SOVIET PHYSICS JETP VOLUME 34, NUMBER 6 JUNE, 1972

Contribution to the Theory of Electron Paramagnetic Resonance Width in Metals

L. A. Zaĭtseva

Institute of Chemical Physics, USSR Academy of Sciences

Submitted May 26, 1971

Zh. Eksp. Teor. Fiz. **61**, 2475–2486 (December, 1972)

The temperature dependence of the electron paramagnetic resonance width in metals is calculated. It is shown that the spin-lattice relaxation time depends on the electron level width due to nonrelativistic interaction of the electrons with phonons and nonmagnetic impurities, and is related to the mean free path.

It is known that at $T \neq 0$ the EPR relaxation time in metals is connected with the spin-orbit interaction of the electrons with the phonons.[1] This interaction causes, mainly, also the damping of spin waves in the nonferromagnetic metals.[2] A qualitative agreement with experiments on the temperature dependence of the EPR width $1/\tau_2 \sim T$ were obtained by Andreev and Gerasimenko.[3] Their calculation, however, was carried out only with logarithmic accuracy. In the formulas obtained by Andreev and Gerasimenko there appear divergences of the same type as the infrared divergences in quantum electrodynamics. According to their assumption, these divergences are eliminated by introducing the quantity $\omega_S = 2\mu_B H$ as the lower limit for integration with respect to energy (μ_B is the Bohr magneton and H is the constant magnetic field).

It will be shown in this paper that besides the parameter ω_S, the lower limit of integration can also be the finite nonrelativistic width of the electron level $1/2\tau_1$. It is necessary here to satisfy the condition $\omega_S \leq 1/2\tau_1$. In addition it is found in this paper that there exists a temperature region where the EPR width becomes proportional to the mean free path and does not contain a logarithmic factor. In this case the temperatures are lower than the Debye temperature T_D and can be such as to satisfy the condition $1/2\tau_1 > 2Tv/u$ (v is the velocity of the electrons in the Fermi surface and u is the velocity of sound). The latter inequality is satisfied only in the case when the mean free path of the electrons $l = \tau_1 v$ is determined by the collision of the electrons with the lattice defect. This leads to a temperature dependence $1/\tau_2 \sim T^2$. Jafet,[4] without taking impurities into account, obtained for the same temperatures $1/\tau_2 \sim T^5$. Such a dependence of the EPR width on the temperature arises only when the magnetic fields H and $\mathbf{h}(\mathbf{r}, t)$ are sufficiently small ($\omega_S \ll 1/2\tau_2$). It is precisely for this case that Jafet obtained his spin-lattice interaction Hamiltonian by using the general properties of Hamiltonians with respect to time reversal and coordinate reflex.

Within the limits of zero temperatures, the EPR width is determined by the spin-orbit interaction of the lattice defect if we confine ourselves to consideration of nonmagnetic impurities. The case when the static scattering amplitude is independent of the momentum (which corresponds to taking the short-range part of the electron-defect interaction into account) was considered in [5]. In the present paper we consider a model in which the interaction with defects has a long-range character. Namely, if expansion or contraction is produced at a certain point by introducing an extraneous atom, then in our case the deformation of the main lattice will decrease with distance like $1/r^3$.[6]

2. INTERACTION HAMILTONIAN

The Hamiltonian of the electron in the lattice is chosen to be in the form

$$\hat{\mathscr{H}}(\mathbf{r}, t) = \frac{1}{2m}\hat{\mathbf{p}}^2 + \mu_B\sigma(\mathbf{h}(\mathbf{r}, t) + \mathbf{H}) + \sum_j V(\mathbf{r} - \mathbf{R}_j + \mathbf{U}) + \sum_j \frac{\mu_B\sigma}{2mc}[\nabla V(\mathbf{r} - \mathbf{R}_j + \mathbf{U})\hat{\mathbf{p}}], \tag{1}$$

where V is the potential of the electric field of the lattice and $\mathbf{h}(\mathbf{r}, t) = h_k \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$ is the alternating field. The displacement $\mathbf{U}(\mathbf{r}, t)$ for the phonons is expressed by the well known formula[7]

$$\mathbf{U}(\mathbf{r}, t) = \sum_{\mathbf{q},\lambda} \sqrt{\frac{1}{2\rho\omega_\lambda(\mathbf{q})}}\, e_{\lambda\mathbf{q}}[b_{\lambda\mathbf{q}}e^{i\mathbf{q}\mathbf{r} - i\omega_\lambda(\mathbf{q})t} + b^+_{\lambda\mathbf{q}}e^{-i\mathbf{q}\mathbf{r} + i\omega_\lambda(\mathbf{q})t}], \tag{2}$$

where λ is the polarization index and $\omega_\lambda(\mathbf{q}) = u_\lambda|\mathbf{q}|$ is the phonon energy. The velocity of sound u_λ will henceforth be assumed, for simplicity, to be the same for all polarizations. The displacement amplitude for the impurities is chosen, in accordance with [6], in the form

$$\mathbf{U}(\mathbf{r}) = \frac{V_0}{4\pi}\frac{\mathbf{r}}{r^3}, \tag{3}$$

where V_0 is a constant with the dimension of volume and characterizes the defect.

We shall assume the main Hamiltonian $\mathscr{H}_0(\mathbf{r}, t)$ to be teh zeroth term of the expansion of the Hamiltonian (1) in the displacement $\mathbf{U}$ and in the field $\mathbf{h}$. The part of Hamiltonian (1) which is linear in the displacement $\mathbf{U}$ is divided into two interaction Hamiltonians, nonrelativistic $\hat{\mathscr{H}}^{(1)}_{\text{in}}(\mathbf{r}, t)$ and relativistic $\hat{\mathscr{H}}^{(2)}_{\text{in}}(\mathbf{r}, t)$.

