Stimulated Scattering of Electromagnetic Waves from a Highly Conducting Surface

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It is shown that for the stimulated scattering from the surface of a highly conducting medium, the theory that is linear in terms of the surface inclinations is not valid, owing to resonance with the surface electromagnetic modes. The linear theory leads to divergent results as the conductivity tends to infinity. Correct results can be obtained by solving the essentially nonlinear self-consistent problem of diffraction by a periodic surface. The initial surface takes on a periodic structure due to the effect of the incident wave and the resultant instability. A threshold minimum occurs at resonance when one of the diffracted spectra grazes along the surface. Hence, owing to the resonant character of the interaction, the threshold becomes much lower than that for scattering by the boundary of a transparent medium. The nonlinearity of the problem leads to the dependence, derived in the paper, of the gain and the threshold on the amplitude of the amplified wave.

STIMULATED scattering of electromagnetic waves from the surface of a highly conducting medium (for example, a liquid metal) should possess a number of important features. Actually, it is known from diffraction theory that the amplitudes of grazing waves, scattered by an ideally conducting periodic surface, is anomalously large[1)]. This is connected with the fact that undamped H waves are propagated along the boundary of an ideally conducting medium.[1] Therefore, resonance develops in diffraction by such a surface[2)] when one of the scattered waves slides along it; the amplitude of this wave increases appreciably.

The occurrence of the resonance increases also the reaction of the scattered and incident waves on the motion of the interface; the threshold of stimulated scattering from surface waves (SSSW) is correspondingly decreased and becomes much less than in stimulated scattering from the boundary of a transparent medium.[3-7] Close to resonance, the problem is essentially nonlinear even for small inclinations of the surface.

1. AMPLITUDES OF THE SCATTERED WAVES. RESONANCE

The scattered waves are determined by the boundary conditions at the excited surface $z = \zeta(\mathbf{r}, t) = a\cos(\Omega t - \mathbf{q}\cdot\mathbf{r})$ (the z axis is orthogonal to the undisturbed surface):

$$\vec{\mathscr{E}}_t = \xi[\mathbf{n}, \vec{\mathscr{H}}_t], \tag{1.1}$$

where $\mathbf{n} = \mathbf{e}_z - \nabla\zeta$ is the outward normal, ξ the surface impedance ($|\xi| < 1$), $\vec{\mathscr{E}}_t$ and $\vec{\mathscr{H}}_t$ are the tangential components of the electric and magnetic fields which, because of the smallness of the inclination $|\nabla\zeta| \ll 1$ we shall seek in the form of an expansion in plane waves:[3)]

$$\begin{pmatrix}\vec{\mathscr{E}}\\ \vec{\mathscr{H}}\end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}\exp(-i\omega t + i\mathbf{kr} + ik_z z) + \sum_{m=-\infty}^{\infty}\begin{pmatrix}\mathbf{E}_m\\ \mathbf{H}_m\end{pmatrix}\exp(-i\omega_m t + i\mathbf{k}_m\mathbf{r} + ik_{mz}z)\right].$$

Here E and H are the amplitudes of the incident plane wave, ω and $\mathbf{k} = (k_x, 0, k_z) = k(\sin\theta, 0, -\cos\theta)$ are its frequency and wave vector; $\mathbf{E}_m$, $\mathbf{H}_m$, $\mathbf{k}_m$, ω_m are the amplitudes, wave vectors and frequencies of the scattered waves, $\mathbf{r} = (x, y, 0)$ is a vector lying in the plane of separation. By virtue of the laws of conservation of energy and impedance,

$$\omega_m = \omega + m\Omega, \quad k_m^{x,y} = k^{x,y} + mq^{x,y}, \quad m = 0, \pm1, \pm2, \dots \tag{1.3}$$

The normal components of the wave vectors are found from the dispersion laws $\omega_m^2/c^2 = k_{mz}^2 + k_{mx}^2 + k_{my}^2$, and since $\Omega \ll \omega$, we have, in the nonrelativistic approximation, $\omega_m^2/c^2 \approx \omega^2/c^2 = k^2$ and, consequently,

$$k_{mz} = [k^2 - (k_{mx}^2 + k_{my}^2)]^{1/2}, \quad \mathrm{Im}\, k_{mz} \geqslant 0, \quad \mathrm{Re}\, k_{mz} \geqslant 0. \tag{1.4}$$

