Additional Absorption and Parallel Pumping in an Antiferromagnetic Substance with Anisotropy of the Easy Plane Type

V. A. KOLGANOV

Moscow Physico-technical Institute

Submitted June 22, 1971

Zh. Eksp. Teor. Fiz. **62**, 333–345 (January, 1972)

Nonlinear dynamic phenomena in an antiferromagnetic substance with weak ferromagnetism are studied in the case when the external variable and stationary magnetic fields are collinear and lie in the basal plane of the crystal. An expression is obtained for the threshold field strength. The behavior of the high-frequency complex susceptibility beyond the instability threshold is studied with the aid of the nonstationary state density matrix. The effect of various processes of unstable oscillation interaction is estimated.

THIS paper deals with phenomena that take place in antiferromagnets of the easy-plane type, in which the spin-wave vector consists of two branches when the alternating magnetic field **h** is applied parallel to the constant field **H** lying in the basal plane of the crystal. The homogeneous field **h** of the indicated polarization excites linearly homogeneous precession[1)] of the antiferromagnetic vector $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$ (the HF branch in the resonant case), which decays into two spin waves of the LF branch. This can be accompanied by intense threshold absorption of the source energy, which sets in when the rate of excitation of the spin wave of the LF branch becomes comparable with the rate of their attenuation. The resultant appreciable growth of the amplitude of the spin waves of the LF branch in the stationary state causes an increase in the rate of relaxation of the homogeneous precession **L** and, as a consequence, a rise in the level of the absorbed power. In analogy with the phenomenon investigated by Suhl[1], which differs from the phenomenon indicated here in that the spin-system oscillation in a ferromagnet correspond to precession of the ferromagnetic vector **M**, this effect can be called "additional absorption" (AA) in the case of nonresonant excitation of the oscillations of **L** and "saturation of the main resonance" in the particular case of resonant excitation.

In the presence of weak ferromagnetism ($MnCO_3$, $CoCO_3$), a direct decay of a photon into two spin waves of the LF branch is also possible. The mechanism of the threshold growth of the absorbed power is considered in analogy with the AA, if account is taken of the fact that the absorbed power is proportional to the number of excited spin waves in the stationary state. This effect admits of an interpretation given by Schlomann for "parallel pumping" (PP)[2] with allowance for the fact that the necessary elasticity of the precession of the magnetic moment M is ensured here by the Dzyaloshinskiĭ field H_D.

The phenomena mentioned above were investigated by Ozhogin[3]. The classical equations of motion of magnetization were used to calculate the threshold amplitude of the alternating magnetic field of "indirect parallel pumping," but the state beyond the threshold was not investigated.

Recalling the obvious interest aroused by the study of the mechanism of spin-wave relaxation, we allow ourselves to dwell once more on the investigation of the indicated effects, and primarily the stationary beyond-threshold regime, on the basis of concrete calculations and numerical estimates. The undisputed stimulus in this investigation were the results of the experimental study of AA in $CsMnF_3$ ($H_D = 0$), performed by Seavey[4] and by Prozorova and Borovik-Romanov[5].

The Hamiltonian of an antiferromagnet with anisotropy of the easy-plane type is written in the form

$$\hat{\mathscr{H}} = \sum_{\mathbf{R}_1\Delta} [-J(\Delta)\hat{\mathrm{S}}_{\mathbf{R}_1}\hat{\mathrm{S}}_{\mathbf{R}_1+\Delta} + 2D(\Delta)(\hat{S}^z_{\mathbf{R}_1}\hat{S}^y_{\mathbf{R}_1+\Delta} - \hat{S}^y_{\mathbf{R}_1}\hat{S}^z_{\mathbf{R}_1+\Delta})] + \sum_{\mathbf{r},\,\mathbf{R}\neq\mathbf{r}} Q(\mathbf{R}-\mathbf{r})\hat{S}_{\mathbf{r}}{}^x\hat{S}_{\mathbf{R}}{}^x + \sum_{\mathbf{R}} [P(\hat{S}_{\mathbf{R}}{}^x)^2 - \mu_B g(\mathbf{H}+\hat{\mathbf{h}})\hat{S}_{\mathbf{R}}],$$

where $h = h_z$, $H = H_z$, and zy is the basal plane. Changing over to the proper coordinate axis of each spin and carrying out in succession the Holstein-Primakoff transformation in analogy with the procedure used in[6], we obtain the quadratic part of the spin Hamiltonian and the interaction Hamiltonian of the spin system with alternating magnetic field:

$$\hat{\mathscr{H}}_s = \sum_{j\mathbf{k}} \varepsilon_{j\mathbf{k}}\hat{c}^+_{j\mathbf{k}}\hat{c}_{j\mathbf{k}} \quad \hat{\mathscr{H}}_s{}^f = \hat{\mathscr{H}}^f_{20} + \hat{\mathscr{H}}^f_{1\mathbf{k},\,1-\mathbf{k}},$$

where the index 1 is connected with the oscillations of **M**, 2 with the oscillations of **L**, and f with the oscillations of **h**.

So as not to clutter the results and to simplify the analysis, we first study the case when there is no Dzyaloshinskiĭ interaction, i.e., $H_D = 0$, a situation realized, for example, in $CsMnF_3$. This makes it possible to investigate the AA in "pure form."

1. CASE $H_D = 0$. ADDITIONAL ABSORPTION

1. The Hamiltonian of excitation by an alternating magnetic field of homogeneous precession L is

$$\hat{\mathscr{H}}_{20}{}^f = \Psi_{20}{}^f\hat{c}_{20}{}^+(t)\hat{b}(t) + \text{h.c.}, \tag{1.1}$$

where b(t) is the photon annihilation operator, and the amplitude of the interaction of the spin system with the

[1)]We are dealing in fact with an almost homogeneous field and "almost homogeneous" precession.

alternating magnetic field is

$$\Psi_{20}{}^{f} = \sqrt{\pi M_0 \varepsilon \varepsilon_{20} / H_E}. \quad (1.2)$$

To describe the stimulated oscillations of L, we the equation of motion for the operator of these oscillations $c_{20}(t)$. Changing over to c-numbers and introducing phenomenologically the attenuation, we arrive at the following equation:

$$i\frac{dc_{20}(t)}{dt} = (\omega_{20} - i\Delta\omega_{20}^{\text{eff}})c_{20}(t) + \frac{\Psi_{20}{}^{f}}{\hbar}b(t), \quad b(t) = be^{-i\omega t}. \quad (1.3)$$

