W-Meson Pair Production in High Energy Electron Collisions

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Total cross sections for $e^{\pm}+e^{-}\rightarrow e^{\pm}+e^{-}+W^{+}+W^{-}$ processes are obtained in the asymptotic region of initial particle energies for cases when the anomalous magnetic moment of the W-meson is zero. The covariant formulation of the Weizsäcker-Williams method, whose validity is ensured by ultrarelativism of the electrons, is employed. The asymptotic behavior of the $e^{+}+e^{-}\rightarrow W^{+}+W^{-}+\mu^{+}+\mu^{-}$ cross section is found.

1. The prediction of W mesons in electron-electron and electron-positron collisions has been considered in a number of studies. The process of annihilation of an electron-positron pair into a pair of W mesons

$$e^{+}+e^{-}\rightarrow W^{+}+W^{-} \tag{1}$$

is discussed in[1-4]. The cross section of the process (1) at $s \gg M^2$ ($s = 4E^2$, E is the electron energy in the c.m.s.; M is the W-meson mass), and for zero anomalous magnetic moment γ of the W meson, is equal to

$$\sigma_1 = \pi\alpha^2 / 3M^2. \tag{2}$$

The processes of single production of W mesons

$$e^{-}+e^{+}\rightarrow W^{\pm}+\mu^{\mp}+\tilde{\nu}_{\mu}(\nu_{\mu}), \tag{3}$$

$$e^{-}+e^{\pm}\rightarrow W^{-}+e^{\pm}+\nu_e \tag{4}$$

were investigated in[5-8]. These processes have appreciably lower characteristic cross sections, $\sim \alpha^2 G/\pi\sqrt{2}$, but their threshold energy is half the threshold energy of reaction (1).

Exact expressions for the integral cross sections of the process

$$e^{+}+e^{-}\rightarrow W^{+}+W^{-}+\gamma \tag{5}$$

were obtained in[9]. Process (5) is the simplest process of W-meson production in electron-positron collisions, whose cross section, even at $\gamma = 0$ increases with increasing s. At $s \gg M^2$ and at photon frequencies $\omega \gg M$ we have

$$\sigma_5 \approx \alpha^3 s / 72M^4. \tag{6}$$

In the present study we investigated the processes

$$e^{-}+e^{-}\rightarrow e^{-}+e^{-}+W^{+}+W^{-}, \tag{7}$$

$$e^{-}+e^{+}\rightarrow e^{-}+e^{+}+W^{+}+W^{-}. \tag{8}$$

The calculations are carried out for $\gamma = 0$ at $s \gg M^2$. We used for the calculation the covariant formulation of the Weizsäcker-Williams (WW) method, developed in[10] and used in this case for two virtual photons. The approximation reduces in essence to the fact that the actual behavior of the cross sections is determined by diagrams with exchange of two virtual photons (Fig. 1a). They are singled out because the invariant masses of both virtual photons, which stand in the denominators of the corresponding matrix elements, can be much smaller than the characteristic energy $\sqrt{s}$ of the process. This circumstance is connected with the ultrarelativism of electrons emitting γ quanta[8,9].

For the case $\gamma \neq 0$, it is impossible to use the WW method to obtain the total cross sections of the processes (7) and (8)[9].

We describe here also the asymptotic behavior of the cross section of the process

$$e^{+}+e^{-}\rightarrow W^{+}+W^{-}+\mu^{+}+\mu^{-}. \tag{9}$$

2. In the lower approximation in the electromagnetic interaction, the process (7) is described by "block" diagrams (Figs. 1 and 2). $P_1(E_1, \mathbf{P}_1)$ and $P_2(E_2, \mathbf{P}_2)$ denote the 4-momenta of the initial electrons, and $P_1'(E_1', \mathbf{P}_1')$ and $P_2'(E_2', \mathbf{P}_2')$ the 4-momenta of the final electrons; The 4-momenta of the W-mesons are designated $P_3(E_3, \mathbf{P}_3)$ and $P_4(E_4, \mathbf{P}_4)$; m is the electron mass and M the W-meson mass.

