CONTRIBUTION TO THE THEORY OF AN EQUILIBRIUM HIGH-FREQUENCY GAS DISCHARGE

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The theory of an equilibrium high-frequency gas discharge in a cylindrical cavity (waveguide) is developed without imposing any restrictions on the penetration depth of the field in the normal skin effect. The structure of the discharge is studied and equations are obtained which determine the dependence of the discharge parameters on the input power. A dependence of the maximum temperature in a hydrogen discharge at atmospheric pressure on the input power is found and explains the presence of two types of discharge in diatomic gases, viz., diffuse and column discharges. The experimental dependence of the discharge current on the power, observed by Kapitza and Filimonov, differs from that calculated theoretically under the assumption of local thermodynamic equilibrium. A theory of an equilibrium high-frequency discharge in a gas stream is developed under the assumption of a strong skin effect. The dependence of the plasma temperature in a discharge on the input power and gas flow velocity is found. The one-dimensional problem of a plane stationary discharge in a given gas stream is solved and can be applied to plasmotrons with helical gas flows. The problem of free propagation of a discharge front, which is similar to the problem of flame propagation during slow combustion, is also solved.

1. EQUILIBRIUM HIGH-FREQUENCY DISCHARGE IN A CYLINDRICAL RESONATOR

KAPITZA[1] investigated experimentally a high-frequency gas discharge in a cylindrical resonator at pressures on the order of atmospheric. The absence of a theory of the low-temperature equilibrium high-frequency discharge makes it difficult to ascertain the degree to which the experimental results of[1] offer evidence of a high electron temperature. The present paper is an attempt to construct a theory of high-frequency discharge in a dense gas assuming local thermodynamic equilibrium. In the limiting case of a strong skin effect, a theory of high-frequency discharges was constructed by Pitaevskiĭ and the author[2]. However, the assumed skin effect does not make it possible to investigate fully the features of a high-frequency discharge. No limitations are imposed in the present paper on the magnitude of the skin effect.

At not too high temperatures (for hydrogen at atmospheric pressure up to 8000°K, so long as electron-ion collisions can be neglected in comparison with the number of electron collisions with neutral atoms), the conductivity of the gas increases very strongly, exponentially, with increasing temperature, and the thermal conductivity is a relatively slow function of the temperature. This makes it possible to develope the discharge theory almost to conclusion, without any adjustment parameters, and to find the dependence of the plasma temperature in the discharge, of its dimensions, etc., on the power input, and also to determine the structure of the discharge. The obtained formulas may turn out to be useful for an experimental determination of the temperature dependence of the thermal conductivity of gases, which at present has not been thoroughly investigated. The obtained dependence of the temperature on the power can explain the presence of two types of discharge in diatomic gases—diffuse and filamentary type, as observed in[1]. The current in the discharge, calculated theoretically assuming an infinitely long cylindrical equilibrium discharge, increases with increasing power much more slowly than in accord with the results of Kapitza and Filimonov[3].

Initial Equations and Their Solution

We consider an infinitely long cylindrical discharge in a cylindrical resonator (or waveguide) of radius R in the stationary state, when the Joule heat released in the plasma by the electromagnetic field is transmitted by heat conduction to the cooled resonator walls. We assume that the gas pressure in the resonator is high enough to obtain local thermodynamic equilibrium in the discharge. The electric conductivity and the thermal conductivity of the gas are then known functions of the equilibrium temperature. Just as in[2], we neglect the conduction of the gas and the radiation losses. Assume, for concreteness, that the electric vector $\mathbf{E}$ is parallel to the resonator axis and depends only on the distance r to the axis, while the nonvanishing component of the vector $\mathbf{H}$ is H_φ. Neglecting the displacement current in comparison with the conduction current, we write down Maxwell's equations and the heat-conduction equation in cylindrical coordinates:

$$\frac{1}{r}\frac{d}{dr}rH=\frac{4\pi}{c}\sigma E, \tag{1.1}$$

$$\frac{dE}{dr}=\frac{i\omega}{c}H, \tag{1.2}$$

$$\frac{1}{r}\frac{d}{dr}r\varkappa\frac{dT}{dr}+\frac{1}{2}\sigma|E^2|=0 \tag{1.3}$$

(a time dependence of the type $e^{i\omega t}$ is assumed).

We confine ourselves to the region of relatively low temperatures, in which the electron concentration is determined from the ionization-equilibrium formulas and the conductivity increases exponentially with the temperature:

$$\sigma \sim \exp(-I/2T),$$

where I is the ionization potential of the gas atoms. We denote by T_m the maximum plasma temperature reached on the discharge axis, and use the condition $T_m \ll I$, under which the conductivity of the gas decreases appreciably when the temperature decreases by a small amount (compared with T_m): $\Delta T = T_m - T \sim T_m^2/I \ll T_m$.

Outside the discharge, in the region $T_m - T \gg T_m^2/I$, the conductivity of the gas is exponentially small, so that we can put $\sigma = 0$ in the heat-conduction equation (1.3), after which its solution takes the form

$$r\varkappa dT/dr = -s_0/2\pi, \tag{1.4}$$

$$\int_{T_0}^{T} \varkappa dT = \frac{s_0}{2\pi}\ln\frac{R}{r}, \quad T_m - T \gg \frac{T_m^2}{I} \tag{1.5}$$

where T_0 is the temperature of the cooled wall of the resonator and s_0 is the power per unit length of the discharge.

