APPEARANCE OF A CURRENT (NEUTRAL) SHEET IN A PLASMA MOVING IN THE FIELD OF A TWO-DIMENSIONAL MAGNETIC DIPOLE

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Submitted April 22, 1971

Zh. Eksp. Teor. Fiz. 61, 1864–1876 (November, 1971)

The two-dimensional problem of stretching out of a dipole magnetic field by a plasma flow is considered in the strong magnetic field approximation. It is assumed that the magnetic dipole field is initially frozen in the plasma and the region is bounded by a uniformly expanding cylindrical surface on which the magnetic flux is conserved. It is shown that doubling of the boundary surface radius results in a neutral point appearing at the region boundary; on further expansion the neutral point produces a current (neutral) sheet. The problem can be regarded as a simplified model of the mechanism of formation of the earth's magnetic tail and of solar coronal streamers or helmet structures.

1. INTRODUCTION

THE significance of the neutral points of the magnetic field and of the current sheets developing from them to the problem of dissipation of magnetic energy and particle acceleration in a plasma has been discussed in [1-3] (see also the literature cited in these papers). It was shown in [3] that in a plasma placed in a strong magnetic field the current sheets occur in those locations where neutral points of the magnetic field should appear in the absence of the plasma. A method of constructing the current sheets and of the magnetic field in their vicinity was indicated in the same paper for planar two-dimensional problems.

In the present paper we apply this method to the case when the current sheet results from the drawing out of a dipole magnetic field by a plasma current. Three possible applications of this problem can be indicated.

One concerns the magnetic tail of the earth's magnetosphere. It is known that the tail of the magnetosphere contains the so-called neutral sheet, which separates magnetic fields of opposite directions.[4,5] For a strong magnetic field and two-dimensional geometry, the concept of current sheet and neutral sheet are identical, since at a negligibly low plasma pressure any current sheet should separate equal and opposite magnetic fields. We can therefore use the two terms on a par.

The causes of the earth's magnetic tail are not fully clear.[5] We shall show below that even a partial penetration of the geomagnetic field into the solar-wind plasma suffices for the production of the current sheet.

The second application of the results may be the corona rays or streamers. Observations of the solar corona during eclipses demonstrate a characteristic structure of the distribution of matter in the corona, in the form of corona rays or streamers, and also helmets and fans.[6,7] At present there is no doubt that these formations are connected with the local large-scale magnetic field on the surface of the sun. This connection is confirmed by direct calculations of the magnetic field in the corona from data on the radial component of the field on the photosphere.[8,9] The calculations are based on two main premises: the magnetic field over the photosphere is potential up to a certain level in the corona, at which level, owing to drawing-out by the solar wind, the magnetic field becomes purely radial. The magnetic-field distributions calculated under these assumptions exhibit a good correlation with the optical structure of the chromosphere and of the corona, with the radio and x-ray pictures of the sun,[10] and also with the structure of the interplanetary field.[11]

Such a correspondence, however, takes place only for a crude picture of the field. It is easy to verify that the magnetic field constructed with the aid of the methods developed in [8,9] should, generally speaking, contain neutral points. No attention was called to this circumstance in [8,9], since the proposed methods were used mainly for approximate numerical calculations. At the same time, as shown in [3], the appearance of neutral points corresponds to a special situation, where the presence of plasma cannot be neglected even in the case of negligibly low density and pressure. Namely, as a result of the high plasma conductivity there should appear, in place of the neutral points, current sheets with corresponding geometry of the magnetic field.

A similar picture—a current sheet with quasiradial fields of opposite direction on either side of it, is apparently also observed in streamers. We note here that formally, the processes of occurrence of coronal streamers and the magnetosphere tail are quite analogous. In both cases the dipole magnetic field is drawn out by the stream of solar-wind plasma: in the corona it is the dipole magnetic field of an extended active region, and in the magnetosphere it is the earth's magnetic field.

We must stipulate at once that the model developed below can serve only as a first rather crude approximation to the real situation, principally owing to the assumption that the field is two-dimensional and strong. Obviously, the tail of the magnetosphere, as well as the streamers, can be regarded as planar two-dimensional formations only by stretching the point greatly. In addition, at large distances from the dipole, the field intensity is low, the energy of the solar wind becomes large compared with the magnetic energy, and the field can no longer be regarded as potential.

