STATISTICS OF ELECTRONS IN SEMICONDUCTORS IN A QUANTIZING MAGNETIC FIELD

B. A. ARONZON and E. Z. MEĬLIKHOV

Submitted March 25, 1971

Zh. Eksp. Teor. Fiz. 61, 1906–1912 (November, 1971)

The effect of a quantized magnetic field on the free electron concentration in semiconductors is studied. Semiconductors with a nonparabolic conduction band, of the n-InSb type, are considered. The free electron concentration in degenerate semiconductors increases with increasing magnetic field; in nondegenerate semiconductors, the effect of the magnetic field on the concentration is insignificant. The magnitude of the concentration depends on the electron Lande g-factor. The results are obtained with a digital computer and are presented graphically. The experiments performed with n-InSb and n-InAs samples in fields up to 150 kOe at 78°K are in good agreement with the theoretical calculations.

We shall consider the influence of a quantizing magnetic field on the concentration of the free carriers in impurity semiconductors. A similar problem was proposed also earlier[1-3], but no account was taken there of the nonparabolicity of the conduction band and, consequently, of the anomalously large Lande g-factor (an important circumstance in cases of practical interest), nor was the statistics of the electrons at the impurity levels taken into account correctly—the spin splitting of the electrons was characterized by g = 2 and the formation of a system of Landau impurity levels was completely ignored. As a result, the conclusions derived in the cited papers are incorrect even qualitatively.

1. ELECTRON STATISTICS IN THE CONDUCTION BAND

According to the Kane model[4], the density of states in a nonparabolic conduction band, in the presence of a magnetic field, can be written in the form[3]

$$Z(\varepsilon)=\frac{(2m^*)^{3/2}}{8\pi^2\hbar^3}\hbar\omega_{H^*}\sum_{N,\sigma=\pm1}\left[B(\varepsilon)-\left(N+\frac{1}{2}\right)\hbar\omega_{H^*}+\sigma G(\varepsilon)\hbar\omega_{H^*}\right]^{-1/2}, \tag{1}$$

where

$$G(\varepsilon)=\left|\frac{1}{2}\frac{m^*}{m_0}\left[1+\left(1-\frac{m_0}{m^*}\right)\frac{\Delta}{3\varepsilon+3\varepsilon_g+2\Delta}\right]\right|, \qquad B(\varepsilon)\cong\varepsilon. \tag{2}$$

In the Kane model $B(\epsilon)\neq\epsilon$, but the actual difference between the two is quite small and in all the cases considered below the resultant errors do not exceed ~15%. In (1) and (2), ϵ is the electron energy, m^* the effective mass of the electron at the bottom of the conduction band, m_0 the mass of the free electron, $\omega_H^* = eH/m^*C$ the cyclotron frequency of the electron in a magnetic field of intensity H, ϵ_g the width of the forbidden band, and Δ the spin-orbit splitting in the valence band.

For $m_0/m^* \gg 1$ and $\epsilon \ll \epsilon_g + \frac{2}{3}\Delta$, Eq. (2) can be rewritten in the form

$$G=\Delta/2(3\varepsilon_g+2\Delta), \tag{3}$$

which yields G = 0.35 for InSb and G = 0.20 for InAs.

The electron concentration n in the conduction band is determined by the expression

$$n=\int_0^\infty\frac{Z(\varepsilon)\,d\varepsilon}{1+\exp[(\varepsilon-\varepsilon_F)/k_0T]} \tag{4}$$

$$=\frac{1}{2\sqrt{\pi}}N_c\mathcal{H}\sum_{N=0}^{\infty}\left\{F_{-1/2}\left[E_F-\left(N+\frac{1}{2}\right)\mathcal{H}+C\mathcal{H}\right]+F_{-1/2}\left[E_F-\left(N+\frac{1}{2}\right)\mathcal{H}-G\mathcal{H}\right]\right\},$$

where $N_c = (2\pi m^* k_0 T)^{3/2}/4\pi^3\hbar^3$ is the effective density of states in the conduction band in the absence of a magnetic field, $\mathcal{H} = \hbar\omega_{H^*}/k_0T$, $E_F = \epsilon_F/k_0T$ (ϵ_F is the Fermi energy), and

$$F_{-1/2}(x)=\int_0^\infty\frac{t^{-1/2}\,dt}{1+\exp(t-x)}$$

is the Fermi integral. We note that E_F is, generally speaking, a function of the magnetic field.

