Light Absorption in Doped Semiconductors in the Presence of a Strong Electromagnetic Field

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The electron Green's functions of a doped semiconductor in the presence of impurity-band transitions induced by a strong electromagnetic field are found. Quasiparticle damping due to electron scattering by impurities and the specific damping due to electron confinement near the impurity by a strong field are taken into account. The spectral dependence of absorption and amplification of a weak electromagnetic field in the presence of a strong one is determined. It is shown that scattering by impurities does not remove the peculiarities of weak wave absorption due to the gap in the electron energy spectrum. Coherent interaction with an electromagnetic field thus becomes noticeable at very small field strengths. The effect of impurity scattering on the shape of the fundamental absorption edge and of the impurity level-band transition absorption edge in a degenerate equilibrium semiconductor under conditions of the Burstein-Moss effect is considered. The impurity-band absorption edge is found to be sharp in contrast to that for the fundamental absorption, which is smeared out to an extent which is proportional to the inverse time between collisions between the electrons and impurities.

INTRODUCTION

A study of the interaction of a strong electromagnetic field with semiconductors is an urgent problem, owing primarily to the high energy density of the electromagnetic field in semiconductor lasers[1,2], The problem lies in taking consistent account of the influence of the interactions between the semiconductor electrons and the impurities, phonons, and electrons on the character of the optical transitions in the strong field[3]. Such a problem was considered for band-band transitions in[4,5]. Of particular interest is the study of transitions between a discrete (acceptor) impurity level and the conduction band, since theoretical and experimental investigations have shown that optical transitions of this type are realized in lasers based on doped semiconductors[6].

The state of a semiconductor in a strong field was considered by one of the authors[7] in the case of impurity-band transitions. It was shown that the energy level of the impurity in the field of a strong monochromatic wave $\mathbf{E}_0 \sin(\Omega_0 t)$ is transformed into an impurity band, since the impurity electrons become delocalized as a result of the impurity—band—impurity transition, and a gap $2\bar{\lambda}$ appears in the energy spectrum of the conduction-band and impurity-band electrons at a quasimomentum value $p_0 = \sqrt{2m_a(\Omega_0 - E_i)}$, where m_a is the effective mass of the electron and E_i is the distance from the level to the edge of the band. In[7], however, the excitation damping due to scattering by impurities and the specific damping due to the confinement of the electron near the impurity by the strong field were neglected.

In the present paper, using a graphical technique of averaging over the impurity coordinates[8,9], we obtain the Green's function with allowance for the aforementioned dampings, and consider impurity absorption of a weak electromagnetic field of frequency Ω in the presence of a strong field. It is assumed that the electron-electron and electron-phonon interactions cause a quasi-equilibrium stationary state, characterized by the Fermi quasilevel of the electrons in the band, to be established. It is assumed also that the quantity $\mu_0 = \Omega_0 - E_i$ greatly exceeds the gap and the reciprocal time $1/\tau_a$ between the collisions of the electron with the impurities. The presence of the small parameters $(\mu_0\tau_a)^{-1}, \bar{\lambda}/\mu_0 \ll 1$ makes it possible to perform the calculations in the ladder approximation[8].

One of the results is that at $(\mu_0\tau_a)^{-1}, \bar{\lambda}/\mu_0 \ll 1$ the influence of the impurity scattering on the absorption of the weak wave can be neglected. The absorption (amplification) turns out to be equal to zero in the interval $|\Omega - \Omega_0| < 2\bar{\lambda}$ and increases strongly at $|\Omega - \Omega_0| \geq 2\bar{\lambda}$.

It should be noted that in band–band transitions the impurity scattering disrupts the gap effectively[5] and it can be shown that the impurity state annihilates in this case the singularities of the weak-signal absorption at impurity concentrations such that $1/\tau_a$ becomes larger than the gap. In this respect, band—band and band—impurity transitions differ noticeably. There is likewise a difference between the corresponding coefficients of absorption of the weak field by a degenerate equilibrium semiconductor, i.e., under the conditions of the Burstein-Moss effect. As shown in the present paper, the impurity—band absorption edge turns out to be abrupt, in contrast to interband absorption, where the edge is smeared out by an amount proportional to the reciprocal time between the collisions of the electron with the impurities.

1. FORMULATION OF PROBLEM. HAMILTONIAN OF SYSTEM

The system "impurity semiconductor in a strong electromagnetic field of frequency Ω_0'' is described approximately by the Hamiltonian

$$H = \sum_{\mathbf{p}} E_{\mathbf{p}} a_{\mathbf{p}}^{+} a_{\mathbf{p}} - E_i \sum_{n} b_n^{+} b_n + \sum_{n\mathbf{p}} \{\lambda_{\mathbf{p}n} a_{\mathbf{p}}^{+} b_n e^{-i\Omega_0 t} + \lambda_{\mathbf{p}n}^{*} b_n^{+} a_{\mathbf{p}} e^{i\Omega_0 t}\} + \sum_{\mathbf{pp}'} U(\mathbf{p}-\mathbf{p}') a_{\mathbf{p}}^{+} a_{\mathbf{p}'} + H_1. \quad (1)$$

