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Photon Production by an External Field

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Photon production by a varying electromagnetic field is considered in the low energy approximation.

PHOTON PRODUCTION BY AN EXTERNAL FIELD

It is well known that the polarization of the vacuum in quantum electrodynamics leads to a nonlinear interaction between electromagnetic fields. The following nonlinear effects have been considered: 1) The scattering of light by light;[1-3] 2) elastic scattering of photons in the field of a nucleus (Delbrück scattering);[4,5] the splitting of a photon into two photons,[6,7] and the coalescence of two photons into one,[8] in a Coulomb field. (Photon splitting in uniform magnetic[9,10] and crossed[11] fields has also been considered recently.) We shall here consider the possibility of photon production by a classical varying electromagnetic field. This field can, for example, be of macroscopic origin or belong to a heavy charged particle when we may neglect the effect of other particles upon this particle. A similar approach to pair production in a number of nuclear processes was developed earlier.[12,13] It is clear, to begin with, that the field which produces photons is subject to the requirement of sufficiently rapid variation (a characteristic parameter being the time required for a quantum to traverse one Compton wavelength of the electron). A similar situation exists in the abrupt change of nuclear charge that accompanies β decay or proton capture.

A Feynman diagram corresponding to the considered process is shown in the accompanying figure (to which all possible permutations of the external lines are to be added). The external field will be assumed to be such that the Fourier components of the tensor $F_{\mu\nu}$ are vanishingly small for frequencies exceeding the electron mass m. Under this condition the matrix elements can be calculated with the aid of the Heisenberg-Euler-Schwinger effective Lagrangian[13-15]

$$L' = (e/m)^4 1/(2\pi)^2 \cdot 720 (14\,\mathrm{Sp}\,F^4 - 5\,\mathrm{Sp}^2 F^2), \tag{1}$$

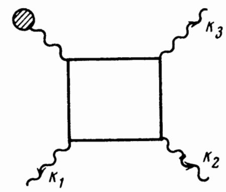
in which we assume $F \rightarrow F + f$, where f corresponds to the fields of real photons.

The four-momenta of the emitted photons are represented by k_1, k_2, and k_3, and the corresponding polarization vectors by e_1, e_2, and e_3. The matrix element of the process, calculated from (1), is given by

$$\begin{aligned} M = \frac{1}{(2\pi)^2 90i}\left(\frac{e}{m}\right)^4 \{14\,\mathrm{Sp}[F(k)f_1f_2f_3 + F(k)f_1f_3f_2 \\ + F(k)f_2f_1f_3] - 5[\mathrm{Sp}(F(k)f_1)\mathrm{Sp}(f_2f_3) \\ + \mathrm{Sp}(F(k)f_2)\mathrm{Sp}(f_1f_3) + \mathrm{Sp}(F(k)f_3)\mathrm{Sp}(f_1f_2)]\}. \end{aligned} \tag{2}$$

Here $F_{\mu\nu}(k) = \int \exp(-ikx) \times F_{\mu\nu}(x) d^4x$, $F^*_{\mu\nu}(k) = F_{\mu\nu}(-k)$; $(f_i)_{\mu\nu} = (k_\mu e_\nu - e_\mu k_\nu)_i$.

The transferred momentum $k = k_1 + k_2 + k_3$ is time-



like. When $k^2 = 0$ the momenta of the created photons are parallel and the corresponding phase volume vanishes.[9] This leads to the following selection rule: The Fourier transform of an external field that produces three photons must differ from zero for $k^2 < 0$ [using the Euclidean metric and the scalar product $(ab) = \mathbf{ab} - a_0b_0$]. The integral over the phase volume can be transformed into an integral over the transferred four-dimensional momentum, after which the total probability of the process is written in the clearly covariant form

$$W_{3\gamma} = \frac{17}{4\cdot 90^3(2\pi)^8}\left(\frac{e}{m}\right)^8 \int (-k^2)^4 F_{\mu\nu}(k)F_{\nu\mu}(-k)\theta(-k^2)\theta(k_0)d^4k, \tag{3}$$

where $\theta(x)$ is a familiar step function.