The problem of the linear response to an alternating magnetic field, as seen from what follows, is best considered by methods wherein the electron field is quantized. We neglect anisotropy effects. Then the most general expression for the interactions between the electrons and the phonons (or impurities), are first order in the displacement $\mathbf{U}(\mathbf{r}, t)$, can be written in the form

$$\hat{\mathscr{H}}^{(1)}_{in}(t) = \lambda_1 \int \psi^+(\mathbf{r}, t)\,\nabla\mathbf{U}(\mathbf{r}, t)\,\psi(\mathbf{r}, t)\,d\mathbf{r}, \tag{4}$$

$$\hat{\mathscr{H}}_{in}^{(2)}(t) = \lambda_2 \lim_{\mathbf{r}'\to\mathbf{r}} \int [\mathbf{U}(\mathbf{r},t)\hat{\mathbf{p}}]\boldsymbol{\sigma}_{\rho\sigma}\cdot$$
$$\times (\psi_\rho{}^+(\mathbf{r}', t)\psi_\sigma(\mathbf{r}, t) - \psi_\rho{}^+(\mathbf{r}, t)\psi_\sigma(\mathbf{r}', t))\,d\mathbf{r}, \qquad (5)$$

where $\psi_\sigma(\mathbf{r}, t)$ and $\psi_\rho^+(\mathbf{r}, t)$ are the second-quantization operators expanded in the eigenfunctions of the Hamiltonian of the free electrons in a constant magnetic field **H**. The redefinition of the Hamiltonians and of the ground-state functions on going to the second-quantization field of the electrons leads to the appearance of the constants λ_1 and λ_2. Using the paper of Andreev and Gerasimenko,[3] we can show that the constant λ_1 of the nonrelativistic electron-phonon (impurity) interaction is of the order of the average electron energy in the crystal, and the constant of the spin-lattice interaction is $\lambda_2 \sim \lambda_1 \Delta g$, where Δg is the deviation of the g-factor from 2.

Andreev and Gerasimenko[3] calculated only the matrix elements of the nonrelativistic Hamiltonian $\mathscr{H}_{in}^{(1)}(\mathbf{r}, t)$. This suffices for an estimate of the order of magnitude of λ_1 and λ_2, since the exact values of these constants will be assumed known from experiment.

3. KINETIC EQUATION

The nonequilibrium magnetization density in the approximation linear in the alternating field h(r, t) is given by the well known formula[8]

$$\delta\mathbf{m}(x) = -i \int dx_1 [\mathbf{m}(x), \mathbf{m}(x_1)\mathbf{h}(x_1)]_- \qquad (6)$$

where x is a four-vector with components r and t, and

$$\mathbf{m}(x) = \mu_B \lim_{x'\to x} \langle \psi_\alpha{}'^+(x)\,\sigma_{\alpha\beta}\psi_\beta{}'(x')\rangle. \qquad (7)$$

Here ψ' and ψ'^+ are operators expanded in the eigenfunctions of the Hamiltonian operator (1) after subtracting the term $\mu_B\sigma h(x)$. The angle brackets in (7) denote also averaging over the Gibbs distribution. The commutator in (6), as is well known, is the retarded two-particle Green's function

$$G^R_{\beta\delta,\alpha\gamma}(x, x_1, x', x_1') = \frac{\langle[\psi_\alpha{}^+(x')\psi_\beta(x), \psi_\gamma{}^+(x_1')\psi_\delta(x_1)S(\infty)]_-\rangle}{\langle S(\infty)\rangle}$$

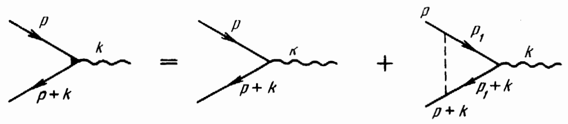
where

$$S(\infty) = \exp\left[-i\int_{-\infty}^{\infty}(\mathscr{H}_{in}^{(1)}(t) + \mathscr{H}_{in}^{(2)}(t))\,dt\right]$$

and ψ and ψ^+ are now the operators of the free particles in the constant magnetic field H.

To be able to use the ordinary diagram technique, we write down the function $G^R_{\beta\delta,\alpha\gamma}$ in the momentum representation. We used furthermore the fact that in the upper complex Ω half-plane (see [7]) the retarded Green's function coincides with the temperature two-particle Green's function with discrete frequencies. The amplitude of the non-equilibrium representation then takes the form

$$\delta\mathbf{m}(\mathbf{k}, i\Omega_m) = \mu_B^2 \mathrm{Sp}\,\boldsymbol{\sigma}\frac{T}{(2\pi)^3}\sum_{\omega_n}\int d\mathbf{p}G(\mathbf{p}+\mathbf{k}, i\omega_n + i\Omega_m)$$
$$\times\Gamma(\mathbf{p}, \mathbf{k}, \mathbf{p}+\mathbf{k}, i\omega_n, i\Omega_m, i\omega_n + i\Omega_m)G(\mathbf{p}, i\omega_n), \qquad (8)$$



In this formula the two-particle electron Green's function has been redefined via the matrix vertex function $\Gamma_{\alpha\beta} = \boldsymbol{\sigma}_{\alpha\beta}\mathbf{h}\Gamma$ and the single-particle electron Green's functions:

$$G(\mathbf{p}, i\omega_n) = [i\omega_n - \varepsilon(\mathbf{p}) - \mu_B\sigma\mathbf{H} - \Sigma(\mathbf{p}, i\omega_n)]^{-1}. \qquad (9)$$

Here $\Sigma(\mathbf{p}, i\omega_n)$ is represented in the form of a sum of self-energy parts. These self-energy parts are due to the nonrelativistic (4) and relativistic (5) interactions of electrons with the phonons and impurities.

