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Some Features of Charged Particle Motion in He^3–He^4 Solutions Located in Strong Electric Fields

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The velocities of positive and negative ions in He^3–He^4 solutions containing up to ~ 1% of He^3 are measured by the time-of-flight technique in electric fields up to 1000 V/cm and temperatures from 0.37 to 0.67°K. It is shown that the positive ion drift velocity dependence on field strength deviates from the linear extrapolation towards higher velocity values, providing the temperature is sufficiently low so that ion scattering by impurity excitations is predominant and the field strength is high. It is found that in solutions as well as in He^4 there exist two values of the critical field and velocity for which the ion motion is accompanied by the formation of quantized vortex rings.

As is well known, rapidly moving ions in superfluid helium form quantized vortex rings. This phenomenon was investigated in detail in pure He^4[1,2] and in very weak He^3–He^4 solutions (~0.01% He^3)[3-6]. It was established that the indicated processes are quite sensitive to the presence of the He^3 impurity. The present study was undertaken for the purpose of obtaining information on the motion of ions in more concentrated solutions (~1 He^3) and in stronger electric fields (~1000 V/cm).

EXPERIMENT AND RESULTS

The ion velocity in the liquid helium was measured by a time-of-flight method which made it possible to determine the time required for the ions to negotiate a certain distance between two pairs of grids that served as shutters in a given electric field[7]. The measuring cell was placed in a cryostat in which a temperature down to ~0.4°K was obtained by pumping off liquid He^3 vapor with an adsorption pump. The temperature was determined from the vapor pressure of the liquid He^3 using the T_{62} scale, with a correction introduced for the thermomolecular pressure difference. The accuracy of the temperature measurement was $\pm 5 \times 10^{-3}$°K. The ions in the liquid helium were produced by bombarding the latter with electrons emitted from a source of titanium tritide. In the measurement of the ion velocity, the scatter from experiment to experiment did not exceed ±5%.

We measured the velocities of the positive and negative ions in solutions containing 0.22, 0.75, and 1.30% He^3 in fields up to 1000 V/cm and in the temperature interval 0.37–0.67°K. A typical plot of the ion drift velocity against the electric field in He^3–He^4 solutions is shown in Fig. 1. Curves 1 and 2 characterize here the motion of the positive and negative ions in a solution containing 0.75% He^3 at 0.37°K. Just as in the case of He^4, in weak electric fields the dependence of the velocity on the electric field is linear. At a certain value of the field the velocity reaches a maximum and subsequently decreases with increasing field. This last region, just as in He^4[2] and in weak solutions[3-6], characterizes the process of formation and motion of the complex consisting of the ion and the vortex ring. It should be emphasized that in the vortex region there is usually a certain increase of the scatter in the measurement of the drift velocity.

In He^3–He^4 solutions, unlike in He^4, a deviation from a linear dependence of v_D on E toward larger values of the ion drift velocity is observed in strong electric fields. This phenomenon is determined to a considerable degree by the temperature: for a solution with a given He^3 content, it occurs below a certain perfectly defined temperature. This is clearly seen from Fig. 2, which shows the dependence of v_D on E for positive and negative ions in the case of a solution with 1.30% He^3 at 0.37 and 0.67°K. In the case of positive ions, this phenomenon was not observed for the indicated solution at T = 0.67°K (curve 2) and is observed at lower temperatures (curve 1). For the solution with 0.75% He^3, this phenomenon was not observed at T = 0.48°K and was observed at lower temperatures (see curve 1 of Fig. 5 below and curve 1 of Fig. 1).

In the case of negative ions, in the range of investigated fields and temperatures, the $v_D(E)$ plots are linear, and the data obtained for the same solution at different temperatures fall on a single straight line within the limits of experimental error (curve 3 of Fig. 2). The plots presented for negative ions pertain to the region ahead of the vortices. The vortex region for these solutions should appear in stronger electric fields. The formation of vortices by negative ions is illustrated in Fig. 3b, which shows the dependence of v_D on E in a solution with 0.22% He^3. One can see clearly the linear section and the following considerable decrease of the velocity, due to vortex formation. This dependence is much stronger than in the case of positive ions (Fig. 3a, curves 1 and 2).

