Excitation of Longitudinal Electromagnetic Waves in a Restricted Plasma by Injection of Relativistic Electron Beams

L. S. Bogdankevich and A. A. Rukhadze

P. N. Lebedev Physics Institute, USSR Academy of Sciences

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A detailed analysis of the nature and conditions of excitation of longitudinal electromagnetic waves in a spatially restricted magnetoactive plasma by a low-density electron beam is presented. The increments, frequency spectra and widths of spectra excited by a plasma wave beam are determined. Nonlinear estimates of the maximal amplitudes of stationary oscillations are presented. The results obtained are discussed from the viewpoint of the possibility of using intense relativistic electron beams for excitation of high-amplitude plasma waves and for the case of probing the earth's ionosphere by a low-intensity relativistic electron beam.

1. INTRODUCTION

THE progress attained in the production of relativistic electron beams with total energy 10^5–10^6 J uncovers great possibilities for a new interesting physical researches. These researches can be arbitrarily broken up into two groups. In the first, use is made of strong-current electron beams with short current pulses, $\tau \lesssim 10^{-7}$ sec, but with high power—electron energy $\mathscr{E} \sim (1-10)$ MeV and current $J \sim (10^5$–$10^6)$ A. Such beams are of interest for the investigation of the interaction of large energy concentrations with matter, for the creation of powerful sources of electromagnetic radiation, for the acceleration of heavy ions, etc. The second group is connected with the use of electron beams of large current-pulse duration, $\tau \lesssim 1$ sec, but low power, $\mathscr{E} \sim (1-10)$ MeV and $J \sim (0.1-1)$ A. Such beams can be used for geophysical research, to sound the ionosphere, to produce artificial auroras, etc.

The present paper is devoted to a theoretical investigation of the interaction of relativistic electron beams with a denser plasma both under laboratory and under ionospheric conditions. Principal attention is paid to the question of exciting the natural electromagnetic oscillations in the plasma by the electron beam. However, before we proceed to a concrete formulation of the problem and to an analysis of the result, let us stop to discuss more general questions of the injection of a relativistic electron beam into a plasma, a question which is of great importance for the entire analysis that follows.

It is known that the limiting vacuum current in a cylindrical electron beam of radius r_0 is of the order of[1]

$$J_0 \approx (mc^3/e)(\gamma^{2/3}-1)^{3/2} \approx 17(\gamma^{2/3}-1)^{3/2}\ \text{kA},$$

where

$$\gamma = (1-u^2/c^2)^{-1/2}$$

is the relativistic factor. The vacuum current is limited by the space charge of the beam electrons and is determined from the equality of the Debye radius to the beam radius $r_0 \approx u\gamma^{1/2}/\omega_{L1}$, where $\omega_{L1} = (4\pi e^2 n_1/m)^{1/2}$ is the Langmuir frequency of the beam and n_1 is the density of the beam electrons. On the other hand, in electron beams with compensated space charge (for example, when beams propagate in a denser plasma), a much larger current should be attained, approximately $u^2\gamma^3/c^3(\gamma^{2/3}-1)^{3/2}$ times larger than the limiting vacuum current.[1] It is easily seen that for the indicated parameters of high-power electron beams, the currents are much larger than the limiting vacuum value, i.e., $u_{L1}^2 \gg u^2\gamma/r_0^2$, but smaller than the critical current of the compensated beam. On the other hand, for beams of low power, the currents are always much smaller than the limiting vacuum current of the beam, $\omega_{L1}^2 \ll u^2\gamma/r_0^2$. Thus, for both high-power and low-power beams with the parameters indicated above, there are no obstacles to injection into a denser plasma from the point of view of the critical currents.

Another possible obstacle to the injection of electron beams into a plasma is the magnetic field of the beam current. In the case of a powerful strong-current beam with a current exceeding the limiting value, the energy of the magnetic field of the current turns out to be so large that the propagation of such a beam is practically impossible. However, as shown in [2,3], when a beam is injected into a denser plasma under conditions when $u_{L2}r_0 \gg u$, where $u_{L2} = (4\pi e^2 n_2/m)^{1/2}$ is the plasma Langmuir frequency and n_2 is the density of the plasma electrons, the magnetic field of the beam current is practically completely cancelled by the current induced in the plasma. Taking this circumstance into account, we consider below the interaction of a relativistic electron beam with a plasma, assuming that the inequalities $n_1 \ll n_2\gamma$ and $\omega_{L2}r_0 \gg u$ are satisfied. This makes it possible to neglect, besides the space charge of the beam electrons, also the magnetic field of the current. We note incidentally that when powerful electron beams, with a current exceeding the limiting vacuum value, interact with a plasma, the last inequality is automatically satisfied. As to low-power beams, in which the current is much smaller than the limiting vacuum current, the energy of the magnetic field in them is negligibly small compared with the kinetic energy of the electrons, and the problem of magnetic neutralization does not arise in their case. We shall nonetheless consider here, too, the interaction of such beams with a denser plasma, assuming the inequality $n_1 \ll n_2\gamma$ to be satisfied.

It should always be borne in mind that magnetic neutralization of the beam takes place within a time[3]

$$\tau < \tau_0 = \nu^{-1}(\omega_{L2} r_0/u)^2,$$

where ν is the plasma-electron collision frequency. This imposes a definite limitation on the duration of the pulsed injection of the electron beam into the plasma, namely, the pulse duration τ should be less than τ_0. Satisfaction of this requirement is important only for strong-current beams, the injection of which into the plasma in the absence of magnetic neutralization is strongly hindered.

We limit ourselves below to an investigation of the interaction of monoenergetic electron beams with a plasma. Actually, the beams cannot be strictly monoenergetic. Even in the case of injection within a time on the order of $1/\omega_{L2}$, i.e., before charge and magnetic neutralization of the beam set in, both the transverse and longitudinal velocities of the beam electrons acquire a certain scatter under the influence of the radial field of the charge; the order of magnitude of this scatter is

$$\frac{\Delta v_\perp}{u} \approx \frac{\Delta v_\parallel}{u} \approx \left(\frac{n_1}{n_2\gamma}\right)^{1/2} \frac{\omega_{L1}}{\gamma^{1/2}} \frac{r_0}{u}.$$

Recognizing that for beams with a current smaller than the critical current of the compensated beam we have $\omega_{L1} r_0 < u\gamma^{1/2}$, we can always neglect this scatter for the problems considered below.