We now investigate the dependence of the temperature on r near the discharge axis. From the condition $E(0) < \infty$ and Eqs. (1.1)–(1.3) it follows that as $r \to 0$

$$H \sim r,\ E(0) - E \sim r^2,\ T_m - T \sim r^2.$$

To solve the problem it is convenient to eliminate E and H from (1.1)–(1.3). Eliminating H from (1.1) and (1.2), we obtain the following equations for E and E*:

$$\frac{1}{r}\frac{d}{dr}r\frac{dE}{dr} - \frac{4\pi i\omega\sigma}{c^2}E = 0, \quad \frac{1}{r}\frac{d}{dr}r\frac{dE^*}{dr} + \frac{4\pi i\omega\sigma}{c^2}E^* = 0.$$

By the same method as in[2], we can obtain from the same equations the relations

$$\begin{gathered}\frac{1}{r}\frac{d}{dr}r(E^*E' - EE^{*\prime}) - \frac{8\pi i\omega\sigma}{c^2}|E^2| = 0,\\ \frac{1}{r}\frac{d}{dr}r\frac{d|E^2|}{dr} - 2|E'|^2 = 0,\\ \frac{1}{r^2}\frac{d}{dr}r^2|E'|^2 + \frac{4\pi i\omega\sigma}{c^2}(E^*E' - EE^{*\prime}) = 0.\end{gathered} \tag{1.6}$$

Eliminating $|E'|^2$ and $E^*E' - EE^{*\prime}$ from (1.6), we obtain an equation for $|E^2|$:

$$\frac{1}{r}\frac{d}{dr}\frac{1}{r\sigma}\frac{d}{dr}r\frac{d}{dr}r\frac{d|E^2|}{dr} - \frac{64\pi^2\omega^2\sigma}{c^4}|E^2| = 0.$$

Substituting here $|E^2|$ from (1.3), we obtain after integrating with respect to r an equation for the temperature:

$$\frac{d}{dr}r\frac{d}{dr}r\frac{d}{dr}\frac{1}{r\sigma}\frac{d}{dr}r\varkappa\frac{dT}{dr} - \frac{64\pi^2\omega^2\sigma}{c^4}r^2\varkappa\frac{dT}{dr} = 0.$$

Just as in[2], we introduce in lieu of T the dimensionless temperature Θ:

$$T = T_m - 2T_m^2\Theta/I,$$

and measure the distance to the discharge axis in units of r^* [1]):

$$r = r^*\zeta,$$

with r^* chosen such as to satisfy as $\zeta \to 0$ the formula

$$\Theta = \zeta^2. \tag{1.7}$$

Denoting by λ the ratio r^*/δ_m (this quantity characterizes the skin effect in the discharge), $\lambda = r^*/\delta_m$, $\delta_m = c\sqrt{8\pi\omega\sigma(T_m)}$, and neglecting the weak temperature dependence of the thermal conductivity in the region $T_m \gg T_m - T$, we obtain the following equation for Θ:

$$\frac{d}{d\zeta}\zeta\frac{d}{d\zeta}\zeta\frac{d}{d\zeta}\frac{e^\Theta}{\zeta}\frac{d}{d\zeta}\zeta\frac{d\Theta}{d\zeta} - \lambda^4\zeta^2e^{-\Theta}\frac{d\Theta}{d\zeta} = 0. \tag{1.8}$$

We are interested in obtaining for (1.8) a solution satisfying the condition (1.7) as $\zeta \to 0$. As $\zeta \to 0$, the function Θ does not depend on any parameter, and Eq. (1.8) contains one parameter λ. Therefore the solution of interest to us is a certain function of ζ, which depends on λ as a parameter:

$$\Theta = \Theta_\lambda(\zeta), \tag{1.9}$$

while $\zeta d\Theta/d\zeta$ is independent of ζ when $\zeta \gg 1$ and is a function of λ only:

$$\zeta d\Theta/d\zeta = \nu(\lambda), \quad \zeta \gg 1. \tag{1.10}$$

The function Θ itself takes for $\zeta \gg 1$ the form

$$\Theta_\lambda(\zeta) = \nu(\lambda)\ln(\zeta/\zeta^*(\lambda)), \quad \zeta \gg 1. \tag{1.11}$$

The functions $\Theta_\lambda(\zeta)$, and also $\nu(\lambda)$ and $\zeta^*(\lambda)$, were obtained by solving (1.8) numerically. The results of the numerical calculation are given in Figs. 1 and 2. Figure 1 shows a family of the functions (1.9) at different λ. Figure 2 shows plots of the functions $\nu(\lambda)$ and $\zeta^*(\lambda)$.

Changing over in (1.4) and (1.5) to dimensionless variables and comparing them in the region $T_m \gg T_m - T \gg T_m^2/I$ with (1.10) and (1.11), we obtain a system of equations determining the maximum temperature T_m in the discharge and the parameter λ of the skin effect as functions of the power input s_0:

$$s_0 = 4\pi\nu(\lambda)\varkappa_m T_m^2/I, \tag{1.12}$$

$$\lambda\zeta^*(\lambda) = \frac{R}{\delta_m}\exp\left(-\frac{2\pi}{s_0}\int_{T_0}^{T_m}\varkappa dT\right). \tag{1.13}$$

The electron concentration as a function of the radius can be represented in the form $N(r)/N_m = \exp(-\Theta_\lambda(r/r^*))$, where N_m is the equilibrium electron concentration at the temperature T_m. Plots of $N(r)/N_m$ against ζ for different λ are shown in Fig. 3.