In addition to the noted regularities, mention should also be made of singularities observed on the frequency dependence of the ion current, which are illustrated in Fig. 4. This figure shows the dependence of the positive-ion current on the frequency of the pulses applied to the shutters, for a solution containing 0.75% He^3 at a temperature T = 48°K. The character of this dependence is determined by the procedure of applying the electric field. In the case of a continuous rapid in-

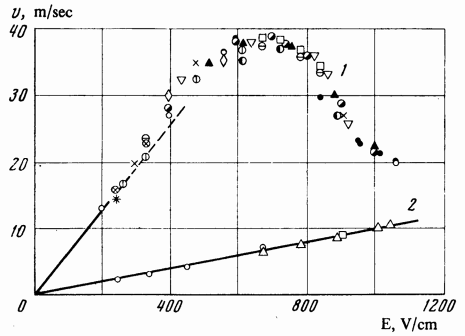

FIG. 1. Dependence of ion drift velocity on the electric field in a solution containing 0.75% He^3 at T = 0.37°K. Curves 1 and 2 correspond to positive and negative ions, respectively. The different symbols correspond to measurements performed on different days.

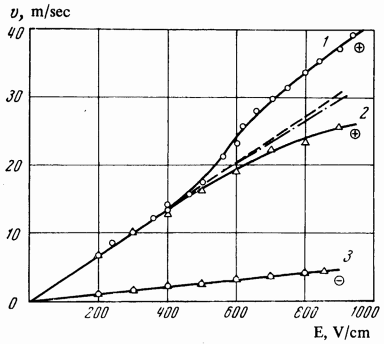

FIG. 2. Dependence of the ion drift velocity on the electric field in an He^3-He^4 solution containing 1.30% He^3. Curves: 1, 2) positive ions, 3) negative ions; O) T = 0.37°K, Δ) T = 0.67°K. Dash-dot line—calculation from formula (4), dashed line—extrapolation of $v_D(E)$ plot in weak electric fields.

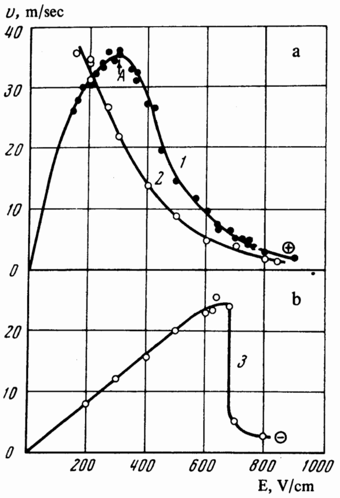

FIG. 3. Drift velocity of positive (a) and negative (b) ions vs. electric field in a solution containing 0.22% He^3: O) T = 0.37°K, ●) T = 0.67°K. The position of the point A is determined by E_{c2} and v_{c2}.

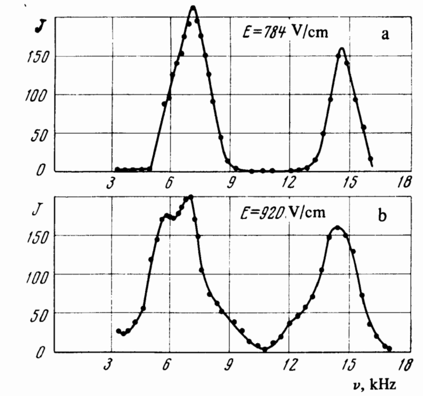

FIG. 4. Dependence of the positive-ion current on the frequency of the pulses applied to the shutters. He^3-He^4 solution with 0.75% He^3, T = 0.48°K; a) rapid increase of electric field, b) stepwise increase of electric field.

