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Charged Impurities in Very Thin Layers

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The interaction of electrons with ionized impurities in thin quantizing layers is considered. The thickness of the layer is assumed to be smaller than the effective Bohr radius in the bulk sample. The donor activation energy is determined, and it is shown that it may appreciably exceed its bulk value. The possibility of the formation of quasi-stationary states is demonstrated. The temperature dependence of the mobility, which also differs substantially from the temperature dependence in the case of a bulk sample, is calculated. Qualitative agreement with experiment is noted.

We consider the interaction of the electrons with ionized impurities in very thin quantizing layers. The thickness of the layer, a, is assumed to be small in comparison with the Bohr radius a_0 of a donor in a bulk sample. At the present time such films of Bi, InSb and InAs[1] have been prepared; in addition the question may involve near-surface conducting layers in a semiconductor under conditions of the field-effect type (for example, an inversion channel). In this case the thickness of the layer may reach a value between 30 and 40 Å. As will become clear from the following, it is necessary to distinguish between two cases: double layers and triple layers. An inversion channel or a film on a substrate whose dielectric constant ϵ having an identical value can serve as an example of the first case. The triple-layer case occurs for a film on a substrate with a different value of ϵ.

Let us begin with the first case. The potential energy of the interaction between an electron and a donor located at the point $(0, 0, z_0)$ is given by

$$U = -\frac{e^2}{\varepsilon_1}\left[\frac{1}{\sqrt{\rho^2+(z-z_0)^2}} + \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\frac{1}{\sqrt{\rho^2+(z+z_0)^2}}\right].$$

Here the cylindrical coordinate system ρ, φ, z has been introduced; the z axis is directed perpendicular to the layer; ϵ_1 and ϵ_2 denote, respectively, the dielectric constants of the semiconductor and of the medium adjoining it. As will be shown below, the distances which are characteristic for the Coulomb problem in the plane of the film are of the order of $a_0 = \hbar^2(\epsilon_1 + \epsilon_2)/2m^*e^2$, where m^* is the effective mass of the electron. By hypothesis the characteristic values of z and z_0 are of the order of a, that is, much smaller than $(\epsilon_1 + \epsilon_2)/2e^2$ (we have set $\hbar = m^* = 1$). Therefore, in the first approximation one can set $U_0 = -\tilde{e}^2/\rho$, where $\tilde{e}^2 \equiv 2e^2/(\epsilon_1 + \epsilon_2)$. The remaining part of the potential is given by

$$V = e^2\left[\frac{2}{(\varepsilon_1+\varepsilon_2)\rho} - \frac{1}{\varepsilon_1\sqrt{\rho^2+(z-z_0)^2}} - \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1(\varepsilon_1+\varepsilon_2)\sqrt{\rho^2+(z+z_0)^2}}\right]$$

and it is substantially different from zero only in the region $\rho \lesssim a \ll a_0$. Solving the Schrödinger equation containing the potential $U_0(\rho)$, we find the energy levels corresponding to bound states of the electron:

$$E_{sn} = \omega_s - \frac{\tilde{e}^4}{2(n+{}^1/_2)^2}, \quad s = 1, 2, \ldots; \; n = 0, 1, 2, \ldots, \tag{1}$$

where ω_s are the quantum levels of the transverse motion, and n is the principal quantum number of the two-dimensional Coulomb problem. The wave functions of these states are given by

$$\psi_{snm} = C_{nm}\varphi_s(z)\frac{e^{im\varphi}}{\sqrt{2\pi}}(2k_n\rho)^{|m|}e^{-k_n\rho}F(-n+|m|, 2|m|+1, 2k_n\rho),$$

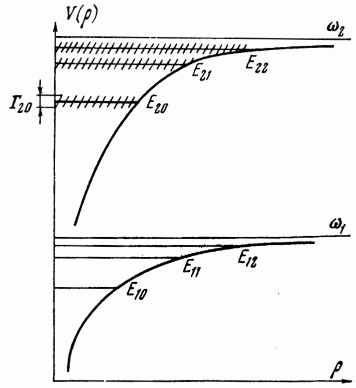
Here

$$m = 0, \pm 1 \ldots \pm n.$$

$$k_n^2 = \sqrt{2(\omega_s - E_{sn})}, \quad C_{nm} = \frac{2k_n}{2|m|!}\sqrt{\frac{(n+|m|)!}{(2|m|+1)(n-|m|)!}}$$

$\varphi_S(z)$ is the normalized function describing the transverse motion: $\int|\varphi_S(z)|^2\,dz = 1$, and F is the confluent hypergeometric function.

