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Proton Electromagnetic Form Factors

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Satisfactory descriptions of the proton electromagnetic form factors are obtained from the available data on elastic ep and μp scattering cross sections in the squared momentum transfer range $0.012 \le q^2 \le 25$ (Gev/c)^2. Each form factor is represented by a sum of poles, with varying pole locations and residues, and also by a product of Γ functions having arguments defined by the variable parameters of the ρ trajectory. It is shown that at large q^2 values the form factors behave as q^{-4}.

1. The present work is a statistical treatment of all available data on ep and μp elastic scattering cross sections, for the purpose of obtaining information about the proton electromagnetic form factors.

It was shown in[1] that the dipole formula describes the behavior of the form factors only in the small q^2 region [$q^2 \le 11$ F^{-2} = 0.43 (GeV/c)^2], where q^2 is the square of the transferred 4-momentum. The expressions used in[1] for the q^2 dependence of the form factors did not permit a statistically satisfactory description of the experimental data in the entire investigated range of q^2 values. We here continue our search for a proton form factor dependence on q^2 that would facilitate a description of all available data on the differential cross sections for elastic ep scattering.

Our method of processing the experimental data has been thoroughly described in[1]. The parameters characterizing the proton form factors were obtained directly from data on the differential cross sections for ep scattering in the entire investigated range of momentum transfers. Norms that allow for the possible systematic errors were introduced as variable parameters.

We considered all the available data on ep scattering in the momentum transfer range $0.012 \le q^2 \le 25$ (GeV/c)^2. Each form factor was represented by a sum of poles and also by Frampton's formula[2] based on the Veneziano model. It was found that all the experimental data can be described with $\chi^2 = 416$ and $\overline{\chi}^2 = 339$ when each proton form factor is represented by a sum of two poles. When Frampton's expressions for the form factors are used, the data are described with $\chi^2 = 414$ and $\overline{\chi}^2 = 341$.

We note that in searching for the best description we did not assume the validity of a scaling law. The bulk of the data pertain to the region of high q^2; therefore the parameters characterizing the electric form factor were determined with less accuracy than those for the magnetic form factor.[1)] Our results indicate that for large q^2 the electric and magnetic form factors behave as q^{-4}. Our analysis included data on muon scattering by protons.[5] By introducing a normalizing factor we were enabled to reconcile the μp scattering cross sections with all available ep scattering data. The accompanying table shows the numbers of all experimental points that were included in the analysis.

2. We shall represent both the magnetic (G_M) and the electric (G_E) form factor of the proton as a sum of contributions from two poles:

$$\frac{1}{\mu_p} G_M(q^2) = \frac{b_1}{1+b_2 q^2} + \frac{1-b_1}{1+b_3 q^2}, \tag{1}$$

$$G_E(q^2) = \frac{a_1}{1+a_2 q^2} + \frac{1-a_1}{1+a_3 q^2}, \tag{2}$$

which can be regarded as a generalization of the dipole formula. In (1), μ_p is the proton magnetic moment (expressed in nuclear magnetons). The parameters a_i and b_i (i = 1, 2, 3) were obtained by minimizing the functional χ^2. For this purpose we used practically all the available data on elastic ep scattering cross sections [358 experimental points in the interval $0.012 \le q^2 \le 25$ (GeV/c)^2]. The totality of ep scattering data is described considerably better by means of (1) and (2) than with the other parametrizations of the proton form factors that were previously considered in[1].

The values of the parameters corresponding to the minimum of χ^2 were

$$\begin{gathered} a_1 = -0.24\pm0.04, \quad b_1 = -0.33\pm0.03, \\ a_2 = (0.37\pm0.05)(\text{GeV/c})^{-2} \quad b_2 = (0.58\pm0.03)(\text{GeV/c})^{-2} \\ a_3 = (2.50\pm0.12)(\text{GeV/c})^{-2} \quad b_3 = (2.42\pm0.07)(\text{GeV/c})^{-2} \end{gathered} \tag{3}$$

Hence we obtain

$$b_2^{-1/2} = (1.31 \pm 0.03)(\text{GeV/c}) \quad b_3^{-1/2} = (0.64 \pm 0.01)(\text{GeV/c}). \tag{4}$$

We note that the first of these values coincides with the mass of the hypothetical ρ' meson.