2. FORMULATION OF PROBLEM. INITIAL EQUATIONS

The interaction of a charged-particle beam with a plasma and the excitation of natural electromagnetic oscillations in a plasma have been the subject of a tremendous number of both theoretical and experimental investigations. These investigations stem from the work of Akhiezer and Faĭnberg and of Bohm and Gross.[4] It is not our purpose to analyze in detail the results of the numerous investigations in this field.[1)] We note only that the overwhelming majority of the theoretical papers consider the interaction of spatially unbounded beams with a plasma; this pertains in particular to relativistic beams. On the other hand, in real experiments, both in the laboratory and in the ionosphere, the electron beam is always bounded, let alone the fact that under laboratory conditions the plasma is also bounded in space. Attempting to adhere as closely as possible to experimental conditions, we shall consider below the interaction of a bounded relativistic electron beam, with radius $r_0 \le R$, passing through a plasma that fills uniformly a metallic waveguide of radius R. An external longitudinal magnetic field $\mathbf{B}_0$ is applied to the system and keeps the beam from spreading radially,

$$B_0^2 > 8\pi n_1 mc^2\gamma. \tag{2.1}$$

This condition enables us also to confine ourselves only to longitudinal electrostatic natural oscillations excited by the electron beam in the plasma.

Actually the condition (2.1) is quite stringent. To keep the beam from spreading it suffices to require that the magnetic pressure exceed the kinetic pressure due to the transverse scatter of the beam-electron velocities. In the case when the ratio of the beam density to the plasma density is small, the system oscillations are longitudinal with a good degree of accuracy. We shall therefore try where possible to avoid using this inequality.

Taking into account the statements made above concerning the beam-electron velocity scatter and being interested in rapid processes in the plasma, we neglect the thermal motion of the particles in both subsystems. For the potential of the small perturbations ($\mathbf{E} = -\nabla\Phi$), having a time and coordinate dependence in the form

$$f(r)\exp(-i\omega t + il\varphi + ik_z z),$$

where ω is the frequency, and l and k_z are respectively the azimuthal and longitudinal wave numbers of the perturbations, the following equation holds true[1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\varepsilon_\perp r\frac{\partial\Phi}{\partial r}\right) + \left(\frac{l}{r}\frac{\partial g}{\partial r} - \varepsilon_\perp\frac{l^2}{r^2} - \varepsilon_\parallel k_z^2\right)\Phi = 0. \tag{2.2}$$

We have introduced here the notation

$$\varepsilon_\perp = 1 - \sum\frac{\omega_L^2\gamma^{-1}}{(\omega - \mathbf{ku})^2 - \Omega^2\gamma^{-2}},$$
$$\varepsilon_\parallel = 1 - \sum\frac{\omega_L^2\gamma^{-3}}{(\omega - \mathbf{ku})^2}, \tag{2.3}$$
$$g = \sum\frac{\omega_L^2\,\Omega\gamma^{-2}}{(\omega - \mathbf{ku})[(\omega - \mathbf{ku})^2 - \Omega^2\gamma^{-2}]},$$

where $\Omega = eB_0/mc$ and the summation extends over the electrons of the plasma and of the beam.

Equation (2.2) is supplemented by boundary conditions obtained from the equation itself by integrating over a narrow layer at the interface between the two media

$$\{\Phi\}|_{r=r_0} = 0 \quad \Phi_{r=R} = 0,$$
$$\left\{\varepsilon_\perp\frac{\partial\Phi}{\partial r} + \frac{l}{r}g\Phi\right\}_{r=r_0} = 0. \tag{2.4}$$

The solution of (2.2) over the regions takes the form

$$\Phi_1 = C_1 I_l(\alpha_1 r) \quad \text{for} \quad r \le r_0,$$
$$\Phi_2 = C_2 I_l(\alpha_2 r) + C_3 K_l(\alpha_2 r) \quad \text{for} \quad r_0 \le r \le R, \tag{2.5}$$

where K and I are Bessel functions of imaginary argument and

$$\alpha_{1,2}^2 = k_z^2\,\varepsilon_{\parallel 1,2}/\varepsilon_{\perp 1,2}.$$

The subscripts 1 and 2 pertain respectively to the regions $r \le r_0$ and $r_0 \le r \le R$.

Substituting (2.5) in the boundary conditions (2.4), we obtain the dispersion equation for the longitudinal and electromagnetic oscillations of the system considered by us:

$$\varepsilon_{\perp 1}\alpha_1 r_0\frac{I_l'(\alpha_1 r_0)}{I_l(\alpha_1 r_0)} + l(g_1 - g_2) + \varepsilon_{\perp 2}\alpha_2 r_0 \cdot\frac{K_l(\alpha_2 R)I_l'(\alpha_2 r_0) - I_l(\alpha_2 R)K_l'(\alpha_2 r_0)}{I_l(\alpha_2 R)K_l(\alpha_2 r_0) - I_l(\alpha_2 r_0)K_l(\alpha_2 R)} = 0. \tag{2.6}$$

In the limit when the radii of the beam and of the plasma coincide, $r_0 = R$, Eq. (2.6) reduces to

$$I_l(\alpha_1 r_0) = 0, \tag{2.7}$$

or

$$k_z^2\varepsilon_{\parallel 1} + \mu_{sl}^2\varepsilon_{\perp 1}/r_0^2 = 0, \tag{2.8}$$

where μ_{sl} are the roots of the Bessel function $J_l(\mu_{sl}) = 0$. In the opposite limit of an unbounded plasma, when $R \to \infty$ (this case corresponds to interaction of a beam with a practically unbounded ionospheric plasma), we

[1)]Such an analysis and a detailed bibliography on the interaction of charged-particle beams with a plasma can be found in Faĭnberg's review[5] and Mikhaĭlovskiĭ's monograph[6].

obtain from (2.6)

$$\varepsilon_{\perp 1}\alpha_1 r_0\frac{I_l'(\alpha_1 r_0)}{I_l(\alpha_1 r_0)}-\varepsilon_{\perp 2}\alpha_2 r_0\frac{K_l'(\alpha_2 r_0)}{K_l(\alpha_2 r_0)}+l(g_1-g_2)=0. \quad (2.9)$$

In the analysis of Eq. (2.6) we shall assume, in addition to the indicated limitations, that the plasma density, as already emphasized in the introduction, greatly exceeds the density of the beam electrons, $\gamma n_2 \gg n_1$. Only under this condition are the spectra of the electromagnetic oscillations of the system determined by the parameters of the plasma itself; the beam, on the other hand, leads to excitation of natural oscillations of the plasma. In addition, this condition ensures narrowness of the spectrum of the plasma oscillations excited by the beam.