We introduce the notation

$$s = (p_1+p_2)^2 = 4E^2,$$
$$q_1 = p_1 - p_1', \quad q_2 = p_2 - p_2', \quad q_1' = p_1 - p_2', \quad q_2' = p_2 - p_1',$$
$$\Delta = p_3 + p_4, \quad t_1 = -q_1^2, \quad t_2 = -q_2^2, \tag{10}$$

where E is the electron energy in the c.m.s. We shall consider the case when the anomalous magnetic moment γ of the W meson is equal to zero. The analysis is carried out in the asymptotic region of the energies of the initial particles ($s \gg M^2$) with logarithmic accuracy (it is assumed that $\ln(s/m^2) \gg 1$. Diagrams a′, b′, and c′ of Fig. 1 are obtained from diagrams a, b, and c, by interchanging the momenta of the final electrons, and are called exchange diagrams. It is quite obvious that the exchange diagrams make the same contribution to the total cross section of the process (7) as the direct diagrams a, b, and c. The interference terms between the direct and exchange diagrams should be suppressed, since an appreciable contribution from these diagrams to the total cross section comes from different regions of the electron scattering angles (exact estimates of the interferences of the diagrams of the direct and exchange types, carried out with the aid of Schwartz inequality, shows that the maximal interference is between diagrams a and a′, but the contribution of this interference to the total cross section is smaller by a factor $\ln^2(s/m^2)$ than the contribution of the diagrams a and a′ themselves).

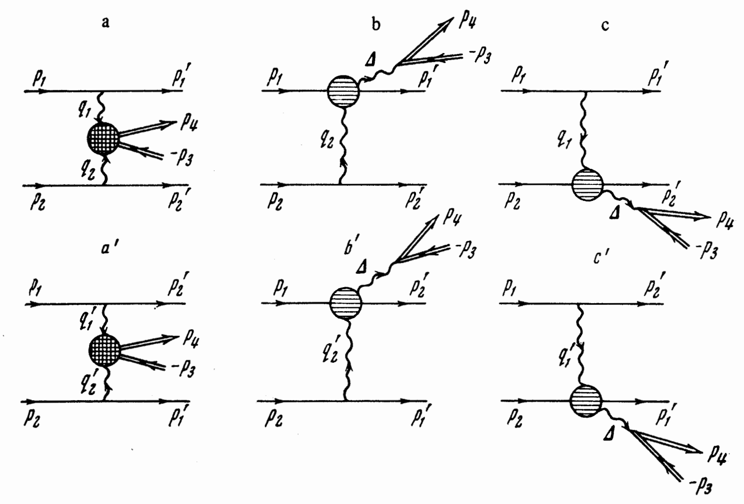

FIG. 1. Feynman diagrams of process (7).

By virtue of the identity of the electrons, the total contribution of the direct and exchange diagrams should be divided by two. This means that we can consider only the diagrams of the direct type and disregard the identity of the electrons. The contribution of the interferences of diagrams a with diagrams b and c to the total cross section vanishes as a result of C-invariance. The contributions of diagrams b and c to the total cross section are equal, and can be calculated with logarithmic accuracy with the aid of the covariant formulation of the WW method[10]. However, inasmuch as the contributions of these diagrams are suppressed by the fact that M^2/s compared with the contribution of the diagram a

$$\left(\sigma_{b,c} \sim \alpha^4 \frac{1}{M^2} \ln^2 \frac{s}{m^2} \ln \frac{s}{M^2}\right),$$

we can neglect this contribution at the assumed accuracy. Thus, we shall calculate in what follows the contribution of diagrams a of Fig. 1 to the total cross section of the process (7).