crease of the electric field, in the region of vortex formation, maxima of regular shape are observed on the frequency characteristic (Fig. 4a). Figure 4b illustrates the same dependence obtained under conditions when the electric field is increased in steps. An additional maximum is observed at lower frequencies besides the main maximum. It can be assumed that this phenomenon is connected with the existence of two species of particles having different velocities. The corresponding velocities were determined from the positions of the maxima on the frequency characteristics, without taking into account the possible shift of the resonant frequencies as a result of superposition of the maxima. The width of the additional peak in the first maximum apparently did not differ strongly from the width of the main peak, while the presence of the additional peak on the second maximum is less clearly pronounced than on the first. A possible reason for this is that the velocity scatter of the moving particles, which determines the relative decrease of the next maximum in comparison with the preceding one, turns out to be different for different species of particles. The relation between the intensities and the maxima remains practically constant in time. Accordingly the dependence of v_D on E in the vortex-formation region is described by two curves, as seen in Fig. 5, which shows the results for a solution containing 0.75% He^3 at 0.48°K. The second branch appears in fields stronger than ~650 V/cm. An analogous picture for a solution having the same concentration was observed in the temperature interval 0.40--0.67°K, and the value of the field at which this splitting took place increased with increasing temperature. It should be noted that a similar phenomenon was also observed in experiments with He^4 [8,9].

DISCUSSION OF RESULTS

a) Strong electric fields. Region ahead of vortices. At sufficiently low temperatures, where the number of thermal excitations is small, the mobilities of the positive ions in weak electric fields in the solutions having the investigated concentrations is determined mainly by the interaction with the impurity excitations. The ion mobility is determined in this case, as shown earlier[7], by the potential of the interaction between the ions and the He^3 impurity, which is of the repulsion type and is given by[1)]

$$U(r)=\frac{\gamma e^2}{2r^4}\left(\frac{V_3}{V}-1\right)\equiv\frac{\varkappa}{r^4},\qquad \varkappa>0, \tag{1}$$

where γ is the atomic polarizability of helium, e is the electron charge, V_3 and V_4 are respectively the volumes per He^3 and per He^4 atom in the solution,

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[1)] The factor 1/2 was erroneously omitted in[7].

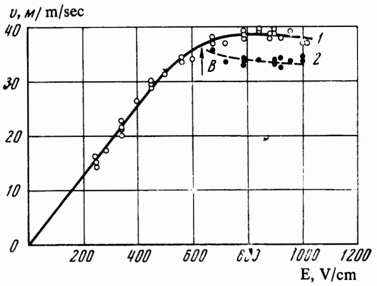

FIG. 5. Positive-ion drift velocity vs. electric field in an He^3-He^4 solution containing 0.75% He^3 at T = 0.48°K. Curves 1 and 2 characterize the simultaneous existence of charged particles with different velocities. The point B corresponds to the values E_{c1} and v_{c1}.

and r is the distance between the ion and the He^3 impurity. A potential of this form leads to a satisfactory agreement between the calculated values of the ion mobility in He^3--He^4 solutions at low temperatures and the experimental values. On the other hand, it is known that if the interaction between the charges and the particles is described by the potential (1), then the dependence of v_D on E should be linear in a wide range of electric fields[10], something not realized for He^3--He^4 solutions, as can be seen, for example, from Fig. 2 (curve 1). The observed dependence of v_D on E for positive ions in solutions at low temperatures can be explained by recognizing that at large ion velocities the potential describing the interaction of the ion with impurity has a complicated character and a noticeable attraction appears in addition to the repulsion forces between the particles.

The presence of attraction between a moving ion and the He^3 impurity can be established by considering the interaction between the velocity field of the superfluid liquid, produced around the moving ion, and the He^3 atom located in the immediate vicinity of the ion. The problem of the interaction of an impurity excitation with a velocity field was solved by Reut and Fisher[11], who obtained for weak solutions and not too low temperatures a thermodynamic estimate of the interaction potential corresponding to attraction forces:

$$U_{\text{eff}}(r) \approx -\left[\frac{m_3^*}{m_3} + \rho_s \frac{\partial}{\partial c}\left(\frac{1}{\rho}\right)\right] m_3 \mathbf{v}_s^2 \equiv -\beta \mathbf{v}_s^2. \tag{2}$$

