SOVIET PHYSICS JETP VOLUME 35, NUMBER 1 JULY, 1972

Energy Spectrum of Multiply Charged Ions in a Laser-Produced Plasma

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Submitted June 9, 1971

Zh. Eksp. Teor. Fiz. **62**, 247–252 (January, 1972)

The energy distributions of differently charged ions are studied theoretically in a plasma produced by laser radiation. Analytic expressions for the ion energy distributions are derived, based upon assumptions regarding the character of the hydrodynamic expansion of the plasma. It is shown that recombination processes play an important part in the formation of the spectra.

IN [1,2] the energy distributions of ions having different charge multiples Z were investigated in a plasma formed by the action of intense laser radiation on heavy condensed targets. The observed energy spectra of the expanding ("dispersing") cloud of vapor are each characterized by the presence of a limit ($\sim$20–40 keV) that is independent of Z. Moreover, each spectrum possesses a peak, which with increasing Z is shifted toward higher energies, and the total number of Z-ions diminishes as Z increases.

To interpret the given results it is necessary to consider an ion acceleration mechanism that will account for both the high upper energy limit and the shape of the spectra. In [1] the energy limit was explained using the concept of quasistationary expansion ("dispersal") of the plasma. It was assumed that ion groups having different charge multiples are formed on a target surface at different moments of time as the laser radiation density varies, and are thereafter accelerated independently. However, the characteristic shape of the observed energy spectra is not accounted for within the framework of the model in [1]. The role of recombination in the formation of the spectra is estimated in [2].

In the present work we consider another physical model for the formation of the spectra of differently charged ions. Making certain assumptions regarding the expansion, we obtain analytic expressions, incorporating all the aforementioned characteristics, for the investigated energy distributions. The proposed model is based on the concept that the formation of the energy spectra of ions with different Z values receives an important contribution from recombination processes during the stage of hydrodynamic expansion after the termination of the laser pulse, and that the plasma formed prior to that time does not necessarily contain the complete set of Z's of the subsequently observed ions. In this model the observed Z-distribution of the ions depends to a considerable degree on ion "diffusion" along the Z axis as the result of recombination during the expansion. Indeed, for plasma density $N_0 \approx 10^{20}$ cm^{-3}, temperature $T \approx 10^2$ eV, and $Z \approx 20$ we have the characteristic photorecombination time $\tau_{\text{ph-r}} \approx 3 \times 10^{-10}$ sec,[3] which is shorter than the hydrodynamic expansion time of the plasma. Consequently, the recombination effect is important.

We shall first consider the qualitative description of energy spectrum formation within the framework of the given model. We shall assume that an expanding cloud of plasma is present when the plasma pulse terminates. In the course of the expansion, within each mass element of the plasma cloud the recombination process generates a determinate Z-distribution of the ions that depends on the changes of density and temperature with time. The final composition of the plasma in a given element also depends on the initial Z-distribution of the ions contained in it. All the differently charged ions contained within a fixed mass element will possess, at least in the last stage of the expansion, an identical velocity that is equal to the hydrodynamic velocity of the given mass element. On the other hand, in the general case all mass elements will contribute to the energy spectrum of ions with a given Z value. The spectral "width," i.e., the set of velocities for each group of Z-ions, will be determined by the plasma velocity distribution along the radial coordinate. The contribution to a given spectral region for ions with fixed Z will depend on the efficiency of recombination in the corresponding mass element.

2. Let us consider a spherically symmetric plasma cloud that is expanding in a vacuum according to a self-similar law. We thus assume that the radial (r) distribution of density ρ as a function of time t is given by

$$\rho(r,t)=\frac{M}{4\pi IR^3(t)}f\left(\frac{r}{R}\right),\quad I=\int_0^1 f(x)x^2\,dx, \tag{1}$$

where M is the total mass of plasma, and R(t) is the coordinate of the boundary between the plasma and the vacuum. In this case the radial coordinate and velocity of a fixed mass element with the Lagrangian coordinate $\eta = m/M$ (where the mass coordinate m is measured from the plasma-vacuum interface) are determined by the relations