Nonetheless, in spite of these limitations, the proposed method demonstrates the physical nature of the mechanism of occurrence of formations similar to streamers and to the geomagnetic tail, and in some cases can even be used for quantitative calculations. This pertains to the third possible application of the results, namely laboratory experiments in which a plasma flows around a planar magnetic dipole.[12] The purpose of these experiments was to stimulate the conditions in the earth's magnetosphere within the framework of two-dimensional geometry. In the latter respect they are closest to the formulation of the problem as discussed below, subject to the limitation, however, that under laboratory conditions the need for ensuring a sufficient degree of freezing-in of the magnetic field in the plasma becomes a serious problem.

2. FORMULATION OF PROBLEM. FIELD IN THE ABSENCE OF PLASMA

We shall use the magnetohydrodynamic approximation of a strong field,[1,3,13] assuming the following conditions to be satisfied:

$$s^2 \ll V_A^2, \quad V^2 \ll V_A^2, \tag{1}$$

where s is the hydrodynamic speed of sound, V the characteristic plasma velocity, and V_A the characteristic Alfven velocity in the problem. In this approximation, as shown in [3], for planar two-dimensional problems the magnetic field in the entire region where the conditions (1) are satisfied should be potential, with the possible exception of individual surfaces (cuts on the complex plane). The latter appear if the magnetic field that is potential in the entire region has singular neutral points inside the region. We call a neutral point of the magnetic field singular if the magnetic field at this point is equal to zero but the electric field differs from zero.

For two-dimensional problems, the electromagnetic field is conveniently described by a vector potential having only a z-component A(x, y, t). Then the condition for the neutral point to be singular is

$$|\mathbf{H}| = |[\nabla A]| = 0, \quad \mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \neq 0. \tag{2}$$

We shall find it convenient, as in [3], to use functions on the complex plane z = x + iy.

As the model of extraction of the field by the solar wind, we consider the following idealized problem.

Let the two-dimensional magnetic dipole be placed in the base of a semicylindrical region on the complex plane (see Fig. 1). We assume that inside this region the conditions (1) are satisfied, and that the magnetic flux is conserved on the boundary of the region R, which expands in accordance with a specified law R = R(t). We assume that the field of the magnetic dipole penetrates partially through the boundary, so that at the initial instant of time the magnetic flux at each point of the surface constitutes a fraction α of the flux in the case of the absence of a boundary.

For the solar corona, such a boundary simulates the region of transition from the chromosphere and the lower corona, in which the strong-field conditions are satisfied, to the upper corona, in which the solar-wind plasma energy dominates. The magnetic field of the active region, which gives rise to the helmets and to the coronal rays, is approximated in this case by the field of a flat magnetic dipole.

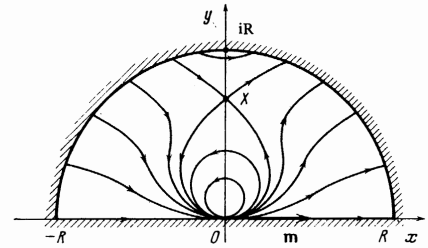


FIG. 1. Geometry of magnetic field of a two-dimensional dipole with force lines frozen into the boundary of a semicylindrical region. The letter X denotes a neutral point (line along the z axis) of the magnetic field.

For a magnetosphere with a stationary magnetic tail, the problem in question can serve as a model for the occurrence of a tail in an initially spherical or quasi-spherical magnetosphere.

In the assumed formulation of the problem, the vector potential A(x, y, t) is defined by the Laplace equation

$$\Delta A = 0, \tag{3}$$

with boundary conditions

$$A(x, y, t) \equiv A(r, \varphi, t) = \begin{cases} 0 & \text{if } y = 0, \ -R \leqslant x \leqslant R \\ +(\alpha m/R_0)\sin\varphi & \text{if } r = R \end{cases} \tag{4}$$

and a singularity of the dipole type at the origin

$$A(r, \varphi, t) \to -\frac{m \sin\varphi}{r} \text{ as } r \to 0. \tag{5}$$

Here r and φ are polar coordinates in the (x, y) plane, R_0 is the initial value of R(t), α is a fraction of the magnetic flux of the dipole penetrating through the boundary, and m is the magnitude of the dipole magnetic moment, henceforth assumed constant.