Neglecting the proper oscillations, we obtain the solution in the form

$$c_{20}(t) \approx \frac{\Psi_{20}^{f}}{\hbar}\frac{b(t)e^{i\delta}}{\sqrt{(\omega_{20}-\omega)^2 + (\Delta\omega_{20}^{\text{eff}})^2}}, \quad \delta = \text{arc tg}\frac{\Delta\omega_{20}^{\text{eff}}}{\omega - \omega_{20}}. \quad (1.4)$$

2. The decay of the homogeneous precession L into two magnons of the LF branch is described by the following term in the Hamiltonian:

$$\hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20} = \sum_{\mathbf{k}} \Psi_{1\mathbf{k},1-\mathbf{k}}^{20}\hat{c}_{1\mathbf{k}}^{+}(t)\hat{c}_{1-\mathbf{k}}^{+}(t)\hat{c}_{20}(t) + \text{h.c.} \quad (1.5)$$

The equations of motion for $c_{20}(t)$, $c_{1\mathbf{k}}(t)$, and $c^{*}_{1-\mathbf{k}}(t)$ then take the form

$$i\frac{dc_{20}(t)}{dt} = (\omega_{20} - i\Delta\omega_{20})c_{20}(t) + \frac{\Psi_{20}{}^{f}}{\hbar}b(t) + 2\sum_{\mathbf{k}}\frac{(\Psi_{1\mathbf{k},1-\mathbf{k}}^{20})^{*}}{\hbar}c_{1\mathbf{k}}(t)c_{1-\mathbf{k}}(t),$$

$$i\frac{dc_{1\mathbf{k}}(t)}{dt} = (\omega_{1\mathbf{k}} - i\Delta\omega_{1\mathbf{k}})c_{1\mathbf{k}}(t) + 2\frac{\Psi_{1\mathbf{k},1-\mathbf{k}}^{20}}{\hbar}c_{1-\mathbf{k}}^{*}(t)c_{20}(t), \quad (1.6)$$

$$-i\frac{dc_{1-\mathbf{k}}^{*}(t)}{dt} = (\omega_{1-\mathbf{k}} + i\Delta\omega_{1-\mathbf{k}})c_{1-\mathbf{k}}^{*}(t) + 2\frac{(\Psi_{1\mathbf{k},1-\mathbf{k}}^{20})^{*}}{\hbar}c_{1\mathbf{k}}(t)c_{20}^{*}(t). \quad (1.7)$$

In the first equation, the summation is over the set of wave vectors **k**, such that the spin waves with these **k** are degenerate in energy. We seek the solution $c_{1\mathbf{k}}(t)$, with allowance for the fact that $\omega_{1\mathbf{k}} = \omega_{1-\mathbf{k}}$ and $\omega = 2\omega_{1\mathbf{k}}$, in the form

$$c_{1\mathbf{k}}(t) = c_{1\mathbf{k}}\exp(-i\omega t/2)\Phi(t),$$

and obtain from the system (1.7), under the condition $\Phi(t) \equiv 1$, the threshold value of the number of elementary excitations with $\mathbf{k} = 0$, corresponding to the homogeneous precession L:

$$n_{20}^{\text{thr}} = (\hbar\Delta\omega_{1\mathbf{k}}/2|\Psi_{1\mathbf{k},1-\mathbf{k}}^{20}|)^2. \quad (1.8)$$

To determine the amplitude of the interaction of the excited oscillations with the spin system, $\Psi_{1\mathbf{k},1-\mathbf{k}}^{20}$, which is a function of the Holstein-Primakoff transformation coefficients u_{ij} and v_{ij}, such that the Fourier components of the spin-deviation operators are represented in the form

$$\hat{a}_{\alpha\mathbf{k}}(t) = \sum_{\nu}[u_{\alpha\nu}\hat{c}_{\nu\mathbf{k}}(t) + v_{\alpha\nu}{}^{*}\hat{c}_{\nu-\mathbf{k}}^{+}(t)], \quad \alpha, \nu = 1, 2, \quad (1.9)$$

we obtain the connection between $\hat{c}_{20}(t)$ and the spin-deviation operators, i.e., the values of u_{i2} and v_{i2} in the case of excitation of homogeneous precession L with arbitrary frequency, whereas u_{i1} and v_{i1} have the usual form in the case $\omega = 2\omega_{i\mathbf{k}}$ (see[6]). Writing down the Hamiltonian in terms of the spin-deviation operators

$$\hat{\mathscr{H}} = {}^{1}/{}_{2}\sum_{\alpha\beta}R_{\alpha\beta}\hat{a}_{\alpha\mathbf{k}}{}^{+}(t)\hat{a}_{\beta-\mathbf{k}}^{+}(t) + \sum_{\alpha\beta}S_{\alpha\beta}\hat{a}_{\alpha\mathbf{k}}{}^{+}(t)\hat{a}_{\beta\mathbf{k}}(t)$$
$$+ {}^{1}/{}_{2}\sum_{\alpha\beta}R_{\alpha\beta}^{*}\hat{a}_{\alpha\mathbf{k}}(t)\hat{a}_{\beta-\mathbf{k}}(t) + \sum_{\alpha}Q_{\alpha}\hat{a}_{\alpha-\mathbf{k}}^{+}(t)\hat{b}(t), \quad (1.10)$$

we obtain from the equations of motion for $\hat{a}_{\alpha\mathbf{k}}(t)$ and $\hat{c}_{\nu\mathbf{k}}(t)$, with allowance for (1.9), an inhomogeneous system of equations for u_{i2} and v_{i2}. Solving this equation, we obtain, knowing $R_{\alpha\beta}$, $S_{\alpha\beta}$, and Q_{α},

$$-u_{22} = u_{12} \approx \frac{\omega}{\omega + \omega_{20}}\left(1 + \frac{\omega_{20}{}^2}{2\omega\gamma H_E}\right)\sqrt{\frac{\gamma H_E}{\omega_{20}}}$$
$$-v_{22} = v_{12} \approx \frac{\omega}{\omega + \omega_{20}}\left(1 - \frac{\omega_{20}{}^2}{2\omega\gamma H_E}\right)\sqrt{\frac{\gamma H_E}{\omega_{20}}}. \quad (1.11)$$

In the case of resonant excitation of L, the expressions for u_{i2} and v_{i2} take the usual form[6]. Knowing u_{i2} and v_{i2}, we obtain for $M_0 \ll H \ll H_E$

$$\Psi_{1\mathbf{k},1-\mathbf{k}}^{20} \approx i\frac{2\mu\omega_{20}\gamma H}{\omega(\omega + \omega_{20})}\sqrt{\frac{\varepsilon_{20}H_E}{2M_0 V}}. \quad (1.12)$$

3. To calculate the effective relaxation rate of the homogeneous precession of the antiferromagnetic moment, $\Delta\omega_{20}^{\text{eff}}$, we turn to the kinetic equations for $n_{1\mathbf{k}}(t)$ and $n_{20}(t)$.