The matrix element corresponding to diagram a can be written in the form

$$M = -\frac{ie^4}{(2\pi)^5} \frac{m^2 S_{\nu\mu}}{\sqrt{4E_1E_2E_1'E_2'E_3E_4}} \frac{(\bar{u}_1'\gamma^\nu u_1)(\bar{u}_2'\gamma^\mu u_2)}{t_1 t_2}. \quad (11)$$

Here the tensor $S_{\nu\mu}$ (see [9]) describes the transformation of two virtual photons (with squared masses q_1^2 and q_2^2 and with polarizations ν and μ) into two vector mesons. For calculations with logarithmic accuracy, we shall use covariant WW techniques[10] (with allowance for the contribution of the magnetic moment of the electron to the emission of the exchange γ quantum). We first employ the WW method, expressing the cross section of the process (7) in terms of the cross section of the photoprocess $q_2 + p_1 \to p_1' + p_3 + p_4$, and then express the cross section of the photoprocess in terms of the process of the transition of two γ quanta into two vector mesons: $q_1 + q_2 \to p_3 + p_4$. As a result we obtain

$$\sigma_\gamma = \frac{\alpha^2}{\pi^2} \int \frac{ds_\gamma}{s_\gamma} \frac{dt_2}{t_2} \frac{d\Delta^2}{\Delta^2} \frac{dt_1}{t_1} \left(1 - \frac{s_\gamma}{s} + \frac{s_\gamma^2}{2s^2}\right) \left(1 - \frac{\Delta^2}{s_\gamma} + \frac{\Delta^4}{2s_\gamma^2}\right) \sigma_{\gamma\gamma}(\Delta^2) \quad (12)$$

here $s_\gamma = (q_2 + p_1)^2$, $\sigma_{\gamma\gamma}(\Delta^2)$ is the total cross section of the process $\gamma + \gamma \to W^+ + W^-$,

$$\sigma_{\gamma\gamma}(\Delta^2) = \frac{\alpha^2}{2\Delta^2} T_{\nu\mu,\nu'\mu'}(q_1, q_2)\, \delta_{\nu\nu'} \delta_{\mu\mu'} \quad (\text{for} \quad q_1^2 = q_2^2 = 0), \quad (13)$$

$$T_{\nu\mu,\nu'\mu'}(q_1, q_2) = \int \sum_{\text{пол}} S_{\nu\mu} S^*_{\nu'\mu'} \delta(q_1 + q_2 - \Delta) \frac{d^3p_3}{2E_3} \frac{d^3p_4}{2E_4}. \quad (14)$$

We note that the approximation parameter $\ln(s/m^2)$ is large because of the ultrarelativism of the radiating electron, and not as a result of the quasiclassical character of its motion, since in our case, unlike the usual cases where the WW method is employed, the energy lost by the electron is not small when compared to the energy itself, so that $s - s_\gamma \sim s_\gamma \sim \Delta^2 \sim s$[8,9].

We note also that the applicability of the WW method is based on the fact that the departure of the γ quanta from the mass shell should not lead to the appearance of additional factors of the type $t_{1,2}/M^2$. Such a situation does indeed take place in the case $\gamma = 0$ and does not occur for $\gamma \neq 0$ (see also[9]).

The vertex of the interaction of a W meson with one photon is given by

$$\gamma^\lambda_{\nu\mu}(p_2, p_1) = (p_1 + p_2)_\lambda \delta_{\mu\nu} - p_{1\nu}\delta_{\mu\lambda} - p_{2\mu}\delta_{\nu\lambda} + \gamma[\delta_{\mu\lambda}(p_2 - p_1)_\nu - \delta_{\nu\lambda}(p_2 - p_1)_\mu], \quad (15)$$

where p_1, μ and p_2, ν are the momenta and polarizations of the initial and final W mesons, respectively. From (15) we get the relation

$$p_{2\nu}\gamma^\lambda_{\nu\mu}(p_2, p_1) p_{1\mu} = \gamma[k^2 p_{2\lambda} - k_\lambda(p_2 k)]. \quad (16)$$