We now calculate the electric current in the discharge. By virtue of (1.1) we have

$$I_0 = 2\pi\int_0^\infty \sigma E r dr = \frac{c}{2}rH|_{r\to\infty}.$$

Using (1.2) and (1.6), we obtain, after simple transformations, the following expression for the mean-squared current

$$\overline{|I_0^2|} = 4\pi\frac{c^2}{\omega}\frac{\varkappa_m T_m^2}{I}\varphi(\lambda) = \frac{4\pi\cdot 10^9}{\omega}\frac{\varkappa_m T_m^2}{I}\varphi(\lambda)\ [a^2], \tag{1.14}$$

where

$$\varphi(\lambda) = \lambda^2\int_0^\infty e^{-\Theta}\frac{d\Theta}{d\zeta}\zeta^2 d\zeta$$

[1])We note that T_m and r^* are not known beforehand. They will be determined below from the condition that the solution of (1.8) coincides with (1.4) and (1.5) in the region $T_m \gg T_m - T \gg T_m^2/I$.

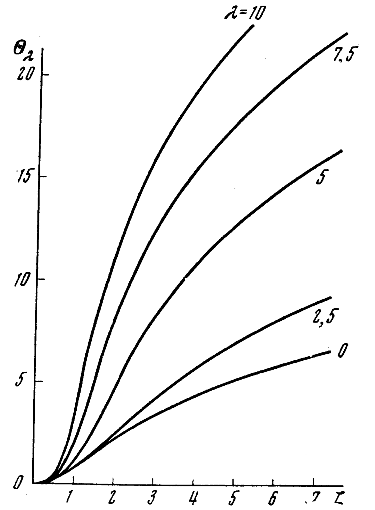

FIG. 1. Dependence of the dimensionless temperature on the dimensionless coordinate at different values of λ.

is a function of the parameter λ and is shown in Fig. 4. Relation (1.14), together with (1.12) and (1.13), makes it possible to find the dependence of the current in the discharge on the power input.

In the limiting case of weak skin effect[2)], $\lambda \ll 1$, we obtain the following expression for the dimensionless temperature Θ as a function of ζ: $\Theta = 2\ln(1 + \zeta^2/2)$, whence $\nu(0) = 4$, $\zeta^*(0) = \sqrt{2}$ and Eqs. (1.12) and (1.13) take the form

$$s_0 = 16\pi\varkappa_m T_m^2 / I,$$

$$\lambda = \frac{R}{\sqrt{2}\delta_m} \exp\left(-\frac{2\pi}{s_0}\int_{T_0}^{T_m} \varkappa dT\right), \quad \lambda \ll 1. \tag{1.15}$$

Noting that $\varphi(\lambda) \approx 2\lambda^2$ for $\lambda \ll 1$, we obtain the following expression for the mean-squared current:

$$\overline{|I_0^2|} = 8\pi \frac{c^2}{\omega} \frac{\varkappa_m T_m^2}{I} \lambda^2, \quad \lambda \ll 1.$$

Finally, in the absence of the skin effect, the electron concentration in the discharge as a function of ζ takes the form $N/N_m = 1/(1 + \zeta^2/2)^2$, $\lambda \ll 1$.

For sufficiently large λ, there is a strong skin effect and the thickness of the transition layer on the discharge surface becomes much smaller than its radius. In this case the boundary of the discharge can be regarded as planar at each point. Let r_0 be the coordinate of the transition layer on the discharge surface, and $S_0 = s_0/2\pi r_0$ the energy flux density through a unit discharge surface. In accord with[2]

$$r_0 = R \exp\left(-\frac{2\pi}{s_0}\int_{T_0}^{T_m} \varkappa dT\right).$$

Comparing this relation with (1.13), we obtain

$$\lambda\zeta^*(\lambda) = r_0/\delta_m, \quad r_0 \gg \delta_m.$$

Expressing s_0 in (1.12) in terms of S_0 and using the following result of the numerical calculation:

$$\nu(\lambda) = 1{,}57 \cdot \lambda\zeta^*(\lambda), \quad \lambda\zeta^*(\lambda) \gg 1, \tag{1.16}$$

we find that in the case of a strong skin effect, formula (1.12) goes over into formula (4.17) of[12]: $S_0 = 3.14\,\varkappa_m T_m^2 / I\delta_m$.

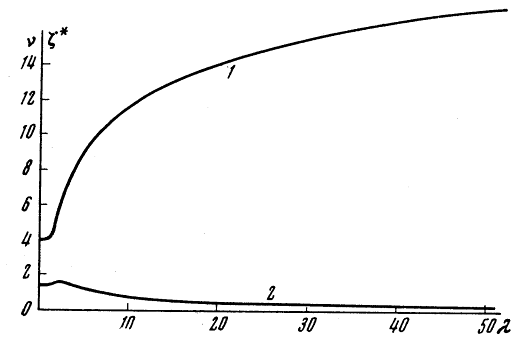

FIG. 2. Plots of the functions ν(λ) (curve 1) and ζ*(λ) (curve 2).

