Radiation Statistics of a Q-Switched Neodymium Laser

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Results are presented of an experimental investigation of a Q-switched neodymium glass laser with a rotating prism and passive shutter. The results are employed for elucidating the correlation between the spectral and time characteristics of the radiation. It is shown that for Q-switching by means of the rotating prism at threshold pumping the radiation is a noise process with Gaussian statistics. For pumping energies exceeding the threshold value when the radiation power is great, the radiation statistics differ from Gaussian as a result of nonlinear effects in the laser. In the case of Q-switching with a finite relaxation time of a passive shutter and under constant pumping conditions, a large scatter of spectral widths and duration of the ultrashort pulses are observed as well as the absence of strict correlation between them.

In connection with the possibility of obtaining ultrashort pulses (USP) of light at high powers from solid-state lasers with passive shutters, much attention is being paid presently to the measurement of the duration of the USP and to comparison of the measurement results with the limiting length $\Delta t \sim (c\Delta\nu)^{-1}$, which is determined by the width of the generation spectrum $\Delta\nu$ in cm^{-1}.[1-3] However, the width and the structure of the radiation spectrum are determined in general both by the amplitude and by the phase modulation of the field, and in the presence of phase modulation it is impossible to determine uniquely, from the width of the spectrum, the duration of the USP and hence also its power. It is therefore of interest to investigate the characteristics of the amplitude and phase modulation in the radiation of real lasers.

For this purpose, we undertook experimental investigations of the radiation characteristics of neodymium-glass lasers, namely a laser Q-switched with a rotating prism and a laser with a passive shutter. As shown by us in [4], for the presently widely used domestic polymethine dyes known under Nos. 3955 and 1000, the relaxation time τ_{rel} of the working-level population is longer than the USP duration Δt, and the emission of lasers with such shutters over the axial period T (T is the time necessary for the photon to cover the closed path between the cavity mirrors) has the form of individual segments of a noise pattern with duration $\sim\tau_{rel}$ much shorter than the axial period, unlike a laser with a rotating prism, where the duration of the noise pattern is equal to T. Therefore the radiation parameters observed experimentally in the investigated lasers and determined both by the amplitude and by the phase modulation are characteristics of noise processes of duration 10^{-11}–10^{-10} sec for lasers with passive shutters and 10^{-8} sec for lasers with rotating prisms. To ascertain the role of phase and amplitude modulation in such processes, it is of interest to investigate the statistical properties of solid-state laser emission and compare them with the characteristics of well known random Gaussian processes.

We note also that an investigation of the statistical properties of laser emission is of interest also for the study of the physical processes in lasers, and particularly for the study of nonlinear effects in the active medium and in the passive shutter.

It is known that the radiation field during the axial period can be written in the form

$$E(t)=\sum_n a_n\cos(2\pi c\nu_n t+\varphi_n),$$

where a_n are the amplitudes, ν_n the frequencies, and φ_n the phases of the laser modes. The emission characteristics depend on the ratio of the phases of the individual modes. Thus, for example, in the case of interference of a large number of modes with random phases φ_n, the distribution is uniform in the interval $(0, 2\pi)$, and the radiation over the axial period of the generation is described by a random Gaussian process. In the general case, however, the statistical independence of φ_n in solid-state lasers is not obvious and calls for additional investigations.

A known property of a Gaussian random process is the connection between the intensity correlation function $\langle I(t)I(t+\tau)\rangle$ (where the angle brackets for the ergodic process denote averaging over a time interval much larger than the correlation interval) and the square of the modulus of the normalized correlation function of the field $\gamma(\tau)$:[5]

$$\langle I(t)I(t+\tau)\rangle=\langle I\rangle^2[1+|\gamma(\tau)|^2], \tag{1}$$

where $\gamma(\tau)$ is connected with the spectral density $S(\nu)$ by the Wiener–Khinchin theorem

$$\gamma(\tau)=\int S(\nu)\exp(2\pi ic\nu\tau)\,d\nu \Big/ \int S(\nu)\,d\nu.$$

It should be noted here that relation (1) is only a necessary condition for the process to be Gaussian, i.e., failure to satisfy it shows that the process does not satisfy the Gaussian statistics, while the process for which (1) is satisfied requires, strictly speaking, additional proof of Gaussian behavior.

The solid-state laser emission resulting from the spontaneous noise in the linear section of lasing development is a Gaussian random process which can subsequently be distorted by different nonlinear effects. Therefore, comparing the experimentally registered spectral density $S(\nu)$ and the intensity correlation function $\langle I(t)I(t+\tau)\rangle$, we can conclude whether the registered radiation is a random Gaussian process.