We shall now discuss the behavior of our form factors for large values of q^2. When the relations hold true, we easily obtain

$$\Delta_M = b_1 b_3 + (1-b_1) b_2 = 0, \quad \Delta_E = a_1 a_3 + (1-a_1)\ a_2 = 0, \tag{5}$$

or

$$G_E(q^2) = (1+a_2 q^2)^{-1}(1+a_3 q^2)^{-1}, \quad G_M(q^2) = \mu_p (1+b_2 q^2)^{-1} \times (1+b_3 q^2)^{-1}.$$

and for large q^2 the form factors $G_{E,M}(q^2)$ behave like q^{-4}. Using the values obtained for the parameters b_i and a_i, we derive

$$\Delta_M = (-0.02\pm0.08)(\text{GeV/c})^{-2}, \quad \Delta_E = (-0.14\pm0.22)(\text{GeV/c})^{-2}. \tag{6}$$

The results of our treatment therefore indicate that for large q^2 the form factors behave like q^{-4}. This result is consistent with the conclusion reached by the authors of

[1)]It has been shown in[3,4,6] that a study of the scattering of polarized electrons by polarized protons would permit improved accuracy in determining the electric form factor.

Laboratory	No. of included points	No. of discarded points	Normalizing factor	Laboratory	No. of included points	No. of discarded points	Normalizing factor
Stanford [8]	77	16	0.951±0.014	Cornell [22]	12	13	0.859±0.017
Stanford [11]	8	0	1.016±0.007	DESY [19, 20]	8	3	0.967±0.023
Cornell [12]	24	4	1.009±0.019	ORSAY [23–25]	10	2	0.992±0.013
Cornell [13]	9	0	0.823±0.029	SLAC [26, 27]	36	7	0.945±0.015
CEA [14–17]	43	3	0.985±0.016	Bonn [7–28]	72	5	0.957±0.014
ORSAY [18]	9	1	0.982±0.010	DESY [29–31]	27	0	0.936±0.015
DESY [19–21]	25	12	0.896±0.019	Brookhaven [5]	63	2	0.919±0.017

the experimental articles[2,7].

The proton form factor expressions (1) and (2) with the parameters (3) are very similar to the dipole formula. If (5) is valid, the ratio

$$y = (1 + a_{\text{dip}}q^2)^2 / (1 + b_2q^2)(1 + b_3q^2)$$

of the function (1) to the dipole formula

$$G_M(q^2) / \mu_p = (1 + a_{\text{dip}}q^2)^{-2} \qquad (a_{\text{dip}} = 1.38 \pm 0.02 \ [1])$$

reaches its minimum, $y(q_m^2) = 0.97$, for

$$q_m^2 = \frac{b_2 + b_3 - 2a_{\text{dip}}}{a_{\text{dip}}(b_2 + b_3) - 2b_2b_3} = 0.16 \pm 0.08,$$

returns to unity for

$$q_1^2 = (b_2 + b_3 - 2a_{\text{dip}}) / (a_{\text{dip}}^2 - b_2b_3) \approx 2q_m^2$$

and increases further to

$$y(\infty) = a_{\text{dip}}^2 / b_2b_3 \approx 1.45.$$

Equations (1) and (2) show that we have not assumed the scaling law

$$G_M = \mu_p G_E. \tag{7}$$

It follows from (3) that the parameters a_1 and b_1, a_2 and b_2 differ by more than twice the error.

If we assume (7), the description of the experimental results does not remain quite so good ($\chi^2 = 429$, $\overline{\chi}^2 = 342$). We then have the parameters

$$\begin{aligned} b_1 &= a_1 = -0.45 \pm 0.03, \\ b_2 &= a_2 = (0.67 \pm 0.02)(\text{GeV/c})^{-2}, \\ b_3 &= a_3 = (2.23 \pm 0.05)(\text{GeV/c})^{-2}. \end{aligned} \tag{8}$$

We also considered a variant, not observing the scaling law, where G_M and G_E are related by

$$G_M = \mu_p G_E / (1 + \alpha q^2), \tag{9}$$

in which G_E is given by (2). For $\chi^2 = 427$, $\overline{\chi}^2 = 341$ we obtained

$$\alpha = (+0.01 \pm 0.01)(\text{GeV/c})^{-2}.$$

We also treated all the available experimental data on ep scattering cross sections, parametrizing the form factors with sums of three and four poles. The parameters of two of the poles were fixed at values corresponding to the masses of the ρ and ρ' mesons. All the remaining parameters were varied.[2)] This procedure yielded no better description; the nominal values determined for some parameters were exceeded by the corresponding errors.

We also treated all the data subject to the assumption

$$G_M/\mu_p = c + \frac{b_1}{1 + b_2q^2} + \frac{1 - b_1 - c}{1 + b_3q^2}. \tag{10}$$

The adequacy of the description was not changed, but $c = (7.7 \pm 1.8) \times 10^4$. Thus the data do not favor the existence of a core in the nucleon.