In the case of a strong skin effect, it is meaningful to introduce the plasma surface resistance R_z, defined by the formula

$$S_0 = R_z \overline{|J_0^2|},$$

where J_0 is the current flowing through a unit surface of the discharge ($J_0 = I_0/2\pi r_0$). Using the relation (1.6), we obtain for R_z the same expression as in[2]:

$$R_z = 3.14\pi \frac{\omega\delta_m}{c^2} = 3{,}14 \cdot \pi \cdot 10^{-9}\omega\delta_m\ [\Omega], \quad \lambda\zeta^*(\lambda) \gg 1.$$

It is easy to perform the same calculations for an inductive high-frequency discharge, when the components E_φ and H_z differ from zero. We do not present the corresponding calculations here since, first, the final results do not differ qualitatively from those presented above, and second, because a strong skin effect frequently takes place in the inductive discharges used in practice, so that the difference between the two cases disappears.

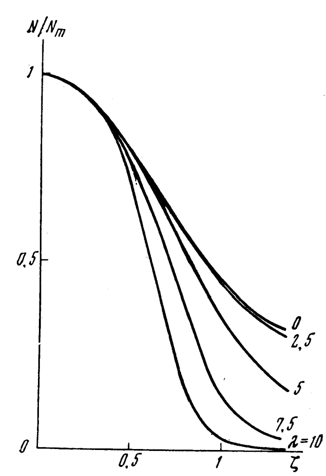

FIG. 3

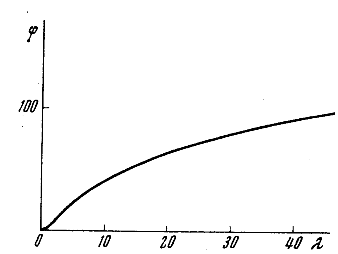

FIG. 4

FIG. 3. Coordinate dependence of the electron concentration in the discharge for different λ.

FIG. 4. Plot of the function φ(λ).

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2)The second term in (1.8) tends rapidly to zero both as $\zeta \to 0$ and as $\zeta \to \infty$, and begins to play a noticeable role only when $\lambda > 1$.

Equilibrium High-frequency Discharge in Hydrogen

To determine the explicit dependence of the discharge parameters on the power input from (1.12) and (1.13), it is necessary to know the temperature dependence of the thermal conductivity of the gas. By way of an example of the use of the theory employed above, we calculate the discharge parameters in hydrogen at atmospheric pressure. The resonator radius and the field frequency are assumed to be $R = 10$ cm and $\omega = 10^{10}$ sec^{-1}, as in Kapitza's experiments.[1] The temperature dependence of the thermal conductivity of hydrogen at atmospheric pressure[4] is shown in Fig. 5. In the temperature region under consideration, the main contribution is made by the thermal conductivity of the neutral atoms and molecules.

To solve the system (1.12) and (1.13) it is convenient first to eliminate s_0 and to find the dependence of the skin-effect parameter on the temperature T_m. Substitution of this dependence into (1.12) yields the dependence of the plasma temperature in the discharge T_m on the power input s_0. For hydrogen at atmospheric pressure, this dependence, shown in Fig. 6, calls for some explanation.

At low temperatures and low powers, there is practically no skin effect and the initial section ABCD of the plot is described by formula (1.15). The multiply-valued dependence of the temperature T_m on the power input on this section of the curve is due to the nonmonotonic dependence of the thermal conductivity of hydrogen on the temperature and to the presence of the maximum characteristic of diatomic gases, which is connected with energy transport by the excited molecules (see Fig. 5). On section CD, the temperature increases with increasing power input, leading to a strong increase of the conductivity and of the heat release. The thermal conductivity increases insignificantly and, starting with a certain temperature, does not provide an outlet for the power released in the discharge. Therefore, starting with the point D, the radius of the discharge increases strongly with increasing power input to the discharge, thereby providing a balance between the energy fed into the discharge and that carried away by heat conduction. On section DE the temperature decreases somewhat with increasing power input to the discharge. Finally, with further increase of the power input, on section EF (see the insert in Fig. 6) and beyond, the discharge continues to come closer to the cooled wall of the resonator, so that the heat is carried away from the discharge, as before, by heat conduction as a result of the large temperature gradient.

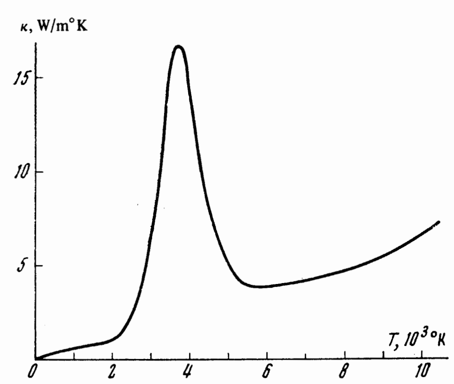

FIG. 5. Temperature dependence of the thermal conductivity of hydrogen at atmospheric pressure.

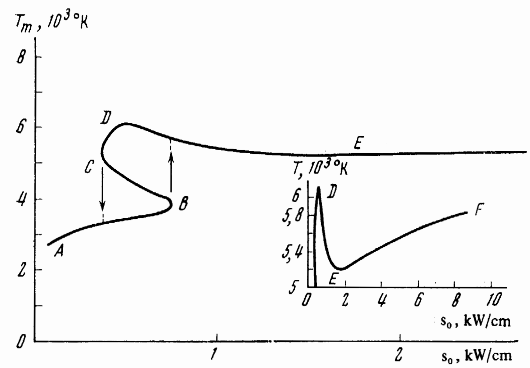

FIG. 6. Dependence of the temperature on the discharge axis on the power for hydrogen at atmospheric pressure. R = 10 cm, $\omega = 10^{10}$ sec^{-1}.