3. EXCITATION OF NATURAL MODES IN A BOUNDED PLASMA BY A POWERFUL ELECTRON BEAM

We start the analysis of Eq. (2.6) with the case of the interaction of a relativistic beam with a bounded plasma, when the radii R and r_0 are of the same order. In such a formulation, this problem is of interest for the development of powerful sources of electromagnetic radiation. We shall therefore concentrate on the parameters of a high-power electron beam, $J \approx 10^5$ A, $r_0 = 1$–3 cm, $n_1 \sim 10^{12}$–10^{13} cm^{-3}, and $\mathscr{E} \approx 1$–10 MeV, assuming that the inequalities $\omega_{L2}^2 \gg \omega_{L1}^2 > u^2\gamma/r_0^2$ are satisfied.

a) $R = r_0$. In the limits when the radii of the beam and of the plasma coincide, Eq. (2.6) reduces to (2.8):

$$k_z^2\left(1-\frac{\omega_{L2}^2}{\omega^2}\right)+k_\perp^2\left(1-\frac{\omega_{L2}^2}{\omega^2-\Omega^2}\right)-\frac{k_z^2\omega_{L1}^2\gamma^{-3}}{(\omega-k_zu)^2}-\frac{k_\perp^2\omega_{L1}^2\gamma^{-1}}{(\omega-k_zu)^2-\Omega^2\gamma^{-2}}=0, \quad (3.1)$$

where $k_\perp^2 \equiv \mu_{sl}^2/r_0^2$. Neglecting the beam contribution in the first approximation, we determine the spectrum of the natural frequencies of the oscillations of a bounded plasma

$$\omega_{1,2}^2(k_z)=\frac{1}{2}(\omega_{L2}^2+\Omega^2)\pm\left[\frac{1}{4}(\omega_{L2}^2+\Omega^2)^2-\frac{k_z^2}{k_z^2+k_\perp^2}\omega_{L2}^2\Omega^2\right]^{1/2}. \quad (3.2)$$

On the other hand, it is seen from (3.1) that the contribution of the beam is maximal near the straight lines

$$\omega=k_zu, \quad \omega=k_zu\pm\Omega\gamma^{-1}. \quad (3.3)$$

The intersection of these straight lines with the spectral curves is precisely the region in which strong interaction of the beam with the plasma takes place and the natural plasma oscillations build up.

The instability at the intersections of the straight line $\omega = k_zu$ with the branches of the natural oscillations $\omega_{1,2}(k_z)$ is customarily called two-stream or Cerenkov instability; the instability of the intersections of $\omega = k_zu \pm \Omega\gamma^{-1}$ and $\omega_{1,2}(k_z)$ is called cyclotron instability.[5] Let us analyze these instabilities separately.

For two-stream instability

$$\omega=k_zu+\delta=\omega_0+\delta, \quad \delta\ll\omega_0, \quad (3.4)$$

where ω_0 coincides with one of the roots of (3.2). It is easy to prove that at $2.4u > r_0 \min(\omega_{L2}, \Omega)$ the development of two-stream instability is possible only on the upper branch of the natural oscillations ω_1; when $2.4u < r_0 \min(\omega_{L2}, \Omega)$ the two-stream instability can develop on both branches ω_1 and ω_2. According to (3.1), the growth increment δ of the two-stream instability satisfies the relation

$$2\frac{\delta}{\omega_0}\left[k_z^2\frac{\omega_{L2}^2}{\omega_0^2}+k_\perp^2\frac{\omega_{L2}^2\omega_0^2}{(\omega_0^2-\Omega^2)^2}\right]-\frac{k_z^2\omega_{L1}^2\gamma^{-3}}{\delta^2}-\frac{k_\perp^2\omega_{L1}^2\gamma^{-1}}{\delta^2-\Omega^2\gamma^{-2}}=0. \quad (3.5)$$

In the case of intense plasma, $\omega_{L2} \gg \Omega$, the frequency spectrum and the maximum growth increment of the oscillations on the upper branch are determined, according to (3.2) and (3.5), by the formulas

$$\omega_1=\omega_{L2}\left[1+\frac{\Omega^2}{2\omega_{L2}}\left(1+\frac{\mu_{sl}^2u^2}{r_0^2\omega_{L2}^2}\right)^{-1}\right]\approx\omega_{L2};$$

$$\operatorname{Im}\delta_{1max}=\begin{cases}\dfrac{\sqrt{3}}{2}\left(\dfrac{n_1\gamma^{-1}}{2n_2}\right)^{1/3}\omega_{L2} & \text{for } \operatorname{Im}\delta_1>\Omega\gamma^{-1},\\[2ex] \dfrac{\sqrt{3}}{2}\left(\dfrac{n_1\gamma^{-3}}{2n_2}\right)^{1/3}\dfrac{\omega_{L2}}{(1+(2,4)^2u^2/r_0^2\omega_{L2}^2)^{1/3}} & \text{for } \operatorname{Im}\delta_1<\Omega\gamma^{-1},\end{cases} \quad (3.6)$$

where short-wave radial modes, $\mu_{sl}u > r_0\omega_{L2}$, are excited in the first of these cases and the fundamental mode, $\mu_{00} = 2.4$, is excited in the second. At the lower branch, in the case of a dense plasma, the oscillations can be excited only if $\Omega > \mu_{sl}u/r_0 \geq 2.4u/r_0$, and the maximum increment and the corresponding oscillation frequency are

$$\omega_{2max}=\frac{\sqrt{3}}{2}\Omega, \quad \operatorname{Im}\delta_{2max}=\frac{3}{8}\left(\frac{n_1\gamma^{-3}}{n_2}\right)^{1/3}\Omega. \quad (3.7)$$