Thus, at the bottom of each sub-band ω_S, energy levels of the electron appear which (in the approximation under consideration) correspond to localized states. The characteristic dimension of the bound state in the plane of the film is of the order of $(n + {}^1/_2)\tilde{e}^{-2} \gg a$. The multiplicity of the degeneracy of the n-th level is equal to $2n + 1$. Taking the perturbation $V(\rho, z)$ into account leads to a shift and a splitting of the levels which have been found.



In addition, all of the levels with $s > 1$ lie in the background of the continuous spectrum associated with the lower sub-bands, and therefore the perturbation $V(\rho, z)$ also leads to their smearing-out. The corresponding transitions are analogous to the predissociation of diatomic molecules and are related to the matrix elements of $V(\rho, z)$ which are off-diagonal with respect to s[2] (see the figure). However, in contrast to predissociation, the motion with respect to the coordinate ρ has a quantum nature, and it is not quasiclassical in nature. Since no crossing or pseudo-crossing of the terms occurs in the situation under consideration, one can calculate the width of the level by using perturbation theory.

Let us calculate the probability for decay of the state with $s = 2$ and $n = 0$ (the ground Coulomb level of the second sub-band). The wave function of the initial state has the form $\psi_i = \sqrt{2/\pi}\, k_0 \exp\{-k_0\rho\}\, \varphi_2(z)$. In the final state the particle is described by the axially symmetric (since the perturbation does not depend on the angle φ) wave function ψ_f, corresponding to the same energy $\omega_2 + E_{20}$, but referring to the first sub-band:

$$\psi_f = e^{-i\varkappa\rho} F\left(1/2 - \frac{ie^2}{\varkappa}, 1, 2i\varkappa\rho\right) \varphi_1(z).$$

Here $\kappa \equiv \sqrt{2(\omega_2 - \omega_1 + E_{20})} \sim 1/a$ and ψ_f is normalized to a δ-function of energy. In evaluating the matrix element V_{if} of the transition, we shall utilize the smallness of the parameter $\tilde{e}^2/\kappa$ and the property that V differs substantially from zero only for small $\rho(\rho \sim a)$, when one can replace $\psi_i(\rho)$ by its value for $\rho = 0$. As a result we obtain the following expression for the decay probability per unit time:

$$\Gamma_{20} = \frac{4\pi^2 \tilde{e}^4 k_0^2}{\varkappa^2 \varepsilon_1^2} \left| \langle (\varepsilon_1 + \varepsilon_2) e^{-\varkappa|z - z_0|} + (\varepsilon_1 - \varepsilon_2) e^{-\varkappa|z + z_0|} \rangle_{12} \right|^2. \tag{3}$$

The angular brackets denote taking the matrix element with respect to the functions $\varphi_s(z)$. In the case of a surface donor center ($z_0 = 0$), one obtains the following result in the model of a rectangular potential well:

$$\varphi_s(z) = \sqrt{\frac{2}{a}} \sin\frac{s\pi z}{a}, \quad \varkappa = \frac{\sqrt{3}\pi}{a}, \quad \Gamma_{20} = \left(\frac{2}{3\pi}\right)^2 \tilde{e}^4 (k_0 a)^2. \tag{4}$$

Thus, the ratio of the levelwidth Γ_{20} to the distance between the levels, $E_{20} - E_{21}$, is given by $(\tilde{e}^2 a/\pi)^2 \sim (a/a_0)^2 \ll 1$.