The intensity correlation function can be measured with the aid of a two-photon luminescence (TPL) procedure.[6] As is well known, the brightness of the TPL track is determined in terms of the intensity correlation function as follows:

$$f(\tau)=1+2\int I(t)I(t+\tau)\,dt \Big/ \int I^2(t)\,dt,$$

where the integration is over the entire time of observation. On the other hand, the spectral density $S(\nu)$ can be determined with the aid of ordinary spectral instruments. We note that $f(\tau)$ is determined only by the amplitude of the modulation in the emission, unlike the spectral density, which is determined both by the amplitude and by the phase modulation.

Figure 1 shows a diagram of the experimental setup for the investigation of the emission characteristics of a laser Q-switched with a rotating prism (Fig. 1a) and of a laser with a passive shutter (Fig. 1b). To eliminate "parasitic" mode selection, the resonant mirrors were coated on wedge-shaped substrates, and the prism rotation axis was tilted somewhat away from the direction perpendicular to the resonator axis, while the end faces of the neodymium rod were cut at the Brewster angle. The prism was rotated at a frequency of 500 Hz. For the laser with the passive shutter we chose a ring resonator scheme, which eliminated the structure, connected with the cell position, from the spectral and from the temporal pictures of the radiation. The active rod (AR) of neodymium glass was 15 mm in diameter and 260 mm long. The dye cell serving as the passive shutter (PS) was 1 mm thick and was also mounted at the Brewster angle to the resonator axis. The cell transmission at the wavelength $\lambda = 1.06\ \mu$ was $T_b = 50\%$. Polymethine dye No. 3955 dissolved in nitrobenzene was used. Diaphragms D_1 and D_2, of 2 mm diameter each, were introduced into the resonator to suppress the radiation of the nonaxial modes. The duration of the axial radiation period in both resonator schemes (Figs. 1a and 1b) was the same at T = 6.7 nsec. Diaphragm D was used to register the radiation of the laser axial modes outside the resonator.

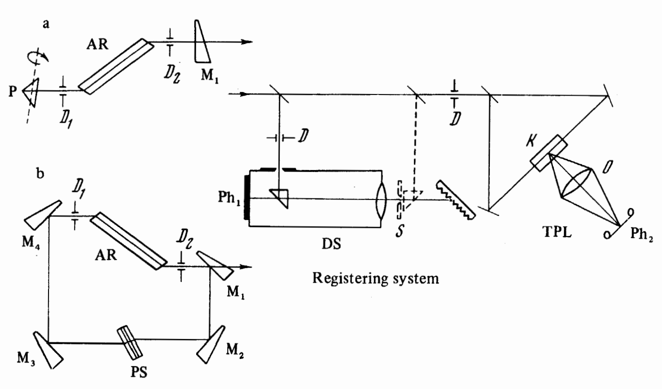

FIG. 1. Diagram of experimental setup. a—laser with rotating prism, b—laser with passive shutter. AR—active rod (neodymium glass 15 mm in diameter by 260 mm long), PS—cell with passive shutter, P—rotating prism, D_1 and D_2—diaphragms 2 mm in diameter, reflection coefficients of mirrors M_1–M_4: $R_1 = 82\%$, $R_2 = R_3 = R_4 = 100\%$, DS—diffraction spectrograph, S—slit producing density markers, TPL—device for registration of two-photon luminescence tracks, K—cell with solution of rhodamine 6Zh, O—objective. Diaphragm D was used to separate axial-mode emission.

The laser emission spectra were investigated with the aid of a diffraction spectrograph DS with resolution 0.15 cm^{-1}. To determine the distribution of the spectral density of the radiation over the wavelength, we converted from photographic density to intensity by the method of photographic photometry. To obtain a density scale on the film, we photographed the diffraction pattern from the slit S together with the spectrum.[7] To eliminate possible effects connected with violation of the reciprocity law, the diffraction picture was obtained with the slit illuminated by laser radiation having the same duration and the same wavelength as that used to register the spectrum. The intensity distribution in the plane Ph_1 as a function of the angle is then given by the well known function $(\sin u/u)^2$. The distribution of the spectral density of the radiation over the wavelength was determined from the microphotographs of the diffraction pattern and of the spectrum.