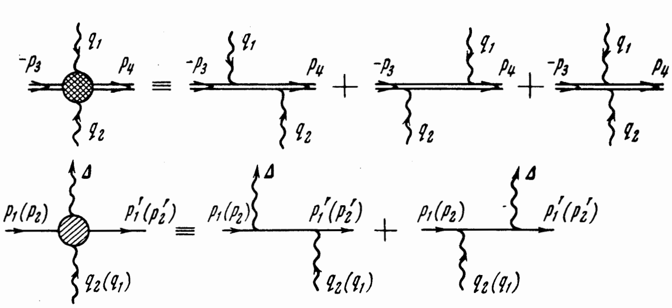

FIG. 2. Feynman diagrams corresponding to the blocks in Fig. 1.

Here $k = p_2 - p_1$ is the momentum of the emitted photon. Using the explicit form of $S_{\nu\mu}$ (see, e.g.,[9]), the relation (16), and the gauge invariance, we can state that if both photons are on the mass shell, then the tensor $T_{\nu\mu,\nu'\mu'}$, and consequently also the cross section $\sigma_{\gamma\gamma}$, is proportional to α^2/M^4 for all γ (with the exception of the case $\gamma = +1$, when, by virtue of additional cancellations, the cross section is given by $\sigma_{\gamma\gamma} \sim \alpha^2/M^2$). In addition, it follows from (16) that the departure of the photons from the mass shell does not lead to the appearance of additional factors of the type t_1/M^2 and t_2/M^2 only in the case when $\gamma = 0$.

We now derive an expression for the total cross section of the process (7). The cross section of the process $\gamma + \gamma \to W^+ + W^-$ at $\Delta^2 \gg M^2$ is given by

$$\sigma_{\gamma\gamma} = \frac{5}{24}\frac{\pi\alpha^2\Delta^2}{M^4}. \tag{17}$$

The invariant variables in (12) vary in the following ranges:

$$\begin{gathered}(2M+m)^2 \leqslant s_\gamma \leqslant (\sqrt{s}-m)^2,\\ t_{2\,min}^{\,max} = -2m^2 + \frac{1}{2s}\{s(s+m^2-s_\gamma)\\ \pm\sqrt{s^2-4m^2s}\sqrt{(s-m^2-s_\gamma)^2-4m^2s_\gamma}\},\\ 4M^2 \leqslant \Delta^2 \leqslant (\sqrt{s_\gamma}-m)^2,\\ t_{1\,min}^{\,max} = -2m^2 + \frac{1}{2s_\gamma}\{(s_\gamma+m^2+t_2)(s_\gamma+m^2-\Delta^2)\\ \pm\sqrt{(s_\gamma-m^2+t_2)^2+4m^2t_2}\sqrt{(s_\gamma-m^2-\Delta^2)^2-4m^2\Delta^2}\}.\end{gathered} \tag{18}$$

When integrating with logarithmic accuracy, we can set the limits equal to

$$\begin{gathered}4M^2 \lessapprox s_\gamma \lessapprox s, \quad \frac{m^2s_\gamma^2}{s^2} \lessapprox t_2 \lessapprox s,\\ 4M^2 \lessapprox \Delta^2 \lessapprox s_\gamma, \quad \frac{m^2\Delta^4}{s_\gamma^2} \lessapprox t_1 \lessapprox s_\gamma.\end{gathered} \tag{19}$$

Integrating formulas (12) within the limits (19), we obtain

$$\sigma_7 = \frac{5s}{54\pi}\left(\ln^2\frac{s}{m^2}\right)\frac{\alpha^4}{M^4}. \tag{20}$$

3. We now proceed to consider the process (8). Process (8) is described by the same diagrams of Figs. 1 and 2, in which it is necessary to make the substitutions

$$p_2 \to -p_2'^+, \quad p_2' \to -p_2^+. \tag{21}$$

Now p_1 is the 4-momentum of the initial electron, p_2^+ is the 4-momentum of the initial positron, p_1' and $p_2'^+$ are the 4-momenta of the final electron and positron, and $s = (p_1 + p_2^+)^2$.