Limiting ourselves for simplicity to the case of plane geometry (the wave vector $\mathbf{q} = (q, 0, 0)$ of the surface wave lies in the plane of incidence (x, z)) and to H-polarization of the incident wave $\mathbf{H} = (0, H, 0)$, we reduce the boundary conditions (1.1), with account of (1.2), to the form[4)]

$$\sum_{m=-\infty}^{\infty} H_m i^m J_{m+l}(s_m)\alpha_{m,m+l} = \lambda_l H, \quad l = 0, \pm1, \pm2, \dots, \tag{1.5}$$

where

$$\begin{gathered}\lambda_0 = \beta_0 - \xi, \quad \lambda_{\pm1} = -\tfrac{1}{2}aq\sin\theta, \quad \lambda_l = 0 \ \text{ for } |l| \geqslant 2;\\ s_m = ak(\beta_m + \beta_0), \quad \beta_m(\varkappa, \theta) = k_{mz}/k,\\ \alpha_{n,m}(\varkappa, \theta) = \beta_n + \xi + \frac{m\varkappa(\sin\theta + n\varkappa)}{\beta_n + \beta_0},\end{gathered} \tag{1.6}$$

[1)]In the case considered by us, the surface becomes periodic because the incident electromagnetic wave induces a flexural wave $\zeta = a\cos(\Omega t - \mathbf{q}\cdot\mathbf{r})$ as a result of the development of an instability.

[2)]L. N. Deryugin was the first to point out the origin of surface resonance in diffraction by a periodic conducting surface.[2]

[3)]A similar problem on the diffraction of a sound wave by a periodic surface was considered by Lysanov[8] (normal incidence, impedance equal to zero), and also by Urusovskii,[9] by whom the general situation was analyzed. In what follows, we shall follow the method proposed in[8].

[4)]It was proved in [8] that a system of the type (1.5), in the case of a vanishing impedance and for normal incidence (0=0), can be solved by truncating it. Here, near resonance, the result depends on the relation between the small parameters $(qa)^2 \ll 1$ and the dimensionless normal component of the wave vector β_m of the sliding spectrum. In the case of a small but nonzero impedance ξ three small parameters $(qa)^2$, $|\xi|$, $|\beta_m|$ appear near resonance and the solution of the system (1.5) strongly depends on the relations among these, although the possibility of truncation itself is associated only with the smallness of the inclination $qa \ll 1$; see also[9].

*$[\mathbf{n}, \vec{\mathscr{H}}_t] \equiv \mathbf{n} \times \vec{\mathscr{H}}_t$.

$\kappa = q/k$ is the dimensionless wave number of the surface wave; J is the Bessel function.

As will be shown below, solution of (1.5) with accuracy up to the principal terms in the small inclinations of the surface $(qa)^2 \ll 1$ has the form

$$H_m/H \sim (qa)^{|m|}[(qa)^2 + \beta_m + \xi]^{-1}$$

and it is impossible to obtain it with the help of ordinary perturbation theory, which leads to $H_m/H \sim (qa)^{|m|}$. Near resonance, when the normal component of one of the the wave vectors of the scattered waves becomes very small ($|\beta_m| = |k_{mz}/k| \ll |(qa)^2 + \xi|$), the amplitude of the sliding spectrum increases materially. Thus, as $\beta_1 \to 0$,

$$H_1/H \sim qa[(qa)^2 + \xi]^{-1}.$$

For very small inclinations $(qa)^2 \ll |\xi|$ the results of ordinary perturbation theory are valid and

$$H_1/H \sim qa/\xi,$$

for in the case of inclinations that are large compared with the impedance, $(qa)^2 \gg \xi$, we have

$$H_1/H \sim 1/qa.$$

Inasmuch as we shall need in the following only the principal terms of the expansion of the fields in the small parameter $(qa)^2$ and corrections to them connected with the small parameter $(qa)^2/|\xi|$ in the case of small inclinations and with $|\xi|/(qa)^2$ in the case of large ones, it suffices to limit ourselves to the five central equations of the system (1.5) ($l = 0, \pm 1, \pm 2$), leaving in them the amplitudes of the scattered spectra H_m up to second order inclusively ($m = 0, \pm 1, \pm 2$). In this case, we obtain the truncated system:

$$\sum_{m=-2}^{2} X_m i^m J_{m+l}(s_m)\alpha_{m,m+l} = \lambda_l, \quad l = 0, \pm 1, \pm 2, \tag{1.7}$$

where $X_m = H_m/H$ are the dimensionless amplitudes of the scattered spectra. Without presenting the straightforward but cumbersome calculations, we write out only the final result.