It follows from (3.6) and (3.7) that in a dense plasma, $\omega_{L2} \gg \Omega$, there will practically always be excited the upper branch of the plasma waves with frequency $\omega_1 \approx \omega_2$, since $\delta_2 \ll \delta_1$; the spectral width of the oscillations excited in this case is determined by (3.6) (recognizing that $\operatorname{Re}\delta_1 = 3^{-1/2}\operatorname{Im}\delta_1$). The situation is different in a rarefied plasma, in which $\Omega \gg \omega_{L2}$. For the upper branch of the oscillations in such a plasma we obtain from (3.2) and (3.5)

$$\omega_1=\Omega\left[1+\frac{\omega_{L2}^2}{2\Omega^2}\left(1+\frac{\mu_{sl}^2u^2}{r_0^2\Omega^2}\right)^{-1}\right]\approx\Omega,$$

$$\operatorname{Im}\delta_{1max}\approx\frac{\sqrt{3}}{2}\left(\frac{n_1\gamma^{-3}}{2n_2}\frac{\omega_{L2}}{4\Omega}\right)^{1/3}\omega_{L2}. \quad (3.8)$$

The maximum growth increment is possessed in this case by oscillations with transverse wavelength $\mu_{sl}/r_0 \approx \Omega/u$. As to the lower branch, the plasma waves can be excited in a rarefied plasma only under the condition

$$\omega_{L2}>\mu_{sl}u/r_0\geqslant 2{,}4u/r_0,$$

with the maximum growth increment and the corresponding oscillation frequency determined by the expressions

$$\omega_{2max}=\frac{\sqrt{3}}{2}\omega_{L2}, \quad \operatorname{Im}\delta_{2max}={}^3/_8(n_1\gamma^{-3}/n_2)^{1/3}\omega_{L2}. \quad (3.9)$$

It is seen from (3.8) and (3.9) that in a rearefied plasma, in which $\Omega \gg \omega_{L2}$, unlike in a dense plasma (with $\omega_{L2} \gg \Omega$), the oscillations at the lower branch should be predominantly excited (it is assumed that $\omega_{L2} > 2.4u/r_0$), since $\delta_2 \gg \delta_1$. The frequency of the excited waves is in this case also of the order of the plasma frequency, $\omega_2 \approx \omega_{L2}$, and the width of the spectrum is of the order of the growth increment and is given by formula (3.9).

Let us examine now the cyclotron instability produced by a relativistic electron beam in a plasma. We write

$$\omega = k_z u \pm \Omega\gamma^{-1} + \delta = \omega_0 + \delta, \qquad \delta \ll \omega_0, \tag{3.10}$$

where ω_0 is one of the roots of (3.2). We then obtain from (3.1)

$$2\frac{\delta}{\omega_0}\left[\frac{k_z^2\omega_{L2}^2}{\omega_0^2} + \frac{k_\perp^2\omega_{L2}^2\omega_0^2}{(\omega_0^2-\Omega^2)^2}\right] - \frac{k_z^2\omega_{L1}^2\gamma^{-3}}{(\delta\pm\Omega\gamma^{-1})^2} - \frac{k_\perp^2\omega_{L1}^2\gamma^{-1}}{\delta(\delta\pm 2\Omega\gamma^{-1})} = 0. \tag{3.11}$$

This relation determines the growth increments of the oscillations.

In the case of a dense plasma, $\omega_{L2} \gg \Omega$, we obtain from (3.11) for the maximum growth increments of the waves in the upper branch of the oscillations, whose frequencies are equal to ω_1 (see (3.6)),

$$\operatorname{Im}\delta_{1max} = \begin{cases} \dfrac{\sqrt{3}}{2}\left(\dfrac{n_1\gamma^{-1}}{2n_2}\right)^{1/3}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta_1 > \Omega\gamma^{-1}, \\ \dfrac{1}{2}\left(\dfrac{n_1}{n_2}\dfrac{\omega_{L2}}{\Omega}\right)^{1/2}\omega_L, & \text{for} \quad \operatorname{Im}\delta_1 < \Omega\gamma^{-1}. \end{cases} \tag{3.12}$$

In the first of these cases the cyclotron instability degenerates into two-stream instability and coincides with (3.6), and in the second it has an essentially different character. It suffices to note that the cyclotron instability is accompanied by excitation of mainly short-wave modes, $\mu_{sl}u > r_0\omega_{L2}$, whereas two-stream instability is characterized by excitation of the fundamental radial oscillation mode. From a comparison of the increments (3.6) and (3.12) we see in this case that under the condition

$$[1 + (2{,}4)^2u^2 / r_0^2\omega_{L2}^2]^{-1/3} > (n_1/n_2)^{1/6}(\omega_{L2}/\Omega)^{1/2},$$

the two-stream instability should predominate over cyclotron instability. In the opposite limiting case cyclotron instability predominates, and therefore the spectrum width of the excited plasma waves and their growth increment are determined by formula (3.12). As to the lower oscillation branch, it can be easily shown that in a dense plasma, $\omega_{L2} > \Omega$, the maximum growth increment of a developing cyclotron instability

$$\operatorname{Im}\delta_{2max} \approx {}^1/_2(n_1/n_2)^{1/2}\Omega, \tag{3.13}$$

is always much smaller than the growth increment of the oscillations on the upper branch. Therefore the cyclotron instability, just like the two-stream instability, should not appear in a dense plasma at the lower branch.

In a rarefied plasma in which $\Omega \gg \omega_{L2}$ and the oscillation frequency on the upper branch, in accord with (3.9), is of the order of $\omega_1 \approx \Omega$, the maximum growth increment of these oscillations is equal to

$$\operatorname{Im}\delta_{1max} \approx \frac{1}{2}\frac{\omega_{L2}}{\Omega}\left(\frac{n_1}{n_2}\right)^{1/2}\left[1 + \frac{\omega_{L2}^4 r_0^2}{(2{,}4u\Omega)^2}\right], \tag{3.14}$$

and corresponds to the fundamental radial mode, $\mu_{00} = 2.4$. This increment is in essence much smaller than (3.8), and therefore in the upper branch of a rarefied plasma the two-stream instability will always predominate over the cyclotron instability. Recognizing in addition that in a rarefied plasma the two-stream instability on the lower branch of oscillations will have a still larger increment (see (3.9)), whereas the cyclotron instability increment is relatively small on this branch:

$$\operatorname{Im}\delta_{2max} = {}^1/_3(n_1/6n_2)^{1/2}(\omega_{L2}/\Omega)^{1/2}\omega_{L2}, \tag{3.15}$$

(the frequency of the excited oscillations is of the order of $\omega_{2max} \approx \omega_{L2}/\sqrt{3}$ in this case), then we arrive at the conclusion that cyclotron instability should practically never appear in a rarefied plasma; the character of the excitation of oscillations in such a plasma is practically always governed by two-stream instability with the spectrum (3.9).

b) $R > r_0$. From the experimental point of view, it is more realistic to consider the interaction of a powerful electron beam with a plasma when $R > r_0$, with R and r_0 of the same order of magnitude. Equation (2.6) can then be investigated analytically only in the opposite limiting cases of long ($\alpha_{1,2}R \ll 1$) or short ($\alpha_{1,2}R \gg 1$) wavelengths.