We note that the magnetic flux through the contour element dl = {dx, dy} is equal to

$$d\Phi = H_x dy - H_y dx \equiv dA,$$

since H = {∂A/∂y, −∂A/∂x, 0}. Therefore the condition (4) on the x axis follows from the symmetry of the problem, owing to which the flux through the x axis is equal to zero, and on the surface r = R(t) the condition (4) corresponds to conservation of the initial (at $r = R_0$) flux through each element of the expanding cylindrical surface.

Just as in [3], we introduce the complex potential

$$F(z, t) = A(x, y, t) + iB(x, y, t), \tag{6}$$

where A(x, y, t) is the sought potential, and B(x, y, t) is the conjugate harmonic function. The magnetic-field intensity vector **H** is by definition equal to

$$\mathbf{H} = H_x + iH_y = -i(dF/dz)^* \tag{7a}$$

or

$$dF/dz = -H_y - iH_x, \tag{7b}$$

where the asterisk denotes the complex conjugate.

The solution of our problem (Eqs. (3)–(5)) is obvious:

$$F(z,t)=\frac{im}{z}-im\frac{(\alpha R-R_0)}{R^2R_0}z. \tag{8}$$

The first term describes the field of a plane dipole, and the second corresponds to a certain effective homogeneous field necessary to satisfy the boundary condition (4).

If the radius of the region R(t) increases to a value larger than $2R_0/\alpha$, then a magnetic-field neutral point of type X appears on the y axis inside the region (Fig. 1). Its coordinate, as is clear from (7) and (8), is

$$z_0=iR\sqrt{\frac{R_0}{\alpha R-R_0}}. \tag{9}$$

The electric field at the neutral point is not equal to zero:

$$E_z=-\frac{1}{c}\frac{\partial A}{\partial t}=+\frac{m}{cR_0}\frac{\alpha R-2R_0}{R^2(\alpha R/R_0-1)^{1/2}}\dot{R}>0 \tag{10}$$

when $\dot{R}>0$ and $R>2R_0/\alpha$. Therefore, the neutral point (9) is singular[3] in the sense of conditions (2) and should be eliminated from the region where the solution F(z, t) is defined, if we are interested in the field in a plasma with good conductivity.

3. SOLUTION WITH CURRENT SHEET

In accordance with the rule indicated in [3], the singular neutral point that appears at $R(t)>2R_0/\alpha$ should be excluded from the region where the solution is defined, by introducing the cut Σ on the complex plane. The cut should run along the y axis from the upper limit of the region in which the neutral point first appears, to a certain height h (Fig. 2a). The length of the cut at a given instant of time (i.e., the distance R − h) should be no smaller than the distance from the boundary to the neutral point (9) at the same instant.

The introduction of the cut takes into account the fact that at the instant when the neutral point appears ($R = 2R_0/\alpha$) a current sheet with a width that increases with increasing R(t) is produced. The problem with the current sheet requires a boundary condition on the edges of the cut. We assume that the magnetic force lines do not cross the cut, i.e.,

$$A|_\Sigma=A_0(t) \tag{11}$$

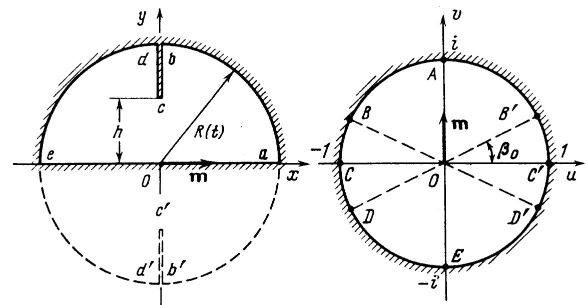
(A(x, y, t) = const at a fixed t is the equation of the family of force lines). Disregarding the possible dissipative processes in the current sheet, we shall assume that $A|_\Sigma$ = const, namely (see (4)),