The temperature dependence of the plasma in a hydrogen discharge on the power, shown in Fig. 6, explains the presence of two types of discharge in a diatomic gas—the diffuse and filamentary discharges observed experimentally by Kapitza[1]. Between the points B and C, each value of the power corresponds to three values of the temperature T_m at once, and apparently only two of them—the smallest and the largest—are stable. In our opinion, the lower branch AB corresponds to a diffuse discharge and the branch CDEF to a filamentary discharge. When the power is varied, the transitions between the two types occur jumpwise at the points B and C, respectively, as shown by the arrows in Fig. 6, so that hysteresis sets in. Figure 7 shows the dependence of the radius $r_{0.5}$ (at which the electron concentration drops to $N_m/2$) on the power input s_0. The branch AB corresponds to the diffuse discharge and the branch CD to the filamentary discharge. Thus, the presence of two types of discharge in a diatomic gas is due to the presence of a characteristic maximum in the temperature dependence of the thermal conductivity.

Figure 8 shows the dependence of the average current in the discharge $(|\overline{I_0^2}|)^{1/2} = I_0/\sqrt{2}$ on the power input s_0. For comparison, the figure shows the experimentally measured current in a deuterium discharge[3] 3). The rapid increase of the discharge current with increasing power, observed in[3], cannot be explained if an equilibrium infinitely long low-temperature discharge is assumed.

2. EQUILIBRIUM HIGH-FREQUENCY DISCHARGE IN A GAS STREAM

In this section we investigate a question of practical importance, that of heat exchange and the structure of an equilibrium high-frequency discharge in a gas

3)It can be shown, with the same accuracy at which the thermal conductivity itself is presently known) that the difference between the thermal conductivities of hydrogen and deuterium does not affect the dependence of the current in the discharge on the power input.

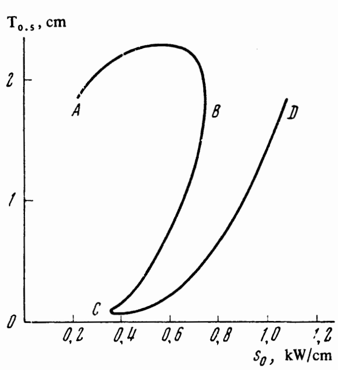

FIG. 7

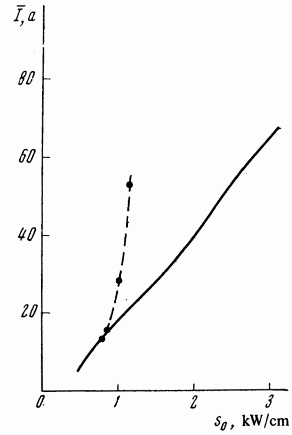

FIG. 8

FIG. 7. Dependence of discharge radius on the power input.

FIG. 8. Dependence of the current in the discharge on the power input. The continuous curve is a plot of (1.14). The points and the dashed curve correspond to the experimental results given in [3].

stream. Without allowance for the exact structure of the discharge, this question was first analyzed correctly by Raĭzer[5].

In the presence of local thermodynamic equilibrium, a high frequency discharge in a gas stream is described by the system of magnetohydrodynamic equations, consisting of Maxwell's equations, the Navier-Stokes equations, the continuity equations, the heat-transfer equation, and the equation of state of the gas. The properties of the medium, besides the equation of state enter into the equation via the kinetic coefficients—the electric conductivity, the thermal conductivity, the specific heat, and the viscosity. The boundary conditions for these equations depend on the construction of the concrete apparatus used to obtain the discharge, and cannot be described in general form.

However, the problem can be solved completely in the important case of a strong skin effect, when the depth of penetration of the field into the discharge is small compared with the dimensions of the discharge. Under these conditions, the conversion of the electromagnetic energy into Joule heat occurs in a thin layer (on the order of the penetration depth) on the discharge boundary. In the region of the boundary, all the quantities can be regarded as functions of one coordinate only. The solution of the one-dimensional problem on the discharge boundary determines the structure of the transition layer, and also the dependence of the temperature in the discharge on the electromagnetic energy flux density and on the velocity of the gas stream. Outside the discharge, the conductivity of the gas decreases rapidly and the complete system of equations breaks up into independent systems of hydrodynamic equations describing the motion of the gas, and Maxwell's equations for the electromagnetic field outside the discharge. By matching the solution of the external hydrodynamic problem to the solution in the transition region it is possible, in principle, to obtain the total distribution of the temperature and consequently to determine the shape of the discharge. By way of an example, a solution (applicable to plasmotrons with helical gas flow) was obtained for a stationary discharge occupying a half-space and situated in a specified gas stream whose velocity is directed at an arbitrary angle to the cooled surface of the vessel. The distance from the cooled wall to the transition layer was determined as a function of the plasma temperature and the gas flow velocity.

A solution was obtained for the problem of free propagation of a plane gas-discharge front, analogous to the problem of flame propagation in slow combustion. The dependence of the velocity of the free propagation of the discharge on the plasma temperature was obtained. The corresponding dimensionless velocity of discharge propagation is small (on the order of T_m/I—the ratio of the plasma temperature to the ionization potential of the gas). For this reason, the dependence of the plasma temperature in the discharge on the power input for free propagation of the discharge does not differ, accurate to terms of the order of T_m/I, from the case of a discharge in a stationary gas.