For small inclinations $(qa/2)^2 \ll \beta_0|\beta_{\pm 1} + \xi|$, we have

$$X_0 = (\cos\theta - \xi)/(\cos\theta + \xi), \tag{1.8}$$

$$X_m = -iak\frac{\cos\theta}{\cos\theta + \xi}\left[\beta_m - \xi + \frac{m\kappa(\sin\theta + m\kappa)}{\beta_m + \xi}\right], \quad m = \pm 1. \tag{1.9}$$

The amplitudes of the spectra of second order are $\sim (qa)^2$ and in what follows they will not be needed. We note that Eqs. (1.8) and (1.9) are applicable both far from resonance, $\beta_{\pm 1} \sim 1$ and also near resonance, for inclinations that are small[5)] in comparison with the impedance, $(qa/2)^2 \ll |\xi| \cos\theta$. It should be noted that to find the principal terms in the amplitudes X_0 and $X_\pm$ under the condition $\beta_0|\beta_{\pm 1} + \xi| \gg (qa/2)^2$, it suffices to consider in place of (1.7) a set of three equations relative to these amplitudes.

[5)]If we use a scattering theory that is linear in the inclination $qa \ll 1$, then we get the expressions (1.8) and (1.9), and when the conductivity of the medium $\sigma \to \infty$ (the impedance $\xi \to 0$) we get for the amplitude of the grazing specturm an expression that diverges at resonance, which demonstrates the inapplicability of the linear theory. The region of its applicability is determined not by the inequality $qa \ll 1$, but by the stronger inequality $(qa/2)^2 \ll |\xi| \cos\theta \ll 1$.

For sufficiently large inclinations $(qa/2)^2 \gg |\xi| \cos\theta$, near resonance, when $|\beta_1| \ll |\xi| \ll |\beta_{-1}|$, the amplitudes are

$$X_0 = \frac{\beta_0 - \beta_2}{\beta_0 + \beta_2}, \quad X_1 = \mp\frac{4i}{qa}\frac{\beta_0\beta_2}{\beta_0 + \beta_2}, \quad X_2 = \frac{2\beta_0}{\beta_0 + \beta_2}; \tag{1.10}$$

$$\beta_2 \approx i[(1 \mp \sin\theta)(3 \mp \sin\theta)]^{1/2}, \quad q \approx (\pm 1 - \sin\theta)k. \tag{1.11}$$

Generally, $X_m \sim (qa)^{|m|-1}$ as $\beta_1 \to 0$, $m = 0, \pm 1, \pm 2, \ldots$.

As will be shown below, the combination of fields $H_1(H_0 + H) - H_2H_1$ enters into the "radiation pressure," and, since $H_0, H_2 \sim H$, $H_1 \sim H/qa$ for large inclinations, it can be expected that $H_1(H_0 + H) - H_2H_1 \sim |H|^2/qa$. But, substituting the fields H_0, H_1, H_2 from (1.10) in this expression, we can verify that $H_1(H_0 + H) - H_2H_1 = 0$ in this approximation, i.e., the principal terms of the expansion of the fields make not contribution to the radiation pressure. In order to find the first nonvanishing term in the pressure, it is necessary to find the subsequent terms in the expansion of the amplitudes of the scattered spectra, the ratio of which to the principal will be $\sim O[(qa)^2] + O(|\beta_1 + \xi|)$. In the case $(qa/2)^4 \ll |\xi| \cos\theta \ll (qa/2)^2$, account of corrections of the type $O(|\xi|)$ for $|\beta_1| \ll |\xi|$, $(qa/2)^2$ and $q \approx k(\pm 1 - \sin\theta)$ gives the nonvanishing result

$$H_1(H_0 + H)^* - H_2H_1^* = -8i\frac{(1 \pm \sin\theta)(3 \mp \sin\theta)}{(1 \mp \sin\theta)^2}\frac{\xi^*}{(ka)^3}|H|^2. \tag{1.12}$$

If $(qa)^4 \gg |\xi| \cos\theta$, then the most important corrections $\sim O[(qa)^2]$ and consequently, $H_1(H_0 + H) - H_2H_1 \sim qa|H|^2$, i.e., of the same order as in the absence of resonance. By virtue of the relative smallness of the radiation pressure, this case represents no interest.