Measurement of the correlation function of the radiation intensity $f(\tau)$ by the TPL procedure was carried out by us in a triangular scheme. We used a solution of rhodamine 6Zh dye in ethyl alcohol. The following method was used to obtain density markers in this case. One of the channels in the triangular TPL scheme was covered, and the rhodamine 6Zh concentration was chosen such that the track brightness remained practically constant along the cell. Further, a screen partly covering the input aperture was placed between the cell K and the objective O, on which a square diaphragm was mounted. It is easy to show that in this case the brightness of the track image in the plane Ph_2 should have a linearly decreasing section, photography of which gave the necessary density markers. The obtained photographs of the TPL tracks and of the pulses encountered on the cell were converted from photographic density to intensity with the aid of the density markers for direct determination of the function $f(\tau)$ of interest to us.

The track diameter, the relative aperture, and the magnification of the TPL optical system were chosen such that the maximum geometrical scattering spot on the plane Ph_2, due to the finite dimension of the track along the optical axis of the objective O, was much smaller than the longitudinal dimensions of the image of the central spot of the TPL track, which characterizes the duration Δt of the USP. We estimated the resolution in the registration of the USP duration to be not worse than 10^{-13} sec. The photometry conditions were chosen such that the systematic error was smaller than the error in the photography of the TPL track.

Thus, the apparatus described above has made it possible to obtain in a single flash both the spectral density $S(\nu)$ and the intensity correlation function $\langle I(t)I(t+\tau)\rangle$ of the laser emission.

Figures 2a and 2b show, by way of an example, typical microphotographs of the simultaneously obtained TPL track and of the emission spectrum of a laser with rotating prism. The reduction of the obtained data has shown that the corresponding intensity correlation func-

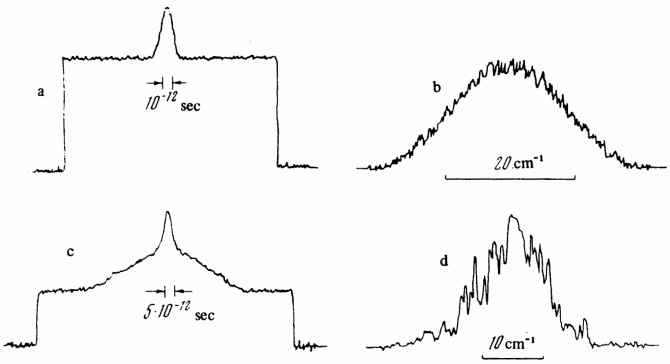

FIG. 2. Microphotographs of simultaneously photographed TPL tracks and emission spectra of a laser with rotating prism (a, b) and a laser with passive shutter (c, d).

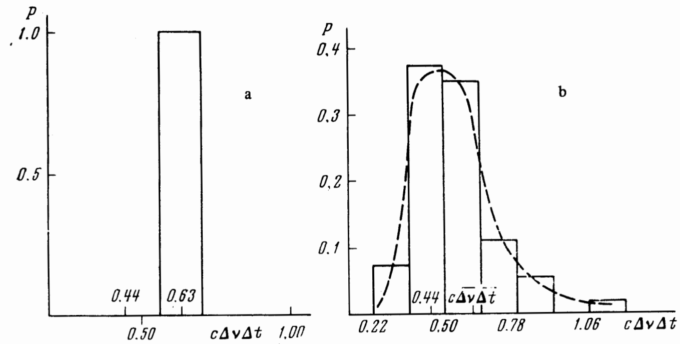

FIG. 3. Experimentally obtained distribution of the probability density of the quantity $c\Delta\nu\Delta t$ for a laser with rotating prism at 15% excess of pump energy over threshold (a) and of a laser with passive shutter (b). The width of the column is equal to double the measurement error.

tion $\langle I(t) I(t+\tau)\rangle$ and the spectral density $S(\nu)$ are well described by curves having a Gaussian shape.

As follows from the Wiener–Khinchin theorem, for a random process with a spectral-density width $\Delta\nu$, assuming that $S(\nu)$ has a Gaussian shape, the width $\Delta\tau$ of the square $|\gamma(\tau)|^2$ of the modulus of the field correlation function should amount to

$$\Delta\tau = 2\sqrt{2}\ln 2 / \pi c\Delta\nu, \qquad (2)$$

if the width of the curves is measured at the level of half the maximum value.

At the same time, the correlation function of the intensity characterizes the average duration Δt of the intensity pulses in the radiation, $\Delta t = \Delta x n\sqrt{2}/c$, where Δx is the dimension of the central spot on the TPL track and n is the refractive index. We then find from (2) that for a Gaussian random process $c\Delta\nu\Delta t = 0.44$.

For a laser with a rotating prism, the duration of the random process is large in comparison with the correlation interval $T \gg \Delta\tau$, and therefore the registered width of the spectrum and the width of the intensity correlation function are average characteristics of the process.