We can set up an equation in diagram form for the integrand of (8). The wavy lines in the diagrams represent the alternating magnetic field, the continuous lines correspond to the complete electron Green's functions (9), and a dashed line denotes a phonon or impurity Green's function

$$D_\Phi(\mathbf{p}-\mathbf{p}_1, \omega_n - \omega_{n_1}) = \frac{1}{\rho}[(i\omega_n - i\omega_{n_1})^2 - \omega^2(\mathbf{p}-\mathbf{p}_1)]^{-1}, \qquad (10)$$

$$D_{\text{пр}}(\mathbf{p}-\mathbf{p}_1, \omega_n - \omega_{n_1}) = 2\pi C|\mathbf{U}(\mathbf{p}-\mathbf{p}_1)|^2(\omega_n - \omega_{n_1}), \qquad (11)$$

where C is the impurity concentration (see [7]) and $\mathbf{U}(\mathbf{p}-\mathbf{p}_1) = -iV_0(\mathbf{p}-\mathbf{p}_1)/(\mathbf{p}-\mathbf{p}_1)^2$ is the Fourier component of the amplitude of the static displacement.

The vertex with the spin-lattice interaction is set in correspondence with the expression

$$\lambda_2 e_{iml}\sigma_{\alpha\beta}{}^i(\mathbf{p}_1' + \mathbf{p}')_l, \qquad (12)$$

where e_{iml} is a unit antisymmetrical tensor and contains the sum of the corresponding electron momenta $\mathbf{p}'$ and $\mathbf{p}_1'$ before and after the interaction of the electron with the lattice, respectively. The nonrelativistic vertex corresponds to the factor

$$\lambda_1\delta_{\alpha\beta}(\mathbf{p}' - \mathbf{p}_1') \qquad (13)$$

which contains the difference between the corresponding electron momenta before and after the scattering. The electromagnetic vertex of first order (the second diagram) corresponds to the product $\sigma_{\alpha\beta}\mathbf{h}_\mathbf{k}$. The first and third diagrams contain the exact vertex function $\Gamma_{\alpha\beta}$.

We note that the present problem is solved in terms of the second-order perturbations connected with the relativistic interaction of the electrons with the lattice. In this approximation it therefore suffices to take the D function of zeroth order in the interaction.

The transition in the equation for the vertex function Γ from the sum over imaginary frequencies to an integral with respect to a real variable is carried out with the aid of the analytic-continuation technique proposed by Eliashberg.[9] This calls for the substitution

$$T\sum_{\omega_n} \to \frac{1}{4\pi i}\int_L dz\,\mathrm{th}\frac{z}{2T},$$

where L is a definite contour in the plane of the complex frequency z. Further calculations, which are connected with the determination of the contour L in this case, are analogous to Holstein's calculations[10] where the same method was applied to the problem of electric resistance. Out of all the possible products of two G-functions of real variables we retain, just as in [9, 10]. only those products of the type $G^R_{\beta\gamma}$ ($\mathbf{p} + \mathbf{k}$, $\omega + \Omega G^A_{\delta\alpha}$

$\times$ (p, ω), which define the present problem; here R and A denote retarded and advanced functions, respectively.

A similar operation of transition from sums to integrals was performed on formula (8). As a result we can represent the nonequilibrium magnetization in the form

$$\delta \mathbf{m}(\mathbf{k}, \Omega) = \mu_B \int d\mathbf{p} \, \mathrm{Sp} \, F(\mathbf{p}, \mathbf{k}, \Omega) \sigma, \tag{14}$$

where F is a matrix relative to spin variables and is determined by the expression

$$F(\mathbf{p}, \mathbf{k}, \Omega) = -\frac{i\mu_B}{2(2\pi)^4} \int d\omega \left(\mathrm{th} \frac{\omega + \Omega}{2T} - \mathrm{th} \frac{\omega}{2T} \right) \times G^R(\mathbf{p} + \mathbf{k}, \omega + \Omega) \Gamma(\mathbf{p}, \mathbf{k}, \mathbf{p} + \mathbf{k}, \omega, \Omega, \omega + \Omega) G^A(\mathbf{p}, \omega). \tag{15}$$

It follows from the definition (14) that F(p, k, Ω) is the amplitude of the Fourier function of the distribution. Expression (14) simplifies if it is assumed that $\Omega \ll T$, and the functions Γ, G^A, and G^R depend smoothly on the frequency ω. Under these conditions we can use the formula

$$\mathrm{th} \frac{\omega + \Omega}{2T} - \mathrm{th} \frac{\omega}{2T} \sim 2\Omega \delta(\omega). \tag{16}$$

We choose the direction of the z axis along the constant magnetic field **H**. Then the electron Green's function can be usually written in the form

$$G^{A,R} = \frac{1}{2}(1 + \sigma^z) G_+^{A,R} + \frac{1}{2}(1 - \sigma^z) G_-^{A,R},$$

$$G_\pm^{A,R} = [\omega - \varepsilon(\mathbf{p}) \mp \mu_B H - \Sigma_\pm^{A,R}]^{-1}. \tag{17}$$

We now redefine the vertex function Γ in accordance with the formula

$$G^R(p + k) \Gamma(p, k, p + k) G^A(p) = \mathscr{T}(p, k, p + k) G^A(p) - G^R(p + k) \mathscr{T}(p, k, p + k). \tag{18}$$

The equation for Γ, corresponding to the diagram equation (see the figure), can then be written in the form

$$G_R^{-1}(p + k) \mathscr{T}(p, k, p + k) - \mathscr{T}(p, k, p + k) G_A^{-1}(p) = \sigma \mathbf{h}_k + \Lambda(p, k, p + k). \tag{19}$$

Here Λ(p, k, p + k) is the sum of the vertex parts. Each of these parts is connected with a relativistic or nonrelativistic interaction between the electrons and the lattice.