Structure of Transition Layer

We shall be interested below in the motion of a gas at velocities much lower than that of sound. In this case the pressure and density changes due to the gas motion are small. In determining the derivatives of the thermodynamic quantities in a nonuniformly heated gas, the pressure can therefore be regarded as constant. At low flow velocities we can neglect the viscosity of the gas as well as the forces exerted on the electrons by the magnetic field. The heat-transfer equation reduces in this case to the heat-conduction equation

$$\rho c_p\left(\frac{\partial T}{\partial t}+\mathrm{v}\,\mathrm{grad}\,T\right)=\mathrm{div}(\varkappa\,\mathrm{grad}\,T)+\frac{1}{2}\sigma|\mathbf{E}^2|,$$

in which the specific heat c_p, the thermal conductivity κ, and the conductivity σ are specified functions of the temperature. In the equation of state, the pressure can also be regarded as constant and the density ρ can be determined from it as a function of the temperature. Knowing the gas density, the hydrodynamic velocity of the gas v and the small pressure changes associated with the gas motion are determined from the continuity equation and the Navier-Stokes equations.

Just as in the case of a discharge in a stationary gas, we confine ourselves to relatively low temperatures, when the conductivity is an exponential function of the temperature. The condition $T_m \ll I$ makes it possible to solve the problem separately in two overlapping regions: in the region outside the discharge $T_m - T \gg T_m^2/I$, where the temperature varies considerably and there is practically no heat release, and in the transition region on the discharge boundary $T_m - T \ll T_m$, in which the temperature differs little from its maximum value inside the discharge.

Let us consider the transition region on the discharge boundary. In the case of a strong skin effect, when the depth of penetration of the field into the plasma is small compared with the dimensions of the discharge, all the quantities in the transition layer can be regarded as dependent on only one coordinate r, and the discharge surface can be regarded as flat. In the one-dimensional case, the equation of heat conduction for a stationary discharge in the gas stream is

$$\rho v_r c_p \frac{dT}{dr} = \frac{d}{dr}\varkappa\frac{dT}{dr} + \frac{1}{2}\sigma|\mathbf{E}^2|. \tag{2.1}$$

The product ρv_r is determined directly from the continuity equation $d(\rho v_r)/dr = 0$, and the equation for $|\mathbf{E}^2|$, when the vector $\mathbf{E}$ is parallel to the discharge surface, takes the form (see[2], formula (4.7))

$$\frac{d}{dr}\frac{1}{\sigma}\frac{d^3|\mathbf{E}^2|}{dr^3} - \frac{64\pi^2\omega^2\sigma}{c^4}|\mathbf{E}^2| = 0. \tag{2.2}$$

The heat-conduction equation (2.1) contains only the velocity component normal to the discharge surface. This makes it possible to disregard the gas motion along the discharge boundary and by the same token dispense with the Navier-Stokes equations. It follows from the continuity equation that $\rho v_r = \text{const} = -\rho_0 u$, where ρ_0 is the gas density at a certain temperature $T = T_0$ and u is the projection of the gas velocity at this temperature on the normal to the discharge surface. The sign is chosen such as to make the velocity $u > 0$ correspond to motion of the gas inside the discharge (in the $r \to -\infty$ direction). Equation (2.1) can be integrated. We denote by $w(T)$ the enthalpy of the gas as a function of the temperature. Recognizing that the Ponyting vector $\mathbf{S}(r)$ and the heat flux $-\kappa dT/dr$ vanish inside the discharge, we obtain ($w_m \equiv w(T_m)$)

$$-\rho_0 u(w_m - w) + \varkappa\frac{dT}{dr} + S(r) = 0. \tag{2.3}$$

In the region $T_m \gg T_m - T \gg T_m^2/I$, the absorption of the electromagnetic energy can be neglected, so that $S = \text{const} = S_0$, and the thermal conductivity and the specific heat can be regarded as constant and equal to their values at $T = T_m$. In this temperature region, the relation (2.3) takes the form

$$\rho_0 u c_p(T_m)(T - T_m) + \varkappa_m dT/dr + S_0 = 0. \tag{2.4}$$

Eliminating $|\mathbf{E}^2|$ from (2.1) and (2.2), we obtain the following equation for the temperature in the transition layer:

$$\left(\frac{d}{dr}\frac{1}{\sigma}\frac{d^3}{dr^3}\frac{1}{\sigma} - \frac{64\pi^2\omega^2}{c^4}\right)\left(\rho_0 u c_p\frac{dT}{dr} + \frac{d}{dr}\varkappa\frac{dT}{dr}\right) = 0. \tag{2.5}$$

Since the temperature in the transition layer changes little, we can put $c_p = c_p(T_m)$ and $\kappa = \kappa_m$. Taking into account the temperature dependence of the conductivity, it is convenient, just as in the case of a stationary gas, to change over to the dimensionless variables Θ and ζ: $T = T_m - 2T_m^2\Theta/I$, $r = -\delta_m\zeta$, where $\delta_m = c/\sqrt{8\pi\omega\sigma(T_m)}$ is the depth of penetration of the field into the plasma with temperature T_m. Denoting by χ the dimensionless velocity of the gas stream

$$\chi = \rho_0 u c_p(T_m)\delta_m / \varkappa_m, \tag{2.6}$$

we obtain after integration

$$\left(\frac{d^3}{d\zeta^3}e^\Theta\frac{d}{d\zeta} - e^{-\Theta}\right)\left(\frac{d\Theta}{d\zeta} - \chi\Theta\right) = 0. \tag{2.7}$$