To conclude this section, we note that

$$X_{-n}(\kappa, \theta) = X_n(-\kappa, \theta) = X_n(\kappa, -\theta),$$

which follows from the system (1.5) with account of

$$\alpha_{n,m}(\kappa, \theta) = \alpha_{-n,-m}(-\kappa, \theta) = \alpha_{-n,-m}(\kappa, -\theta) = \alpha_{n,m}(-\kappa, -\theta),$$

and

$$\beta_{-n}(\kappa, \theta) = \beta_n(-\kappa, \theta) = \beta_n(\kappa, -\theta).$$

This circumstance allows us to limit our consideration to only one of the resonances of first order $\beta_1 \to 0$, since the case $\beta_{-1} \to 0$ is easily obtained by means of the corresponding substitution ($n \to -n$, $\theta \to -\theta$) in the final formulas. We shall not consider resonances of order higher than first, since it is not difficult to show that they do not lead to an increase in the reaction of the incident and scattered spectra on the motion of the boundary in comparison with the nonresonant situation.

2. THE DISPERSION EQUATION FOR WAVES ON THE SURFACE IN THE PRESENCE OF AN ELECTROMAGNETIC FIELD

The mechanical action of a strong electromagnetic wave on the motion of the interface is associated with the transfer of momentum from the field to the medium. The force acting on a unit of surface:

$$F_i = \Pi_{ik}n_k|_{z=\zeta(x,t)}, \tag{2.1}$$

where

$$\Pi_{ik} = -\frac{1}{4\pi}\left[\mathscr{E}_i\mathscr{E}_k + \mathscr{H}_i\mathscr{H}_k - \frac{1}{2}\delta_{ik}(\mathscr{E}^2 + \mathscr{H}^2)\right]$$

is the Maxwell stress tensor.[6)] This force, which is equal to the momentum flux of the field through the surface of the metal, was taken into account in the boundary conditions for the equations of motion of the medium,[10] and one can assume the force to be a surface one if the depth of the skin layer $\delta \sim |\xi|/k$ is small in comparison with the penetration depth of the surface wave $\sim 1/q \gtrsim 1/k$. Then the derivation of the dispersion equation of surface waves is completely analogous to the one given previously for transparent media (see[4] for a liquid, and[5] for an isotropic solid), therefore, we give only the final result.[7)]

For capillary-gravitational waves on the surface of the liquid metal,

$$\Omega(q) = \pm\Omega_0(q) - i\Gamma(q) \mp \frac{iq^2P}{2\rho\Omega_0(q)}\frac{|H|^2}{8\pi}, \qquad (2.2)$$

where $\Omega_0(q) = (gq + \alpha q^3/\rho)^{1/2}$ is the dispersion law of the surface wave; $\Gamma(q) \sim q^2$ in the low-frequency region is its damping; α, ρ, and g are the surface tension, liquid density, and acceleration due to gravity. The quantity P in (2.2) is the dimensionless radiation pressure at the frequency of the surface wave, defined by the equality

$$(p_\mathrm{r})_{q\Omega} = (F_n)_{q\Omega} = -iq\zeta_{q\Omega}P|H|^2/8\pi, \qquad (2.3)$$

where F_n is the component of the force (2.1) normal to the surface[8)], $\zeta_{q\Omega} = a/2$ is the Fourier component of the depression. In the case of a dielectric $P \sim 1$,[4,5] while at resonance $P \sim 1/\xi$. An explicit expression for P is given below ((2.7), (2.9), (2.10)). The dispersion law for Rayleigh waves has the same form as in (2.2), except that $\Omega_0(q) = c_R q$, where c_R is the velocity of the Rayleigh wave, and there is a factor of the order of unity in front of the latter component.

For a sufficiently high intensity of the incident radiation, $I = c|H|^2/8\pi \geq I_0$, an instability arises in the system, manifesting itself in the growth of the amplitude of the depression with increment

$$\Omega''(q) = \frac{q^2|\mathrm{Re}\,P|}{2\rho c\Omega_0(q)}I - \Gamma(q). \qquad (2.4)$$

The threshold intensity is

$$I_0(q) = 2\rho c\Omega_0(q)\Gamma(q)/q^2|\mathrm{Re}\,P|. \qquad (2.5)$$

It is seen from (2.4), (2.5) that the threshold of SSSW depends essentially on the value of Re P.