We substitute in (19) explicit expressions for the complete G-functions (9). As a result we obtain

$$[\Omega - \varepsilon(\mathbf{p} + \mathbf{k}) + \varepsilon(\mathbf{p})] \mathscr{T}(p, k, p + k) - \mu_B H [\sigma^z, \mathscr{T}(p, k, p + k)]_- = \sigma \mathbf{h}_k + J(p, k, p + k), \tag{20}$$

where the term describing the interaction of the electrons with the lattice is of the form

$$J(p, k, p + k) = \Lambda(p, k, p + k) + \Sigma^R(p, k) \mathscr{T}(p, k, p + k) - \mathscr{T}(p, k, p + k) \Sigma^A(p). \tag{21}$$

The vertex function $\mathscr{T}$ is connected with the distribution function F by formulas (15) and (18). Therefore, using these relations, we can obtain from (20) the analog of the kinetic equation for the matrix distribution function F.

The solution of Eq. (21) in the zeroth approximation in the interactions of the electrons with the lattice is made up of the following values of the vertex functions:

$$\mathscr{T}_z{}^0 = \frac{2h_z}{\Omega - \mathbf{kv}} \quad \mathscr{T}_{21}{}^0 = \frac{2h_+}{\Omega + \omega_s - \mathbf{kv}}, \quad \mathscr{T}_{12}{}^0 = -\frac{2h_-}{\Omega - \omega_s - \mathbf{kv}}, \tag{22}$$

where $h^\pm = (h_k)_x \pm i(h_k)_y$ and h_z are the transverse and longitudinal (with respect to the direction H_z = H) components of the amplitude of the alternating magnetic field. We see therefore that the transverse vertex functions $\mathscr{T}_{21}$ and $\mathscr{T}_{12}$ become infinite when

$$\Omega = \mp \omega_s + \mathbf{kv}. \tag{23}$$

At the value

$$\Omega = \mathbf{kv} \tag{24}$$

the longitudinal vertex function $\mathscr{T}_z$ becomes infinite.

At these resonant values of the frequency of the alternating field, the following equation is satisfied in the zeroth approximation in the spin-lattice interaction:

$$\mathscr{T}(p, k, p + k) G(p) = G(p + k) \mathscr{T}(p, k, p + k). \tag{25}$$

The self-energy nonrelativistic parts $\Sigma^{(1)}$ which enter in G are calculated here for a zero field **H**. This means that we have discarded quantities of the order ω_S / vq_D or $\omega_S / 2vp_0$, where q_D is the Debye momentum and p_0 is the Fermi momentum. $\Sigma^{(1)}(p, \omega)$ is calculated at the values $|p| = p_0$ and at $\omega = 0$. Now $\Sigma^{(1)}(p_0)$ is not a matrix quantity. The quantity Re $\Sigma^{(1)}(p_0)$ corresponds to renormalization of the chemical potential in the G functions that enter in (25), and

$$\mathrm{Im}\, \Sigma^{(1)}(p_0) = 1/2\tau_1,$$

where τ_1 is the momentum relaxation time. The relativistic width of the electronic level is of the same order as the EPR width $1/\tau_2$. Therefore neglect of the relativistic interactions in the functions of (25) presupposes smallness of the quantities of order $1/\tau_2$ in comparison with $1/\tau_1$ and ω_S.

From Eq. (20) for the vertex function $\mathscr{T}$ we can obobtain with the aid of the definitions (14), (15), and (18) an equation similar to that of Bloch[2] for the nonequilibrium magnetization. The difference between this equation and that obtained by Silin[2] lies in the fact that here only the spin-lattice interaction is taken into account, and not the spin-spin interaction. As is well known, the width due to the spin-spin interaction in metals is small compared with the spin-lattice width.

4. WIDTH OF ELECTRON PARAMAGNETIC RESONANCE

We shall assume that the effective width of the skin layer is much larger than the free path length $\tau_1 v$ and the Larmor radius v/ω_S. Without account of the electron diffusion, the reciprocal spin-lattice relaxation time $1/\tau_2$ is the width of the paramagnetic resonance. In the case of spin waves, when the spatial inhomogeneity determines the effect itself, the imaginary part of the dispersion equation is completely determined by τ_2.

We define the quantity $1/\tau_2$, in accordance with formulas (14), (15), (16), (20), and (25), by the equation

$$\left(\frac{1}{\tau_2} \right)_{ij} \delta m^j = \frac{\mu_B{}^2 \Omega}{(2\pi)^4} \int d\mathbf{p} \, \mathrm{Sp} \, \sigma^i \, \mathrm{Im} \, J(\mathbf{p}, \Omega) [G^A(\mathbf{p}) - G^R(\mathbf{p})], \tag{26}$$

where only terms of zero order in **k** are taken into account.