$$A|_\Sigma=+\alpha m/R_0. \tag{12}$$

It is thus necessary to solve the Dirichlet problem for Eq. (3) with conditions (5) and

$$A(x,y,t)=\begin{cases}0 & \text{if } y=0,\ -R\leqslant x\leqslant R\\ +(\alpha m/R_0)\sin\varphi & \text{if } r=R,\ 0\leqslant\varphi\leqslant\pi.\\ +\alpha m/R_0 & \text{if } x=0,\ h\leqslant y\leqslant R\end{cases} \tag{13}$$

To solve this problem we use the symmetry principle (see [14], p. 143), namely, we supplement the region in which the solution is defined by a similar region that is



a b

FIG. 2. a) Elimination of the singular neutral point with the aid of a cut on the complex plane and symmetrical expansion of the region of applicability of the solution; the shading shows the position of the new boundary. b) Mapping of the symmetrized region on the interior of the unit circle.

symmetrical about the x axis (dashed lines in Fig. 2a). Accordingly we take in lieu of the boundary condition (13)

$$A(x,y,t)=\begin{cases}-\alpha m/R_0 & \text{on } \Sigma'\\ +(\alpha m/R_0)\sin\varphi & \text{at } r=R.\\ +(\alpha m/R_0) & \text{on } \Sigma\end{cases} \tag{14}$$

We seek a solution of the problem in the form

$$F(z,t)=+im/z+f(z,t), \tag{15}$$

where

$$f(z,t)=a(x,y,t)+ib(x,y,t) \tag{16}$$

is an unknown analytic function. By virtue of (14), the real part of this function should satisfy the boundary condition

$$a(x,y,t)=\begin{cases}+\alpha m/R_0-m/y & \text{on } \Sigma\\ +(\alpha m/R_0-m/R)\sin\varphi & \text{at } r=R.\\ -\alpha m/R_0-m/y & \text{on } \Sigma'\end{cases} \tag{17}$$

With the aid of the function (see [14], p. 149)

$$w(z,t)=u+iv=\frac{i\cos\beta_0}{2}\left\{\left(\frac{z}{R}-\frac{R}{z}\right)-\left[\left(\frac{z}{R}-\frac{R}{z}\right)^2+\frac{4}{\cos^2\beta_0}\right]^{1/2}\right\} \tag{18}$$

where

$$\cos\beta_0=\frac{2\gamma}{1+\gamma^2},\qquad \gamma=\frac{h}{R}, \tag{19}$$

we map conformally the symmetrized region with the cuts (Fig. 2a) on the interior of the unit circle (Fig. 2b). The boundary condition (17) is thereby transformed into

$$a(\beta)=a_1(\beta)+a_2(\beta), \tag{20}$$

where

$$a_1(\beta)=+\frac{m}{R}\left\{\frac{\cos\beta}{\cos\beta_0}+\left[\frac{\cos^2\beta}{\cos^2\beta_0}-1\right]^{1/2}\right\} \tag{21a}$$

if $|\beta|\leq\beta_0$, $|\pi-\beta|\leq\beta_0$,

$$a_1(\beta)=+\frac{m}{R}\frac{\cos\beta}{\cos\beta_0} \tag{21b}$$

if $\beta_0<\beta<\pi-\beta_0$, $\pi+\beta_0<\beta<2\pi-\beta_0$ and

$$a_2(\beta)=\begin{cases}+\dfrac{\alpha m}{R_0} & \text{if } |\pi-\beta|\leqslant\beta_0\\ -\dfrac{\alpha m}{R_0}\dfrac{\cos\beta}{\cos\beta_0} & \text{if } \beta_0<\beta<\pi-\beta_0,\ \pi+\beta_0<\beta<2\pi-\beta_0;\\ -\dfrac{\alpha m}{R_0} & \text{if } |\beta|\leqslant\beta_0\end{cases}\tag{22}$$

β is the polar angle in the plane w = u + iv.

