$\frac{\partial^2\Delta^{(1)}}{\partial\mathbf{r}^2} = 2ie\mathbf{A}\left[2\frac{\partial\Delta_1}{\partial\mathbf{r}} + \frac{\langle\Delta\rangle}{C_0}\frac{\partial C}{\partial\mathbf{r}}\right]. \tag{15}$$

In the approximation linear in the field, the expression for the current takes the form

$$\mathbf{j} = -4eC\nu\left\{2e\mathbf{A}\Delta^2 - i\left(\Delta^{(1)}\frac{\partial\Delta}{\partial\mathbf{r}} - \Delta\frac{\partial\Delta^{(1)}}{\partial\mathbf{r}}\right)\right\}. \tag{16}$$

Here Δ is the solution of Eq. (4) when there is no magnetic field. When obtaining Eq. (16) for the current we used the fact that Δ is a real and $\Delta^{(1)}$ an imaginary quantity. Close to T_c the quantity A changes little and we can assume it to be constant when averaging Eq. (16) for the current.

Using Eq. (15) for the averaged current we find

$$\mathbf{j} = -8e^2C_0\nu\mathbf{A}\left\{\langle\Delta^2\rangle - \frac{4}{3}\langle\Delta_1^2\rangle - \frac{\langle\Delta^2\rangle\langle(C - C_0)^2\rangle}{3C_0^2}\right\} = -\frac{\mathbf{A}}{4\pi\lambda^2}. \tag{17}$$

Substituting Eqs. (3), (7), and (11) into Eq. (17) we find for the penetration depth λ

$$\lambda^{-2} = 32\pi e^2 C_0 \nu \Delta_0^2\left\{1 - \frac{11\varphi_0}{12\pi C_0^{3/2}\sqrt{2\tau}} - \frac{\chi(0)}{3C_0^2}\right\}. \tag{18}$$

Weak inhomogeneities in the mean free path do not change the temperature dependence of the penetration depth and lead only to a renormalization of the coefficient. Inhomogeneities in the interaction constant lead to relative corrections to the penetration depth which increases when one approaches T_c. In the region of the spreading of the transition, determined by Eq. (14), these corrections become large.

3. VORTEX ENERGY IN A FILM

Inhomogeneities lead to a change in the energy of vortices occurring in a type II superconductor in an external magnetic field. When approaching the transition temperature there arise corrections to the average vortex energy of the same form as the corrections to the specific heat and to the penetration depth. Outside the region (14) where the phase transition is smeared out these corrections are small and they do not lead to qualitatively new effects, and we shall therefore not consider them in what follows.

What is important is that inhomogeneities lead to a dependence of the energy of a vortex on its position in the superconductor. In a sufficiently thin film we can neglect the curvature of the vortex and give the position of the vortex by the two coordinates of its center. The energy of the vortex turns out to be a random function of these coordinates. We can write the energy of one vortex in the form: $F = \langle F\rangle + F_1$. In first order in the inhomogeneities

$$F_1(\mathbf{r}) = \nu \int d\mathbf{R} \Big\{ A_1(\mathbf{r}+\mathbf{R}) (|\Delta(\mathbf{R})|^2 - \Delta_0^2) + (C(\mathbf{r}+\mathbf{R}) - C_0) \times \Big| \Big(\frac{\partial}{\partial \mathbf{R}} - 2ie\mathbf{A}(\mathbf{R}) \Big) \Delta(\mathbf{R}) \Big|^2 \Big\}, \qquad (19)$$

where $\Delta(\mathbf{R})$ and $\mathbf{A}(\mathbf{R})$ are the solution of the set (4) for one vortex in the homogeneous case. The correlation function $\langle F_1(\mathbf{r}) F_1(0) \rangle$ can be expressed through Eq. (19) in terms of the correlation functions φ and χ defined by Eq. (3). We need in what follows the average of the square of the force acting upon the vortex, $\langle (\nabla F_1)^2 \rangle$:

$$D_1^2 = \langle (\nabla F_1)^2 \rangle = -\nu^2 \int d\mathbf{R}\, d\mathbf{R}' \Big\{ \frac{\partial^2}{\partial \mathbf{R}^2} \varphi(\mathbf{R}-\mathbf{R}') [|\Delta(\mathbf{R})|^2 - |\Delta_\infty|^2] \times [|\Delta(\mathbf{R}')|^2 - |\Delta_\infty|^2] + \frac{\partial^2}{\partial \mathbf{R}^2} \chi(\mathbf{R}-\mathbf{R}') \Big| \Big(\frac{\partial}{\partial \mathbf{R}} - 2ie\mathbf{A}(\mathbf{R}) \Big) \Delta(\mathbf{R}) \Big|^2 \times \Big| \Big(\frac{\partial}{\partial \mathbf{R}'} - 2ie\mathbf{A}(\mathbf{R}') \Big) \Delta(\mathbf{R}') \Big|^2 \Big\}. \qquad (20)$$