3. The data on elastic μp scattering obtained by Lederman's group[5] were included in the analysis. As we know, this group obtained differential cross sections for μp scattering that differ somewhat from the corresponding cross sections for ep scattering. These investigators concluded that the difference does not indicate a violation of μe universality and is associated with small systematic errors. They compared the proton form factors obtained from μp scattering data with the form factors obtained in[8] from ep scattering data.

We arrived at the same conclusion from a direct comparison of μp scattering cross sections with all the data on ep scattering cross sections in the entire investigated range of momentum transfers. The normalizing factor for the μp data is 0.919 ± 0.017. In our opinion, this result strengthens the conclusion in[5] regarding the existence of μe universality.

We note that the data on electron and muon scattering by protons are described by (1) and (2) with $\chi^2 = 479$, $\overline{\chi}^2 = 401$, and thus somewhat better than the data on ep scattering alone. In this case the parameters a_i and b_i do not differ from those given in (3).

Our table gives all the normalizing factors for two-pole parametrization. The different parametrizations have the same normalizing factors within error limits.

4. We also treated all ep scattering data statistically, using Frampton's[2] expressions for the proton form factors:

$$\frac{G_{E,M}(t)}{G_{E,M}(0)} = \frac{\Gamma(1 - \alpha(t))\Gamma(r_{E,M} + 1 - \alpha(0))}{\Gamma(r_{E,M} + 1 - \alpha(t))\Gamma(1 - \alpha(0))} \tag{11}$$

Here $t = -q^2$, $G_E(0) = 1$, $G_M(0) = \mu_p$, and

$$\alpha(t) = \alpha(0) + \alpha'(0)t, \tag{12}$$

is the ρ trajectory.

The parameters r_E, r_M and $\alpha(0)$, $\alpha'(0)$ were obtained by minimizing the χ^2 functional. For $\chi^2 = 413$, $\overline{\chi}^2 = 341$ we obtained

$$\alpha(t) = (0.76 \pm 0.02) + (0.59 \pm 0.04)t, \tag{13}$$

$$r_M = 2.34 \pm 0.03, \quad r_E = 2.40 \pm 0.06. \tag{14}$$

These values of $\alpha(0)$ and $\alpha'(0)$ differ from the ρ trajectory parameters that have been determined from high-energy hadronic processes.[9] When the latter values are used [$\alpha(t) = 0.483 + 0.885t$] the ep scattering data are described much less adequately.

2)Data on form factors in the momentum transfer range from 0.08 to 3.9 $(\text{GeV/c})^2$ were processed similarly in[7].

We also note that our value for r_M is close to the result obtained by Frampton,[2] who used only the data for the magnetic form factor.

5. It is well known that Rosenbluth's formula, on the basis of which all the data were analyzed, was derived in a one-photon approximation. This formula is usually tested by a small amount of data obtained for identical values of q^2, but at different scattering angles, and also by (less precise) data on the polarization of recoil photons; or the e^-p and e^+p scattering cross sections are compared. In each instance only a small portion of the total data is used. We were able to test the one-photon formulation for all the data within the framework of Gourdin and Martin's work.[10] These authors showed that after making certain assumptions the interference of one-photon and two-photon diagrams makes the following contribution to the e^-p scattering cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{2\gamma}=\sigma_{N_s}\left\{\frac{\alpha}{\pi}c(q^2)\,\mathrm{tg}\frac{\theta}{2}\left[\mathrm{tg}^2\frac{\theta}{2}+\frac{1}{1+\tau}\right]^{1/2}\right\}. \qquad (15)$$

Here $\alpha = 1/137$, θ is the electron scattering angle (lab system), σ_{N_S} is the Mott scattering cross section, and $\tau = q^2/4M^2$ (M is the nucleon mass). Equation (15) was added to Rosenbluth's equation when χ^2 was minimized.

With regard to $c(q^2)$ we assumed

$$c(q^2)=\beta' \quad \text{or} \quad c(q^2)=\beta'' q^2/4M^2, \qquad (16)$$

where β' and β'' are constants. The description was not improved, and the values of the parameters characterizing the form factors were not changed. We obtained the parameter values