Now let us determine the shift and splitting of the Coulomb levels, ΔE_{nm}. It is a priori clear that the influence of the perturbation $V(\rho, z)$ on the levels with $m \neq 0$ will be less important than on the levels with $m = 0$, because the wave functions of the bound states tend to zero as $\rho \to 0$ if $m \neq 0$. In first-order perturbation theory we find

$$\Delta E_{snm} = \frac{k_n^2}{2} \frac{\tilde{e}^4 |\langle z \rangle_{ss}|^2}{|m|(m^2 - 1/4)}, \quad m \neq 0. \tag{5}$$

Formula (5) again refers to a surface donor at $z_0 = 0$, but the order of magnitude of the splitting, $\Delta E_{nm} \sim k_n^2 (a/a_0)^2$, remains for any value of z_0 between 0 and a.

The levels with $m = 0$ are shifted by an amount which is of first-order of smallness in the parameter a/a_0: $\Delta E_{sn0} = 4\tilde{e}^2 k_n^2 \langle z \rangle_{ss}$. One can determine the position of the ground level of the donor with a great deal of accuracy if one utilizes a straightforward variational method. Choosing the trial function in the form $\varphi_s(z) e^{-\lambda\rho}$, we obtain

$$E_s(\lambda) = \omega_s + \lambda^2/2 - 4\tilde{e}^2 \lambda^2 \langle 1/2\, \pi z [H_1(2\lambda z) - N_1(2\lambda z)] - z \rangle_{11},$$

where H and N denote the Struve and Neumann functions, respectively. Let us minimize $E_s(\lambda)$, assuming that $2\lambda z < 2\lambda a \ll 1$. As a result we find

$$E_{s00} \approx \omega_s - 2\tilde{e}^4 / (1 + 8\tilde{e}^2 \langle z \rangle_{ss}) \equiv \omega_s - \delta(a). \tag{6}$$

Formula (6) for $s = 1$ gives the distance of the donor level from the bottom of the conduction band, which was shifted upward by an amount ω_1 as a consequence of the quantization of the transverse motion. In other words, the quantity $\delta(a)$ for $s = 1$ is the donor activation energy in a thin film. It is interesting to note that this quantity may substantially exceed the activation energy of the same donor in a bulk sample. For example, for InSb we have $m^* = 0.014\, m_e$, $\epsilon_1 = 17$; assuming $\epsilon_2 = 1$, $a = 150$ Å, and $\varphi_s = \sqrt{2/a}\, \sin(\pi z/a)$, we obtain

$$\delta(a) = 2\tilde{e}^4 (1 + 4a\tilde{e}^2)^{-1} \approx 37^\circ \mathrm{K}.$$

The bulk value δ_V of the activation energy amounts to 7°K.

The increase of the donor's activation energy with a reduction of the film thickness should lead to a decrease in the carrier concentration (at a fixed temature). If the density of acceptors and holes is neglected, then the number of carriers in an n-type film in the region $\exp(-\delta/T) \ll \pi N/T \ll \exp(\delta/T)$ is given by $\sqrt{NT/\pi}\, \exp(-\delta/2T)$, where N denotes the total number of donors per unit surface area of the film. For a bulk sample, the corresponding expression is proportional to $T^{3/4} \exp\{-\delta/2T\}$ (see, for example, [3]). A second consequence of the increased value of $\delta(a)$ in comparison with δ_V is the possibility of lifting the degeneracy of the electrons (for fixed values of the temperature and donor concentration) upon changing from thick films to thin ones. In addition to the reduction in the radius of the bound state, which is proportional to $[\delta(a)]^{-1/2}$, the point is that for thin layers the criterion for overlap of the electron orbits may turn out to be more stringent than for the case of a bulk sample. This criterion has the form $\max(n_s a_0^2, n_V a a_0^2) > 1$, where n_s and n_V denote the surface and volume concentrations, respectively, of donors in the layer; the quantity N introduced above is given by: $N = n_s + a n_V$. However, in the case of a bulk sample it is necessary to require $n_V a_0^3 > 1$, which can be satisfied for smaller values of n_V.