We write down in explicit form that part of the expression $J(\mathbf{p}, 0, \mathbf{p}+0, 0, \Omega, 0+\Omega)$ (21) which corresponds to relativistic interaction of the electrons with the phonons. To this end we note that the relativistic vertex part $\Lambda^{(2)}$ is represented by the third diagram of the figure and consists of the function D_{ph}, the vertex function Γ, and two G-functions. The quantities $\Sigma^{(2)}$ consist, as is well known, of one G-function and the function D_{ph}. In addition, $\Lambda^{(2)}$ and $\Sigma^{(2)}$ contain two spin-lattice vertices (12) each. Following the analytic continuations,[9, 10] the relativistic part J_2 of expression (21) is represented in the form of integrals of the retarded and advanced Green's functions:

$$
\begin{aligned}
J_2(\mathbf{p}, 0, \mathbf{p}+0, 0, \Omega, 0+\Omega) = &-\frac{i\lambda_2^2}{6(2\pi)^4}\int d\mathbf{p}_1 \int d\omega_1 (\mathbf{p}_1+\mathbf{p})^2 \cdot \\
&\times \{(2N(\omega_1)+1)(D_{\mathrm{ph}}^{A}(p-p_1) - D_{\mathrm{ph}}^{R}(p-p_1)) \\
&\times [(\sigma^j\mathcal{T}_1 - \mathcal{T}\sigma^j)G^A(p_1)\sigma^j - \sigma^j G^R(p_1, \Omega)(\mathcal{T}_1\sigma^j - \sigma^j\mathcal{T})] \\
&+ (2f(\omega_1)-1)D_{\mathrm{ph}}^{A}(p-p_1) \cdot \\
&\times [(\sigma^j\mathcal{T}_1 - \mathcal{T}\sigma^j)G^A(p_1)\sigma^j - (\sigma^j G^R(p_1, \Omega)\mathcal{T}_1\sigma^j \\
&- \mathcal{T}\sigma^j G^R(p_1)\sigma^j)] - (2f(\omega_1+\Omega)-1)D_{\mathrm{ph}}^{R}(p-p_1) \cdot \\
&\times [(\sigma^j\mathcal{T}_1 G^A(p_1)\sigma^j - \sigma^j G^A(p_1, \Omega)\sigma^j\mathcal{T}) \\
&- \sigma^j G^R(p_1, \Omega)(\mathcal{T}_1\sigma^j - \sigma^j\mathcal{T})]\},
\end{aligned}
\tag{27}
$$

where

$$
G^A(p_1) = G^A(\mathbf{p}_1, \omega_1), \quad G^R(p_1, \Omega) = G^R(\mathbf{p}_1+0, \omega_1+\Omega),
$$

$$
D_{\mathrm{ph}}^{A,R}(p-p_1) = D_{\mathrm{ph}}^{A,R}(\mathbf{p}-\mathbf{p}_1, 0-\omega_1).
$$

$$
\mathcal{T} = \mathcal{T}(\mathbf{p}, 0, \mathrm{p}+0, 0, \Omega, 0+\Omega) = \mathcal{T}(\mathbf{p}, \Omega),
$$

$$
\mathcal{T}_1 = \mathcal{T}(\mathbf{p}_1, 0, \mathbf{p}_1+0, \omega_1, \Omega, \omega_1+\Omega) = \mathcal{T}_1(\mathbf{p}_1, \omega_1, \Omega),
$$

Here $f(\omega_1)$ and $N(\omega_1)$ are Fermi and Bose distributions, which are connected with the analytic-continuation functions $\tanh(\omega_1/2T)$ and $\coth(\omega_1/2T)$ by the formulas

$$
f(\omega_1) = \frac{1}{2}\left(1 - \mathrm{th}\frac{\omega_1}{2T}\right), \quad N(\omega_1) = \frac{1}{2}\left(\mathrm{cth}\frac{\omega_1}{2T} - 1\right).
$$

The vertex function $\mathcal{T}^0$, as seen from (22), does not depend on the angles when $k \to 0$ and is a function of the magnetic fields $\mathbf{H}$ and $\mathbf{h}$ only. In this case the relation

$$
\mathcal{T}_1^0(\mathbf{p}_1, \omega_1, \Omega) = \mathcal{T}^0(\mathbf{p}, \Omega) = \mathcal{T}(\Omega) \tag{28}
$$

is satisfied. Bearing this circumstance in mind, we can easily verify that the nonrelativistic part J_1 of J (21) vanishes. This is a consequence of the Ward theorem for the nonrelativistic interaction of electrons with the lattice. In the presence of a constant field $\mathbf{H}$, Ward's theorem is valid if the resonance condition (25) is satisfied.

To calculate $1/\tau_2$ by using (26), we note that the difference between the retarded and advanced Green's functions can be represented in the form

$$
G_\pm{}^A(\mathbf{p}) - G_\pm{}^R(\mathbf{p}) = 2i\pi\delta(\varepsilon(\mathbf{p}) \pm \mu_B H). \tag{29}
$$

This is true if

$$
1/\tau_1 \ll 2|\varepsilon(\mathbf{p}) \pm \mu_B H|. \tag{30}
$$

In the opposite case, such a difference of the G-functions should be written in the form

$$
G_\pm{}^A(\mathbf{p}) - G_\pm{}^R(\mathbf{p}) = \frac{i}{\tau_1}\left[(\varepsilon(\mathbf{p}) \pm \mu_B H)^2 + \frac{1}{4\tau_1^2}\right]^{-1}. \tag{31}
$$

We use the fact that the momenta of the conduction electrons are close to the Fermi momentum. It then follows from an analysis of formula (26) that the essential region of integration over the initial momenta of the electrons turns out to be of the order of $1/\tau_1 v$. Therefore, even if inequality (30) is not satisfied, the use of a δ-like formula for ImG in the initial integration leads to correct results. The essential region of the repeated integration varies, depending on how the virtual momentum enters in the lattice vertices (12) or (13). The spin-lattice interaction (12) and (5) forces us to use in the calculation of the remaining integral in formula (26) the exact expression (31) for the imaginary parts of the G-functions.