The behavior of the system of magnons of the LF branch in response to a time-dependent perturbation will be described with the aid of a nonstationary state density matrix $\hat{\rho}'_{1\mathbf{k}}(t)$,[2)] which is a solution of the equation

$$\frac{\partial}{\partial t}\hat{\rho}_{1\mathbf{k}}'(t) = \frac{1}{i\hbar}[\hat{\mathscr{H}}_s + \hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20}(t), \hat{\rho}_{1\mathbf{k}}'(t)] \quad (1.13)$$

under the conditions

$$\hat{\rho}_{1\mathbf{k}}'(-\infty) = \hat{\rho}_{1\mathbf{k}}, \quad [\hat{\mathscr{H}}_s, \hat{\rho}_{1\mathbf{k}}] = 0.$$

Solving (1.13) by the iteration method, we obtain[8]

$$\hat{\rho}_{1\mathbf{k}}'(t) = \hat{\rho}_{1\mathbf{k}} + \sum_{m=1}^{\infty}\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t),$$

$$\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t) = (-i\hbar)^{-m}\int_{-\infty}^{t}d\tau_1\int_{-\infty}^{\tau_1}d\tau_2\ldots\int_{-\infty}^{\tau_{m-1}}d\tau_m \quad (1.14)$$
$$\times[\hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20}(\tau_1)[\hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20}(\tau_2),\ldots,[\hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20}(\tau_m),\hat{\rho}_{1\mathbf{k}}]]\ldots],$$

$$\hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20}(\tau) = \exp\{i\hat{\mathscr{H}}_s\tau/\hbar\}\hat{\mathscr{H}}_{1\mathbf{k},1-\mathbf{k}}^{20}\exp\{-i\hat{\mathscr{H}}_s\tau/\hbar\}. \quad (1.15)$$

Averaging the equation of motion for $\hat{n}_{1\mathbf{k}}(t)$ with the aid of $\hat{\rho}'_{1\mathbf{k}}(t)$ represented by formulas (1.14) and (1.15), we obtain the answer in the form of a series in powers of the perturbation. Neglecting in the odd terms of the series the quantity $\bar{n}_{1\mathbf{k}}$ in comparison with $n_{20}(t)$, where $\bar{n}_{1\mathbf{k}} = \text{Tr}(\hat{\rho}_{1\mathbf{k}}, \hat{n}_{1\mathbf{k}})$, we represent the right-hand side of the average equation of motion, with allowance for (1.8), in the form

$$\mathscr{L}^{(2m-2)}(t) = 0, \quad m = 1, 2\ldots,$$
$$\mathscr{L}^{(2m-1)}(t) = \frac{i^{2m-2}}{(2m-1)!}2\Delta\omega_{1\mathbf{k}}\bar{n}_{1\mathbf{k}}\left(\frac{n_{20}(t)}{n_{20}^{\text{thr}}}\right)^m + \ldots, m = 1, 2\ldots \quad (1.16)$$

In the same manner we obtain for $n_{1\mathbf{k}}(t)$

$$n_{1\mathbf{k}}^{(0)} \equiv \bar{n}_{1\mathbf{k}}, \quad n_{1\mathbf{k}}^{(2m-1)}(t) = 0, \quad m = 1, 2\ldots,$$
$$n_{1\mathbf{k}}^{(2m)}(t) = \frac{i^{2m}}{(2m)!}2\bar{n}_{1\mathbf{k}}\left(\frac{n_{20}(t)}{n_{20}^{\text{thr}}}\right)^m + \ldots, \quad m = 1, 2\ldots \quad (1.17)$$

We express $\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t)$ in terms of $\Delta^{(m-1)}\hat{\rho}_{1\mathbf{k}}(t)$, integrate with respect to τ_1, and after summing over m we obtain

---

[2)]The idea of using the nonstationary density matrix in the analysis of parametric effects was advanced in[7].

$$\sum_{m=1}^{N}\Delta^{(m)}\hat{\rho}_{1k}(t)=\frac{1}{-i\hbar\Delta\omega_{1k}}\left[\mathscr{H}^{20}_{1k,1-k}(t),\ \sum_{m=0}^{N-1}\Delta^{(m)}\hat{\rho}_{1k}(t)\right].\quad(1.18)$$

We use this relation for the averaging, noting that the summation over m in the left-hand side of the equation begins with m = 1, since $\mathscr{G}^{(0)} = 0$, and the right-hand side it begins with m = 0, since $n^{(0)}_{1k} \neq 0$. In the right-hand side, the upper limit of summation N − 1 can be replaced by N, for whereas $\Delta^{(N)}\rho_{1k}(t)$ contributes to $\mathscr{G}(t)$, it does not contribute to $n_{1k}(t)$. We can use here also the convergence of the series (1.16) and (1.17). Taking the limit as $N \to \infty$, we average the equation of motion for $\hat{n}_{1k}$ on the basis of relation (1.18). Introducing the phenomenological damping, we obtain a kinetic equation, which we write out here for the case of arbitrary ω and ω_{1k}:

$$\frac{dn_{1k}(t)}{dt}=-2\Delta\omega_{1k}(n_{1k}(t)-\bar{n}_{1k})+\frac{8|\Psi^{20}_{1k,1-k}|^2}{\hbar^2}\cdot$$
$$\times\frac{\Delta\omega_{1k}+\Delta\omega_{1-k}}{(\omega-\omega_{1k}-\omega_{1-k})^2+(\Delta\omega_{1k}+\Delta\omega_{1-k})^2}\cdot\quad(1.19)$$
$$\times[n_{20}(t)(n_{1k}(t)+n_{1-k}(t)+1)-n_{1k}(t)n_{1-k}(t)].$$

Analogously, considering the perturbation of a system of elementary excitations corresponding to homogeneous precession **L**, we use a relation similar to formula (1.18). As a result we obtain a kinetic equation for $n_{20}(t)$:

$$\frac{dn_{20}}{dt}=-2\Delta\omega_{20}(n_{20}(t)-\bar{n}_{20})+\left(\frac{\Psi^f_{20}}{\hbar}\right)^2\frac{\Delta\omega^{\mathrm{eff}}_{20}(t)}{(\omega-\omega_{20})^2+(\Delta\omega^{\mathrm{eff}}_{20}(t))^2}n_f$$
$$-\sum_k\frac{8|\Psi^{20}_{1k,1-k}|^2}{\hbar^2}\frac{\Delta\omega_{1k}+\Delta\omega_{1-k}}{(\omega-\omega_{1k}-\omega_{1-k})^2+(\Delta\omega_{1k}+\Delta\omega_{1-k})^2}[n_{20}(t)(n_{1k}(t)$$
$$+n_{1-k}(t)+1)-n_{1k}(t)n_{1-k}(t)].\quad(1.20)$$