Here m_3 is the mass of the He^3 atom, m_3^* is the effective mass of the He^3 impurity in the solution, c is the concentration, ρ_s is the density of the superfluid component, ρ is the density of the liquid helium, and $\beta = 2.5 \times 10^{-23}$ erg-sec^2/cm^2. For an approximate estimate of (2), the positive ion can be regarded as a small sphere of radius R_i. Then the velocity v_s of the superfluid component near the ion is of the order of $v_s \sim v_D R_i^3/r^3$, where r is the distance from the center of the ion[12]. Hence, in accord with (2), the potential of the interaction between the He^3 impurity and the field v_s near the moving ion is equal to

$$U'_{\text{eff}}(r) \sim -\beta v_D^2 R_i^6/r^6. \tag{3}$$

At $R_+ \approx 6$ Å, $v_D = 30$ m/sec, and r = 10 Å we obtain $U'_{\text{eff}}(r) \sim -0.1$°K, which yields a noticeable contribution of opposite sign to the potential (1) and leads, for the same value of r, to $U(r) \approx 0.5$°K. Naturally this should increase the mobility at higher ion velocities.

At low velocities, as already noted (Fig. 2) the mobilities of the positive ions at different temperatures coincide, thus evidencing that in weak electric fields the influence of the rotons is quite small. The situation changes in strong electric fields, where the thermal excitations begin to play an important role, and decrease the mobility greatly. Our experiments show that the influence of He^3 on the ion mobility in a roton gas is much stronger at higher velocities. Thus, whereas in weak electric fields μ_r in a solution containing 1.30% He^3 decreases by an approximate factor of 4 in comparison with the analogous quantity in He^4, in strong fields μ_r in the solution decreases by a factor of 30 compared with the same quantity in He^4. The data for the values of μ_r in He^4 in strong fields were taken here from the paper of Strayer et al.[13], and the comparison of μ_r in a solution was carried out for equal values of the velocities.

The features of ion motion in a roton gas at large drift velocity were considered by Strayer, Donnelly, and Roberts[13]. According to the model assumed in this work, the velocity field of a superfluid liquid around the moving ion leads to localization of the rotons. The quasiparticles localized near the ion execute vibrational motion and transfer a fraction of the momentum, by collision with other excitations, from the moving ion to the excitation gas, and this leads to additional dissipative losses. An approximate calculation performed in[13] leads to the following relation between the ion drift velocity v_D and the electric field E:

$$E = \frac{v_D}{\mu} + v_D \left(\frac{f}{e}\right) \rho_{nr} \left(\frac{kT}{\mu_0}\right)^{1/2} R_i \varepsilon_r, \tag{4}$$

where μ is the mobility in weak electric fields, ρ_{nr} is the density of the normal component and is due to the rotons, μ_0 is the effective mass of the roton, and ϵ_r is a tabulated function that depends on v_D and T. The parameter f = 0.060 was chosen to reconcile in best fashion the expression (4) with the experimental $v_D(E)$ dependence in He^4 at 0.65°K. The plot of v_D against E for positive ions, obtained with the aid of (4) for solutions containing 1.30% He^3 (dash-dot line in Fig. 2) indicates that the agreement between the calculated and experimental curves is poor. The deviation of $v_D(E)$ from linearity in the direction of stronger fields, obtained experimentally at v_D = 25 m/sec, is approximately 10 times larger than the deviation that follows from (4). Such a discrepancy apparently calls for a considerable refinement of the model describing the interaction of the ions with the rotons in the presence of He^3 impurities at large drift velocities.

b) Vortex region. The mechanism of vortex formation by moving ions in superfluid helium cannot be regarded at present as completely clear. Arnold et al.[8] and Cusolo and Maraviglia[9] have observed that in He^4 at T < 0.8°K, in electric fields that do not exceed greatly the critical values, quantized vortex rings are produced not by all the ions, but only by a fraction (approximately 40%). Vortices are produced by the other ions only after a higher field value is reached. Thus, two concrete vortex-production fields are observed, E_{c1} and E_{c2}, with corresponding two critical temperatures v_{c1} and v_{c2}. The temperature dependences of E_{c1} and E_{c2} apparently indicate that the vortex production processes are connected, at not too low temperatures, with rotons in the first case and with phonons in the second. The results enable us to determine the values of the corresponding critical fields and velocities in solutions. The values of E_{c1} and v_{c1} are naturally determined by the values of the electric field and the drift velocity at which branching of the v_D vs. E curves takes place (see Fig. 5), and E_{c2} and v_{c2} have the meaning of the values of the field and velocity on the main curve, at which stable vortex rings are produced (for example point A on curve 3a).