In the limiting case of large size inhomogeneities, $L \gg \xi(T)$, the main contribution to the integrals comes from the region $R, R' \gg \xi(T)$. With logarithmic accuracy, we find for D_1

$$D_1^2 = -\Big[2\pi d \nu \Delta^2 \ln \Big(\frac{L}{\xi(T)} \Big) \Big]^2 \Big[\xi^{-4}(T) \Big(\frac{\partial^2 \varphi}{\partial \rho^2} \Big)_{\rho=0} + \Big(\frac{\partial^2 \chi}{\partial \rho^2} \Big)_{\rho=0} \Big], \qquad (21)$$

where d is the film thickness, $\boldsymbol{\rho}$ a two-dimensional vector in the plane of the film. Equation (21) is applicable at all temperatures if the last term is multiplied by $[(2T/\Delta)\tanh(\Delta/2T)]^2$ and if we understand by the pair size $\xi(T)$ the expression

$$\xi^2(T) = v l_{tr} \Big(\mathrm{th} \frac{\Delta}{2T} + \frac{\Delta}{2T} \mathrm{ch}^{-2} \Big(\frac{\Delta}{2T} \Big) \Big) \Big/ 12\Delta^3 T \sum_\omega (\omega^2 + \Delta^2)^{-3/2}. \qquad (22)$$

Near T_c Eq. (22) gives the usual expression $\xi^2(T) = C/\tau$.

In the opposite limiting case, $L \ll \xi(T)$, the main contribution in Eq. (20) comes from the region $\rho \sim \xi(T)$. Using the fact that the functions φ and χ differ from zero in the region $R \sim L \ll \xi(T)$, we get

$$D_1^2 = 2\pi \nu^2 d \Delta^4 \{ \varphi_0 I_1 + \xi^{-4}(T) \chi_0 I_2 \}, \qquad (23)$$

where I_1 and I_2 are numbers of order unity, equal to

$$I_1 = \int_0 \rho \Big(\frac{\partial f^2}{\partial \rho} \Big)^2 d\rho, \quad I_2 = \int_0 \rho \Big\{ \frac{\partial}{\partial \rho} \Big[\Big(\frac{\partial f}{\partial \rho} \Big)^2 + \frac{f^2}{\rho^2} \Big] \Big\}^2 d\rho, \qquad (24)$$

while $f(\rho) = \Delta(\rho\xi)/\Delta_\infty$.

As before $\varphi_0 = \int \varphi(\mathbf{r}) d^3\mathbf{r}$, $\chi_0 = \int \chi(\mathbf{r}) d^3\mathbf{r}$. If the dimension L of the inhomogeneities is small compared to the film thickness d, φ_0 and χ_0 will be independent of the film thickness. For very thin films, $d \ll L$, φ_0 and χ_0 are proportional to d.

Other averages are evaluated similarly. For instance, in the limit of small size inhomogeneities, $L \ll \xi$, we have

$$D_2^2 \equiv \langle (\nabla^2 F_1)^2 \rangle = 2\pi \nu^2 d \frac{\Delta^4}{\xi^2(T)} [\varphi_0 I_3 + \xi^{-4}(T) \chi_0 I_4], \qquad (25)$$

where

$$I_3 = \int_0^\infty \rho \Big(\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} f^2 \Big)^2 d\rho; \quad I_4 = \int_0^\infty \rho \Big(\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} \Big[\Big(\frac{\partial f}{\partial \rho} \Big)^2 + \frac{f^2}{\rho^2} \Big] \Big)^2 d\rho.$$

In the limit of large inhomogeneities, $L \gg \xi$, the expression for D_2^2 differs from Eq. (21) for D_1^2 only by the substitution

$$\frac{\partial^2 \varphi}{\partial \rho^2} \to \frac{\partial^2}{\partial \rho^2} \frac{\partial^2}{\partial \rho^2} \varphi, \quad \frac{\partial^2 \chi}{\partial \rho^2} \to \frac{\partial^2}{\partial \rho^2} \frac{\partial^2}{\partial \rho^2} \chi.$$

The inhomogeneities lead thus to the appearance of effective potential wells for the vortices. The size of these wells, r_c, is determined by the size L of the inhomogeneities or the size of the core of the vortex $\xi(T)$ depending on which of these quantities is the larger.