According to (2.1), (2.3) and (1.2), we get in the case of incidence of a plane H-polarized wave H = (0, H, 0):

$$(p_\mathrm{r})_{q\Omega} = \frac{|H|^2}{8\pi}\left\{\sum_{n=-\infty}^{\infty}\left[\frac{k^2-k_xk_{nx}}{k^2}i^{n-1}J_{n-1}(s_n)X_n + \frac{k^2-k_xk_{nx}}{k^2}(-i)^{n+1}J_{n+1}(s_n^*)X_n^*\right]\right.$$

$$\left. + \sum_{\substack{n,m,\nu,\nu' \\ n-m-\nu+\nu'=1}} \frac{k^2-k_{nx}k_{mx}}{k^2}i^{n-m+1}J_\nu(s_n)J_{\nu'}(s_m^*)X_nX_m^*\right\}. \qquad (2.6)$$

For small inclinations $(qa/2)^2 \ll |\xi|\cos\theta$, and also far from resonance, in accord with the previous section, $X_n \sim (qa)^{|n|}$. Consequently, to find the principal term in the radiation pressure, it suffices to limit ourselves to the amplitudes X_0, $X_{\pm 1}$, which give a contribution $\sim qa$ in (2.6). Using (1.8), (1.9), (2.3), and (2.6), we find P:

$$P = 2\left|\frac{\cos\theta}{\cos\theta+\xi}\right|^2\left[\cos^2\theta\frac{\beta_1-\beta_{-1}^*}{\varkappa} + \frac{\varkappa}{\beta_1+\xi} - \frac{\varkappa}{\beta_{-1}^*+\xi^*}\right]. \qquad (2.7)$$

As is seen from (2.7), far from resonance, $P \sim 1$ and at resonance ($|\beta_1| \lesssim |\xi|$) the pressure P increases to a value $\sim 1/|\xi| \gg 1$.

Close to resonance ($\beta_1 \to 0$, $q = k(\pm 1 - \sin\theta)$) for large inclinations $(qa/2)^2 \gg |\xi|\cos\theta$ we have $X_m \sim (qa)^{|m|-1}$ and

$$(p_\mathrm{r})_{q\Omega} = (|H|^2/16\pi)(1 \mp \sin\theta)[X_1(X_0^*+1) - X_2X_1^*]. \qquad (2.8)$$

The calculation of Eqs. (2.8) has already been performed (see (1.12)). Thus, for $(qa/2)^2 \gg |\xi|\cos\theta \gg (qa/2)^4$,

$$P = 8\frac{(1\pm\sin\theta)(3\mp\sin\theta)}{(1\mp\sin\theta)^2}\frac{\xi^*}{(ka)^4}. \qquad (2.9)$$

In the case of very large inclinations $(qa/2)^4 \gg |\xi|\cos\theta$, the pressure $P \sim 1$ in correspondence with the remark made in Sec. 1.).

For dimensionless radiation pressure P, it is not difficult to find the interpolation formula which qualitatively describes the behavior of the function $P(\kappa, \theta, (qa)^2, \xi)$ and which leads to the correct results in the limiting cases considered:

$$P = \varkappa\left\{\frac{B(\varkappa)(\beta_1+\xi)^*\cos^2\theta + (qa/2)^4B_1}{|\beta_1+\xi|^2\cos^2\theta + (qa/2)^4} - \frac{B^*(-\varkappa)(\beta_{-1}+\xi)\cos^2\theta + (qa/2)^4B_1^*}{|\beta_{-1}+\xi|^2\cos^2\theta + (qa/2)^4}\right\}; \qquad (2.10)$$

$$B(\varkappa) = 2\left|\frac{\cos\theta}{\cos\theta+\xi}\right|^2\left[1 + \frac{(\beta_1^2-\xi^2)\cos^2\theta}{\varkappa}\right]\frac{|\beta_1+\xi|\cos\theta}{|\beta_1+\xi|\cos\theta + (qa/2)^2} - \frac{(qa/2)^2\beta_2^2(\varkappa+\sin\theta)}{2\varkappa[|\beta_1+\xi|\cos\theta + (qa/2)^2]}, \quad B_1 \sim 1, \qquad (2.11)$$