We consider two limiting cases. In the absence of degeneracy (i.e., when $\frac{1}{2}\mathcal{H} - G\mathcal{H} - E_F \gg 1$) $F_{-1/2}(x) \cong \sqrt{\pi}e^x$ and (4) can be reduced to the form

$$n=N_c\mathcal{H}\frac{\operatorname{ch}(G\mathcal{H})}{2\operatorname{sh}(\mathcal{H}/2)}\exp E_F. \tag{5}$$

In the case of strong degeneracy in the quantum limit ($E_F < (\frac{1}{2} + G)\mathcal{H}$), when all the electrons are at the lowest sublevel of the lower (N = 0) Landau level, Eq. (4) takes the form

$$n=\frac{1}{2\sqrt{\pi}}N_c\mathcal{H}F_{-1/2}\left[E_F-\left(\frac{1}{2}-G\right)\mathcal{H}\right]. \tag{6}$$

2. STATISTICS OF ELECTRONS AT IMPURITY LEVELS

We consider the so-called shallow impurity levels. In semiconductors with a small effective mass, the radius of the Bohr orbit of the electron located at such a center is very large, and the regions of localization of the electrons at the impurity atoms overlap in the absence of a magnetic field. This leads to formation of an impurity band that can overlap with the conduction band. A strong magnetic field leads to localization of the wave functions of the electron at the impurities, and as a result the binding energy ϵ_D of the electron to the impurity center increases and the band overlap is eliminated[5-8]. We shall henceforth assume the magnetic field to be "strong" in the required sense[8].

When $\epsilon_D \gtrsim k_0T$, the increase of the ionization energy in the magnetic field leads to the effect of "magnetic freezing out"[5]; we consider the opposite case: $\epsilon_D \ll k_0T$ (for InSb and InAs this means $T \gtrsim 20°K$ at $H \sim 100$ kOe), when this effect is missing.

In a magnetic field, the energy spectrum of an electron at an impurity is a system of Landau impurity levels. In a quantizing magnetic field ($\hbar\omega_{H^*} \gg k_0T$) almost all the electrons are at the zero Landau level, and we shall therefore take only this level into account. Within the framework of the hydrogen-like model we have[5]

$$\varepsilon_{\pm} = -\varepsilon_D(H) + \tfrac{1}{2}\hbar\omega_H^* + \sigma G'\hbar\omega_H^*.$$

Here $\epsilon_\pm$ is the electron energy (reckoned from the bottom of the conduction band at H = 0), $\sigma = \pm 1$, and G′ is the reduced Lande g-factor of the electron at the impurity. If the cyclotron-rotation energy is much larger than the binding energy of the localized electron, its interaction with the magnetic field can be regarded in the same manner as for a free electron. We shall therefore assume G′ = G[9].

By regarding the impurity center as a subsystem with two nondegenerate energy levels $\epsilon_\pm$, we can write down its thermodynamic potential Ω in the following manner[10]:

$$\Omega / k_0T = \ln[1 + \exp(E_F - E_+) + \exp(E_F - E_-)]. \quad (7)$$

Here $E_\pm = \epsilon_\pm/k_0T$. The concentration of the nonionized impurity atoms is determined by the expression

$$N_D^- = -N_D\left[\frac{\partial(\Omega/k_0T)}{\partial E_F}\right], \quad (8)$$

where N_D is the concentration of the impurity atoms.

From (7) and (8) we obtain the concentration N_D^+ of the ionized impurity atoms

$$N_D^+ = N_D - N_D^- = \frac{N_D}{1 + 2\exp[E_F - \tfrac{1}{2}\mathcal{H} + \varepsilon_D/k_0T]\operatorname{ch} G\mathcal{H}}. \quad (9)$$

We took into account here the fact that G′ = G. Since we are considering the case $\epsilon_D \ll k_0T$, we can rewrite (9) in the form

$$\frac{N_D^+}{N_D} = \frac{1}{1 + 2\exp(E_F - \tfrac{1}{2}\mathcal{H})\operatorname{ch} G\mathcal{H}}. \quad (10)$$