$$\beta'=0.19\pm0.14,\ \beta''=0.10\pm0.06. \qquad (17)$$

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[1] S. I. Bilen'kaya, Yu. M. Kazarinov, and L. I. Lapidus, Zh. Eksp. Teor. Fiz. **60**, 460 (1971) [Sov. Phys. JETP **33**, 247 (1971)].
[2] P. H. Frampton, Phys. Rev. D **1**, 3141 (1970).
[3] N. Dombey, Phys. Lett. B **29**, 588 (1969).
[4] A. I. Akhiezer and M. P. Rekalo Dokl. Akad. Nauk SSSR **180**, 1081 (1968) [Sov. Phys. Dokl. **13**, 572 (1968)].
[5] L. Camilleri, J. H. Christenson, M. Cramer, L. M. Lederman, Y. Nagashima, and T. Yamanouchi, Phys. Rev. Lett. **23**, 153 (1969).
[6] A. I. Akhiezer, L. N. Rozentsveĭg, and I. M. Shmushkevich, Zh. Eksp. Teor. Fiz. **33**, 765 (1957) [Sov. Phys. JETP **6**, 588 (1958)].
[7] Chr. Berger, V. Burkert, G. Knop, B. Langenbeck, and K. Rith, Preprint 1-075, Phys. Institut, Universität Bonn, Juli 1969.
[8] T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. **142**, 922 (1966).
[9] C. Lovelace, Phys. Lett. **28**, 264 (1968)
[10] T. A. Griffy and L. I. Schiff, in High-Energy Physics, edited by E. H. S. Burhop, Academic Press, New York, 1967, Vol. 1, p. 341.
[11] D. J. Drickey and L. N. Hand, Phys. Rev. Lett. **9**, 521 (1962).
[12] K. Berkelman, M. Feldman, R. M. Littauer, G. Rouse, and R. R. Wilson, Phys. Rev. **130**, 2061 (1963).
[13] D. N. Olson, H. F. Schopper, and R. R. Wilson, Phys. Rev. Lett. **6**, 286 (1961).
[14] K. W. Chen, J. R. Dunning, Jr., A. A. Cone, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. **141**, 1267 (1966).
[15] J. R. Dunning, Jr., K. W. Chen, N. F. Ramsey, J. R. Rees, W. Shlaer, J. K. Walker, and R. Wilson, Phys. Rev. Lett. **10**, 500 (1963).
[16] K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. F. Frank, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Lett. **11**, 561 (1963).
[17] M. Goitein, R. J. Budnitz, L. Carroll, J. R. Chen, J. R. Dunning, Jr., K. Hanson, D. Imrie, C. Mistretta, and R. Wilson, Phys. Rev. D **1**, 2449 (1970).
[18] D. Frèrejacque, D. Benaksas, and D. Drickey, Phys. Rev. **141**, 1308 (1966).
[19] H. J. Behrend, F. W. Brasse, J. Engler, H. Hultschig, S. Galster, G. Hartwig, H. F. Schopper, and E. Ganssauge, Nuovo Cimento A **48**, 140 (1967).
[20] W. Albrecht, H.-J. Behrend, F. W. Brasse, W. Flauger, H. Hultschig, and K. G. Steffen, Phys. Rev. Lett. **17**, 1192 (1966).
[21] W. Albrecht, H.-J. Behrend, H. Dorner, W. Flauger, and H. Hultschig, Phys. Rev. Lett. **18**, 1014 (1967).
[22] R. M. Littauer, H. F. Schopper, and R. R. Wilson, Phys. Rev. Lett. **7**, 141 (1961).
[23] P. Lehmann, R. Taylor, and R. Wilson, Phys. Rev. **126**, 1183 (1962).
[24] B. Dudelzak, A. Isakov, P. Lehmann, and R. Tchapoutian, in Proc. XII Int. Conf. on High Energy Physics, Dubna, 1964 (Atomizdat, Moscow, 1966), Vol. I, p. 916.
[25] D. J. Drickey, B. Grossetete, and P. Lehmann, in Proc. Sienna Int. Conf. on Elementary Particle Physics, 1963, p. 493.
[26] J. Litt, G. Buschhorn, D. H. Coward, H. De Staebler, L. W. Mo, R. E. Taylor, B. C. Barish, S. C. Loken, J. Pine, J. I. Friedman, G. C. Hartmann, and H. W. Kendall, Phys. Lett. B **31**, 40 (1970).
[27] D. H. Coward, H. De Staebler, R. A. Early, J. Litt, A. Minten, L. W. Mo, W. K. H. Panofsky et al., Phys. Rev. Lett. **20**, 292 (1968).
[28] Chr. Berger, E. Gersing, G. Knop, B. Langenbeck, K. Rith, and F. Schumacher, Phys. Lett. B **28**, 276 (1968).
[29] W. Bartel, F.-W. Büszer, W.-R. Dix, R. Felst, D. Harms, H. Krehbiel, P. E. Kuhlmann, J. McElroy, and G. Weber, Phys. Lett. B **33**, 245 (1970).
[30] W. Bartel, B. Dudelzak, M. Krehbiel, J. M. McElroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmidt, and G. Weber, Phys. Rev. Lett. **17**, 608 (1966).
[31] W. Bartel, B. Dudelzak, M. Krehbiel, J. M. McElroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmidt, and G. Weber, Phys. Lett. B **25**, 236 (1967).

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