We seek a solution in the form of a sum of two functions

$$f(w,t)=f_1(w,t)+f_2(w,t).\tag{23}$$

The first of the functions, satisfying the boundary condition (21), is determined immediately by any of the methods of complex-variable theory (for example, with the aid of the Schwartz integral, see [14], p. 200) and is equal to

$$f_1(w,t)=+\frac{2m}{R\cos\beta_0}w-\frac{m}{2R\cos\beta_0}\left\{\left(w+\frac{1}{w}\right)-\left[\left(w+\frac{1}{w}\right)^2-4\cos^2\beta_0\right]^{1/2}\right\}.\tag{24}$$

We seek the function $f_2(w, t)$ in the form of a power series

$$f_2(w,t)=\sum_{n=1}^{\infty}C_nw^n.\tag{25}$$

Since $a_2(\beta)$ is an even function, the coefficients C_n are real and are given by (see [14], p. 345)

$$C_n=\frac{2}{\pi}\int_0^{\pi}a_2(\beta)\cos n\beta\,d\beta,\tag{26}$$

which yields

$$C_n=-[1-(-1)^n]\left\{\frac{\sin n\beta_0}{n}-\frac{1}{2\cos\beta_0}\left[\frac{\sin(n+1)\beta_0}{n+1}+\frac{\sin(n-1)\beta_0}{n-1}\right]\right\}\frac{\alpha m}{\pi R_0}-\frac{\alpha m\delta_{1n}}{R_0\cos\beta_0},\tag{27}$$

where δ_{1n} = 1 at n = 1 and δ_{1n} = 0 at n ≠ 1. The series (25) with the coefficients (27) breaks up into three series:

$$f_2(w,t)=-\frac{\alpha m}{R_0\cos\beta_0}w+\frac{2\alpha m}{\pi R_0\cos\beta_0}(r_1+r_2)-\frac{4\alpha m}{\pi R_0}r_3,\tag{28}$$

$$r_1=\sum_{k=0}^{\infty}\frac{\sin 2(k+1)\beta_0}{2(k+1)}w^{2k+1},\tag{29}$$

$$r_2=\sum_{k=0}^{\infty}\frac{\sin 2k\beta_0}{2k}w^{2k+1},\tag{30}$$

$$r_3=\sum_{k=0}^{\infty}\frac{\sin(2k+1)\beta_0}{2k+1}w^{2k+1}.\tag{31}$$

The series (29)–(31) can be summed by differentiating them with respect to the parameter β_0. For example,

$$\frac{\partial r_3}{\partial\beta_0}=\sum_{k=0}^{\infty}\cos(2k+1)\beta_0\cdot w^{2k+1}=\mathrm{Re}'\sum_{k=0}^{\infty}(we^{i\beta_0})^{2k+1}.\tag{32}$$

The prime denotes here that the real-part symbol Re pertains only to the coefficient $\exp[i\beta_0(2k+1)]$ of w^{2k+1}. The series under the Re′ sign is summed as the usual geometric progression and converges inside the unit circle. Integrating with respect to β_0 and separating Re′, we obtain

$$r_3=\frac{1}{2}\operatorname{Arg}\frac{1+we^{i\beta_0}}{1-we^{i\beta_0}}.\tag{33}$$

We calculate analogously the series (29) and (30).

After substituting the sums of the series (29)–(31) in (28) and making simple transformations, we obtain

$$f_2(w,t)=\frac{-\alpha m}{R_0\cos\beta_0}w-\frac{\alpha m}{R_0\cos\beta_0}\frac{2}{\pi}\left\{\frac{i}{2}\cos\beta_0\ln\frac{(1-w^2)-2iw\sin\beta_0}{(1-w^2)+2iw\sin\beta_0}+\frac{1}{2}\left(w+\frac{1}{w}\right)\frac{i}{2}\ln\frac{1-w^2e^{-2i\beta_0}}{1-w^2e^{2\beta_0}}-\beta_0w\right\}.\tag{34}$$

4. PROPERTIES OF THE SOLUTION

Thus, the solution of the problem with current sheet takes the form

$$F(w(z),t)=f_0(w,t)+f_1(w,t)+f_2(w,t),\tag{35}$$

where

$$f_0(w,t)=+\frac{im}{z}=\frac{-2m\cos\beta_0}{R}\left\{\left(w+\frac{1}{w}\right)-\left[\left(w+\frac{1}{w}\right)^2-4\cos^2\beta_0\right]^{1/2}\right\}^{-1},\tag{36}$$

the functions $f_1(w, t)$ and $f_2(w, t)$ are given by formulas (24) and (34), respectively, and the function w(z, t) is given by formulas (18) and (19). All these functions depend on the time as a parameter via the R(t) dependence.