Using (29), we can easily obtain an explicit expression for $1/\tau_2$ from the definition (26). For this purpose it should be noted that in the approximation of zeroth order in the interaction the non-equilibrium magnetization equals, according to formulas (14), (15), (18), (22), (25), and (29),

$$
\delta m_0{}^\pm = \chi_0 \frac{\Omega}{\Omega \pm \omega_s} h_k{}^\pm = \chi^\pm h_0{}^\pm, \quad \delta m_0{}^z = \chi_0 h_k{}^z = \chi^z h_0{}^z, \tag{32}
$$

where $\chi_0 = \mu_B^2 m p_0/\pi^2 \ll 1$ is the static susceptibility. Here the amplitude of the alternating magnetic field $\mathbf{h}_k$ inside the metal is connected with the amplitude of the field $\mathbf{h}_0$ outside the metal by the formula

$$
h_k{}^{+,-,z} = (1 + 4\pi\chi^{+,-,z})h_0^{+,-,z},
$$

where $\chi^{+,-,z}$ is the non-equilibrium susceptibility.

Let us calculate the EPR widths $1/\tau_\parallel$ and $1/\tau_\perp$, measured along the magnetic field $\mathbf{H}$ and perpendicular to it, taking first into account the interaction of the electrons only with impurities. An expression for $\mathrm{Im}J_2$ (see (26)) as applied to the impurity can be easily obtained from (27). Since the impurity is static, it suffices to make the formal substitution

$$
\mathrm{cth}\frac{\omega_1}{2T} \to 1, \quad \mathrm{th}\frac{\omega_1}{2T}, \quad \mathrm{th}\frac{\omega_1+\Omega}{2T} \to 0
$$

and to use for the D functions the impurity functions D_{imp}. For the resonance conditions (see (23)–(25)) we have

$$
\begin{aligned}
\frac{1}{\tau_\parallel} &= \frac{\lambda_2^2 p_0^2 C V_0^2}{3\pi^3}\frac{1}{2\tau_1}\int\frac{d\mathbf{q}}{q^2}\left[\frac{1}{Z_+(\mathbf{qv})} + \frac{1}{Z_-(\mathbf{qv})}\right], \\
\frac{1}{\tau_\perp} &= \frac{\lambda_2^2 p_0^2 C V_0^2}{12\pi^3}\frac{1}{2\tau_1}\int\frac{d\mathbf{q}}{q^2}\left[\frac{2}{Z_0(\mathbf{qv})} + \frac{1}{Z_+(\mathbf{qv})} + \frac{1}{Z_-(\mathbf{qv})}\right], \\
Z_\pm(x) &= (x \pm \omega_s)^2 + \frac{1}{4\tau_1^2}, \quad Z_0(x) = x^2 + \frac{1}{4\tau_1^2},
\end{aligned}
\tag{33}
$$

where $\mathbf{q} = \mathbf{p} - \mathbf{p}_1$ and $|\mathbf{q}|$ and ω_S have been discarded everywhere in comparison with p_0.

It is seen from these formulas that neglect of the finite lifetime of the given state (and at the same time the use of δ functions for ImG throughout) leads to divergences of the infrared type.

When $1/2\tau_1 \ll 2p_0 v$ the integrals (29) can be evaluated exactly and their values are

$$
1/\tau_\parallel = \beta[\ln 4p_0\tau_1 v - {}^1\!/_2 \ln(1 + (2\tau_1\omega_s)^2)],
$$

$$
1/\tau_\perp = \beta[\ln 4p_0\tau_1 v - {}^1\!/_4 \ln(1 + (2\tau_1\omega_s)^2)],
$$

where $\beta = 16\lambda_2^2 C V_0^2 p_0 m/3\pi$.

In the limiting case when $1/2\tau_1 < \omega_S \ll 2p_0 v$, the longitudinal and transverse times of the residual spin-lattice relaxation are respectively equal to

$$
\frac{1}{\tau_\parallel} = \beta\ln\frac{2p_0 v}{\omega_s}, \quad \frac{1}{\tau_\perp} = \beta\ln 2p_0 v\left(\frac{2\tau_1}{\omega_s}\right)^{1/2}. \tag{34}
$$

At $\omega_s < 1/2\tau_1 \ll 2p_0v$, the longitudinal and transverse residual EPR widths are equal to each other and take on the value

$$1/\tau_{\parallel} = 1/\tau_{\perp} = \beta \ln 4p_0\tau_1 v. \tag{35}$$

The case when $1/2\tau_1 > 2p_0v$ and $1/\tau_{\parallel} = 1/\tau_{\perp} = 4\beta\tau_1 vp_0$ is not realized in practice.

The longitudinal and transverse EPR widths due to the spin-lattice interaction of the electrons with the phonons, are connected via the definition (26) with the imaginary part of (27) and can be reduced to the form

$$\frac{1}{\tau_{\parallel}} = \frac{2\lambda_2^2p_0^2}{3\pi^3\rho u}\frac{1}{2\tau_1}\int\frac{dq}{q}\left(\operatorname{cth}\frac{uq}{2T} - \operatorname{th}\frac{uq}{2T}\right)$$
$$\times\left[\frac{1}{Z_+(uq+\mathbf{vq})} + \frac{1}{Z_-(uq+\mathbf{vq})}\right],$$
$$\frac{1}{\tau_{\perp}} = \frac{\lambda_2^2p_0^2}{12\pi^3\rho u}\frac{1}{2\tau_1}\int\frac{dq}{q}\left(4\operatorname{cth}\frac{uq}{2T} - 2\operatorname{th}\frac{uq}{2T}\right.$$
$$\left. - \operatorname{th}\frac{uq+\omega_s}{2T} - \operatorname{th}\frac{uq-\omega_s}{2T}\right)\left[\frac{2}{Z_0(uq+\mathbf{vq})} + \frac{1}{Z_+(uq+\mathbf{vq})}\right.$$
$$\left. + \frac{1}{Z_-(uq+\mathbf{vq})}\right]. \tag{36}$$