Inhomogeneities in the film thickness lead to the same effects. The effective depth and size of the wells which appear can, as above, be expressed in terms of the correlation function of the deviations of the film thickness from its average.

4. CRITICAL CURRENT

When a current flows along a superconductor in the mixed state a Lorentz force arises which acts on the vortices. When there are no inhomogeneities this force causes a motion of the vortices which leads to the dissipation of energy. Defects pin the vortices and therefore the existence of metastable current states is possible. The vortex-defect interaction force is determined by the random energy F_1. Equating this force to the Lorentz force we get an equation describing the vortex distribution

$$dF_1(\mathbf{r}_i)/d\mathbf{r} = \Phi_0 [\mathbf{n} J(\mathbf{r}_i)], \qquad (26)$$

where $\Phi_0 = \pi/e$ is a flux quantum, $\mathbf{n}$ a unit vector parallel to the field, and $\mathbf{J}(\mathbf{r}_i)$ the current density at the center of the i-th vortex multiplied by the film thickness d. This density depends on the coordinates of all other vortices which are also determined by Eq. (26).

Equation (26) has a solution corresponding to a vanishing average current. States with a current are metastable and at non-zero temperatures the current is damped when thermal fluctuations are taken into account. This damping is slow and will be neglected in what follows. The current distribution across the sample depends on the prehistory. Below we consider the case when a current is transmitted along a film which was in a non-conducting state with an equilibrium vortex distribution. The vortices are then shifted and at the edges of the film there appears a current state which penetrates the deeper the stronger the current. The current and field distribution which then occurs can be found in the simplest case when the distance between the vortices is small compared to the effective penetration depth, $\lambda_{eff} = \lambda^2/d$, but large compared to the average size of the well. If the depth of the well is not too small we can use for the solution of Eq. (26) the self-consistent field approximation and assume that the right-hand side of (26) is a self-averaged quantity. The current is in that case determined by the average vortex density distribution, i.e., their average displacement from the equilibrium positions in the non-conducting state.

The connection of the current density and the vortex distribution is found below from the London equation. Here we find the average vortex displacement for a given magnitude of the current density in the center of this vortex. For a low current this displacement is small but when the current density approaches some critical value J_c the vortex displacement tends to infinity. In order to express the vortex displacement in terms of the current density we must solve Eq. (26) and average the solution over the different vortices. We consider first the case of low currents when the vortex displacement is small compared to the average size of the well. In that case the average displacement is proportional to the current and equal to

$$R_y = \langle (\partial^2 F_1 / \partial y^2)^{-1} \rangle \Phi_0 J_x. \tag{27}$$

The average is here evaluated under additional conditions corresponding to the fact that the vortex was at the bottom of the well, i.e., $dF_1/dx = dF_1/dy = 0$, while the second derivatives satisfy the inequalities $F_{1xx} > 0$, $F_{1yy} > 0$, $F_{1xx}F_{1yy} > F_{1xy}^2$. Assuming that the random quantity F_1 has a Gaussian distribution we get

$$R = \frac{2(\sqrt{3}+1)\ln(\sqrt{3}+\sqrt{2})}{\sqrt{\pi}} \frac{\Phi_0 J_x}{D_2} \tag{28}$$

where D_2 is determined by Eq. (25).

Of more interest is the case of strong currents. When the current increases the vortices penetrate into even deeper wells. The magnitude of the critical current J_c is determined from the condition that the density of deep wells free to contain vortices becomes equal to the vortex density. For a Gaussian distribution of the random quantity F_1 the distance between deep wells in which $F_1' > \Phi_0 J$ is equal to

$$R^2 = r_c^2 \frac{\Phi_0 J}{D_1} \exp\left(\frac{\Phi_0 J}{D_1}\right)^2, \tag{29}$$

where the average size of the well $r_c = \max(\xi(T), L)$, while D_1 is determined by Eqs. (20), (21), and (23). Equating the distance (29) to the average distance between vortices we get an expression for the critical current

$$J_c = \frac{D_1}{\Phi_0} \ln^{1/2}\left[\frac{1}{nr_c^2 \ln^{1/2}(1/nr_c^2)}\right], \tag{30}$$

where n is the vortex density.