The temperature dependences of the indicated quantities in solutions are much weaker than in He^4. Thus, whereas in He^4, according to the data of[9], the value of E_{c1} increase approximately by 5 times when the temperature rises from 0.48 to 0.67°K, in a solution containing 0.75% He^3 this value increases only by 30%. Similarly, whereas E_{c2} in He^4 increases by approximately 2.5 times when the temperature is increased from 0.37 to 0.47°K, in the solution this quantity increases by ~ 25%. Finally, whereas in He^4 there is observed also a certain temperature dependence of the quantities v_{c1} and v_{c2}, in a solution containing 0.75% He^3 this quantities are practically independent on the temperature in the measured temperature interval, and amount to $v_{c1} = 35$ m/sec and $v_{c2} = 38$ m/sec. It is interesting to note that v_{c1} in the solution exceeds the analogous quantity in He^4, while v_{c2}, to the contrary, is smaller. Apparently the weaker temperature dependences of the critical fields observed in this investigation, together with the independence of v_{c1} and v_{c2} of the temperature, give grounds for assuming that in He^3-He^4 solutions of not too low concentration the processes of vortex formation by moving ions are strongly influenced by the He^3 impurity.

When they move in superfluid helium, the vortex rings interact with the elementary excitations. To describe the magnitude of this interaction, it is convenient to introduce the coefficient α, which defines the energy loss of the vortex ring on a path length of 1 cm. The coefficient α can be represented in the form[1]

$$\alpha = kF'/2v. \tag{5}$$

Here $k = h/m_4$ is the circulation, where m_4 is the mass of the He^4 atom, F' the friction force exerted by the excitation gas on a unit length of vortex line, and v is the velocity of the vortex ring. This velocity is connected with the electric field by the relation[2]

$$\ln\frac{v}{E} = -\frac{eE}{\alpha} - \frac{1}{4} + \ln\frac{2ke}{\pi\alpha a_0}, \tag{6}$$

where a_0 is the radius of the vortex core.

The experimental dependence of the vortex-ring velocity v on E makes it possible to determine with the aid of (6) the value of α, which in a solution containing 0.22% He^3 at T = 0.34°K turned out to be $\alpha = 132 \pm 7$ eV/cm and increased slightly at T = 0.67°K to $\alpha = 144 \pm 7$ eV/cm. In the case when different types of excitations do not interact with one another, their contribution to the coefficient α can be regarded as additive. As shown by estimates using the data of Rayfield and Reif[1], in a solution with 0.22% He^3 the contribution of the phonons to α can be neglected, and therefore

$$\alpha = \alpha_3 + \alpha_r, \tag{7}$$

where α_3 and α_r are the contributions made to α by the impurity excitations and the rotons, respectively. At T = 0.3% K we have $\alpha_r < 10^{-3}$ eV/cm and therefore $\alpha = \alpha_3$. When the temperature is increased to 0.67°K, according to the data of[1], we have $\alpha_r \approx 3$ eV/cm in pure He^4 and $\alpha_r \approx 5$ eV/cm in a solution containing 0.22% He^3, with allowance for the change in the energy-spectrum parameters[14,7,15]. It follows thus from the foregoing estimates that α_3 depends very little on the temperature.