$$r(\eta,t)=F(\eta)R(t),\quad v(\eta,t)=\dot r(\eta,t)=F(\eta)\dot R(t), \tag{2}$$

where the function $F(\eta)$ is given by the expression

$$\eta=\frac{1}{I}\int_F^1 f(x)x^2\,dx. \tag{3}$$

Furthermore, at time t the number of Z-ions $dN_Z(\eta,t)$ in a volume element r_η, $r_\eta + dr$ is

$$dN_Z(\eta,t)=x_Z(\eta,t)N_0(\eta,t)4\pi r_\eta^2dr, \tag{4}$$

where $x_Z(\eta,t) = N_Z(\eta,t)/N_0(\eta,t)$ is the degree of ionization of Z-ions in a plasma element with the coordinate η, $N_0 = \rho(\eta,t)/m_i$, and m_i is the ionic mass; then

$$\sum_{Z=0}^{Z=Z_0(\eta)} x_Z(\eta,t)=1,$$

where $Z_0(\eta)$ is the maximum ionic charge in the η element of the plasma at time $t = 0$.

On the other hand, in accordance with (2) we have

$$\frac{r_\eta(t)}{R(t)} = \frac{\dot{r}_\eta(t)}{\dot{R}(t)} = F(\eta), \quad d\left[\frac{r_\eta(t)}{R(t)}\right] = d\left[\frac{\dot{r}_\eta(t)}{\dot{R}(t)}\right] = dF(\eta),$$

i.e., the radial distribution of the ions has a single-valued relation to their velocity distribution. Therefore, in virtue of the relations

$$N_0(\eta, t) = \frac{M}{4\pi I m_i R^3} f\left(\frac{r_\eta}{R}\right) = \frac{M}{4\pi I m_i R^3} f(\beta), \quad \eta = \eta\left(\frac{r_\eta}{R}\right) = \eta(\beta),$$

from (4) we obtain (using the notation $\alpha = r/R$, $\beta = v/\dot{R}$)

$$dN_Z(\beta, t) = \frac{M}{4\pi I m_i} x_Z(\eta, t) 4\pi f(\alpha)\alpha^2 d\alpha$$

$$= \frac{M}{I m_i} x_Z(\beta, t) f(\beta) \frac{dv}{\dot{R}} = \frac{M}{m_i} \varphi_Z(v, t)\, dv, \tag{5}$$

where

$$\varphi_Z(v, t) = \frac{1}{I} x_Z(\beta, t) f(\beta)\beta^2 \frac{1}{\dot{R}}.$$

It is obvious that

$$\int_0^\infty \Bigg[\sum_{Z=0}^{Z=Z_0(\beta)} \varphi_Z(v, t)\Bigg] dv = \frac{1}{I}\int_0^\infty f(x) x^2\, dx = 1, \tag{6}$$

which corresponds to conservation of the total number of ions of all charges, whereas

$$\int_0^\infty \varphi_Z\, dv < 1,$$

i.e., the number of ions with a given value of Z decreases as a result of recombination during the expansion.

3. To determine the form of the function $x_Z(\eta, t) = x_Z(\beta, t)$ we now write the kinetic equation of the recombination process in a given plasma element with the coordinate η. Confining our analysis to photorecombination alone (which plays the principal role for $T \approx 10^2$ eV, $N_0 \approx 10^{19}$–10^{20} cm^{-3}, and not too large Z), we obtain

$$\frac{dx_Z}{dt} = N_0(t, \eta)\, b(T)\, (\Sigma x_Z Z)\, [(Z+1)^2 x_{Z+1} - Z^2 x_Z], \tag{7}$$

where b(T) is the photorecombination coefficient of a singly charged ion.