As was shown above, charged impurities in thin films possess quasi-stationary energy levels, a fact related to the small probability of a transition between sub-bands upon scattering of an electron by an impurity center. These levels can be observed either spectroscopically or else by their influence on electron scattering in which resonances appear. The absorption spectrum of light obliquely incident on the film and polarized in the plane of incidence should contain lines corresponding to the transitions $E_{snm} \to E_{s'nm}$. For example, the probability for the transition 100 → 200, that is, between the ground levels of the first and second subbands, is given by

$$W_{12} = 2e^2 \mathscr{E}_z^2 |\langle z \rangle_{12}|^2 \frac{\Gamma_{20}}{(\omega - \Delta)^2 + \Gamma_{20}^2}, \tag{7}$$

where Γ_{20} is defined in (4), $\Delta = E_{200} - E_{100}$, ω denotes the frequency of the light, and ϵ_z denotes the z-component of the electric field of the wave. The quantity W_{12} should be compared with the probability W_i for ionization of the level E_{100} by light of the same frequency (i.e., with the intensity of the background on which the narrow absorption line is superimposed). Elementary calculations lead to the formula

$$W_i = \frac{2\pi^2 k_0{}^4 e^2 \mathscr{E}_z{}^2 |\langle z\rangle_{\|}|^2}{(\omega-\delta)^3}.$$

One obtains the following results for the above indicated values of the parameters for InSb films: the resonance frequency $\omega = \Delta \approx 4\times 10^{14}$ $\sec^{-1}$, the line width $\Gamma_{20} \approx 3\times 10^{11}$ $\sec^{-1}$, and the ratio of the absorption intensity at the center of the line to the background intensity $W_{12}/W_i \approx 1.5\times 10^4$. Of course, for the existence of the described states it is necessary that the orbits of neighboring donors should not overlap, that is, one must have $n_S a_0^2$, $n_V a a_0^2 < 1$. We note that upon fulfillment of this condition the electrons remain nondegenerate at any temperature.

As to the resonance scattering by quasidiscrete levels, this only changes the kinetic characteristics of the film to a small degree. In fact, a resonance in the scattering leads to the result that the phase $\eta(E)$ in the asymptotic form of the wave function reaches a value of the order of unity. Therefore, the scattering amplitude, containing the factor $(e^{2i\eta}-1)$, increases by a factor $1/\eta_0(E)$ in comparison with the case when no resonance is present ($\eta_0 \ll 1$ is the phase of the potential scattering). It is easy to verify that in the energy range corresponding to the resonances, $E_{S00} < E < \omega_S$, the phase of the potential scattering is of the order of $\tilde{e}^2/\sqrt{E} \sim a/a_0$; therefore the scattering cross section $\sigma(E)$ increases by $(a_0/a)^2$ times. The electron's mobility is determined by the integral $\int \sigma(E)^{-1}\exp(-E/T)\,dE$. Since the width of the resonance region is not larger than $\tilde{e}^4$, the influence of the resonance scattering is exponentially small for $T \ll a^{-2}$, and for $T \gtrsim a^{-2}$ its relative contribution is of the order of $\tilde{e}^4/T \lesssim \tilde{e}^4 a^2 \ll 1$. (In the case of a degenerate system the kinetic characteristics would strongly depend on the position of the Fermi level relative to the quasi-stationary levels, and the scattering resonances would play an essential role. However, the degeneracy is related to the overlapping of the orbits and is therefore incompatible with the existence of quasi-stationary states.)

We shall now consider the scattering of electrons by charged centers without taking the resonances into account and, for the time being, neglecting the perturbation $V(\rho, z)$. The question is, therefore, the derivation of the two-dimensional analog of Rutherford's formula. The problem can be solved exactly by introducing two-dimensional parabolic coordinates: $\xi = \rho + x$, $\eta = \rho - x$. The variables ξ and η vary from 0 to ∞, the infinitesimal element of length is given by $dl = \frac{1}{2}\sqrt{(1+\eta/\xi)d\xi^2 + (1+\xi/\eta)d\eta^2}$, and the area element is given by $dS = [(\xi+\eta)/4\sqrt{\xi\eta}]\,d\xi\,d\eta$. The Schrödinger equation containing the potential $-\tilde{e}^2/\rho$ has the form

$$-\frac{2}{\xi+\eta}\left[\sqrt{\xi}\frac{\partial}{\partial\xi}\left(\sqrt{\xi}\frac{\partial\psi}{\partial\xi}\right)+\sqrt{\eta}\frac{\partial}{\partial\eta}\left(\sqrt{\eta}\frac{\partial\psi}{\partial\eta}\right)\right]-\frac{2\tilde{e}^2}{\xi+\eta}\psi = E\psi. \tag{8}$$