It is seen from (2.10) that the SSSW threshold (2.5) reaches a minimum as a function of the wave number q of the excited surface wave when one of the scattered first order spectra grazes the surface, which corresponds to the onset of resonance in the surface electromagnetic modes.[1] Here the amplitude of the resonantly scattered spectrum

$$H_m \sim qa[\beta_m + \xi + (qa/2)^2]^{-1}H$$

increases significantly.[9)]

When ordinary perturbation theory ($(qa/2)^2 \ll |\xi|\cos\theta$) is valid, it is not difficult to obtain an expression for the dimensionless radiation pressure P for arbitrary linear polarization of the incident wave $\mathbf{E} = E(-\sin\varphi\cos\theta, \cos\varphi, -\sin\varphi\sin\theta)$ and arbitrary

[6)]We note specially that all the calculations are carried out for an ideally conducting medium, and the finite conductivity is taken into account only in the resonance terms, where it leads to a finite (nonzero) width of the resonance and consequently to a finite value of the force at resonance. See also footnote 9 below.

[7)]Although the medium is absorbing, one can show that in a number of cases it is legitimate to neglect thermal effects in the derivation of the dispersion equation (2.2).

[8)]We note that the force exerted on the surface by the electromagnetic wave also has tangential components, but they are small in proportion with the smallness of the impedance ξ.

[9)]The width of the resonance is determined both by the original damping of the surface H mode $\sim \xi_0$ and by its scattering on the rough surfaces, which leads to an additional damping $\sim (qa)^2$. We also note that the considered resonance is analogous to scattering from a quasistationary level[11] (cf. (1.8) and (1.9) with the Breit-Wigner formula).

orientation of the wave vector $\mathbf{q} = q(\cos\psi, \sin\psi, 0)$:

$$P = -2\Big[\frac{\beta_{-1}^* - \beta_1}{\varkappa}\cos^2\theta + \varkappa\Big(\frac{1}{\beta_{-1}^* + \xi^*} - \frac{1}{\beta_1 + \xi}\Big)(\sin\varphi\cos\psi - \cos\varphi\sin\psi\cos\theta)^2\Big]. \quad (2.7')$$

Here[10] $\varphi = \cos^{-1}(E_y/E)$, $\psi = \tan^{-1}(q_y/q_x)$, $0 \le \varphi \le \pi/2$. By comparing Eqs. (2.7) and (2.7′), we can establish the fact that consideration of the general case leads only to the appearance in the resonance components of a factor of the order unity, which depends on the angles φ and ψ. This factor goes to zero and, consequently, the resonance disappears in this case when waves that are H-polarized in the scattering plane are missing from the scattered waves.

We shall not give the expressions for the amplitudes of the scattered spectra in the general case of small inclinations, since they are easily obtained from the corresponding formulas of the work of Gavrikov et al.[5] by means of the limit transition $\epsilon \to i\infty$.

The dimensionless radiation pressure $P \sim 1$ far from resonance (the same as in scattering by a dielectric[4,5]) and at resonance increases to a value $P \sim 1/\xi$ for $(qa/2)^2 \ll |\xi|\cos\theta$ and $P \sim |\xi|/(qa)^4$ for $(qa/2)^4 \ll |\xi|\cos\theta \ll (qa/2)^2$. Consequently, the increment decreases with increase in the amplitude of the amplified surface wave (the threshold increases, see the drawing); the minimum of the SSSW threshold $I_0 \sim 2\rho c\Omega_0\Gamma|\xi|/q^2$ is reached for small inclinations $(qa/2)^2 \ll |\xi|\cos\theta$.[11] The expression for the threshold close to resonance is materially simplified. Thus, for $|\beta_{-1}| \gg |\beta_1|$ and $\kappa/|\beta_1 + \xi| \gg \beta_0^2|\beta_1 - \beta_1^*|/\kappa$, we need keep in (2.7) only the first component and

$$I_0 = \frac{\rho c\Gamma(q)\Omega_0(q)}{q^2|\varkappa|}\left|\frac{\beta_0+\xi}{\beta_0}\right|^2\left|\frac{(\beta_1+\xi)^2}{\mathrm{Re}(\beta_1+\xi)}\right|. \quad (2.12)$$