In (7) we transform to the dimensionless "local" time characterizing the recombination efficiency in the η-element:

$$\tau = \int_0^t \sum_0^{Z_0} x_Z(t, \eta)\, b(T) N_0(t, \eta)\, dt,$$

so that

$$dx_Z / d\tau = (Z+1)^2 x_{Z+1} - Z^2 x_Z. \tag{8}$$

We shall also assume that in the initial state each plasma element contains ions with a narrow Z distribution characterized by the effective value $Z_0(\eta)$:

$$x_Z(0, \eta) = \begin{cases} 1, & Z = Z_0(\eta), \\ 0, & Z \neq Z_0(\eta). \end{cases} \tag{9}$$

The system of equations (8) is easily solved for arbitrary initial conditions. However, physical interest attaches to the solution for a later stage of the expansion, corresponding to the experimentally observed picture. For $\tau Z \gg 1$ the solution of (8) with the initial conditions (9) can be represented in the form

$$x_Z(t, \eta) = \left[\frac{Z_0(\eta)!}{Z!}\right]^2 \frac{(2Z)!}{(Z_0 - Z)!(Z_0 + Z)!} \exp\{-Z^2\tau(\eta, t)\}. \tag{10}$$

We note that in the case of three-body recombination the definition of the local time $\tau(\eta, t)$ and the coefficients in (8) will be different.

For the inverse transformation from τ to t, i.e., to calculate the function $S(\tau, \eta) = \sum x_Z Z$ representing the number of electrons per ion, we go from the discrete equations (8) to an equation for the continuous function $f(Z, \tau)$:

$$x_Z(\tau) = \int_{Z-1/2}^{Z+1/2} f(Z, \tau)\, dZ \approx f(Z)\Delta Z, \quad \Delta Z = 1;$$

then in place of (8) and (9) we have, respectively,

$$\frac{df}{d\tau} = \frac{\partial}{\partial Z}[Z^2 f(Z)], \tag{8'}$$

$$f(Z, 0) = \delta[Z - Z_0(\eta)]. \tag{9'}$$

The solution of (8) with the initial condition (9) is

$$f(Z, \tau) = \frac{1}{(\tau Z - 1)^2}\delta\left[\frac{Z}{1 - Z\tau} - Z_0(\eta)\right],$$

from which we obtain

$$x_Z(\tau, \eta) = \begin{cases} 1, & Z = Z_0(\eta)/(1 + \tau Z_0(\eta)), \\ 0, & Z \neq Z_0(\eta)/(1 + \tau Z_0(\eta)), \end{cases} \tag{11}$$

$$S(\tau, \eta) = \sum x_Z Z = \frac{Z_0(\eta)}{1 + \tau Z_0(\eta)}. \tag{12}$$

In the approximation represented by (8′) and (9′) we ignore the "blurring" of the function $f(Z, \tau)$; this is obviously unimportant in calculating the average number of electrons. Substituting (12) into the expression for τ, we now obtain

$$\tau + \frac{1}{2}\tau^2 Z_0(\eta) = Z_0(\eta)\int_0^t b(T) N_0(t, \eta)\, dt. \tag{13}$$

Assuming that $\eta = \eta(\beta)$ and that $Z_0(\eta) = Z_0$ is a given function determined by the initial radial distribution of the effective ionic charge, we obtain

$$\tau + \frac{1}{2}\tau^2 Z_0(\beta) = \frac{Z_0(\beta) M}{4\pi I m_i}\int_0^t \frac{f(\beta)\, b(T)}{R^3(t)}\, dt$$

$$\approx \frac{M Z_0}{4\pi I m_i} f(\beta) \int \frac{b\, dt}{R^3}. \tag{13'}$$

Combining (6) and (10), we write the expression for the spectrum in the case of $t \to \infty$ and $\tau Z \gg 1$. [We note that for $t \to \infty$ we have the "local" time $\tau \to$ const when $\eta \neq 0$; for $\eta \to 0$, i.e., on the "tail" of the energy spectrum, ($v \to \dot{R}$), we have $\tau \to 0$.] We now have

$$\varphi_Z(v) = \frac{1}{\dot{R} I}\left[\frac{Z_0(\beta)!}{Z!}\right]^2 \frac{(2Z)!}{[Z_0(\beta) + Z]!\,[Z_0(\beta) - Z]!} e^{-Z^2\tau} f(\beta)\beta^2, \quad Z \leqslant Z_0(\beta). \tag{14}$$

For $\tau \to 0$, i.e., when $v \to \dot{R}$, we obtain

$$x_Z = \left[\frac{Z_0(\beta)!}{Z!}\right]^2 \frac{\tau^{Z_0(\beta) - Z}}{[Z_0(\beta) - Z]!},$$

$$\varphi_Z(v) = \frac{1}{I\dot{R}}\left[\frac{Z_0(\beta)!}{Z!}\right]^2 \frac{\tau^{Z_0(\beta) - Z}}{[Z_0(\beta) - Z]!} f(\beta)\beta^2. \tag{15}$$

Thus our problem is solved, in principle, by (13), (14), and (15).