3. CALCULATION OF THE CONDUCTION-ELECTRON CONCENTRATION

Supplementing (4) and (10) with the neutrality equation $n = N_D^+$, we obtain the following system:

$$\eta = A\mathcal{H}\sum_{N=0}^{\infty}\{F_{-1/2}[E_F - (N + \tfrac{1}{2} + G)\mathcal{H}] + F_{-1/2}[E_F - (N + \tfrac{1}{2} - G)\mathcal{H}]\}, \quad (11)$$

$$\eta = \frac{1}{1 + 2\exp(E_F - \tfrac{1}{2}\mathcal{H})\operatorname{ch} G\mathcal{H}}; \quad (12)$$

Here $\eta = n/N_D$ is the dimensionless concentration of the conduction electrons, and $A = N_C/2\sqrt{\pi}N_D$ is a parameter characterizing the degree of degeneracy of the electron gas at H = 0, with $A \ll 1$ corresponding to strong degeneracy and $A \gg 1$ to the absence of degeneracy.

Before we proceed to discuss the results of the numerical solution of this system, let us consider two limiting cases. For a nondegenerate electron gas we can rewrite (11) in a form similar to (5). We then obtain for $\mathcal{H} \gg 1$

$$\eta = 1 - 1/A\sqrt{\pi}\mathcal{H}. \quad (13)$$

We see therefore that although the concentration increases with increasing $\mathcal{H}$, but since $A \gg 1$ and $\mathcal{H} \gg 1$, the relative change of η is very small and is therefore difficult to observe experimentally.

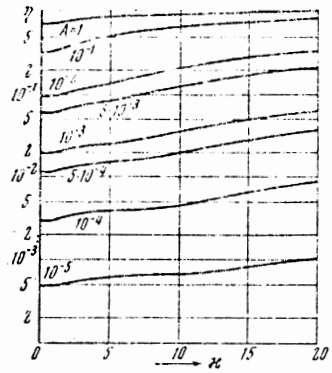

FIG. 1. Conduction electron concentration η in impurity semiconductors, at different degrees of degeneracy A, vs. the magnetic field $\mathcal{H}$ at G = 0.35.

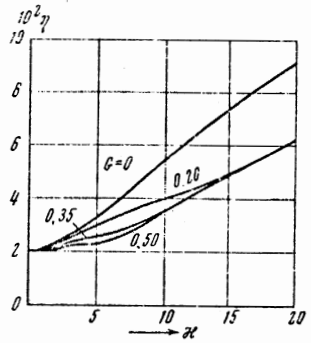

FIG. 2. Concentration of degenerate conduction electrons η in impurity semiconductors at different values of the Lande factor G vs. the magnetic field $\mathcal{H}$ at $A = 10^{-3}$.

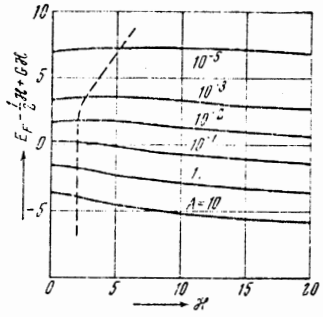

FIG. 3. Fermi energy E_F in impurity semiconductors at different degrees of degeneracy vs. the magnetic field $\mathcal{H}$ at G = 0.35.

In the case of the quantum limit, (11) can be written in a form analogous to (6), while (12), subject to the additional condition $G\mathcal{H} \gg 1$, takes the form

$$\eta = \frac{1}{1 + \exp(E_F - \tfrac{1}{2}\mathcal{H} + G\mathcal{H})}. \quad (14)$$

From (14) with account taken of (6) we obtain

$$\eta = A\mathcal{H}F_{-1/2}[\ln(1/\eta - 1)]. \quad (15)$$

We see that $\eta \to 1$ as $\mathcal{H} \to \infty$, and since $\eta(0) \ll 1$, the relative growth of the electron concentration is very large; we note that η is independent of G in this case.

The system (11) and (12) was solved for the general case with a computer. The most important result of these calculations is that the concentration of the free electrons increases monotonically with increasing $\mathcal{H}$. The main cause of this effect is the sublinear dependence of the Fermi energy E_F on the magnetic field $\mathcal{H}$. The energy gap between the Fermi level and the bottom of the conduction band $E_F - \tfrac{1}{2}\mathcal{H} + G\mathcal{H}$ decreases with increasing $\mathcal{H}$ (see Fig. 3). This causes a decrease in