Diagrams a, b, and c describe the production of a pair of W mesons in scattering of an electron by a positron, and diagrams (a′, b′, c′) will be called diagrams of the annihilation type. The analysis of the diagrams a, b, and c is perfectly analogous to that given in the preceding section for process (7). Their contribution to the total cross section is determined by diagram a and is given by formula (20).

We consider now the contribution of the diagrams a′, b′, and c′. It is of independent interest to analyze them separately, since these diagrams describe the process $e^+ + e^- \to W^+ + W^- + \mu^+ + \mu^-$. The interference of the diagram a′ with diagrams b′ and c′ vanishes because of C-invariance. Let us examine the contribution of diagram a′ to the total cross section of the processes (8) and (9).

A rigorous analysis shows that, with logarithmic accuracy, the contribution of diagram a′ to the cross section can be represented in the form

$$d\sigma_{a'} = \int dk^2 f(k^2)\, d\sigma_\gamma(\Delta^2), \tag{22}$$

where $d\sigma_\gamma(\Delta^2)$ is the contribution made to the cross section of the process $e^+ + e^- \to W^+ + W^- + \gamma$ by diagrams with emission of a quantum by the final particle[9], $k^2 = (p_1' + p_2'^+)^2$ is the square of the invariant mass of the produced pair of leptons, and

$$f(k^2) = \frac{\alpha}{3\pi}\frac{k^2+2m^2}{k^4}\sqrt{\frac{k^2-4m^2}{k^2}}. \tag{23}$$

The function $f(k^2)$ has a peak near the lower limit with respect to k^2. The peak has a simple physical meaning—in the region of the peak, the components of the pair travel in parallel, and therefore the invariant mass of the pair is small, so that it can be stated that we are dealing with a photon converted into a pair of particles[11-12]. Integrating formula (22) over the variation region of the variables[12],

$$\begin{gathered}4m^2 \leqslant k^2 \leqslant (\sqrt{s}-2M)^2,\\ 4M^2 \leqslant \Delta^2 \leqslant (\sqrt{s}-\sqrt{k^2})^2,\end{gathered} \tag{24}$$

we obtain with logarithmic accuracy

$$\sigma_{a'} = \frac{\alpha^4}{216\pi}\frac{s}{M^4}\ln\frac{s}{m^2}, \tag{25}$$

where m is the mass of the electron for the case of process (8) and the mass of the muon for the case of process (9).

Estimates of the contribution of diagrams b′ and c′ show that they are of the order of

$$\sigma_{b'} \sim \frac{\alpha^4}{M^2}\ln^3\frac{s}{m^2}, \quad \sigma_{c'} \sim \frac{\alpha^4}{M^2}. \tag{26}$$

It follows therefore that in the initial-particle energy region considered by us ($s \gg M^2$) we can neglect the contribution of diagrams b′ and c′, and also the contribution made to the cross section of processes (8) and (9) by their interference with one another and with the other diagrams. The cross section of process (9) then coincides with the contribution of diagram a′

$$\sigma_9 = \sigma_a'. \tag{27}$$

As to the cross section of process (8), it is determined by the diagram a and its cross section is given by formula (20):

$$\sigma_8 = \sigma_7. \tag{28}$$

We note here that the interference of diagrams a and a′, as follows from estimates based on the Schwartz inequality, does not contain any logarithmic terms.