4. Equations (14) and (15) lead to a number of conclusions regarding the form of the energy spectrum without having specific forms of the functions $f(\beta)$ and $b(T)$. Indeed, it follows that $\varphi_Z(v) = 0$ for $v = 0$ and $v = \dot{R}$, because $f(1) = 0$. The velocity limit $v = \dot{R}$ is here independent of Z. In addition, the role of recombination is enhanced as we move into the low-energy portion of the spectrum, i.e., into deeper layers of the plasma, but is unimportant near the "tail" of the distribution.

We now present a concrete example, using the assumptions

$$f(\alpha) = (1-\alpha^2)^q, \quad R = R_0 + (\dot{R})_{max}t \ [^3],$$
$$Z_0(\eta) = \text{const}, \quad b(T) = b_0 = \text{const}.$$

The last of these assumptions corresponds physically to the case of isothermal expansion, which can be realized, in principle, through additional heating resulting either from the absorption of laser radiation in the stage of developed gas dynamics or from the release of heat during recombination.

In this case, for $\beta < 1$ we have

$$\tau = \left[\frac{M}{2\pi I m_i} f(\beta)\, b_0 \frac{1}{R_0 \dot{R}_{max}}\right]^{1/2},$$
$$I = \frac{\pi^{1/2} q \Gamma(q)}{(2q+3)(2q+1)\Gamma(q+1/2)}, \quad q > 1$$

and (14) is transformed into

$$\varphi_Z(v) = B(Z,q)(1-\beta_{max}^2)^q \beta_{max}^2 \exp\{AZ^2(1-\beta_{max}^2)^{q/2}\},$$
$$B(Z,q) = \frac{1}{I\dot{R}_{max}}\left(\frac{Z_0!}{Z!}\right)^2 \frac{(2Z)!}{(Z_0+Z)!(Z_0-Z)!},$$
$$A = \left[\frac{Mb_0}{2\pi m_i R_0 \dot{R}_{max}}\right]^{1/2}, \quad \beta_{max} = \frac{v}{\dot{R}_{max}} \qquad (16)$$

From (16) we find that the maximum value of $\varphi_Z(v)$ is reached when

$$\beta^2 \approx 1 - (2/AZ^2)^{2/q}, \qquad (17)$$

i.e., with increasing Z the maximum is shifted into the region of large v. Substituting (17) into (16), we find that the maximum value

$$[\varphi_Z(v)]_{max} \sim \frac{(2Z)!}{(Z!)^2(Z_0+Z)!(Z_0-Z)!Z^4} \sim \frac{(1-Z/Z_0)^{Z-Z_0}}{Z^{9/2}(Z_0/4)^Z(1+Z/Z_0)^{Z+Z_0}} \qquad (18)$$

diminishes as Z increases. For $v \to \dot{R}$, i.e., at the edge of the plasma cloud, recombination is insignificant and, as follows from (15), the initial composition of the plasma does not change. Thus the model developed here incorporates many of the characteristics observed in the ionic spectra. A detailed numerical comparison of the analytic equations with the known experimental data does not appear to be possible, because these data do not include a complete set of the parameters required for such a comparison. Thus, for example, the more detailed studies[1,2] do not give the radial distribution of plasma density, do not determine the energy limits of high-Z ions with sufficient accuracy, do not contain data regarding the diameter of the focal spot, etc.

We mention, in conclusion, a recent article[4] that contains numerical calculations of laser-produced plasma decay taking recombination and ionization into account. These calculations were performed for specific conditions of a LiH plasma and are limited to small values of Z.

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Translated by I. Emin
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