In the limit when $1/2\tau_1 \ll 2Tv/u$, the integrals can be easily calculated and the reciprocal times of longitudinal and transverse relaxation take the form

$$\frac{1}{\tau_{\parallel}} = \alpha T\left\{\ln\operatorname{th}\frac{up_0}{T} - \left[\ln\frac{u}{4T\tau_1 v} + \frac{1}{2}\ln(1+(2\omega_s\tau_1)^2)\right]\right\},$$
$$\frac{1}{\tau_{\perp}} = \alpha T\left\{\ln\operatorname{th}\frac{up_0}{T} - \left[\ln\frac{u}{4T\tau_1 v} + \frac{1}{4}\ln(1+(2\omega_s\tau_1)^2)\right]\right\}, \tag{37}$$

where $\alpha = 16\lambda_2^2mp_0/3\pi\rho u^2$, and it was assumed that $1/2\tau_1 \ll 2p_0v$ and $2p_0 < q_D$.

At low temperatures, when the inequalities $1/2\tau_1 < \omega_s < \omega_s v/u < 2Tv/u < 2p_0v$ are satisfied, we get from (37)

$$\frac{1}{\tau_{\parallel}} = \alpha T\ln\left(\frac{2T}{\omega_s}\frac{v}{u}\right),\quad \frac{1}{\tau_{\perp}} = \alpha T\ln\left[2T\frac{v}{u}\left(\frac{2\tau_1}{\omega_s}\right)^{1/2}\right]. \tag{38}$$

For the case of temperatures larger than the Debye temperature, when the inequalities

$$1/2\tau_1 < \omega_s < \omega_s v/u < 2p_0v < 2Tv/u,$$

are satisfied, formulas (37) go over into

$$\frac{1}{\tau_{\parallel}} = \alpha T\ln\frac{2p_0v}{\omega_s},$$
$$\frac{1}{\tau_{\perp}} = \alpha T\ln\left[2p_0v\left(\frac{2\tau_1}{\omega_s}\right)^{1/2}\right] \tag{39}$$

In the limit when the nonrelativistic width of the electronic levels becomes larger than ω_s, the temperature width of the EPR (just like the residual width) does not depend on the mutual placements of the constant at alternating magnetic fields. Then

$$\frac{1}{\tau_{\parallel}} = \frac{1}{\tau_{\perp}} = \alpha T\ln 4T\tau_1\frac{v}{u} \tag{40}$$

for

$$\omega_s < \frac{1}{2\tau_1} \lesseqgtr \omega_s\frac{v}{u} < 2T\frac{v}{u} < 2p_0v$$

and

$$\frac{1}{\tau_{\parallel}} = \frac{1}{\tau_{\perp}} = \alpha T\ln 4\tau_1 vp_0 \tag{41}$$

for

$$\omega_s < \frac{1}{2\tau_1} \lesseqgtr \omega_s\frac{v}{u} < 2p_0u < 2T\frac{v}{u}.$$

In the unlikely case when $1/2\tau_1 > 2p_0v$ we have

$$\frac{1}{\tau_{\parallel}} = \frac{1}{\tau_{\perp}} = \alpha T\frac{4p_0v\tau_1}{\pi}.$$

Direct calculation of $\operatorname{Im}\Sigma_{\text{ph}}^{(1)}(p_0)$ leads to the well known dependence of the nonrelativistic width on the temperature:

$$\frac{1}{2\tau_1^{\Phi}} = \frac{\lambda_1^2}{2\pi\rho u^4 v}\begin{cases} T_D^2T, & \text{if}\quad T > T_D, \\ 7\zeta(3)T^3, & \text{if}\quad T < T_D. \end{cases} \tag{42}$$

We see therefore that $1/2\tau_1^{\text{ph}} < 2Tv/u$ at all temperatures. At low temperatures, the nonrelativistic width connected with the electron-phonon interaction is smaller than the nonrelativistic impurity width. The nonrelativistic width of the electron level, due to the interaction of the electrons with the lattice defects, is proportional to the impurity concentration C:[7]

$$1/2\tau_1^{\text{пр}} = \lambda_1^2V_0^2Cp_0^2/2\pi v.$$

This means that at sufficiently high impurity concentrations there can realize the inequality $1/2_1^{\text{imp}} > 2Tv/u$. Since we stipulated at the very beginning the condition $\omega_s < T$, such an impurity width is $1/2\tau_1^{\text{imp}} > \omega_s$. Under these conditions, the expressions in (36) go over into

$$\frac{1}{\tau_{\parallel}} = \frac{1}{\tau_{\perp}} = \alpha\frac{2T}{\pi}\int_0^{2p_0u/T} d\xi(\operatorname{cth}\xi - \operatorname{th}\xi)\operatorname{arctg}\frac{4\tau_1 vT}{u}\xi,$$

where $\xi = uq/2T$. At $T \ll 2p_0u < T_D$ we have

$$\frac{1}{\tau_{\parallel}} = \frac{1}{\tau_{\perp}} = \alpha T^2\tau_1^{\text{imp}}\frac{v}{u}\pi + \alpha T\frac{\pi}{2}\exp\left\{-\frac{u}{4T\tau_1^{\text{imp}}v}\right\}. \tag{43}$$

The second term of this expression is smaller than the first and is compared with it if $1/2\tau_1^{\text{imp}} \sim 2Tv/u$.