We shall henceforth put $\omega = 2\omega_{1k}$ and take into account the fact that $n_{1k} = n_{1-k}$, $n_{1k} \gg \bar{n}_{1k}$, and $n_{20} \gg \bar{n}_{20}$. The main result of this is that in the stationary state ($\dot{n}_{1k} = 0 = \dot{n}_{20}$) we get from (1.19) and (1.20)

$$\Delta\omega^{\mathrm{eff}}_{20}=\Delta\omega_{20}+\sum_k\Delta\omega_{1k}\frac{n_{1k}}{n_{20}},\quad n_{1k}=2(n_{20}-n^{\mathrm{thr}}_{20}),\quad(1.21)$$
$$n_{20}=\left(\frac{\Psi^f_{20}}{\hbar}\right)^2\frac{n_f}{(\omega_{20}-\omega)^2+(\Delta\omega^{\mathrm{eff}}_{20})^2}.$$

The last equation, with allowance for (1.8), (1.12), and $n_f = h^2V/8\pi\epsilon$, yields the threshold value of the amplitude of the alternating magnetic field

$$h_z^{\mathrm{thr}}=\frac{\Delta\omega_{1k}\omega(\omega+\omega_{20})\sqrt{(\omega_{20}-\omega)^2+\Delta\omega_{20}^2}}{\gamma^2H\omega_{20}^2}.\quad(1.22)$$

The solution of the system (1.21) is simplest to obtain in the case $\Sigma \gg \omega_{20}$ (where $\Sigma = \sum_k(\Delta\omega_{1k})$. Then, accurate to $\omega_{20}\Sigma^{-1}$, we have $n_{20} \approx n^{\mathrm{thr}}_{20}$. To determine n_{1k} and $\Delta\omega^{\mathrm{eff}}_{20}$ we obtain the following relations:

$$\Delta\omega_{20}^{\mathrm{eff}}\approx\sqrt{(\omega_{20}-\omega)^2+\Delta\omega_{20}^2}\sqrt{p-\frac{(\omega_{20}-\omega)^2}{(\omega_{20}-\omega)^2+\Delta\omega_{20}^2}},\quad(1.23)$$
$$n_{1k}\approx n_{20}^{\mathrm{thr}}\frac{\Delta\omega_{20}^{\mathrm{eff}}-\Delta\omega_{20}}{\Sigma},$$

where $p = (h/h^{\mathrm{thr}})^2$.

Finally, taking into account the expressions

$$\delta\hat{M}_z(t)=-i\sqrt{\frac{M_0\varepsilon_{20}}{2H_EV}}\hat{c}_{20}(t)+\mathrm{h.c.},\ \hat{h}_z(t)=i\sqrt{\frac{2\pi\varepsilon}{V}}\hat{b}(t)+\mathrm{h.c.}$$

and changing over to c-numbers, we obtain for the

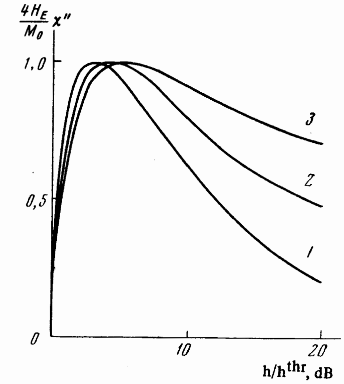

FIG. 1. Dependence of the imaginary part of the susceptibility on h at $|\omega_{20}-\omega| \gg \Delta\omega_{20}$ and $H_D = 0$, for σ equal to zero (curve 1), 0.1 (2), and 0.2 (3).

susceptibility $\chi_{zz}(\omega)$, with allowance for (1.4),

$$\chi(\omega,p)=\frac{M_0}{2H_E}\frac{\omega_{20}}{(\omega_{20}-\omega)-i\Delta\omega^{\mathrm{eff}}_{20}(p)},$$

which can be represented, using (1.23), in the form

$$\chi''(\omega,p)\approx\chi''(\omega,1)\frac{\sqrt{(\omega_{20}-\omega)^2+\Delta\omega_{20}^2}}{\Delta\omega_{20}p}\sqrt{p-\frac{(\omega_{20}-\omega)^2}{(\omega_{20}-\omega)^2+\Delta\omega_{20}^2}},$$
$$\chi'(\omega,p)\approx\chi'(\omega,1)p^{-1}.\quad(1.24)$$

In the case considered above, the expressions for n_{1k} and $\Delta\omega^{\mathrm{eff}}_{20}$ do not depend on the form of the collision integrals, in view of the fact that under the condition $\Sigma \gg \omega_{20}$ the considerable increase of the effective relaxation rate $\Delta\omega^{\mathrm{eff}}_{20}$ as a result of the decay of the homogeneous precession **L** into spin waves of the LF branch leads to "quenching" of the amplitude of the homogeneous precession **L** near the threshold value, and this determines uniquely the stationary state of the system. The value of $\Delta\omega^{\mathrm{eff}}_{20}$ is then obtained from the relation $n_{20} \approx n^{\mathrm{thr}}_{20}$, and n_{1k} is obtained from the first equation of (1.21) with $n_{20} \approx n^{\mathrm{thr}}_{20}$ taken into account.