The first experiments aimed at investigating the statistical properties of lasers with rotating prisms were performed under conditions close to threshold. It turned out then that the experimentally measured quantity $c\Delta\nu\Delta t = 0.44 \pm 0.05$, i.e., it is close to the value of $c\Delta\nu\Delta t$ for the case of Gaussian statistics. The data were reduced for 20 flashes, and under fixed laser-operation conditions the values of $c\Delta\nu\Delta t$ for each individual flash were within the limits of the indicated measurement error, $\approx 12\%$.

Thus, the experimentally obtained data indicate that the radiation of a laser with rotating prism at threshold pumps obeys Gaussian statistics. This means that the field envelope of such a process has a Rayleigh distribution, and the phase of the process is uniformly distributed in the interval $(0, 2\pi)$.

To ascertain the influence exerted by possible nonlinear effects in the laser, which depend on the radiation power density, on the radiation statistics, we performed analogous measurements of $\Delta\nu$ and Δt at pump energies exceeding the threshold by 15 and 25%. It was found that when the pump energies increased in comparison with the threshold energy by 15%, the value of $c\Delta\nu\Delta t$ increases to $c\Delta\nu\Delta t = 0.63 \pm 0.08$, and when the pump energy is increased by 25% we get $c\Delta\nu\Delta t = 0.73 \pm 0.09$. For a constant pump level above threshold, the experimental values of $c\Delta\nu\Delta t$, as before, lie within the limits of the measurement errors (Fig. 3a).

Thus, even a slight excess of the pump energy over threshold, i.e., a relatively small increase of the radiation power (which nevertheless leads to a noticeable increase of the TPL track brightness), leads to a significant increase of $c\Delta\nu\Delta t$, which can be interpreted as a distortion of the statistical characteristics of the laser emission.

We note that the conclusions drawn from a comparison of the experimentally obtained values of $c\Delta\nu\Delta t$ with the value $c\Delta\nu\Delta t = 0.44$ indicating that the statistical properties of the radiation are altered are valid only if the spectral density $S(\nu)$ is described by a Gaussian curve, and therefore we determined not only the spectral width but also its shape. It turned out that in all the investigated pump regimes, from threshold values to 25% excess above threshold, the shape of the emission density was Gaussian accurate to within the photometry error, which in our case did not exceed 4%. Therefore the increase of the value $c\Delta\nu\Delta t$ at pumps exceeding the threshold value cannot be attributed to a change in the shape of the spectral density.

One of the possible nonlinear effects that depend on the radiation power density and can lead to a distortion of the Gaussian statistics of the radiation may be frequency self-moduiation.[8] As shown by estimates, at an active rod length l and a giant-pulse duration ΔT, the power density W necessary for this effect to become manifest is given by the relation

$$\frac{\Delta T}{T} l = \frac{\lambda}{2\pi}[n_2|E|^2]^{-1} = \frac{\lambda}{2\pi}[h_2 W]^{-1},$$

where $n_2|E|^2$ is the nonlinear increment to the refractive index. The quantity h_2 for glass is of the order of 10^{-9} cm^2/MW. Therefore the minimum power density W_{min} necessary for this effect to become manifest at $l = 26$ cm, $\Delta T = 30$ nsec, and $T = 6.7$ nsec is $W_{min} \approx 135$ MW/cm^2; at the same time, the average power density in our generator, at 15% above pump threshold, was $W = 200\ MW/cm^2$, which is close to W_{min}.

It can thus be assumed that for a laser with rotating prism the statistical characteristics of the radiation depend on the power density, namely, when the power is increased the random process describing the emission ceases to be Gaussian.

We investigated the statistical properties of the radiation also for a laser with passive shutter, in which the relaxation time was $\tau_{rel} > \Delta t$. The noise process describing the radiation has in this case a characteristic duration $(20-60) \times 10^{-12}$ sec.[4]

Another feature of a laser with passive shutter, unlike a laser with rotating prism, is the presence of a section in which the shutter becomes bleached, and in which a nonlinear interaction of the initially Gaussian radiation takes place with the medium of the shutter at relatively low power densities, as a result of which the statistical properties of the radiation may change. During the succeeding section of the active-medium gain saturation, different nonlinear effects that depend on the power density can also lead to an appreciable change in the radiation statistics by introducing, for example, additional phase modulation.