It is necessary to find the solution of Eq. (8) which has the form of a plane wave for negative values of x and for large values of ρ

$$\psi \sim e^{ikx} \text{ for } -\infty < x < 0,\ \rho\to\infty,\ k \equiv \sqrt{2E},$$

or in terms of the coordinates ξ and η:

$$\psi \sim \exp\frac{ik}{2}(\xi-\eta) \quad \text{for } \eta\to\infty \text{ and for all } \xi.$$

The coordinates ξ and η determine only the absolute value of the Cartesian coordinate y: $y = \sqrt{\xi\eta}$. This difficulty, however, is unimportant since the Hamiltonian is even with respect to y, and the wave function that describes the scattering also possesses obvious symmetry with respect to the x axis.

The following is completely analogous to the derivation of the "three-dimensional" Rutherford formula (see [2]). Separating the variables in Eq. (8), we find the exact solution of the Schrödinger equation describing the scattering:

$$\psi = \frac{1}{\sqrt{\pi}}\exp\left\{\frac{\pi\tilde{e}^2}{2k}\right\}\Gamma\left(\frac{1}{2}-\frac{i\tilde{e}^2}{k}\right)\exp\left\{i\frac{k}{2}(\xi-\eta)\right\}\cdot F\left(\frac{i\tilde{e}^2}{k},\frac{1}{2},ik\eta\right). \tag{9}$$

The signs in Eq. (9) correspond to an attraction field. Let us define the scattering amplitude $f(\varphi)$ in the two-dimensional case according to the asymptotic expression for the wave function:

$$\psi \sim e^{ikx} + \frac{f(\varphi)}{\sqrt{\rho}}e^{ik\rho}. \tag{10}$$

From Eq. (9) we obtain

$$f(\varphi) = \frac{e^{-i\pi/4}\Gamma\left(\frac{1}{2}-\frac{i\tilde{e}^2}{k}\right)\exp\{2i(\tilde{e}^2/k)\ln|\sin\varphi/2|\}}{\sqrt{2k}\,\Gamma(i\tilde{e}^2/k)\,|\sin\varphi/2|}. \tag{11}$$

The two-dimensional analog of the scattering cross section $|f(\varphi)|^2$ determines the flux of particles scattered at the angle φ:

$$d\sigma(\varphi) = |f(\varphi)|^2 d\varphi = \frac{\tilde{e}^2}{2m^*v^2}\operatorname{th}\frac{\pi\tilde{e}^2}{\hbar v}\frac{d\varphi}{\sin^2\varphi/2}. \tag{12}$$

Formula (12) is written down in terms of the usual units; $v = \hbar k/m^*$ denotes the particle's velocity. In contrast to the "three-dimensional" formula, the "two-dimensional" Rutherford formula explicitly contains the quantum constant, as is clear from expression (12). It is obvious that the classical, quantum, and Born expressions for $d\sigma(\varphi)$ do not coincide. The Born approximation corresponds to the case $\tilde{e}^2/\hbar v \ll 1$. In this connection the resulting formula

$$\frac{d\sigma}{d\varphi} = \frac{\pi\tilde{e}^4}{2m^*\hbar v^3}\operatorname{cosec}^2\frac{\varphi}{2}$$

was utilized in article [4], where the case of a strongly degenerate electron-impurity system was treated. As is well known, the criterion for strong degeneracy of a system with a Coulomb interaction coincides with the condition for the applicability of the Born approximation to the scattering. The opposite limiting case, $\tilde{e}^2/\hbar v \gg 1$, corresponds to the applicability of the quasiclassical approximation to the Coulomb field. Actually, in this case formula (12) gives

$$\frac{d\sigma}{d\varphi} = \frac{\tilde{e}^2}{2m^*v^2}\operatorname{cosec}^2\frac{\varphi}{2},$$

which agrees with the result of classical mechanics.