In the region far from the boundary inside the discharge we have an asymptotic expression for Θ:

$$\Theta = e^{-(\zeta-\zeta_0)}, \qquad \zeta - \zeta_0 \gg 1. \tag{2.8}$$

Making relation (2.4) dimensionless, we find that outside the discharge Θ satisfies the equation

$$\frac{d\Theta}{d\zeta} - \chi\Theta = -\frac{S_0 I\delta_m}{2\varkappa_m T_m^2}, \qquad \zeta_0 - \zeta \gg 1. \tag{2.9}$$

Equation (2.7) does not contain ζ explicitly. Consequently, its solution, the asymptotic form of which is (2.8), is a certain function of $\zeta - \zeta_0$, which depends on χ as a parameter:

$$\Theta = \Theta_\chi(\zeta - \zeta_0), \tag{2.10}$$

and $\Theta' - \chi\Theta$ in the region $\zeta_0 - \zeta \gg 1$ does not depend on ζ and is a certain function of χ:

$$\frac{d\Theta}{d\zeta} - \chi\Theta = -\mu(\chi), \qquad \zeta_0 - \zeta \gg 1. \tag{2.11}$$

Integrating (2.11), we get

$$\Theta = \frac{\mu(\chi)}{\chi}(1 - e^{\chi(\zeta-\zeta_0)}), \qquad \zeta_0 - \zeta \gg 1. \tag{2.12}$$

The function $\mu(\chi)$ obtained by numerical calculation is shown in Fig. 9. Figure 10 shows the family of functions (2.10) for different χ.

Comparing (2.9) with (2.11), we find the high-frequency energy flux density necessary to heat the plasma in the discharge to the temperature T_m:

$$S_0(T_m) = 2\mu(\chi)\varkappa_m T_m^2 / I\delta_m. \tag{2.13}$$

In the limiting case of an immobile gas $\chi = 0$, $\mu(0) = 1.57$, and (2.13) goes over into formula (4.17)[2].

Planar Discharge in a Homogeneous Gas Stream

The constant ζ_0, which determines the position of the transition layer, is obtained by matching the solution in the transition layer to the solution outside the discharge. The solution of the external hydrodynamic problem cannot be written out in general form, since the boundary conditions depend on the construction of the concrete apparatus used to obtain the discharge. We note, however, that in the plasma burners used in practice, with helical gas flow, the distance from the surface of the discharge to the nearest wall of the vessel can be much smaller than the transverse dimen-

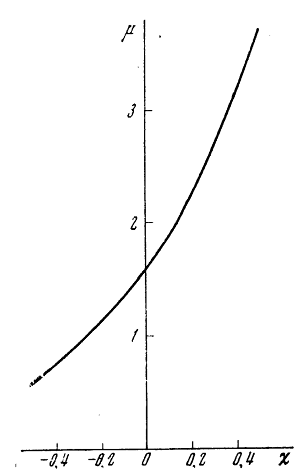

FIG. 9. Plot of the function $\mu(\chi)$.

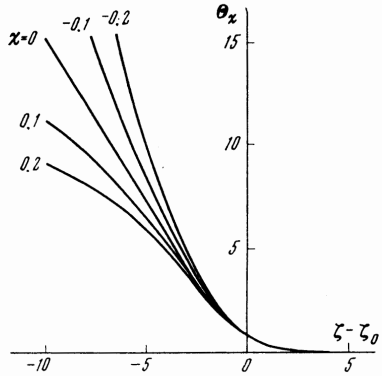

FIG. 10. Family of functions (2.10) for different χ.

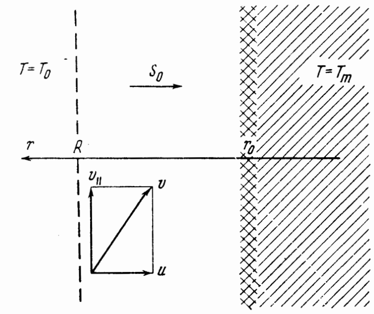

FIG. 11. Diagram of planar discharge in a gas stream. The dashed line represents a grid cooled to a temperature T_0. The doubly-hatched region corresponds to the transition layer on the discharge boundary.

sions of the discharge itself. In this case the following problem becomes meaningful.

Assume that an infinite flat grid (see Fig. 11) cooled to the temperature T_0 is located at $r = R$, and that a homogeneous gas flows through the grid and has a velocity component u normal to the grid. Assume further that the high-frequency flux density S_0 (outside the discharge) is specified. We consider a stationary discharge produced in the region $r \lesssim r_0 = -\delta_m\zeta_0$ and denote the maximum plasma temperature in the discharge by T_m. The temperature distribution outside the discharge in the region $T_m - T \gg T_m^2/I$ is determined from the heat-conduction equation

$$\rho_0 u c_p \frac{dT}{dr} + \frac{d}{dr}\varkappa\frac{dT}{dr} = 0.$$

Reckoning the gas enthalpy from its value at $T = T_0$

$$w(T) = \int_{T_0}^{T} c_p\, dT$$

and denoting by J the heat flux at $r = R$: $J = -\kappa dT/dr$, we write down the first integral of the heat-conduction equation in the form

$$\rho_0 u w + \varkappa\frac{dT}{dr} + J = 0, \qquad T_m - T \gg \frac{T_m^2}{I}. \tag{2.14}$$

From the condition that (2.14) must coincide with (2.3) in the region $T_m \gg T_m - T \gg T_m^2/I$ we obtain the heat flux J:

$$J = S_0 - \rho_0 u w_m. \tag{2.15}$$

Integrating (2.14) with the boundary condition $T = T_0$ at $r = R$, we obtain in implicit form the coordinate dependence of the temperature outside the discharge:

$$R - r = \int_{T_0}^{T} \frac{\varkappa\, dT}{J + \rho_0 u w}, \qquad T_m - T \gg \frac{T_m^2}{I}.$$