5. CONCLUSION

It is seen from formulas (38) and (39) that in the case when $\tau_1\omega_s > 1$ the longitudinal phonon width of paramagnetic resonance is smaller than the transverse width. It is precisely this limit which corresponds to the presence of spin waves in nonferromagnetic metals.[2] If we assume that the mean free path in Na and T = 300° K is connected with electron-phonon interaction, then we can determine from formula (42) the constant λ_1 of the nonrelativistic electron-lattice interaction. At this temperature we have from [11] $\tau_1^{\text{ph}}v = 3.5 \times 10^{-6}$ cm, $v = 6 \times 10^7$ cm/sec, and $u = 5 \times 10^5$ cm/sec; we then have $\lambda_1 \approx 3.4 \times 10^{-12}$ erg. We shall also assume that the width of the spin waves includes the phonon time τ_1^{ph}. Then $1/2\tau_1^{\text{ph}}$ reaches the value ω_s (at $H = 10^4$ Oe) when T ~ 10°K, and the longitudinal and transverse relativistic widths become equal (Eqs. (40) and (41)). It is precisely at this temperature and in such a field H that the vanishing of spin waves is observed.[12] No spin waves were observed above the Debye temperature. The fields required to satisfy the condition $\omega_s\tau_1 \sim 1$ at such temperatures are stronger by three orders of magnitude than those used at low temperatures. This causes the smallness condition $\omega_s < T$ which was used essentially in the derivation of expressions for the transverse EPR waves, to be violated at $T \lesssim 10^3$ °K.

The residual longitudinal and transverse EPR widths are also different precisely for the case of spin waves (see (34)).

In the region of ordinary EPR, where $\omega_s \tau_1 < 1$, the width connected with the relativistic interaction of the electrons with the phonons prevails over the residual relativistic width, starting with which $T < T_D$. The dependence of this width on the temperature is of the form $\sim T^2$. This is connected with the fact that up to temperatures that are not too far from the Debye temperature the nonrelativistic width of the electron level is determined by the interaction of the electrons with the impurities and not with the phonons. When $T < T_D$, the action of formula (40), where $1/\tau_2 \sim T \ln (1/T^2)$, is limited in insufficiently purified metals. With increasing temperature, the EPR width acquires at $T > T_D$ a linear temperature dependence in the form $T \ln (1/T)$. In this case the nonrelativistic width of the electron level is due to the phonons. The EPR width becomes strictly linear with respect to T when the number of impurities is sufficient to cause the nonrelativistic level width to be determined only by the interaction of the electrons with the impurities. In the case of very strongly contaminated metals, the reciprocal relaxation time $1/\tau_2$ does not depend on the temperature (see (35)).

The experimental temperature dependence of the EPR width in Na samples[11] is well described by a parabolic formula up to $T \sim 70$°K. In the comparison it was assumed that the mean free path is due to the interaction of the electrons with the impurities and is of the order of 4.5×10^{-8} cm, the constant $\alpha \approx 0.76 \times 10^{-6}$, and the Debye temperature T_D was assumed to be 150°K. With the same constant α, the theoretical values obtained with the aid of the logarithmic formula with a phonon mean free path are close to the experimental data near 120°K. At the value of α given above, the spin-lattice interaction constant is $\lambda_2 \approx 15.2 \times 10^{-16}$ erg if Δg is assumed to be $\approx 4.5 \times 10^{-4}$.

The minimum value of the phonon (impurity) momentum which enters in $1/\tau_2$ is connected with the particle dimensions d in the sample only in the case when d is smaller than the mean free path. This occurs, in particular, when $d = 7 \times 10^{-6}$ cm[11] at $T > 70$°K, in the case of small mean free paths determined by the electron-phonon collisions. At lower temperatures, when the mean free path is large and is due to the interaction of the electrons with the lattice defects, $1/\tau_2$ does not depend on the dimensions of the particles in the samples used in the experiment[11, 12] (their diameter was equal to 7×10^{-6}, 6×10^{-5}, and 3×10^{-4} cm). The large difference in the experimental EPR widths in [11] and [13] at low temperatures is apparently connected not so much with the unequal amount of impurities in the differently-prepared samples as with the diffusion of the magnetization inside the metal.

In conclusion I am grateful to A. S. Kompaneets for useful remarks and am indebted to M. A. Kozhushner for interest in the work and valuable discussions.

---

[1] A. W. Overhauser, Phys. Rev. **89**, 689 (1953).

[2] V. P. Silin, Fiz. Metal. Metalloved. **4**, 1 (1970).

[3] V. V. Andreev and V. I. Gerasimenko, Zh. Eksp. Teor. Fiz. **35**, 1209 (1958) [Sov. Phys. JETP **8**, 848 (1959)].

[4] J. Jafet, Solid State Phys. **14**, 1 (1963).

[5] P. Fulde and A. Luther, Phys. Rev. **175**, 337 (1968).

[6] Van Buren, Defects in Crystals (Russ. transl.), IIL, 1962.

[7] A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics) Fizmatgiz, 1962, pp. 73, 410, 427 [Pergamon, 1965].

[8] R. Kubo, J. Phys. Soc. Jap. **12**, 570 (1957).

[9] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys. JETP **11**, 696 (1960)].

[10] T. Holstein, Ann. Phys. (N.Y.) **29**, 410 (1964).

[11] V. I. Petinov and A. Yu. Ardashev, Fiz. Tverd. Tela **11**, (1), (1969) [Sov. Phys. Solid State **11**, (1), (1969)].

[12] S. Schultz and G. Dunifer, Phys. Rev. Lett. **18**, 283 (1967).

[13] F. Vescial, N. Vanderven, and R. Shumacher, Phys. Rev. A (1964–1965) **134**, 1286 (1964).

Translated by J. G. Adashko
257