The derivative of the potential is

$$\frac{dF}{dw}=\frac{-1}{\cos\beta_0}\left(\frac{\alpha m}{R_0}-\frac{2m}{R}\right)-\frac{m}{R\cos\beta_0}\left(1-\frac{1}{w^2}\right)-\frac{\alpha m}{R_0\cos\beta_0}\frac{2}{\pi}\left\{i\cos\beta_0\left[\frac{w-i\sin\beta_0}{(1-w^2)+2iw\sin\beta_0}-\frac{w+i\sin\beta_0}{(1-w^2)-2iw\sin\beta_0}\right]+\frac{1}{2}\left(1-\frac{1}{w^2}\right)\frac{i}{2}\ln\frac{1-w^2e^{-2i\beta_0}}{1-w^2e^{2i\beta_0}}+\frac{i}{2}\left(w+\frac{1}{w}\right)w\left[\frac{e^{2i\beta_0}}{1-w^2e^{2i\beta_0}}-\frac{e^{-2i\beta_0}}{1-w^2e^{-2i\beta_0}}\right]-\beta_0\right\}.\tag{37}$$

Figure 3 shows schematically the picture of the magnetic force lines corresponding to the obtained solution. The magnetic field vanishes on the surface of the current sheet at the point X_* with coordinates x = 0 and y = h_*, where h_* is determined from the condition

$$\frac{dF}{dw}(e^{i\beta_*})=0.\tag{38}$$

The neutral point X_* differs from the singular neutral point X obtained in Sec. 2 in that it is a nonanalytic singular point lying on the boundary of the region of applicability of the solution. The potential in the vicinity of this singular point is given by

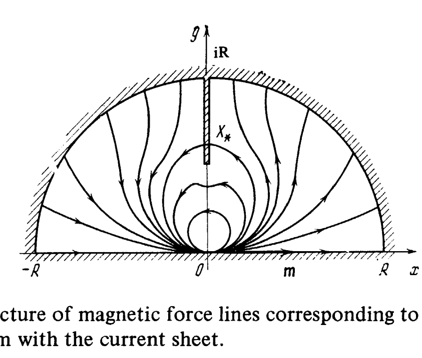


FIG. 3. Picture of magnetic force lines corresponding to the solution of the problem with the current sheet.

$$F(z,t) \approx \pm {}^1\!/_2 c_1 (z - ih_*)^2 e^{-i\pi/4} + c_2, \quad x = \pm 0, \tag{39}$$

where c_1 and c_2 are constants which, generally speaking, depend on the time; $x = +(-)0$ corresponds to the right-hand (left-hand) edge of the cut.

The neutral point X_* separates the region of the forward current ($x = 0$, $h_* < y \le R$) from the region of the backward current ($x = 0$, $h \le y \le h_*$). In the general case, the solution lies between the following two limiting regimes: 1) the backward current is equal to zero, 2) the total current in the cut is equal to zero: the backward current cancels the forward one.

The first case corresponds to the shortest length of the current sheet, i.e., to the largest height h. To find this height, it is necessary to solve the equation

$$\frac{dF}{dw}(-1) = 0 \tag{40}$$

with respect to β_0. With the aid of (37) we find that β_0 depends only on the combination $\alpha R/R_0$ and is equal to

$$\beta_{01}\left(\frac{\alpha R}{R_0}\right) = \frac{\pi}{2}\left(1 - \frac{2R_0}{\alpha R}\right) \tag{41}$$

The index 1 indicates that the formula pertains to the first case. From (19) and (41) we find

$$h_1\left(\frac{\alpha R}{R_0}\right) = R \operatorname{tg} \frac{\pi R_0}{2\alpha R}. \tag{42}$$

The corresponding picture of the force lines is shown in Fig. 4. The magnetic field vanishes on the lower end of the current sheet at the point Y. Near this point we have

$$F(z,t) = d_1 (z - ih)^{3/2} + d_2, \tag{43}$$

where d_1 and d_2 are constants that depend on the time.