In the more general case $\Sigma^2 \gg p\Delta\omega^2_{20}$, when the amplitude of the homogeneous precession L can increase beyond the threshold, we obtain for the susceptibility at $p > 1$ and $\omega_{20} = \omega$

$$\chi''(p)\approx\chi''(1)\left[\frac{\Delta\omega_{20}}{4\Sigma}+\sqrt{\left(\frac{\Delta\omega_{20}}{4\Sigma}\right)^2+\frac{1}{p}}\right],\quad(1.25)$$
$$\chi'(p)\equiv 0.$$

For $|\omega_{20}-\omega| \gg \Delta\omega_{20}$ we get (see Fig. 1)

$$\chi''(\omega,p)\approx\chi''(\omega,1)\frac{\eta}{1+4\sigma^2}\frac{\sigma(p-2)+\sqrt{\sigma^2p^2+(p-1)}}{p},$$
$$\chi'(\omega,p)\approx\chi'(\omega,1)\frac{1+2\sigma^2p+\sqrt{\sigma^2p^2+(p-1)}}{(1+4\sigma^2)p}\quad(1.26)$$

where

$$\eta=|\omega_{20}-\omega|/\Delta\omega_{20},\quad\sigma=|\omega_{20}-\omega|/4\Sigma.$$

It is of interest to estimate the parameter σ. Changing over in σ from summation to integration, for a sample dimension $l \gg \pi/2k$, in the case of a low degree of energy degeneracy of the spin waves with different **k** we obtain at $\omega_{20} \gg \omega$

$$\sigma\approx\pi^2\omega_{20}/2V\Delta\omega_{1k}k_0^2\delta k,$$

where k_0 is obtained from the equation $\epsilon = \epsilon_{1k}$, i.e.,

$$\varepsilon\approx2\sqrt{\varepsilon_{10}^2+\Theta_c^2(k_0a)^2},\quad(1.27)$$

and δk is obtained from the condition that the energy degeneracy of the spin waves is determined only by the field of the spin wave[6], it being assumed that $\delta k \ll k_0$.

The final expression for σ takes the form

$$\sigma \approx \frac{\pi\omega_{20}a^3}{V\Delta\omega_{1\mathbf{k}}}\frac{H_E}{M_0}\left(\frac{\Theta_C}{\varepsilon}\right)^3\left[1-\left(\frac{\varepsilon_{10}}{\varepsilon}\right)^2\right]^{-1/2}.$$

Let us estimate σ for $CsMnF_3$ ($H_E \approx 3 \times 10^5$ Oe, $\omega_{20} \approx 7 \times 10^{11}$, $\Theta_C^x \approx 5.3 \times 10^{-15}$ erg[4]) at f = 36 GHz and H = 3×10^3 Oe:

$$\sigma^x \sim 30 / V\Delta\omega_{1\mathbf{k}}.$$

As shown above, if $\Sigma \gg \omega_{20}$ the behavior of the susceptibility beyond the threshold does not depend on the form of the collision integral. Let us consider now the influence of magnon collisions on the magnitude of the instability threshold. The threshold for the scattering of two elementary excitations corresponding to the homogeneous precession **L**, with formation of two magnons of the LF branch, exceeds under ordinary conditions the threshold of the instability considered here by several orders of magnitude. Three-magnon processes with only LF-branch magnons taking part are forbidden. The probability of processes in which two LF-branch magnons and one phonon take part, as shown by calculation, turns out to be very small. The coalescence of two magnons of the LF branch with approximately equal quasimomenta into a single magnon corresponding to the oscillations of **L**, as follows from the energy and quasimomentum conservation laws and from the fact that the value of k for each H is determined from (1.27), is possible only when $\omega \geq \omega_{20} \geq 2\omega_{10}$. In the latter case, the relaxation rate of the LF-branch magnons obtains an increment that does not depend on the pumping power.

We note that in the foregoing calculations it is necessary to take formal account of four-magnon interaction of pairs of excited magnons of the LF branch. It can be shown, however, that for the antiferromagnets investigated here the influence of the indicated interaction on the stationary state is negligibly small.

2. CASE $H_D \neq 0$

1. In this case the action of the alternating magnetic field on the spin system is not limited to excitation of the homogeneous precession **L**. The new terms in the Hamiltonian, describing the decay of a photon of the radiation incident on the crystal into two LF-branch magnons, has the following form:

$$\hat{\mathscr{H}}^{f}_{1\mathbf{k},1-\mathbf{k}} = \sum_{\mathbf{k}} \Psi^{f}_{1\mathbf{k},1-\mathbf{k}}\hat{c}_{1\mathbf{k}}^{+}(t)\hat{c}^{+}_{1-\mathbf{k}}(t)\hat{b}(t) + \text{э. с.}, \qquad (2.1)$$

where

$$\Psi_{1\mathbf{k},1-\mathbf{k}} = -i\sqrt{\frac{2\pi\varepsilon}{V}}\frac{\mu^2 H_D}{4\varepsilon_{1\mathbf{k}}}, \quad H \ll H_E.$$

Defining the threshold of the excitation of spin waves of the LF branch by the condition $c_{1k}(t) = c_{1k}e^{-i\omega t/2}$, which denotes equality of the relaxation and excitation rates, we obtain at $\omega = 2\omega_{1k}$, from the condition that the system of equations of motion for $c_{1k}(t)$ and $c^*_{1-k}(t)$ have a solution, the following expression for the threshold field

$$h_z^{\text{thr}} = \frac{2\Delta\omega_{1\mathbf{k}}\omega(\omega+\omega_{20})\left[(\omega-\omega_{20})^2+\Delta\omega_{20}^2\right]^{1/2}}{\gamma^2\{[\omega_{20}^2\cdot 2H+(\omega_{20}^2-\omega^2)H_D]^2+(\omega+\omega_{20})^2\Delta\omega_{20}^2H_D^2\}^{1/2}}. \qquad (2.2)$$

In the limiting cases of low and high frequency and in the case of saturation of the main resonance, the expression for the threshold field coincides with the formula obtained[3], with allowance for the fact that here $\Delta\omega_{iq}$ has the meaning of the line half-width.

2. To analyze the threshold processes that occur in the spin system under the action of a time-dependent perturbation, we use, in analogy with the case $H_D = 0$, the nonstationary density matrix defined by formulas (1.14) and (1.15). When considering the perturbation of the system of magnons of the LF branch, we use a relation analogous to (1.18):

$$\sum_{m=1}^{N}\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t) = \frac{1}{-i\hbar\Delta\omega_{1\mathbf{k}}}\left[\hat{\mathscr{H}}^{20}_{1\mathbf{k},1-\mathbf{k}}(t)+\hat{\mathscr{H}}^{f}_{1\mathbf{k},1-\mathbf{k}}(t),\ \sum_{m=0}^{N-1}\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t)\right]. \qquad (2.3)$$