In the region $T_m \gg T_m - T \gg T_m^2/I$ it is convenient to rewrite this formula, with allowance for (2.15), in the form

$$R - r = \int_{T_0}^{T_m} \frac{\varkappa\, dT}{S_0 - \rho_0 u(w_m - w)} - \frac{\varkappa_m}{\rho_0 u c_p(T_m)} \int_{T}^{T_m} \frac{dT}{S_0/\rho_0 u c_p(T_m) - (T_m - T)}$$

From which we obtain after calculating the integral

$$T_m - T = \frac{S_0}{\rho_0 u c_p(T_m)}\left(1 - \exp\left\{\frac{\chi}{\delta_m}\left(R - r - \int_{T_0}^{T_m} \frac{\varkappa\, dT}{S_0 - \rho_0 u(w_m - w)}\right)\right\}\right) \tag{2.16}$$

Changing over in (2.16) to dimensionless variables and comparing with (2.12), we obtain

$$r_0 = -\delta_m\zeta_0 = R - \int_{T_0}^{T_m} \frac{\varkappa\, dT}{S_0 - \rho_0 u(w_m - w)}. \tag{2.17}$$

Formula (2.17), together with (2.13), determines the position of the discharge boundary as a function of T_m or S_0 at a specified gas-stream velocity u.

We note that the formulas obtained above remain in force also for the case $u < 0$, when the gas flows from the discharge into the cold region. For the existence of a stationary state of the discharge, the gas flow velocity u into the discharge must not exceed the velocity of the free propagation of the discharge. In the opposite case the cold gas entering the discharge will not have time to be heated by heat conduction.

Free Propagation Velocity of Discharge

In the case of free propagation of the discharge front, the heat flux $-\kappa dT/dr$ tends to zero with increasing distance from the transition layer. Consequently, so long as the discharge is sufficiently far from the wall, we can put

$$J = 0, \quad T = T_0, \quad r \to \infty. \tag{2.18}$$

We change over to a reference frame in which the discharge is immobile. The formulas derived above for the stationary discharge in a gas stream are valid in this reference frame.

Under the condition (2.18), the relation (2.15) holds only for one value of the velocity, $u = u_0$. It is this relation which determines the connection between the discharge free propagation velocity u_0 and the electromagnetic energy flux density S_0 and the temperature T_m: $u_0 = S_0/\rho_0 w_m$. Putting $\chi = \chi_0$ at $u = u_0$, we obtain a system of two equations determining the dimensionless velocity of free propagation of the discharge χ_0 and the plasma temperature T_m as functions of the incoming high-frequency energy flux S_0:

$$\chi_0 = 2\mu(\chi_0)\frac{c_p(T_m)T_m^2}{w_m I}, \qquad S_0 = 2\mu(\chi_0)\frac{\varkappa_m T_m^2}{I\delta_m}. \tag{2.19}$$

Noting that $c_p(T_m)T_m \sim w_m$ and that $T_m \ll I$, we conclude from (2.19) that the dimensionless velocity of the free propagation of the discharge is low:

$$\chi_0 \approx 2\mu(0)\frac{c_p(T_m)T_m^2}{w_m I} \sim \frac{T_m}{I} \ll 1$$

and thus, accurate to terms of order T_m/I, the dependence of the plasma temperature T_m on the electromagnetic energy flux density does not differ from the case of a discharge in an immobile gas: $S_0 = 3.14\kappa_m T_m^2/I\delta_m$. For the free propagation velocity of the discharge we obtain the expression $u_0 = 3.14\kappa_m T_m^2/\rho_0 w_m \delta_m I$. The table lists the values of S_0 and u_0 for air and argon at atmospheric pressure ($T_0 = 273°K$). Figure 12 shows the dependence of the free-propagation velocity of a plane discharge front in hydrogen at atmospheric pressure.

T_m, °K	S_0, w/cm²	u_0, cm/sec	T_m, °K	S_0, w/cm²	u_0, cm/sec
	Air			Argon	
7000	100	3.1	7000	5.3	0,8
8000	200	4.1	8000	16.4	2,3

Finally, the dimension of the region $\tilde{r}$ in front of the discharge, in which the temperature decreases to a certain value $T_1 > T_0$, is determined in the same manner as the coordinate of the transition region (2.17). We obtain

$$\tilde{r} = \frac{1}{\rho_0 u_0}\int_{T_1}^{T_m} \frac{\varkappa\, dT}{w}.$$

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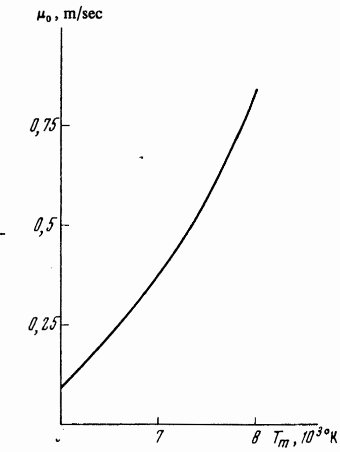

FIG. 12. Dependence of the velocity of the free propagation of the discharge in hydrogen at atmospheric pressure.

Note added in proof (28 September 1971). For approximate calculations of the discharge structure in Ar at higher temperatures, using interpolation formulas, see also [6].

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