Let us consider the second limiting case. The vanishing of the total current means vanishing of the circulation along the contour Γ enclosing the cut Σ. Replacing the integration along the contour Γ by integration along the left and right edges of the cut, and recognizing that, by definition $H_y = -\partial A/\partial x = -\partial B/\partial y$, we find that this condition is equivalent to the condition

$$B(0, ih) - B(0, iR) = 0 \tag{44}$$

or to the following condition in the plane $w = u + iv = \rho e^{i\beta}$

$$\operatorname{Im} F(e^{i(\pi - \beta_0)}) - \operatorname{Im} F(-1) = 0. \tag{45}$$

The latter leads to the transcendental equation with respect to β_{02}:

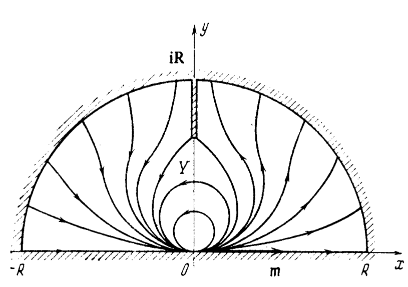


FIG. 4. Picture of the magnetic force lines in the particular case when there are no backward currents (the length of the cut is minimal).

$$\beta_{02} + {}^1\!/_2 \operatorname{ctg} \beta_{02} \ln \cos \beta_{02} = \beta_{01} \tag{46}$$

At small β_{02} (i.e., short lengths of the cut or R differing little from $2R_0/\alpha$) we have in the linear approximation

$$\beta_{02}\left(\frac{\alpha R}{R_0}\right) \approx \frac{4}{3}\beta_{01}\left(\frac{\alpha R}{R_0}\right). \tag{47}$$

The picture of the force lines is the same as in the schematic Fig. 3.

5. ASYMPTOTIC BEHAVIOR OF THE SOLUTION

With increasing R, the solution rapidly reaches its asymptotic form, and to find the latter it is necessary to take in (35) the limit as $\alpha R/R_0 \to \infty$. As a result we get

$$F(w,t) = \frac{m}{2h}\left(w - \frac{1}{w}\right) - \frac{2\alpha m i}{\pi R_0} \ln \frac{1 - iw}{1 + iw}, \tag{48}$$

$$w(z,t) = -i\left\{\frac{h}{z} + \left[\frac{h^2}{z^2} + 1\right]^{1/2}\right\}, \tag{49}$$

$$\cos \beta_0 = 0, \quad \beta_0 = \pi/2. \tag{50}$$

In the case when the backward current is equal to zero, it follows from (41) and (42) that

$$\beta_{01} = \frac{\pi}{2}, \quad h_1 = \frac{\pi}{2\alpha} R_0. \tag{51}$$

In the second limiting case, when the total current is equal to zero, β_{02} is determined by the equation