Let us estimate the order of magnitude of the ratios of the cross sections of the processes (7), (8), and (1) at large values of s/M^2. It follows from (2) and (20) that at M = 5 GeV $\eta = \sigma_8/\sigma_1$ is approximately equal to 0.4 at E = 50 GeV ($s/M^2 = 400$) and to 1 at E = 80 GeV ($s/M^2 \approx 10^3$). Thus, at sufficiently large s the cross sections of the processes (1) and (8) become comparable. We emphasize that the obtained asymptotic formulas describe the behavior of the cross sections of the corresponding processes in the lowest approximation in the electromagnetic interaction. Since the electrodynamics of W mesons is not renormalizable, the region of applicability of the perturbation-theory formulas is bounded by the energies Λ, above which the corresponding quantities (vertex functions, process amplitudes) cannot be described by the first approximation in α without contradicting the fundamental principles of quantum field theory[13-17]. According to estimates made in[15-16], it can be assumed that $\Lambda^2/M^2 \sim 10/\alpha$ at $\gamma = 0$.

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[1]N. Cabibbo and R. Gatto, Nuovo Cimento **20**, 185 (1961); Phys. Rev. **124**, 1577 (1961).

[2]A. A. Komar, Trudy soveshchaniya po slabym vzaimodeĭstviyam (Proceedings of Conference on Weak Interactions), Dubna, 1961.

[3]Nguyen Van Hieu, Zh. Eksp. Teor. Fiz. **42**, 1611 (1962) [Sov. Phys. JETP **15**, 1118 (1962)].

[4]A. D. Dolgov and V. V. Solov'ev, Yad. Fiz. **1**, 860 (1965) [Sov. J. Nucl. Phys. **1**, 615 (1965)].

[5]S. M. Darbinyan and Yu. G. Shakhnazaryan, Yad. Fiz. **3**, 1079 (1966) [Sov. J. Nucl. Phys. **3**, 788 (1966)].

[6]É. A. Choban, Yad. Fiz. **7**, 375 (1968) [Sov. J. Nucl. Phys. **7**, 265 (1968)]; Yad. Fiz. **13**, 624 (1971) [Sov. J. Nucl. Phys. **13**, 354 (1971)].

[7]F. A. Berends and G. B. West Phys. Rev. D **1**, 122 (1970).

[8]A. M. Altukhov and I. B. Khriplovich, Yad. Fiz. **13**, 633 (1971) [Sov. J. Nucl. Phys. **13**, 359 (1971)]; A. M. Altukhov, Yad. Fiz. **13**, 637 (1971) [Sov. J. Nucl. Phys. **13**, 362 (1971)].

[9]N. L. Ter-Isaakyan and V. A. Khoze, Yad. Fiz. **15**, 87 (1972) [Sov. J. Nucl. Phys. **15**, 51 (1972)].

[10]V. N. Gribov, V. A. Kolkunov, L. B. Okun', and V. M. Shekhter, Zh. Eksp. Teor. Fiz. **41**, 1839 (1961) [Sov. Phys. JETP **14**, 1308 (1962)].

[11]V. N. Baĭer and V. A. Khoze, Yad. Fiz. **2**, 287 (1965) [Sov. J. Nucl. Phys. **2**, 205 (1966)].

[12]V. N. Baĭer, V. S. Fadin, and V. A. Khoze, Zh. Eksp. Teor. Fiz. **50**, 156 (1966) [Sov. Phys. JETP **23**, 104 (1966)].

[13]L. D. Landau, Zh. Eksp. Teor. Fiz. **10**, 718 (1940).

[14]J. Oppenheimer, Phys. Rev. **59**, 462 (1941).

[15]S. Bludman and J. Young, Phys. Rev. **126**, 303 (1962).

[16]B. L. Ioffe and M. V. Terent'ev, Zh. Eksp. Teor. Fiz. **47**, 744 (1964) [Sov. Phys. JETP **20**, 496 (1965)].

[17]B. L. Ioffe, Zh. Eksp. Teor. Fiz. **47**, 975 (1964) [Sov. Phys. JETP **20**, 654 (1965)].

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