However, unlike the case $H_D = 0$, here $\mathscr{L}^{(2m)}(t) \neq 0$ (m = 1, 2, ...), as determined by the coupling of two spin-system excitation channels. To take this coupling into account, we also carry out averaging using the following formula:

$$\sum_{m=2}^{N}\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t) = \frac{1}{(-i\hbar)^2}\int_0^\infty\int_0^\infty d\tau_1\,d\tau_2\exp\left(-i\frac{\hat{\mathscr{H}}_s}{\hbar}(\tau_1+\tau_2)\right)$$
$$\times[\hat{\mathscr{H}}^{20}_{1\mathbf{k},1-\mathbf{k}}(t-\tau_1-\tau_2)+\hat{\mathscr{H}}^{f}_{1\mathbf{k},1-\mathbf{k}}(t-\tau_1-\tau_2)$$
$$+\hat{\mathscr{H}}^{f}_{20}(t-\tau_1-\tau_2),\ \exp\left(-i\frac{\hat{\mathscr{H}}_s}{\hbar}\tau_1\right)\left[\hat{\mathscr{H}}^{20}_{1\mathbf{k},1-\mathbf{k}}(t-\tau_1)+\hat{\mathscr{H}}^{f}_{1\mathbf{k},1-\mathbf{k}}(t-\tau_1)\right.$$
$$\left.+\hat{\mathscr{H}}^{f}_{20}(t-\tau_1),\ \sum_{m=0}^{N-2}\Delta^{(m)}\hat{\rho}_{1\mathbf{k}}(t)\right]\exp\left(i\frac{\hat{\mathscr{H}}_s}{\hbar}\tau_1\right)]\exp\left(i\frac{\hat{\mathscr{H}}_s}{\hbar}(\tau_1+\tau_2)\right), \qquad (2.4)$$

where, in accordance with the statements made above, the summation in the left-hand side begins with the minimal nonzero even index. The averagings on the basis of relations (2.3) and (2.4) do not duplicate each other, since the former yields only $\mathscr{L}^{(i)}(t)$ with odd i, and the latter with even i. In other words, averaging with the density matrix $\rho_{1k}(t)$ obtained after summing a series of the type (1.15) is equivalent to averaging on the basis of relations (2.3) and (2.4) with $N \to \infty$.

In the analysis of the perturbation of a system of elementary excitations corresponding to homogeneous precession **L**, it is necessary to take into account in exactly the same manner the coupling of two excitation channels of the spin system on the basis of relation (2.4), with $\Delta^{(m)}\hat{\rho}_{1k}(t)$ replaced by $\Delta^{(m)}\hat{\rho}_{20}(t)$. The system of kinetic equations have the following form:

$$\frac{dn_{1\mathbf{k}}(t)}{dt} \approx -2\Delta\omega_{1\mathbf{k}}n_{1\mathbf{k}}(t) + \frac{4|\Psi^{f}_{1\mathbf{k},1-\mathbf{k}}|^2}{\hbar^2\Delta\omega_{1\mathbf{k}}}(2n_{1\mathbf{k}}(t)n_f - n^2_{1\mathbf{k}}(t))$$
$$+\frac{4|\Psi^{20}_{1\mathbf{k},1-\mathbf{k}}|^2}{\hbar^2\Delta\omega_{1\mathbf{k}}}(2n_{1\mathbf{k}}(t)n_{20}(t) - n^2_{1\mathbf{k}}(t)) + \frac{8|\Psi^{20}_{1\mathbf{k},1-\mathbf{k}}||\Psi^{f}_{1\mathbf{k},1-\mathbf{k}}|\Psi^{f}_{20}}{\hbar^3\Delta\omega_{1\mathbf{k}}}$$
$$\times\frac{(\omega_{20}-\omega)}{(\omega_{20}-\omega)^2+(\Delta\omega^{\text{thr}}_{20}(t))^2}(2n_{1\mathbf{k}}(t)n_f - n^2_{1\mathbf{k}}(t)),$$
$$\frac{dn_{20}(t)}{dt} \approx -2\Delta\omega_{20}n_{20}(t) + \left(\frac{\Psi^{f}_{20}}{\hbar}\right)^2\frac{\Delta\omega^{\text{eff}}_{20}(t)}{(\omega_{20}-\omega)^2+(\Delta\omega^{\text{eff}}_{20}(t))^2}n_f \qquad (2.5)$$
$$-\sum_{\mathbf{k}}\frac{4|\Psi^{20}_{1\mathbf{k},1-\mathbf{k}}|^2}{\hbar^2\Delta\omega_{1\mathbf{k}}}(2n_{1\mathbf{k}}(t)n_{20}(t)-n^2_{1\mathbf{k}}(t))$$
$$-\sum_{\mathbf{k}}\frac{8|\Psi^{20}_{1\mathbf{k},1-\mathbf{k}}||\Psi^{f}_{1\mathbf{k},1-\mathbf{k}}|\Psi^{f}_{20}}{\hbar^3\Delta\omega_{1\mathbf{k}}}\frac{(\omega_{20}-\omega)}{(\omega_{20}-\omega)^2+(\Delta\omega^{\text{eff}}_{20}(t))^2}(2n_{1\mathbf{k}}(t)n_f - n^2_{1\mathbf{k}}(t)).$$

From the condition that $n_{1k}(t)$ must increase exponentially with time at $t \sim 0$ we obtain the instability threshold (2.2).

In the investigation of the stationary state, we use the following notation:

$$\bar{n}_f = \left(\frac{\hbar\Delta\omega_{1\mathbf{k}}}{2|\Psi^{f}_{1\mathbf{k},1-\mathbf{k}}|}\right)^2, \quad \bar{n}_{20} = \left(\frac{\hbar\Delta\omega_{1\mathbf{k}}}{2|\Psi^{20}_{1\mathbf{k},1-\mathbf{k}}|}\right)^2, \quad \zeta = \left(\frac{h^{\text{tp}}}{h^{\text{thr}}}\right)^2, \qquad (2.6)$$

where

$$h^{tp} = 2\omega\Delta\omega_{1k} / \gamma^2 H_D.$$

Confining ourselves to the case

$$1+\eta^2 \ll [\xi(\zeta-p)(\zeta-1)/\zeta^2]^{-1/2}, \tag{2.7}$$

where $\xi = 2\sum_k \Delta\omega_{1k}\Delta\omega_{20}^{-1}$ and for $H > 10^{-2}H_D\omega^{1/2}\omega_{20}$, recognizing that $\tilde{n}_f \gg \tilde{n}_{20}$, we obtain the stationary solution of the system (2.5):

$$n_{1k} = 2\tilde{n}_{20}\frac{\zeta-p}{\xi(\zeta-1)}\left(\frac{\Delta\omega_{20}^{\mathrm{eff}}}{\Delta\omega_{20}}-1\right),$$
$$\Delta\omega_{20}^{\mathrm{eff}} = \Delta\omega_{20}\left[\frac{\zeta(\eta^2+1)-1}{\zeta-1}\left(p-\frac{\eta^2\zeta}{\zeta(\eta^2+1)-1}\right)\right]^{1/2}. \tag{2.8}$$