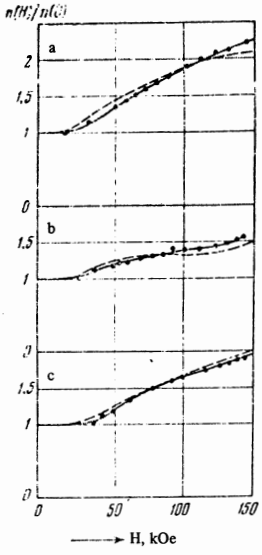

FIG. 4. Experimental plots (continuous lines) of the free-electron concentration in n-InSb and n-InAs vs. the magnetic field. Dashed lines—corresponding theoretical plots. a—n-InSb, $n(0) = 2.2 \times 10^{15}$ cm^{-3}, $A = 10^{-1}$, $G = 0.35$; b—n-InSb, $n(0) = 5.5 \times 10^{16}$ cm^{-3}, $A = 10^{-5}$, $G = 0.35$; c—n-InAs, $n(0) = 1.5 \times 10^{16}$ cm^{-3}, $A = 10^{-2}$, $G = 0.2$.

the degree of degeneracy of the electron gas, leading to an increase of the concentration η. Another cause, not decisive under our conditions, of the change of η is the increase of the electron state density in the conduction band. The results of the calculations are shown in Figs. 1, 2, and 3.

Figure 1 shows plots of $\eta(\mathcal{H})$ for different values of the parameter $A = 1$—10^{-5} at $G = 0.35$ (corresponding to a wide range of electron concentrations in InSb in the absence of a magnetic field: $n(0) = 8 \times 10^{14}$—6×10^{16} cm^{-3}). The value $\mathcal{H} = 1$ corresponds to $H = 11$ kOe for InSb and $H = 17$ kOe for InAs at $T = 77°K$.

Figure 2 shows plots of $\eta(\mathcal{H})$ for different values of the parameter G ($0 \leq G < 0.5$) for $A = 10^{-3}$. When $\mathcal{H} \leq 20$, the rate of change of η decreases with increasing G, and when $\mathcal{H} \gtrsim 20$, as noted above, η is independent of G (for $G \neq 0$).

Figure 3 shows plots of $E_F - \frac{1}{2}\mathcal{H} + G\mathcal{H}$ (the energy gap between the bottom of the conduction band and the Fermi level) against the magnetic field $\mathcal{H}$ for $A = 10$—10^{-5} and $G = 0.35$. We see that the magnetic field decreases the degree of degeneracy of the electrons. The dashed line bounds the region of the quantum limit.

4. EXPERIMENTAL RESULTS

An experimental verification of the notions developed above concerning the change of the conduction-electron concentration in semiconductors in quantizing magnetic fields was carried out by measuring the Hall effect on n-InSb and n-InAs samples. It is known[10] that in a quantizing magnetic field the Hall constant $r_H = 1/n(H)ec$ does not depend on the electron-scattering mechanism or on the field intensity, and therefore Hall-effect measurements under these conditions make it possible to determine the n(H) dependence directly.

It should be noted that Hall measurements of such a type were also undertaken earlier[11-13]. The results of these investigations, however, are quite contradictory even qualitatively, namely in regard to the character of the variation (growth or decrease) of $R_H(H)$. The possible causes of this are the inhomogeneity of the electron concentration in the employed samples[14] and the difficulties connected with measuring small Hall emf's under conditions of pulsed magnetic fields of high intensity.

The measurements were performed in pulsed magnetic fields up to 150 kOe (field half-period 1 msec) at liquid-nitrogen temperature, corresponding to values $\mathcal{H} \leq 13.5$ for InSb and $\mathcal{H} \leq 9$ for InAs. To decrease the inhomogeneity of the electron-concentration distribution, the samples, which measured $7 \times 3 \times 1$ mm, were cut in a plane perpendicular to the crystal growth axis. The initial electron concentration n(0) was measured in a weak ($\mathcal{H} \ll 1$) constant magnetic field.