The minimum value of the threshold corresponds to such wave numbers $q = \kappa k$ for which $\beta_1(\kappa, \theta) = -i\,\mathrm{Im}\,\xi$ and $\kappa = \pm 1 - \sin\theta \pm (\mathrm{Im}\,\xi)^2/2$; if $\mathrm{Im}\,\xi < 0$, and is equal to

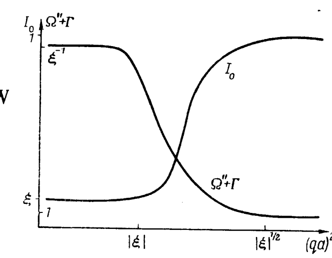
$$I_0 = \frac{\rho c\Gamma(q)\Omega_0(q)}{|\varkappa|q^2}\left|\frac{\beta_0+\xi}{\beta_0}\right|^2\mathrm{Re}\,\xi. \quad (2.13)$$

Formula (2.13) is valid for angles of incidence not very close to 0 and $\pi/2$ (normal and grazing incidence): $\theta \gg (\mathrm{Im}\,\xi/2)^2$ and $(\pi/2 - \theta)^2 \gg (\mathrm{Im}\,\xi)^2/2$. As is seen from (2.13), the SSSW threshold depends weakly on the angle of incidence for mean values of $\theta \sim 1$, while the threshold, as a function of the angle of incidence, reaches a minimum for grazing incidence in the case of scattering on capillary waves and is close to normal incidence in the other cases (scattering from Rayleigh waves at low $T \ll T_D$ and high $T \gg T_D$ (T_D is the Debye temperature) and from gravitational waves). It must be noted that for normal incidence, the threshold goes to infinity because of the vanishing of Re P, as follows from (2.7) in the case of small inclinations.[12]

[10]The factors $\cos\theta/(\cos\theta + \xi)$, which are of the order of unity far from grazing incidence, are omitted in (2.7′).

[11]We note that for thermal fluctuations, the inequality $(qa/2)^2 \ll |\xi|\cos\theta$ is virtually always satisfied (cf[5]).

[12]Starting out from the expression (2.6) and the relations $\beta_{in} = \beta_n X_n = X - n$, we can show that Re P=0 in the case of normal incidence for arbitrary relation between the inclination qa and the impedance ξ.



Schematic dependence of SSSW threshold I_0 and of the growth increment $\Omega'' + \Gamma$ on the amplitude of the amplified surface wave.

The previous analysis pertained to the instability threshold for SSSW. As was remarked earlier,[5], it will correspond to the threshold of appearance of SSSW in the experiment only for sufficiently large effective interaction distances R (for example, R is the dimension of the irradiated region) and long times τ_i of the incident radiation:

$$\tau_i \gg N/\Gamma, \quad R \gg v_{gr}N/\Gamma, \quad (2.14)$$

where $N \sim 10$, $v_{gr} = \partial\Omega_0/\partial q$. If (2.14) is not satisfied, then the threshold of observation of SSSW exceeds the instability threshold:

$$I \geqslant I_0[1 + N/\Gamma\tau]; \quad \tau^{-1} = \tau_i^{-1} + v_{gr}/R. \quad (2.15)$$

Estimates of the SSSW threshold on the surface of a highly conducting medium show that in a number of cases this effect can evidently be observed experimentally. The situation is seen to be especially favorable in the UHF range. Thus, in the case of scattering by gravitational waves on surfaces of liquid metals, $\mathrm{Re}\,\xi \sim 10^{-4}$ and for $k \approx 2\ \mathrm{cm}^{-1}$ the threshold of instability is $I_0 \sim 1\ \mathrm{W/cm}^2$, while the threshold of observation (2.15) for $R \gtrsim 10$ cm, $\tau_r \gtrsim 1$ sec is of the order of 100 W/cm^2. In scattering on Rayleigh waves at low temperatures $T \ll T_D$ under conditions of the anomalous skin effect, the threshold of instability is also small. We note that the possibilities of observation of stimulated scattering are limited by heating of the medium from absorption. Effects associated with local heating of the medium, which are especially important at low temperatures, can make difficult the observation of SSSW. However, estimates which we shall not give because of insufficient space show that long duration pulses are necessary to assure heat transfer. It should also be taken into account that the SSSW threshold depends weakly on the angle of incidence and that the fraction of absorbed power is materially reduced upon increase in the angle of incidence, since the reflection coefficient is increased in this case. Consequently, in the transition to grazing angles of incidence, the heat release decreases without increase in the SSSW threshold.

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