To determine the susceptibility $\chi_{zz}(\omega)$ we calculate the projection of the magnetic moment $M = M_1 + M_2$ on the z axis in the second-quantization representation. We represent the result in fields $H \ll H_E$ in the form

$$\delta\hat{M}_z = \delta\hat{M}_z^{\mathrm{I}} + \delta\hat{M}_z^{\mathrm{II}},$$
$$\delta\hat{M}_z^{\mathrm{I}}(t) = -i\sqrt{\frac{M_0\varepsilon_{20}}{2H_E V}}\hat{c}_{20}(t) + \mathrm{h.c.}, \tag{2.9}$$
$$\delta\hat{M}_z^{\mathrm{II}}(t) = \sum_k \frac{\mu^2 H_D}{2\varepsilon V}\hat{c}_{1k}(t)\hat{c}_{1-k}(t) + \mathrm{h.c.}$$

The quantity $c_{20}(t)$ is determined by the expression (1.4). In the calculation of $c_{1k}(t)c_{1-k}(t)$ we take into account the four-magnon interaction of the excited pairs of magnons of the LF branch[9] with k inside an interval k near k_0 as given by (1.27). The threshold of the four-magnon process of production of new pairs of unstable magnons cannot be attained at real pump powers, owing to the smallness of n_{1k} in the stationary state. The equation of motion for $c_{1k}(t)c_{1-k}(t)$ is

$$i\frac{d}{dt}c_{1k}(t)c_{1-k}(t) = 2(\omega_{1k} - i\Delta\omega_{1k})c_{1k}(t)c_{1-k}(t)$$
$$-(c_{1k}^*(t)c_{1k}(t) + c_{1-k}^*(t)c_{1-k}(t))\frac{2}{\hbar}(\Psi_{1k,1-k}^{20}c_{20}(t) + \Psi_{1k,1-k}b(t))$$
$$+\sum_q (c_{1k}^*(t)c_{1k}(t) + c_{1-k}^*(t)c_{1-k}(t))\frac{2}{\hbar}\Psi_{1k,1-k}^{1q,1-q}c_{1q}(t)c_{1-q}(t). \tag{2.10}$$

Let us estimate the contribution of the four-magnon interaction with allowance for the quantity $\Psi_{1k,1-k}^{1q,1-} \sim \mu^2 H_E/16M_0V$[6] and formulas (1.12) and (2.8):

$$\sum_q (n_{1k} + n_{1-k})\frac{2}{\hbar}\Psi_{1k,1-k}^{1q,1-} \sim 10^{-10}\Delta\omega_{1k}^2.$$

Since the condition $\Delta\omega_{1k} < 10^8$ sec^{-1} is certainly satisfied in the cases of interest, the indicated interaction makes a negligibly small contribution to the effect. The solution of (2.10) at $\omega = 2\omega_{1k}$ in the stationary state is then

$$c_{1k}(t)c_{1-k}(t) = (c_{1k}^*c_{1k} + c_{1-k}^*c_{1-k})\sqrt{\frac{2\pi}{\varepsilon V}}\frac{\mu^2 H_D}{2\hbar\Delta\omega_{1k}}b(t)e^{i\delta'},$$

$$\delta' = \mathrm{arctg}\frac{2\Delta\omega_{20}^{\mathrm{eff}}\omega_{20}^2 H/H_D}{(\Delta\omega_{20}^{\mathrm{eff}})^2(\omega+\omega_{20}) + (\omega_{20}-\omega)[2\omega_{20}^2 H/H_D + (\omega_{20}^2-\omega^2)]}. \tag{2.11}$$

Taking formulas (1.4), (1.21), (2.9), and (2.11) into account, we obtain for the susceptibility

$$\chi(\omega, p) = \chi^{\mathrm{I}}(\omega, p) + \chi^{\mathrm{II}}(\omega, p),$$
$$\chi^{\mathrm{I}}(\omega, p) = \frac{M_0}{2H_E}\frac{\omega_{20}}{(\omega_{20}-\omega) - i\Delta\omega_{20}^{\mathrm{eff}}(p)}, \tag{2.12}$$
$$\chi^{\mathrm{II}}(\omega, p) = \sum_k \frac{\hbar\gamma^4 H_D^2}{2V\omega^2\Delta\omega_{1k}}\exp\{i(\delta'(p) - \pi/2)\}n_{1k}(p).$$

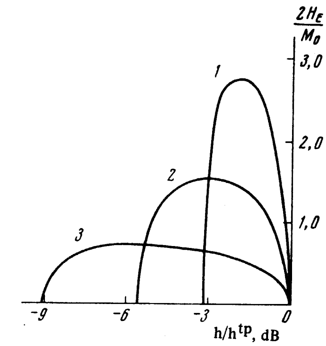

FIG. 2. Dependence of the imaginary part of the susceptibility on h at $\omega \ll \omega_{20}$ and $H_D = 4.4$ kOe and H equal to 1 kOe (curve 1), 2 kOe (2), and 4 kOe (3).

Substituting here formulas (2.8) and (2.11), we obtain the final expressions for the susceptibility, which we present here in two limiting cases. For $\omega \ll \omega_{20}$ we get (see Fig. 2)

$$\mathrm{Re}\,\chi^{\mathrm{I}}(\omega, p) = \frac{M_0}{2H_E}\frac{[(2H+H_D)^2 - H_D^2 p]}{[(2H+H_D)^2 - H_D^2]p},$$
$$\mathrm{Im}\,\chi^{\mathrm{I}}(\omega, p) = \frac{M_0}{8H_E}\frac{H_D(2H+H_D)}{H(H+H_D)}\sqrt{\left(\frac{2H+H_D}{H_D}\right)^2 - p}\,\frac{\sqrt{p-1}}{p},$$
$$\mathrm{Re}\,\chi^{\mathrm{II}}(\omega, p) = \frac{M_0}{128H_E}\frac{H_D^3(2H+H_D)^2}{H^3(H+H_D)^2}\left[\left(\frac{2H+H_D}{H_D}\right)^2 - p\right]\frac{(p-1)}{p}, \tag{2.13}$$
$$\mathrm{Im}\,\chi^{\mathrm{II}}(\omega, p) = \frac{M_0}{64H_E}\frac{H_D^3(2H+H_D)}{H^3(H+H_D)}\sqrt{\left[\left(\frac{2H+H_D}{H_D}\right)^2 - p\right](p-1)}$$
$$\times\left[1 + \frac{(2H+H_D)^2 - H_D^2 p}{2pH_D(H+H_D)}\right].$$