As already noted, Hall measurements under conditions of strong pulsed magnetic fields are very difficult. This is connected primarily with the noise induced by the time-varying magnetic field in the measuring circuit. The frequency spectrum of such parasitic emf's overlaps the useful-signal spectrum and makes frequency selection of the latter impossible. Practical measures towards eliminating and cancelling out the noise are frequently insufficient for exact measurements. For a radical elimination of the foregoing difficulties, our Hall measurements were carried out with alternating current of frequency $f_0 = 20$ MHz. High-frequency Hall measurements are made possible by the smallness of the settling time τ_H of the Hall field. In analogy with the Maxwellian relaxation time, we can write $\tau_H = \kappa_0/4\pi\sigma_{xy}$, where $\sigma_{xy} = nec/H$ is the Hall component of the conductivity tensor and κ_0 is the dielectric constant of the lattice. It is easy to see that $\tau_H \ll (2\pi f_0)^{-1}$ at $H \sim 100$ kOe and $n \gg 10^{12}$ cm^{-3}. This inequality is certainly satisfied under experimental conditions and makes it possible to use a high-frequency measurement procedure. The Hall signal was registered with a selective amplifier with resonant frequency f_0 and a bandwidth 100 kHz, which (in conjunction with transformer coupling of the receiver) eliminated completely the noise of low frequency (compared with $f_0 = 20$ MHz).

The electron concentrations n(H) measured in this manner as functions of the magnetic field are represented by the continuous curves in Fig. 4 for three different samples: InSb ($n(0) + 2.2 \times 10^{15}$ cm^{-3}), InSb ($n(0) = 5.5 \times 10^{16}$ cm^{-3}), and InAs ($n(0) = 1.5 \times 10^{16}$ cm^{-3}). We see that n(H) increases monotonically with increasing magnetic field intensity and at $H = 150$ kOe we have $n(H)/n(0) \sim 1$, i.e., the effect is quite appreciable. The dashed curves in the same figure show the theoretically calculated n(H) dependences and, in accord with the considerations advanced earlier, we assumed for the construction of these plots appropriate values of the parameter A (ensuring the required value of n(0), and also used the values $G = 0.35$ for InSb and $G = 0.20$ for InAs. The theoretical curves are in good agreement with the experimental data.

We have thus shown that the statistical properties of the electrons in impurity semiconductors such as n-InSb lead to an increase in the concentration of the free electrons in a quantizing magnetic field. This effect (and the associated dependence of the Fermi level on the magnetic field) must be taken into account when certain kinetic phenomena in quantizing magnetic fields are considered.

The authors are deeply grateful to S. Yu. Meĭlikhova for the computer calculations.

[1] M. I. Klinger and P. I. Voronyuk, Zh. Eksp. Teor. Fiz. 33, 77 (1957) [Sov. Phys.-JETP 6, 59 (1958)].

[2] A. I. Ansel'm and B. M. Askerov, Fiz. Tverd. Tela 2, 2821 (1960) [Sov. Phys.-Solid State 2, 2512 (1961)].

[3] B. M. Askerov, Kineticheskie éffekty v poluprovodnikakh (Kinetic Effects in Semiconductors), Nauka, 1970.

[4] O. Kane, J. Phys. Chem. Solids, 8, 38 (1959).

[5] Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Solids, 1, 137 (1956).

[6] Yu. M. Burdukov, O. V. Emel'yanenko, N. V. Zotova, E. P. Kesamanly, É. É. Klotyn'sh, T. S. Lagunova, D. N. Nasledov, V. G. Sidorov, G. N. Talalakin, and V. E. Shcherbatov, Izv. AN SSSR, ser. fiz. 28, 951 (1964).

[7] V. F. Bannaya, E. M. Gershenzon, and L. B. Litvak-Gorskaya, Fiz. Tekh. Poluprov. 4, 200 (1970) [Sov. Phys.-Semicond. 4, 159 (1970)].

[8] L. J. Neuringer, IX Mezhdunarodnoĭ konf. po fizike poluprovodnikov (Ninth International Conference on Semiconductor Physics), Vol. 2, p. 757, Nauka, 1969.

[9] W. S. Boyle and A. D. Brailsford, Phys. Rev. 107, 903 (1957).

[10] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964.

[11] E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids, 10, 254 (1959).

[12] E. Braunersreuther, F. Kuhrt, and H. Lippmann, Z. Naturforsch., 15a, 795 (1960).

[13] Kh. I. Amirkhanov, R. I. Bashirov, and Yu. Shch. Zakiev, Dokl. Akad. Nauk SSSR 132, 793 (1960) [Sov. Phys.-Doklady 5, 556 (1960)].

[14] H. Hieronymus and H. Weiss, Solid State Electronics, 5, 71 (1962).

[15] A. C. Beer, J. Appl. Phys., 32, 2107 (1961).

Translated by J. G. Adashko
198