The conditions for the applicability of the self-consistent field used in the derivation of Eqs. (28) and (30) can be written as a restriction on the vortex density n. These conditions have the form

$$n\lambda_{\text{eff}} \gg 1, \quad nr_c^2 \ll 1, \quad J_c \gg J_{c0} nr_c \xi(T), \tag{31}$$

where J_{c0} is the critical pair-breaking current,

$$J_{c0} = \frac{8\pi}{3\sqrt{3}\Phi_0} \frac{C\nu\Delta^2 d}{\xi(T)} = \frac{\Phi_0}{12\sqrt{3}\pi^2 \lambda_{\text{eff}} \xi(T)}. \tag{32}$$

The first condition (31) means that the range of the interaction between the vortices is large compared to the distance between them. The second and third conditions mean that when there is no average current each vortex is near the bottom of its own well. In contrast to the specific heat and the penetration depth where all corrections were determined by the correlation functions φ and χ the magnitude of the critical current depends on the distribution function of the random function F_1. We obtained Eqs. (28) and (30) for a Gaussian distribution. We may assume that the assumption of a Gaussian distribution is justified when in the core of the vortex there is a large number of defects, $\xi(T) \gg L$. If $L \gg \xi(T)$ the distribution of the random function F_1 depends on the detailed form of the defects. In that case Eqs. (28) and (30) are valid as to order of magnitude.

5. CURRENT DISTRIBUTION IN THE FILM

The second equation connecting the current density with the vortex distribution can be obtained by solving an electrodynamic problem. If the vortex density is sufficiently high we can characterize the vortex distribution by an average density n. The equation connecting the vortex density n with the current density follows from the London equation which for a thin film, $d \ll \lambda_L$, after averaging over the vortex positions takes the form

$$4\pi\lambda_{\text{eff}} \frac{\partial J}{\partial y} = H - \Phi_0 n - 2\int_0^b \frac{dy_1 J(y_1)}{y_1 - y}, \tag{33}$$

where $\lambda_{\text{eff}} = \lambda_L^2/d$ is the effective penetration depth in a thin film. The last term in this equation is equal to the magnetic field produced by the current, and H is the external magnetic field. If the vortices are in equilibrium and the current density vanishes the vortex density is determined by the external field and equal to

$$n = H/\Phi_0. \tag{34}$$

In the other limiting case when the critical current J_c flows in the whole film one can easily find the vortex distribution if we bear in mind that J_c is, with logarithmic accuracy, independent of the vortex density n and hence of the y-coordinate. It then follows from Eq. (33) that

$$n\Phi_0 = H - 2J_c \ln[(b-y)/y]. \tag{35}$$

The total current is then equal to $J_c b$.

If is of interest to find the current density distribution in the film when its magnitude is less than the critical one. For weak currents the current density along the whole width of the film is less than the critical density J_c. In that case the vortices are practically not shifted and their density is determined by the magnitude of the magnetic field H_0 which existed until the current was excluded. The current distribution is then the same as when there are no vortices. If $b \ll \lambda_{\text{eff}}$ we can neglect in Eq. (33) the last term and the current density depends linearly on the coordinates. In the other limiting case of a thick film, $b \gg \lambda_{\text{eff}}$, we can neglect the left-hand side of Eq. (33). The solution in that case has the form

$$J(y) = \frac{4J_{\text{tot}} - (H - H_0)(b - 2y)}{4\pi[y(b-y)]^{1/2}} \tag{36}$$

where J_{tot} is the total current flowing in the film.

Equation (36) ceases to be valid at distances of the order of λ_{eff} from the boundaries of the film. It is at those distances impossible to neglect the left-hand side of Eq. (33), but we can consider the film as semi-infinite. Matching the solution of Eq. (33) for a semi-infinite film with expression (36) which is valid inside the film, we get for $y \ll \lambda_{\text{eff}}$

$$J = \frac{4J_{\text{tot}} - (H - H_0) b}{4(2\pi\lambda_{\text{eff}} b)^{1/2}} \left[1 - \frac{y}{2\pi\lambda_{\text{eff}}} \ln\left(\frac{8\lambda_{\text{eff}}}{\gamma y}\right)\right], \tag{37}$$

where $\ln\gamma = 0.559$ is Euler's constant.