In the long-wave limit Eq. (2.6) takes the form

$$\varepsilon_{\perp 1}[l + {}^1/_2\,\alpha_1^2 r_0^2] + l(g_1 - g_2) + \eta_l\varepsilon_{\perp 2} = 0. \tag{3.16}$$

where

$$\eta_l = \begin{cases} \ln^{-1}(R/r_0) & \text{for} \quad l = 0, \\ l[1 + (R/r_0)^{2l}][-1 + (R/r_0)^{2l}]^{-1} & \text{for} \quad l \neq 0. \end{cases} \tag{3.17}$$

Substituting in (3.16) the explicit expressions (2.3), we obtain for axially-symmetrical modes with $l = 0$

$$1 - \frac{\omega_{L2}^2}{\omega^2} + \frac{2}{k_z^2 r_0^2\ln(R/r_0)}\left(1 - \frac{\omega_{L2}^2}{\omega^2-\Omega^2}\right) - \frac{\omega_{L1}^2\gamma^{-3}}{(\omega - k_z u)^2} = 0. \tag{3.18}$$

It is easy to see that in the absence of the beam the frequency spectrum of the plasma oscillations in question coincides with (3.2) if $k_\perp^2$ is studied mean $2/r_0^2\ln(R/r_0)$.

Allowance for the two-stream term in (3.18) is significant only under conditions of Cerenkov instability, when $\omega_0 = k_z u$, where ω_0 is one of the roots of (3.2), and the growth increment (and with it also the width of the excited-oscillation spectrum) is determined by the expression

$$\operatorname{Im}\delta = \frac{\sqrt{3}}{2}\left(\frac{n_1\gamma^{-3}}{2n_2}\right)^{1/3}\omega_0\left[1 + \frac{2u^2}{r_0^2\ln(R/r_0)}\frac{\omega_0^2}{(\omega_0^2-\Omega^2)^2}\right]^{-1/3}. \tag{3.19}$$

This formula, like (3.2), is general and describes the build up of the oscillations on both the upper and lower branches. It follows from it that in a dense plasma, $\omega_{L2} \gg \Omega$, the more probable is the excitation of the upper mode with frequency $\omega_0 = \omega_1 \approx \omega_{L2}$ (see (3.6)) and with growth increment

$$\operatorname{Im}\delta_1 \approx \frac{\sqrt{3}}{2}\left(\frac{n_1\gamma^{-3}}{2n_2}\right)^{1/3}\omega_{L2}\left[1 + \frac{2u^2}{r_0^2\ln(R/r_0)}\frac{1}{\omega_{L2}^2}\right]^{-1/3}. \tag{3.20}$$

On the other hand, in a relatively rare plasma, $\Omega \gg \omega_{L2}$, under the condition $\omega_{L2}^2 > 2u^2/r_0^2\ln(R/r_0)$ (this is the only case to be considered for strong beams) excitation of the lower plasma mode is more probable, with

$$\omega_2^2 = \omega_{L2}^2 - \frac{2u^2}{r_0^2\ln(R/r_0)}, \qquad \operatorname{Im}\delta_2 = \frac{\sqrt{3}}{2}\left(\frac{n_1\gamma^{-3}}{2n_2}\right)^{1/3}\omega_2. \tag{3.21}$$

Let us consider now the axially-asymmetrical modes with $l \neq 0$. Equation (3.16) for such modes, upon substitution of expressions (2.3), reduces to

$$1 - \frac{\omega_{L2}^2}{\omega^2-\Omega^2} - \frac{\omega_{L1}^2\varkappa\gamma^{-1}}{\omega - \mathbf{ku} \pm \Omega\gamma^{-1}}\frac{1}{\omega - \mathbf{ku}} = 0, \tag{3.22}$$

where $2\kappa = 1 - (r_0/R)^{2l}$. We see easily that the spectrum of the oscillation frequencies coincides in this case with the upper hybrid frequency, $\omega_0^2 \approx \omega_{L2}^2 + \Omega^2$, and their growth increment, both in the development of two-stream instability and of cyclotron instability, is determined by the expressions

$$\operatorname{Im}\delta = \begin{cases} \frac{\sqrt{3}}{2}\left(\frac{\varkappa n_1\gamma^{-1}}{2n_2}\right)^{1/3}\omega_{L2} & \text{for} \quad \delta > \Omega\gamma^{-1} \\ \left(\frac{n_1}{2n_2}\frac{\varkappa\omega_{L2}}{\omega_0\Omega}\right)^{1/2}\omega_{L2} & \text{for} \quad \delta < \Omega\gamma^{-1}. \end{cases} \tag{3.23}$$

Comparing (3.20), (3.21), and (3.23), we reach the conclusion that in a dense plasma, $\omega_{L2} \gg \Omega$, under the condition

$$(\varkappa n_1 / 2n_2)^{1/3}\omega_{L2} > \Omega\gamma^{-2/3},$$

excitation of modes with $l = 0$ is preferred if $\kappa < \gamma^{-2}$. In the opposite case, excitation of modes with $l \neq 0$ is more probable. If

$$(\varkappa n_1 / 2n_2)^{1/3}\omega_{L2} < \Omega\gamma^{-2/3},$$

then the modes with $l = 0$ will be predominantly excited, provided that

$$(n_1 / 2n_2)^{1/6}(\varkappa\omega_{L2} / \Omega)^{1/2} < \gamma^{-1};$$

In the case of the opposite limit, on the other hand, the modes with $l \neq 0$ predominante. In a rarefied plasma, $\omega_{L2} \ll \Omega$, under the condition

$$\omega_{L2}^2 > 2u^2 / r_0^2 \ln(R / r_0),$$

the increment (3.21), which corresponds to symmetrical oscillations with $l = 0$, will practically always be maximal.