The momentum relaxation time τ, determining the mobility of the electrons, is expressed in terms of the transport scattering cross section:

$$\frac{1}{\tau} = (n_s + an_v)v\int(1-\cos\varphi)\,d\sigma = \frac{2\pi\tilde{e}^2(n_s+an_v)}{m^*v}\,\mathrm{th}\,\frac{\pi\tilde{e}^2}{\hbar v}. \quad (13)$$

Thus, in contrast to the three-dimensional case, the transport cross section turns out to be finite even without taking the screening of the Coulomb field into account (this property was already indicated in [4]). We note that for a nondegenerate gas the reciprocal screening radius is equal to $2\pi\tilde{e}^2c/T$, where c denotes the number of electrons per unit surface area of the film (see [4]). In the problem under consideration, this quantity is always substantially smaller than the momentum $\hbar k \sim m^* e^2/\hbar$ which is characteristic of the scattering, that is, it is not necessary to take screening into account.

Let us estimate the corrections to formula (12) which are related to the perturbation $V(\rho, z)$. The wave functions of the zero-order approximation have the form $\psi_{\mathbf{k}}(\rho, z) = \varphi_s(z)\exp\{i\mathbf{k}\cdot\boldsymbol{\rho}\}F(i\tilde{e}^2/k, 1/2, i(k\rho - \mathbf{k}\cdot\boldsymbol{\rho}))$. In calculating the matrix elements $V_{\mathbf{kk}'}$ of the perturbation for momenta $k, k' \sim \tilde{e}^2$ one can take $\psi_{\mathbf{k}}$ outside of the integral over $d\boldsymbol{\rho}$ at the point $\boldsymbol{\rho} = 0$, since $V(\rho, z)$ is appreciably different from zero only in the region $\rho \lesssim a$. After doing this, one can easily verify that the correction to $\psi_{\mathbf{k}}$ has a relative order of magnitude a/a_0 and does not exert any substantial influence on the scattering amplitude.

Let us calculate the temperature dependence of the mobility $\mu(T)$ in scattering by charged impurities. Averaging formula (13) over the velocities, we obtain

$$\mu(T) = \frac{e}{m^*}\langle\tau(T)\rangle,$$

$$\langle\tau(T)\rangle = \frac{\sqrt{2m^*}}{\pi\tilde{e}^2(n_s+an_v)}\sqrt{T}\,M\left(\pi\sqrt{\frac{\tilde{e}^4m^*}{2\hbar^2T}}\right), \quad (14)$$

where

$$M(z) = \int_0^\infty e^{-x^2}\,\mathrm{cth}\,\frac{z}{x}\,x^2\,dx.$$

At low temperatures $(T \ll m^*\tilde{e}^4/\hbar^2)$ $\mu(T) \sim \sqrt{T}$, and at high temperatures $(T \gg m^*\tilde{e}^4/\hbar^2)$ $\mu(T) \sim T$. Thus, in films which are thinner than the Bohr radius, the temperature behavior of the impurity mobility is substantially different from the bulk case ($\mu_V(T) \sim T^{3/2}$).

Now let us consider the triple-layer problem. Let ϵ_1 as before denote the dielectric constant of the semiconductor, and let ϵ_2 and ϵ_3 pertain to the dielectrics adjacent to it. By solving Poisson's equation with the appropriate boundary conditions, one can calculate the potential created by the impurity center (see [5,6]). In the general case, however, this expression is very cumbersome. We shall consider the situation in which $\epsilon_1 \gg \epsilon_2, \epsilon_3$, and we shall be interested in the region of distances from the center which are substantially larger than the film's thickness; the Fourier component of the potential with respect to the coordinates x and y is given by

$$U_0(k, z, z_0) \approx -4\pi e^2 / [\varepsilon_1 ka^2 + (\varepsilon_2+\varepsilon_3)k]. \quad (15)$$

The case when ϵ_2 or ϵ_3 are of the same order of magnitude as ϵ_1 is not of theoretical interest, since then $U_0(k) \approx -4\pi e^2/(\epsilon_2+\epsilon_3)k$, that is, the problem reduces to the one already considered.