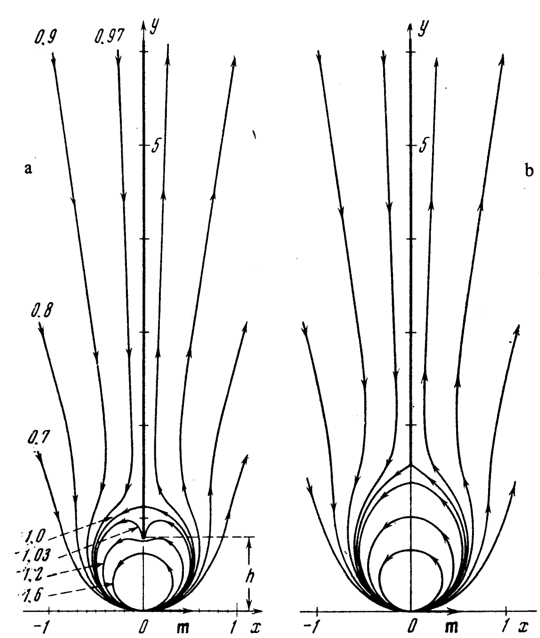


FIG. 5. a) Force lines corresponding to the asymptotic solution (48) at $\alpha = 1$ and $h = 0.8R_0$ (R_0 is taken as the unit of length). The numbers alongside the lines indicate the values of potential A, with m/R_0 taken as unity. The region of backward currents causing the characteristic inflection of the magnetic force lines lies below the point X_* on the cut. b) The same magnetic force lines but in the particular case when there are no backward currents, when the length of the cut is minimal ($\alpha = 1$, $h = h_1 = \pi/2$, $R_0 = 1$.

$$\beta_{02} + {}^1/_2 \operatorname{ctg} \beta_{02} \ln \cos \beta_{02} = \pi/2, \quad (52)$$

whence

$$\beta_{02} = \pi/2, \quad h_2 = 0. \quad (53)$$

The pictures of the force lines in the case of $\alpha = 1$ for $h = 0.8R_0$ and $h = h_1 = \pi R_0/2$ are shown in Fig. 5. Near the dipole, the magnetic field has the same structure as in the general case. At large distances, the magnetic force lines tend to become radial straight lines.

6. CONCLUSION

The model considered has the advantage that it can be calculated to conclusion without changing the fundamental physical meaning of the phenomenon. As a result, the conditions under which neutral layers are produced when a plasma flows in a dipole magnetic field become clear. Three such fundamental conditions can be indicated:

1) A sufficiently high conductivity of the plasma, so that the magnetic field can be regarded as frozen into the plasma.

2) The existence of a boundary or of a sufficiently narrow transition layer between the region where magnetic stresses are predominant (magnetic cavity) and the region where the plasma energy dominates (solar wind).

3) The width of this layer should be such as to satisfy the third condition—penetration of the magnetic field from the magnetic cavity into the region of the wind.

The first two conditions are obvious; as to the third condition, it calls for a special investigation. We confine ourselves here to several remarks.

Thus, in the case of the solar corona, the "capture" of the field by the solar wind occurs, as it were, "from the interior" of the field itself: the matter flows slowly along the force lines in the strong-field region, and then, as the field becomes weaker, this flow is transformed into a radial solar wind that carries the external part of the field away with it. As a result, a quasistationary picture of outflow of matter along the coronal rays is established for the long-lived active region. A rigorous calculation of the steady state should be based on a consistent analysis of the magnetic field and of the solar wind.[15]

In the case of the magnetosphere, the capture of the magnetic field by the solar wind is most probably connected with the instability of the separation surface between the wind and the magnetosphere. Such an instability can be due, for example, to the instability of the tangential velocity discontinuity.[16]

As is clear from the foregoing, the current sheet occurs following any weak penetration of the field into the region of the wind, i.e., for arbitrarily small α in (48). For quasistationary processes such as the magnetospheric tail and the coronal rays, there is apparently realized a situation close to the limiting case when there are no backward currents, since the latter have time to attenuate in the case of slow development of the sheet.[3] In such a case, in accord with (51), the starting point of the sheet moves farther away from the magnetic dipole with decreasing α. We emphasize, however, that the employed strong-field approximation becomes unsuitable for real problems at large distances, owing to the decrease of the field with distance.

We note in conclusion that whereas for the magnetosphere the existence of the neutral sheet has been demonstrated by direct observation,[4] the existence of the sheet for streamers can be inferred for the time being only from their shape and from their connection with the photospheric magnetic fields. Great interest attaches in this connection to radar observations of the sun[17] which, in principle, can fix the current sheets as regions of developed plasma turbulence. As shown in [18], at low frequency of the Coulomb collisions, the current sheet is unstable against excitation of ion-acoustic oscillations and its occurrence is inevitably connected with the development of plasma turbulence.

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Translated by J. G. Adashko
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