Let us consider, finally, short-wave radial oscillations $\alpha_2 R \gg 1$ and $\alpha_2 r_0 \gg 1$. In this limit, Eq. (2.6) takes the form

$$\varepsilon_{\perp 1}\alpha_1 r_0 I'_l(\alpha_1 r_0) / I_l(\alpha_1 r_0) + l(g_1 - g_2) + \varepsilon_{\perp 2}\alpha_2 r_0 \operatorname{cth}\alpha_2(R - r_0) = 0. \tag{3.24}$$

When $\alpha_2 r_0 \gg 1$ the roots of this equation coincide with high degree of accuracy with the roots of Eq. (2.7) (or, which is the same, (2.8) and (3.1)). Therefore the foregoing analysis of the beam stability when the waveguide is completely filled remains in force also in the present case of shortwave oscillations. This makes it possible to compare formulas (3.20), (3.21), and (3.23) with formulas (3.6)–(3.9) and (3.12)–(3.15). It follows from such a comparison that in a dense plasma, $\omega_{L2} \gg \Omega$, the growth increment of the short-wave oscillations are always larger than of the long-wave ones. The difference between them, however, is not very large, of the order of $\gamma^{2/3}$ or $\gamma^{1/3}$; therefore both short-wave and long-wave modes can appear in fact. On the other hand, in a rarefied plasma, $\Omega \gg \omega_{L2}$, the growth increments of the short-wave modes, to the contrary, are smaller than those of the long-wave modes. But even here their ratio is of the order of $\sqrt{3}/2^{5/3} \approx 0.5$, and therefore excitation of both long-wave and short-wave modes should actually be regarded as practically equally probable.

4. STABILITY OF A BOUNDED LOW-POWER BEAM IN AN UNBOUNDED PLASMA

We shall devote this section to the interaction between a relativistic electron beam of low power and a spatially unbounded plasma. Having in mind the application of the results to the sounding of the ionosphere, we shall concentrate on the following beam and plasma parameters: $r_0 \approx 10^2$ cm, $n_1 \approx 10^4$ cm^{-3}, $J_0 \approx 1$ A, $\mathscr{E} \approx (1–10)$ MeV, and $n_2 \approx 10^5–10^6$ cm^{-3}. The inequalities ω_{L1}^2, $\omega_{L2}^2 \ll u^2/r_0^2$ are satisfied in this case for both the beam and the plasma.

The dispersion equation (2.9) of the oscillations in such a system, after substituting the explicit expressions (2.3), can be rewritten in the form

$$1 - \frac{\omega_{L2}^2}{\omega^2 - \Omega^2} - \frac{\omega_{L1}^2\gamma^{-1}}{(\omega - \mathbf{ku})^2 - \Omega^2\gamma^{-1}}\frac{\varphi_{1l}(\alpha_1 r_0)}{\varphi_{1l}(\alpha_1 r_0) + \varphi_{2l}(\alpha_2 r_0)} \times \left[1 - \frac{l\gamma^{-1}\Omega}{(\omega - \mathbf{ku})\varphi_{1l}(\alpha_1 r_0)}\right] = 0, \tag{4.1}$$

where

$$\varphi_{1l}(\alpha_1 r_0) = \alpha_1 r_0 I_l'(\alpha_1 r_0) / I_l(\alpha_1 r_0),$$
$$\varphi_{2l}(\alpha_2 r_0) = -\alpha_2 r_0 K'_l(\alpha_2 r_0) / K_l(\alpha_2 r_0). \tag{4.2}$$

It is easy to see that the spectrum of the oscillation frequencies in the absence of a beam coincides with the upper hybrid frequency

$$\omega_0^2 = \omega_{L2}^2 + \Omega^2. \tag{4.3}$$

The presence of the beam leads to a buildup of these oscillations as a result of the development of either two-stream (Cerenkov) or cyclotron instability. It is obvious that the growth increment of the oscillations is maximal under conditions when the coefficient in the two-stream term of (4.1), $\varphi_{1l}/(\varphi_{1l} + \varphi_{2l})$, reaches a maximum. In the analysis of this relation, it is necessary to distinguish between two possibilities.

a) Surface-type oscillations, $\alpha > 0$. It is easy to show that for axially-symmetrical surface-oscillation modes ($l = 0$), the most favorable buildup conditions are obtained in the short-wave limit, when $\alpha_{1,2} r_0 \gg 1$ and $\varphi_{10}/(\varphi_{10} + \varphi_{20}) \to 1/2$. The growth increment, and consequently also the width of the spectrum of the oscillations excited by the beam in this case, is determined by the expression

$$\operatorname{Im}\delta = \begin{cases} \frac{\sqrt{3}}{2}\left(\frac{n_1\gamma^{-1}}{4n_2}\right)^{1/3}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta > \Omega\gamma^{-1}, \\ \frac{1}{2}\left(\frac{n_1}{2n_2}\right)^{1/2}\left(\frac{\omega_{L2}^2}{\Omega\omega_0}\right)^{1/2}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta < \Omega\gamma^{-1}. \end{cases} \tag{4.4}$$

For asymmetrical modes with $l \neq 0$, more favorable buildup conditions, to the contrary, prevail in the long-wave region $\alpha_{1,2} r_0 \ll 1$. The dispersion equation (4.1) in this limit coincides with (3.22) at $\kappa = 1/2$, and the growth increment of such oscillations is therefore given by the expression (compare with (3.23))

$$\operatorname{Im}\delta = \begin{cases} \frac{\sqrt{3}}{2}\left(\frac{n_1\gamma^{-1}}{4n_2}\right)^{1/3}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta > \Omega\gamma^{-1}, \\ \frac{1}{2}\left(\frac{n_1\omega_{L2}^2}{n_2\Omega\omega_0}\right)^{1/2}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta < \Omega\gamma^{-1}. \end{cases} \tag{4.5}$$

The increments (4.4) and (4.5) are of the same order of magnitude. Consequently, short-wave symmetrical and long-wave asymmetrical modes of surface oscillations should appear actually with practically equal probability.

b) Excitation of volume oscillations. Volume oscillations are defined as those for which α_1^2 or α_2^2 is negative. It can be shown that α_1^2 and α_2^2 cannot be negative simultaneously (this is possible only for identical media, i.e., strictly in the absence of a beam). Of the two remaining possibilities, greater interest attaches obviously to the case when $\alpha_1^2 < 0$ and $\alpha_2^2 > 0$. This corresponds to oscillations generated in the region of the beam and damped outside this region. Only such waves can grow in the system considered by us.