To calculate α_3 it is necessary to know the friction force F' acting on a unit length of a vortex line moving with velocity v. As shown by Reut and Fisher[11], an interaction potential of the type $-g/r^2$, corresponding to attraction forces, acts between the vortex line and the He^3 impurity. Depending how close the He^3 particles move past the vortex line, they can be either captured by the latter, or scattered through a certain angle. A numerical calculation with allowance for the two indicated processes leads to the following expression for α_3:

$$\alpha_3 \approx 0.4\pi k(2gm_3^*)^{1/2}N_3. \tag{8}$$

where N_3 is the number of He^3 atoms per cm^3. We see that α_3 does not depend on the temperature. For a solution containing 0.22% He^3, the calculated value is $\alpha_3 = 80$ eV/cm, which is approximately 1.5 times smaller than the experimental value. Recognizing that the thermodynamic estimate of g given in[11] is approximate, such an agreement with experiment can apparently be regarded as satisfactory.

As follows from our results and those of others[5,6], the dependence of the vortex-ring velocity on the electric field at $v > 25$ m/sec (corresponding to a vortex-ring radius $R < 15$ Å), becomes steeper in comparison with the analogous dependence at $v < 25$ m/sec. According to (6), this is evidence that α decreases in this velocity region. Inasmuch as the radius of the vortex ring becomes comparable with the radius of the ion at sufficiently high velocities, it can be assumed that the decrease of α is due to the presence, inside the vortex, of an ion whose influence on the motion of the vortex ring should become particularly strong when the dimension of the ring becomes comparable with the dimension of the ion. The decrease of α is at first glance unexpected, since introduction of an additional scattering center into the vortex ring should increase the scattering cross section of the complex made up of the ion and the vortex ring. However, inasmuch as the He^3 impurity is attracted by the vortex ring and repelled by the positive ion, the total interaction between the complex and the He^3 impurity can decrease if an ion is present inside the ring, and this leads precisely to a decrease of the effective scattering cross section of the complex, and consequently a decrease of the friction force. Unfortunately, there is still no detailed description of the scattering of impurity excitations by a complex consisting of an ion and a vortex ring with allowance for the velocity field v_s around the ion.

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[1] G. W. Rayfield and F. Reif, Phys. Rev. A (1964–1965) **136,** 5 (1964); Phys. Rev. **136,** 1194 (1964).

[2] G. Careri, S. Cunsolo, P. Mazzoldi, and M. Santini, Phys. Rev. Lett. **15,** 392 (1965).

[3] G. W. Rayfield, Phys. Rev. Lett. **19,** 1371 (1967).

[4] G. W. Rayfield, Phys. Rev. Lett. **20,** 1467 (1968).

[5] D. A. Neeper and L. Meyer, Phys. Rev. **182,** 223 (1969).

[6] M. Kuchnir, J. B. Ketterson, and Pat. R. Roach, Phys. Rev. Lett. **26,** 879 (1971).

[7] B. N. Esel'son, Yu. Z. Kovdrya, and V. B. Shikin, Zh. Eksp. Teor. Fiz. **59,** 64 (1970) [Sov. Phys. JETP **32,** 37 (1971)].

[8] A. J. P. T. Arnold, M. V. Ricci, and J. S. Rosenshein, Proceedings of the Tenth International Conference on the Physics of Low Temperatures, Moscow, VINITI, 1967.

[9] S. Cunsolo and B. Maraviglia, Phys. Rev. **187,** 292 (1969).

[10] B. M. Smirnov, Dokl. Akad. Nauk SSSR **168,** 322 (1966) [Sov. Phys. Dokl. **11,** 429 (1966)].

[11] L. S. Reut and I. Z. Fisher, Zh. Eksp. Teor. Fiz. **55,** 722 (1968) [Sov. Phys. JETP **28,** 375 (1969)].

[12] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Media), Gostekhizdat, 1954.

[13] D. M. Strayer, R. Donnelly, and P. H. Roberts, Phys. Rev. Lett. **26,** 165 (1971).

[14] N. E. Dyumin, B. N. Esel'son, É. Ya. Rudavskiĭ, and I. A. Serbin, Zh. Eksp. Teor. Fiz. **56,** 747 (1969) [Sov. Phys. JETP **29,** 406 (1969)].

[15] V. I. Sobolev and B. N. Esel'son, Zh. Eksp. Teor. Fiz. **60,** 240 (1971) [Sov. Phys. JETP **33,** 132 (1971)].

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