Using Maxwell's equations for the external field,

$$iF_{\mu\nu}(k)k_\nu = 4\pi j_\mu(k), \tag{4}$$

and the relation

$$k^2F_{\mu\nu}(k)F_{\nu\mu}(-k) = 2k_\mu F_{\mu\nu}(k)F_{\nu\lambda}(-k)k_\lambda, \tag{5}$$

the probability (3) can be expressed by means of the density of the current that excites the external field:

$$W_{3\gamma} = \frac{34}{90^3(2\pi)^6}\left(\frac{e}{m}\right)^8 \int (-k^2)^3 j_\mu(k)j_\mu(-k)\theta(-k^2)\theta(k_0)d^4k. \tag{6}$$

Equations for the total emitted energy are obtained by addition of the factor k_0 to the integrands in (3) and (6).

As a concrete example we shall consider a pulsed electric field, which for simplicity is assumed to be spatially uniform: $\mathbf{E} = \mathbf{E}_0 \exp(-t^2/\tau^2)$. Integrating in (3) over the transferred momentum, we obtain the following expression for the probability per unit volume that three photons are produced:

$$w_{3\gamma} = \frac{17\cdot 105}{180^3(2\pi)^{7/2}} e^6 \left(\frac{E_0}{E_{\mathrm{cr}}}\right)^2 \left(\frac{\tau}{\tau_0}\right)^{-7} \lambda\!\!\!^{-}{}^{-3}. \tag{7}$$

Here $E_{cr} = m^2/e = 4.41 \times 10^{13}$ Oe, $\lambda\!\!\!^{-}$ is the Compton wavelength of the electron, τ_0 is the time required for a quantum to travel one Compton wavelength. The probability is here seen to depend on a high power of the ratio between the pulse length τ and the time τ_0.

In the case of the abruptly changing electric field

$$\mathbf{E} = \mathbf{E}_0 / (1 + e^{t/\tau})$$

analogous calculations yield

$$w_{3\gamma} = \frac{17}{120 \cdot 90^3 (2\pi)^4} e^6 \left(\frac{E_0}{E_{cr}}\right)^2 \left(\frac{\tau}{\tau_0}\right)^{-7} \lambda^{-3}. \tag{8}$$

These examples show that the considered effect can actually be realized only in rapidly varying fields. It must be kept in mind, however, that the derived equations are applicable only for $\tau \lesssim \tau_0$, because the low-energy approximation (1) was used at the outset. We can also similarly consider diagrams in which two or three lines correspond to the external field. In the first case photon production is possible if for $k^2 < 0$ the Fourier transforms of quadratic combinations consisting of components of $F_{\mu\nu}$ do not vanish. In the second case it is required that the Fourier transforms of the products of three tensors should not vanish for $k^2 = 0$. We shall now give the result calculated for the production of a single photon by an external field comprising the superimposition of a screened Coulomb field $\varphi(r) = Zer^{-1} \exp(-r/R)$ on the field of a linearly polarized plane wave, $E = E_0 \cos(\kappa x)$.

The differential probability per unit time that this field will produce a single photon with the doubled frequency $2\kappa_0$ is

$$\frac{dw}{d\Omega} = \frac{Z^2 e^6}{45^2 \pi^3} \varkappa_0^5 \left(\frac{E_0}{E_{cr}}\right)^4 \frac{4 \sin^2\theta (1 - \cos\theta)^2 (49 - 33\cos^2\varphi)}{[R^{-2} + 8\varkappa_0^2 (1 - \cos\theta)]^2} \tag{9}$$

Here θ is the angle between the wave vectors of the initial wave (κ) and the created photon ($\mathbf{k}$); φ is the angle between the polarization vector of the initial wave and the projection of $\mathbf{k}$ on a plane perpendicular to κ.

Integrating over the angles of photon emission, we obtain the total probability per unit time:

$$w = \frac{65}{9} \frac{Z^2 e^6 \varkappa_0}{2\pi^2 60^2} \left(\frac{E_0}{E_{cr}}\right)^4 \frac{1}{x^3} \left[\frac{2}{3} x^3 - 4x - 2x^2 + (3x + 2) \ln(1 + 2x)\right], \quad x = 8\varkappa_0^2 R^2. \tag{10}$$

From a physical point of view this process is identical with the coalescence of two photons into one photon in the field of a nucleus, which has been considered previously in [8, 16]. In our case, however, there is the difference that the two original photons are identical and we do not average over their polarizations. The case of circular polarization has been considered by Yakovlev;[17] the corresponding probability differs from (10) by the factor 9/65.

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