The dispersion equation for volume oscillations is obtained from (4.1) by analytic continuation into the region of negative $\alpha_1^2 = -\beta_1^2$. In this case

$$\varphi_{1l}(\alpha_1 r_0) \to \varphi_{1l}(\beta_1 r_0) = \beta_1 r_0 J'_l(\beta_1 r_0) / J_l(\beta_1 r_0). \tag{4.6}$$

The frequency spectrum of the volume oscillations excited by the beam, just as that of the surface oscillations, is determined by formula (4.3). The oscillation growth increments are determined by the most favorable conditions of their buildup. For axially-symmetrical modes (l =0) such conditions are obtained in the limit as $\beta_1 r_0 \to \infty$, with (compare (4.4))

$$\operatorname{Im}\delta = \begin{cases} \dfrac{\sqrt{3}}{2}\left(\dfrac{n_1\gamma^{-1}}{2n_2}\right)^{1/3}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta > \Omega\gamma^{-1}, \\ \dfrac{1}{2}\left(\dfrac{n_1\omega_{L2}^2}{n_2\Omega\omega_0}\right)^{1/2}\omega_{L2} & \text{for} \quad \operatorname{Im}\delta < \Omega\gamma^{-1}. \end{cases} \tag{4.7}$$

In addition to the exact solution (4.7), the dispersion equation can have the approximate solutions $\beta_1 r_0 = \mu_{nl}$ near the frequencies (4.3). The analysis of this equation (which coincides with (2.7) and (3.1)), carried out in the preceding section, shows that the growth increments of two-stream instabilities with excitation of the oscillations on the upper hybrid frequency[2)] do not exceed (4.7). The growth increments of the long-wave modes ($\beta_1 r_0$, $\alpha_2 r_0 \ll 1$) of axially-asymmetrical oscillations ($l \neq 0$) are also of the order of (4.7). The dispersion equation and the growth increments of such volume oscillations are the same as of the surface ones, i.e., they are determined by formulas (4.5). It follows thus from the foregoing analysis that the characteristics of the excitation (the frequency spectra, the growth increments, and the build-up conditions) of both surface and volume oscillations by a bounded beam in an unbounded plasma are approximately the same. This is seen from formulas (4.4), (4.5), and (4.7). In a real experiment, therefore, all the oscillation modes with the same frequency (4.3) should appear with equal probability.

5. DISCUSSION OF RESULTS

Having obtained general formulas for the frequencies of the oscillations excited by a relativistic beam in a plasma, their growth increments, and their excitation conditions, we proceed to apply the results to the aforementioned two concrete problems: a) excitation of plasma waves by a high-power electron beam, and b) sounding of the ionosphere by a low-power electron beam.

a) Plasma-wave excitation. In this problem we concentrate on the following high-power beam parameters: $J \approx 10^5$ A, $\mathcal{E} \approx 1$–10 MeV, $r_0 \approx 1$–3 cm, and $\tau \lesssim 10^{-7}$ sec. In such beams, the electron density is $n_1 \approx 10^{12}$–10^{13} cm^{-3} and the inequality $\omega_{L1}^2 > u^2\gamma/r_0^2$ is satisfied. For the plasma density we assume the values $n_2 \gtrsim 10^{13}$–10^{16} cm^{-3}, which ensures satisfaction of the condition $n_2\gamma \gg n_1$. In addition, to avoid direct action of the high-power beam on the walls of the waveguide in real experiments, one should choose $R/r_0 \approx 2$–3, and the longitudinal magnetic field that focuses the beam should, in accord with (2.1), satisfy the inequality $B_0 \gtrsim 10^4$ Oe. We note incidentally that the condition (2.1) ensures the satisfaction of one more inequality:

$$\Omega^2 \gtrsim \omega_{L1}^2\gamma > u^2\gamma^2 / r_0^2.$$

The foregoing limitations greatly reduce the number of different possibilities in the character of the interaction of the electron beam with the plasma. From the analysis of Sec. 3b it follows that in a dense plasma, in which $\omega_{L2} \gg \Omega$, such a beam always excites the upper branch of the oscillations with frequency $\omega_0 = \omega_{L2}$. The plasma density should be sufficiently high in this case, $n_2 \gtrsim 10^{14}$ cm^{-3}, and the excited oscillations correspond to electromagnetic waves of the millimeter and submillimeter bands. If the magnetic field is in this case so weak, or else if the beam is of sufficiently high energy and density, so that $(n_1/4n_2)^{1/3}\omega_{L2} > \Omega\gamma^{-2/3}$, then the growth increment of the oscillations turns out to be of the order of $\delta \approx (n_1\gamma^{-1}/2n_2)^{1/3}\omega_{L2}$; on the other hand, if $(n_1/4n_2)^{1/3}\omega_{L2} < \Omega\gamma^{-2/3}$, then $\delta \approx (n_1\gamma^{-3}/2n_2)^{1/3}\omega_{L2}$. For an ultrarelativistic beam (i.e., $\gamma \gg 1$), the growth increment of the waves in the first of these cases is much larger than in the second; as a result the spectrum width of the excited plasma waves is also larger, which is not desirable from the practical point of view.

In our opinion, to excite longitudinal waves of large amplitude in a dense plasma, the optimal conditions are given by

$$\omega_{L2} > \Omega > (n_1\gamma^2 / 2n_2)^{1/3}\omega_{L2}.$$

Under such conditions it is possible to excite sufficiently narrow lines of plasma waves. Thus, at $n_2 \approx 10^{15}$ cm^{-3}, $\mathcal{E} \approx 2$ MeV (i.e., $\gamma \approx 5$), $n_1 \approx 10^{12}$ cm^{-3}, and $B_0 \approx 1 \times 10^{14}$ Oe, there are excited in the plasma millimeter-band oscillations with relative spectrum width $\approx 1/30$. The instability has in this case a two-stream character, which makes it possible to estimate the amplitude of the amplitude of the steady-state nonlinear wave in the plasma:[7]

$$\frac{E_0^2}{8\pi n_1 mc^2\gamma} = \frac{1}{2^{4/3}}\left(\frac{n_1}{2n_2}\right)^{1/3}\gamma\frac{u^2}{c^2}. \tag{5.1}$$

For the plasma and beam parameters assumed above we obtain from this $E_0 \approx 10^6$ V/cm, and the efficiency of conversion of the beam energy into plasma-wave energy is 15%. At $r_0 \approx 0.3$ cm, the current in the beam is $J_0 \approx 10^3$ A, and its power amounts to 2×10^9 W, of which 15% goes over into the energy of the plasma oscillations.