From formula (15) it follows that two ranges of values of ρ exist, in which the electron's potential energy depends on the distance in different ways. In the region $a \ll \rho \ll \epsilon_1 a/(\epsilon_2+\epsilon_3)$ we have

$$U_0(\rho) = -\frac{2e^2}{a\varepsilon_1}\left(\ln\frac{2\varepsilon_1 a}{(\varepsilon_2+\varepsilon_3)\rho} - C\right),$$

where C is Euler's constant. However, if $\rho \gg a\epsilon_1/(\epsilon_2+\epsilon_3)$, then Coulomb's law is obtained, $U_0(\rho) = -2e^2/\rho(\epsilon_2+\epsilon_3)$. The exact solution of the Schrödinger equation with the potential (15) is not known. Calculations according to the variational method with a trial function $e^{-\lambda\rho}$ lead to the following result for the activation energy δ of the ground donor level. In the case $16ae^2\epsilon_1/(\epsilon_2+\epsilon_3)^2 \equiv p \ll 1$ the logarithmic region in $U_0(\rho)$ is unimportant and the already-known formulas are obtained; in these formulas, however, it is necessary to replace $\tilde{e}^2$ by $2e^2/(\epsilon_2+\epsilon_3)$ (that is, by a larger quantity): $\delta = 8e^4/(\epsilon_2+\epsilon_3)^2$. In the opposite limit, $p \gg 1$, we find

$$\delta(a) = \frac{e^2}{a\varepsilon_1}(\ln 2p - 3) \equiv \delta_0(\ln 2p - 3). \quad (16)$$

In the intermediate region $p \sim 1$, several values of δ were obtained by numerical methods: $\delta(p = 1) = 0.6\,\delta_0$, $\delta(p = 2) = 0.8\,\delta_0$. The quantity ae^2 is also bounded from above by the condition $2ae^2 \ll \epsilon_1$, which means that small distances $\rho \lesssim a$ are unimportant in the formation of a bound state. For a = 150 Å, $\epsilon_1 = 17$, $\epsilon_2 = 1$, $\epsilon_3 = 5$, and $m^* = 0.014\,m_e$, one obtains $p \approx 33$, $2ae^2/\epsilon_1 \approx 0.5$, and $\delta \approx 68$°K.

Thus, in this case the donor's activation energy is increased by an order of magnitude in comparison with the bulk sample. In this connection, we note that in experiments[7] with InSb films, an abrupt decrease in the concentration of electrons was observed upon a reduction of the thickness to the range $a \lesssim 200$ Å. We also mention the transition from a degenerate electron gas to a nondegenerate gas in that same range of thicknesses. This is in agreement with the theory discussed in the present article. Just like in the double-layer case, the existence of quasi-stationary levels near the higher sub-bands can be observed in the absorption of light.

Finally, let us discuss the kinetic characteristics of the three-layer system. In view of the absence of an exact solution to the scattering problem, let us analyze the different limiting cases. For small energies of the particle, $E \ll (\epsilon_2+\epsilon_3)^2/(a\epsilon_1)^2$, only the asymptotic (Coulomb) region of the potential is important. In this case we have to deal with scattering in a Coulomb field which is distorted over small distances, where the de Broglie wavelength of the electron is much larger than the dimensions of the region of distortion. Such a problem can be solved exactly in the asymptotic limit (see, for example, [2], Sec. 136). The corrections to the purely Coulomb scattering cross section have a relative order of magnitude $E(\epsilon_2+\epsilon_3)^2/e^4$, that is, in the indicated energy range these corrections are unimportant and formula (12) is obtained in its quasiclassical limit (it is only necessary to replace $\tilde{e}^2$ by $2e^2/(\epsilon_2+\epsilon_3)$).

Thus, in the temperature range $T \ll (\epsilon_2 + \epsilon_3)^2/(a\epsilon_1)^2$ the mobility depends on the temperature according to the law $\mu \sim \sqrt{T}$. The Born approximation is applicable for energies $E \gg e^4/(\epsilon_2 + \epsilon_3)^2$. In this case the relaxation time is given by

$$\tau(k) = (\varepsilon_2 + \varepsilon_3)\varepsilon_1 a k^3 / 8\pi e^4 (n_s + a n_v).$$

Hence it is seen that for $T \gg e^4/(\epsilon_2 + \epsilon_3)^2$ the temperature dependence of the mobility is described by the bulk law $\mu \sim T^{3/2}$; however, the coefficient of proportionality depends on the thickness of the film.

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259