We can apply Eqs. (36) and (37) if $J < J_c$ everywhere. When the current or the magnetic field increases the current density at the edge of the film, determined by Eq. (37), reaches the value J_c. When the current further increases there appear near the boundaries of the film regions where the current is equal to the critical value and the vortex density distribution changes appreciably. We consider the case when the width of these regions, y_0, is small compared to the film thickness b, but large compared to λ_{eff}. Inside the film the current density is as before described by Eq. (36). When $y \sim y_0$ it is necessary that the critical current flows in the layer $y < y_0$. Solving Eq. (33) in the region $y_0 < y \ll b$, we get

$$J(y)=\frac{2J_c}{\pi}\arcsin\left(\frac{y_0}{y}\right)^{1/2}. \quad (38)$$

Matching this solution with the solution (36) in the region $y_0 < y \ll b$ we get an expression for the penetration depth y_0 of the critical current in terms of the total current flowing in the film and the change in the external magnetic field, $H - H_0$:

$$y_0=\frac{1}{4bJ_c^2}\left(J_{tot}-\frac{H-H_0}{4}b\right)^2. \quad (39)$$

The change in the vortex density n in the layer $y < y_0$ is determined by Eq. (33) in which we must substitute Eq. (38) for the current j(y) in the region $y > y_0$, while in the region $y < y_0$ the current $J = J_c$. The change in the external field H can be neglected compared to the field produced by the currents if $y_0 \ll b$. As a result we get

$$n(y)=-\frac{2J_c}{\Phi_0}\ln\left[\frac{(y_0)^{1/2}+(y_0-y)^{1/2}}{(y_0)^{1/2}-(y_0-y)^{1/2}}\right]. \quad (40)$$

As in the case described by Eq. (35) the vortex density at the edge of the film tends logarithmically to infinity. This behavior ceases to be valid at distances from the edge of the order of the distance between vortices.

CONCLUSION

The magnitude of the critical current in a thick film can be expressed in terms of correlation functions characterizing the inhomogeneities. These inhomogeneities on the other hand determine the magnitude of the smearing-out of the phase transition.

The simplest connection between the magnitude of the critical current and the width of the smearing-out of the phase transition is obtained in the case when the critical current is determined by the inhomogeneities in the electron-electron interaction while the size of the inhomogeneities is small compared to the pair size $\xi(T)$. Equating the critical current determined by Eqs. (23) and (30) to the critical pair-breaking current J_{c0} of (32) which occurs in a film of small dimensions, we get in that case

$$\left(\frac{J_c}{J_{c0}}\right)^2=\frac{81}{4\pi^2}\frac{\varphi_0 T}{v l_{tr}\tau d}I_1\ln\left(\frac{1}{nr_c^2}\right)$$

Close to the region where the phase transition is smeared out which is determined by Eq. (14a) the critical current is thus of the same order of magnitude as the pair-breaking current. When the temperature is lowered the critical current increases like $T_c - T$ and becomes smaller than the pair-breaking current which increases as $(T_c - T)^{3/2}$. If the current is determined by the inhomogeneities in the mean free path it is proportional to $(T_c - T)^2$ near T_c. The pair-breaking current is then only comparable with the critical current in the case when the amplitude of the relative changes in the mean free path is of the order of unity while the size L of the inhomogeneities is of the order of $\xi(T)$. In all cases the dependence of the critical current on the size L of the inhomogeneities has a maximum for $L \sim \xi(T)$. When $L \gg \xi(T)$ the critical current is proportional to L^{-1} and when $L \ll \xi(T)$ proportional to $L^{3/2}$.

In the case considered when the vortex density is small compared to the defect density the magnitude of the critical current depends weakly on the vortex density. The critical current is therefore distributed almost uniformly over the film. If the total current is less than the critical one, the current distribution turns out to be non-uniform. There can be regions near the edges where the current density is equal to the critical value. Outside these regions the current decreases slowly into the film.

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[1] A. I. Larkin, Zh. Eksp. Teor. Fiz. 58, 1466 (1970) [Sov. Phys.-JETP 31, 784 (1970)].

[2] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950) [English translation in D. ter Haar, Men of Physics, L. D. Landau, Vol. I, Pergamon, Oxford, 1965, p. 138].

[3] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. 37, 1407 (1959) [Sov. Phys.-JETP 10, 998 (1960)].

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