To excite oscillations of longer wavelength, for example in the centimeter band, it is more convenient to make use of the interaction of the electron beam with a rarefied magnetized plasma in which $\Omega \gg \omega_{L2}$. As indicated in Sec. 3, in such a plasma there is preferred excitation of long-wave oscillations at the lower branch, and their frequency, according to (3.21), is of the order of $\omega_0 \approx \omega_{L2}$ (since we assume that $\omega_{L2}^2 \gg u^2/r_0^2$), and the growth increment, which characterizes also the width of the spectrum of the excited wave, is $\delta \approx (n_1\gamma^{-3}/2n_2)^{1/3}\omega_{L2}$. At $n_2 \approx 10^{13}$ cm^{-3}, $n_1 \approx 10^{12}$ cm^{-3}, $\mathcal{E} \approx 5$ MeV (i.e., $\gamma \approx 10$), $r_0 \approx 1$ cm, $J_0 \approx 10^4$ A, and $B_0 \geq 5 \times 10^4$ Oe, centimeter-band oscillations are excited with a relative spectrum width $\approx 1/30$. Here, too,

2)We note, incidentally, that no lower-branch oscillations at $\omega_{L2} < u/r_0$ can be excited.

the instability has a two-stream character and the radiation power can be estimated from formula (5.1). We then find that $E_0 \approx 5 \times 10^6$ V/cm, the efficiency of energy conversion is on the order of unity, and consequently, the radiation power reaches the value of the power of the beam itself.[3)]

It should be noted that in a rarefied plasma, as indicated in Sec. 3, short-wave oscillation modes are excited, with practically equal probability, in addition to the indicated plasma wave. In a real experiment, this can become manifest as broadening of the spectrum of the waves excited in the plasma. From the foregoing estimates it follows that the interaction of powerful strong-current electron beams with a plasma can serve as a rather effective method of exciting large-amplitude plasma waves suitable for particle acceleration.[9]

b) Ionosphere sounding with an electron beam. To sound the ionosphere and to produce artificial auroras, as already noted, one uses electron beams of low power with parameters $n_1 \approx 10^4$ cm^{-3}, $J_0 \lesssim 1$ A, $\mathscr{E} \approx 1$–10 MeV, and $r_0 \approx 10^2$ cm; the ionospheric-plasma parameters are $n_2 \approx 10^5$–10^6 cm^{-3} and $B_0 \approx 0.1$–0.5 Oe, so that we always have $\omega_{L2} > \Omega$ and $(\omega_{L2}^2, \omega_{L1}^2, \Omega^2) < u^2\gamma/r_0^2$.

Whereas the instability of the beam is a useful effect in the excitation of plasma waves, in the sounding of the ionosphere the instability of the beam can lead to a number of undesirable effects that interfere with the sounding, such as turbulization of the plasma, anomalous scattering of the beam electrons by the turbulent pulsations, enhanced diffusion and as a consequence anomalous broadening of the beam, the appearance of an additional scatter of the electron velocities, etc. On the other hand, since the parameters of the ionospheric plasma (its density and the magnetic field intensity B_0) change considerably as the beam moves along the forced lines of the earth's magnetic field, the conditions for the instability development turn out to be different along the path of the electron beam. Naturally, the instability developed predominantly in places where the growth increment is maximal. As a result, the sources of the plasma oscillations excited by the beams are localized, as are also the places where there is considerable scattering and reflection of the electrons. An analysis of the plasma oscillations and of the character of the beam-electron reflection makes it possible therefore to determine the parameters of the ionospheric plasma at the places where beam instabilities develop. By varying the beam parameters it is possible to sound the entire ionosphere.

It was shown in Sec. 4 that when an electron beam interacts with a spatially unbounded plasma under conditions when $\Omega^2 < \omega_{L2}^2 < u^2/r_0^2$ (as is the case in the ionosphere), oscillations are excited only at the upper hybrid frequency $\omega_0^2 \approx \Omega^2 + \omega_{L2}^2 \approx \omega_{L2}^2$. The oscillation growth increment is then, in accordance with (4.4) and (4.7),

$$\delta \approx \frac{\sqrt{3}}{2}\left(\frac{n_1}{4n_2}\gamma^{-1}\right)^{1/3}\omega_{L2}, \quad \text{if} \quad \left(\frac{n_1}{4n_2}\right)^{1/3}\omega_{L2} > \Omega\gamma^{-2/3},$$

or

$$\delta \approx \frac{1}{2}(n_1\omega_{L2}/n_2\Omega)^{1/2}\omega_{L2}, \quad \text{if} \quad (n_1/4n_2)^{1/3}\omega_{L2} > \Omega\gamma^{-2/3}.$$

For ultrarelativistic electron beams, when $\mathscr{E} \gtrsim 2$ MeV (i.e., $\gamma \gg 1$), the first of these cases is realized at the plasma parameters indicated above, and by beam sounding it is possible to determine the plasma density at the points of maximum increment along the path of the electron beam. On the other hand, beams of relatively low energy $\mathscr{E} \lesssim 1$ MeV (i.e., $\gamma \approx 1$) make it possible, in principle, to determine also the intensity of the magnetic field. This is done by measuring the amplitudes of the steady-state nonlinear plasma oscillations, which is connected directly with their growth increment by the relation [7]

$$\frac{E^2}{8\pi n_1 mc^2\gamma} \approx \frac{u^2}{c^2}\gamma^5\frac{n_2}{n_1}\frac{\delta^4}{\omega_{L2}^4}. \quad (5.2)$$

This is precisely why it seems to us that weakly-relativistic or even nonrelativistic electron beams are more suitable for the sounding of the ionosphere.

[3)]Strictly speaking, formula (5.1) cannot be used in this case. The numerical analysis performed in[8] shows that the energy conversion efficiency reaches in this case 30–40%.

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