It is quite difficult to estimate the radiation power of a laser with a passive shutter in the general case, for this requires additional information concerning the number and durations of the USP groups separated by the passive shutter. We can, however, estimate the maximum power density developed in the generator, by assuming that one group of USP is separated with duration $\sim 40 \times 10^{-12}$ sec.[4] Such an estimate gives a maximum power $W_{max} = 3\ GW/cm^2$, at which the appearance of nonlinear effects such as frequency self-modulation is perfectly feasible. Consequently, the radiation will consist of groups of USP which are segments of a noise process that differs from Gaussian. When such radiation is registered, it is necessary to expect a scatter of the experimental data, caused by the short duration of the separated radiation segments, and the deviation from the mean should be determined by the statistics of the final noise process.

To clarify this question, we used the described procedure to measure the values of $\Delta\nu$ and Δt for a large number of flashes. Typical microphotographs of the simultaneously obtained TPL track and of the emission spectrum of the laser with passive shutter are shown in Figs. 2c and 2d. The results of the statistical reduction of the data on these flashes indicate that, unlike a laser with rotating prism, a feature of a laser with a passive shutter is considerable scatter in the values of $\Delta\nu$ and Δt at identical experimental conditions. Thus, for example, the spectrum width $\Delta\nu$ change from flash to flash in a range from 2.5 to 13.2 cm^{-1}, and the USP duration Δt ranges from 1.2×10^{-12} to 6.2×10^{-12} sec (Figs. 4a and 4b). In addition, from flash to flash we observed a change in the form of the envelope of the spectral density of the radiation, and also a deviation of the value of $c\Delta\nu\Delta t$ for each individual flash from $c\overline{\Delta\nu}\,\overline{\Delta t}$, where $\overline{\Delta\nu}$ and $\overline{\Delta t}$ are the values obtained by averaging over 100 flashes (in this case $c\overline{\Delta\nu}\,\overline{\Delta t} = 0.61 \pm 0.09$). The number of observations of $c\Delta\nu\,\Delta t$, expressed in relative units, is shown in Fig. 3b. As follows from the presented data, the value of $c\Delta\nu\Delta t$ changes from flash to flash in a range from 0.3 to 1.1.

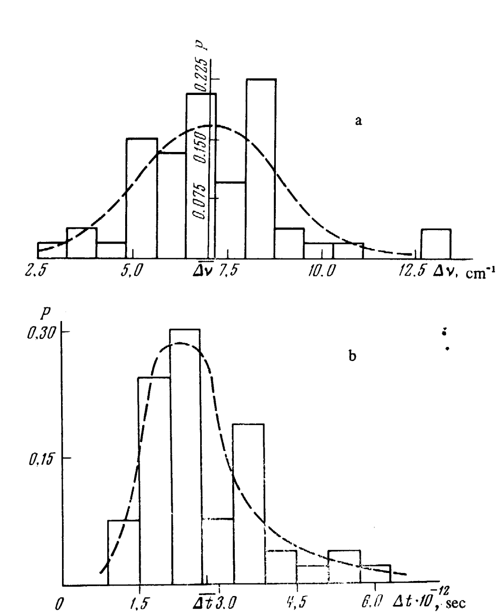

FIG. 4. Experimentally obtained probability distribution density of the width of the emission spectrum $\Delta\nu$ (a) and of the duration Δt of the USP (b) of a laser with passive shutter. The width of each column is equal to double the measurement error.

As already noted, the observed scatter of the experimental values of $\Delta\nu$, Δt, and $c\Delta\nu\Delta t$ for each individual flash are characteristic features of a laser with passive shutter having a finite relaxation time. Inasmuch as $\tau_{rel} > \Delta t$ for the presently used dyes, this scatter should always appear, and there should be no rigorous correlation between the width of the spectrum and the USP duration, as was indeed confirmed by the experiments. The results of [2], in which it is stated that a strict correlation exists between the quantities $\Delta\nu$ and Δt for each individual flash, therefore seem unexpected. Unfortunately, the conditions under which these quantities were measured are not indicated in the cited paper, but it can be concluded from the microphotograph of the TPL given in the paper that the radiation constituted groups of USP, and consequently $\tau_{rel} > \Delta t$, in which case there should be no rigorous correlation between $\Delta\nu$ and Δt.

The results obtained by us indicate that the statistical properties of the laser radiation depend on nonlinear effects that come into play during different stages of generation development. A particularly strong distortion of the initial Gaussian process occurs in a high-power laser. For a laser with a passive shutter, unlike a laser with rotating prism or a laser operating in the free-running mode, the width of the spectral density and the width of the radiation intensity correlation function have a scatter at unchanged experimental conditions, and this scatter reveals the statistical nature of the radiation.

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