For the case $\omega = \omega_{20}$, when the investigated effect is a superposition of the saturation of the main resonance and parallel pumping (PP), we obtain at $p \ll (\omega_{20}H)^2 \times (\Delta\omega_{20}H_D)^{-2}$

$$\mathrm{Re}\,\chi^{\mathrm{I}}(p) \equiv 0, \quad \mathrm{Im}\,\chi^{\mathrm{I}}(p) = \frac{M_0}{2H_E}\frac{\omega_{20}}{\Delta\omega_{20}}\frac{1}{\sqrt{p}},$$
$$\mathrm{Re}\,\chi^{\mathrm{II}}(p) = \frac{M_0}{4H_E}\frac{H_D}{H}\frac{\sqrt{p}-1}{\sqrt{p}}, \tag{2.14}$$
$$\mathrm{Im}\,\chi^{\mathrm{II}}(p) = \frac{M_0}{4H_E}\frac{\Delta\omega_{20}}{\omega_{20}}\left(\frac{H_D}{H}\right)^2(\sqrt{p}-1).$$

At $h > h^{tp}$ given by formula (2.6), there is no stationary solution. In the case $\omega \gg \omega_{20}$, the region of existence of the stationary solution narrows down practically to zero in view of the fact that $h^{thr} \approx h^{tp}$.

We note that the condition (2.7) means $n_{20} \approx n_{20}^{thr}$. The character of the change of $\chi''(p)$, if n_{20} increases beyond the threshold, can be estimated with the aid of formulas (1.26) and (1.27), obtained for the case $H_D = 0$.

At $\omega = \omega_{20}$ there exists a real stationary solution of Eq. (2.5) at $h > h^{tp}$, which, however, is not realized when account is taken of the saturation of $\delta M_z(t)$. A calculation accurate to n_{jk}^3 yields for $H \ll H_E$

$$\langle M_z^2\rangle \approx \frac{(H+H_D)^2}{4H_E^2}\left[2M_0 - \mu\sum_k\left(\frac{\mu H_E}{\varepsilon_{1k}}n_{1k} + \frac{\mu H_E}{\varepsilon_{2k}}n_{2k}\right)\right]^2$$
$$-\sum_k 2\mu M_0\frac{\varepsilon_{2k}}{\mu H_E}n_{2k} + \frac{\mu^2}{4}\sum_{k,q}\frac{(\mu H_E)^2}{\varepsilon_{1k}\varepsilon_{2q}}n_{1k}n_{2q} - \frac{\mu^2}{4}\sum_{k,q}n_{2k}n_{2q}. \tag{2.15}$$

From this we get

$$\max n_{20} \approx \frac{2M_0\varepsilon_{20}}{\mu^2 H_E}, \quad \max\sum_k n_{1k} \approx \frac{2M_0\varepsilon_{1k}}{\mu^2 H_E}.$$

On the other hand, the already mentioned solution for

n_{1k} and n_{20} at $p \neq \zeta$ is of the form

$$n_{1k} \approx \tilde{n}_f(p-\zeta), \qquad n_{20} \approx \left(\frac{\Psi_{20}{}'}{\hbar}\right)^2 \frac{n_f^{thr}\tilde{n}_f}{\tilde{n}_{20}}\left(\frac{p-\zeta}{\zeta-1}\right) \gg \frac{2M_0\varepsilon_{20}}{\mu^2 H_E}.$$

We note also that at $\omega = \omega_{20}$ threshold scattering of two elementary excitations of the HF branch with $k \approx 0$ is possible, with formation of two elementary excitations of the HF branch with **q** and −**q**. Since in this case the value of q is small and the sum over **q** is accordingly small, one should expect the contribution of this process to the collision integral $\mathcal{I}(n_{20})$ to be negligibly small.

As to the PP in ferromagnets, when account is taken of the density of the final states, the kinetic equation contains formally a term describing the "reaction" on the pump, which, however, is suppressed by the terms responsible for the four-magnon interaction of the pairs of excited magnons. In the case of AA in ferromagnets, all the indicated terms are generally speaking, significant in the kinetic equation.

DISCUSSION OF RESULTS

In the case of AA, the intense absorption is attributed to the increase of the phase difference between the alternating magnetic field and the homogeneous precession **L**, this being due to the increase of the effective rate of relaxation of the homogeneous precession **L**, the cause being the threshold decay into spin waves of LF branch. However, besides the increase in the phase difference, a decrease takes place in the amplitude of the homogeneous precession **L**, which is also due to the increase of $\Delta\omega_{20}^{eff}$. As a result, the susceptibility reaches a maximum and then decreases.

In the case of a superposition of AA and PP, the latter effect (PP) leads to a considerable increase of $\Delta\omega_{20}^{eff}$, the result of which has a decrease in the number of spin waves in the stationary state, and consequently also a decrease of the susceptibility to zero at $h = h^{tp}$. As to the stability of the solution, as indicated above, when the stationary state is investigated the four-magnon interaction can be neglected. No question of stability arises in this case.

A comparison of the experimental dependence of the imaginary part of the susceptibility on the amplitude of the alternating magnetic field, obtained in[5] for f = 36 GHz and H = 2–4 kOe, gives satisfactory agreement with the first formula of (1.26) at $\sigma \approx 0.1$–0.2. The order of magnitude of the calculated imaginary part of the susceptibility also agrees with the estimates of the experimental value $\chi''(\omega, p) \sim 10^{-5}$.

In conclusion, the author is deeply grateful to A. S. Borovik-Romanov, M. I. Kaganov, and M. A. Savchenko for useful discussions.

[1] H. Suhl, J. Phys. Chem. Solids **1**, 209 (1957).
[2] E. Schlömann, J. Appl. Phys. **33**, 527 (1962).
[3] V. I. Ozhogin, Zh. Eksp. Teor. Fiz. **58**, 2079 (1970) [Sov. Phys. JETP **31**, 1121 (1970)].
[4] M. Seavey, J. Appl. Phys. **40**, 1597 (1969); Phys. Rev. Lett. **23**, 132 (1969).
[5] L. A. Prozorova, A. S. Borovik-Romanov, Zh. Eksp. Teor. Fiz., Pis'ma Red. **10**, 316 (1969) [JETP Lett. **10**, 201 (1969)].
[6] V. I. Ozhogin, Zh. Eksp. Teor. Fiz. **48**, 1307 (1965) [Sov. Phys. JETP **21**, 874 (1965)].
[7] M. A. Savchenko and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. **51**, 482 (1966) [Sov. Phys. JETP **24**, 325 (1967)].
[8] R. Kubo, J. Phys. Soc. Jap. **12**, 570 (1957).
[9] J. Bierlein and P. Richards, Phys. Rev. B **1**, 4342 (1970).

Translated by J. G. Adashko
45