Here E_p^a is the law governing the electron dispersion in the band, the energies reckoned from the edge of the band,

$$\lambda_{pn} = \frac{eE_0}{2\Omega_0}\left\langle \Psi_p^* \frac{\hat{p}}{m}\Psi_n \right\rangle, \quad \hbar = c = 1,$$

b_n and a_p are the operators for the annihilation of the electron at the impurity level and in the band, $U(p - p')$ is the Fourier component of the impurity potential

$$U(q) = \frac{1}{V}\sum_k \int dr\, e^{iqr} u_k(r - r_k),$$

the summation over n extends over the impurities that give the level $-E_i$, and the summation over k is over all the impurities and defects. The term H_1 describes the electron-phonon and electron-electron interactions that cause the establishment of a quasi-equilibrium state characterized by the Fermi quasilevel (the direction of going around the poles in the Green's function is specified). The same term leads to damping of the excitations, which turns out to be weak in the presence of a Fermi surface[5], to a screening of the impurity potential, and consequently to a shift of the impurity level, and even to its vanishing at a sufficiently high free-electron concentration[10]. In addition, owing to the electrostatic repulsion, it is impossible to place on the impurity level more than one electron even in the presence of spin-connected degeneracy.

We assume that the screening radius is larger than the radius of the bound state, so that the impurity does not vanish. In the case of a hydrogen-like acceptor level and Debye screening, this condition is satisfied if

$$(e^2/\varepsilon v_a)^{1/2} > m_c/m_v, \tag{2}$$

m_v is the effective mass of the hole, ϵ is the static dielectric constant, and v_a is the electron velocity on the Fermi surface. H_1 can be omitted from now on if the ground state is chosen to be a quasi-equilibrium stationary state and if $u_k(r)$ is assumed screened.

We have neglected in (1) recombination, the nonresonant transitions in the field of the strong wave (the corresponding estimates are given in[5] and[7]), and also the scatter of the levels of the individual impurity atoms. The role of this scatter is discussed in the Conclusion.

Since the Hamiltonian (1) does not contain separately the number of particles in the band and on the impurity, it is convenient to change over to a new variable with the aid of the canonical transformation

$$\hat{U}(t) = \exp\left\{-i\mu_a t\sum_p a_p^+ a_p - i\mu_b t\sum_n b_n^+ b_n\right\},$$

where μ_a is the Fermi quasilevel of the electron in the band, and μ_b is the level at the impurity. From the requirement that the transformed Hamiltonian be stationary it follows that $\mu_a - \mu_b = \Omega_0$.

Further, in accordance with the notions developed in[7] concerning the impurity band, we introduce the operator

$$b_p = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} b_n e^{-ipr_n} \tag{3}$$

with commutation relations

$$[b_p b_{p'}]_+ = 0, \qquad [b_p b_{p'}^+]_+ = \Delta_{pp'},$$

$$\Delta_{pp'} = \frac{1}{N}\sum_{n=1}^{N} e^{-i(p-p')r_n},$$

N is the number of impurities with the level $-E_i$. If the impurity atoms are located at lattice sites, then λ_{pn} can be represented in the form

$$\lambda_{pn} = e^{-ipr_n}\lambda_p, \tag{4}$$

λ_p does not depend on the position of the impurity. For example, for acceptor–conduction band transitions we have

$$\lambda_p = \frac{eE_0V_{av}}{2\Omega_0}\frac{M}{\sqrt{N}}, \quad M = \frac{8}{(1+p^2a_B^2)^2}\left(\frac{\pi N a_B^3}{V}\right)^{1/2}, \tag{5}$$

where V_{av} is the matrix element of the interband transitions, and a_B is the radius of the bound state. If the strong-field frequency lies in the impurity absorption region $|p_0 < a_B^{-1}|$, then λ_p can be regarded as independent of p[11]. In the case of forbidden interband transitions or in acceptor–valence band (donor–conduction band) transitions, λ_p depends on the angle between p and E_0.

When (3) and (4) are taken into account, the Hamiltonian (1) takes the form

$$H = \sum_p \left\{\xi_p a_p^+ a_p - \eta\delta\mu b_p^+ b_p + \bar{\lambda}_p a_p^+ b_p + \bar{\lambda}_p^* b_p^+ a_p + \sum_{p'} U(p-p') a_p^+ a_{p'}\right\}, \tag{6}$$

where

$$\xi_p = E_p^a - \mu_a, \quad \delta\mu = \mu_a - \mu_b, \quad \bar{\lambda}_p = \sqrt{N}\lambda_p$$

and we have used the relation

$$b_p = \eta\sum_{p'} \Delta_{pp'} b_{p'},$$

$\eta = N/N_0$ is the percentage impurity concentration, and N_0 is the number of unit cells in the crystal, equal to the number of states in the Brillouin band. We note that η can be regarded as the statistical weight of a state with momentum p in the impurity band produced as a result of the transitions in the strong field. Unlike in[7], we use exact commutation relations and take impurity scattering into account.

The system is described by four Green's functions[7]:

$$G^a_{pp'}(\omega) = -i\int e^{i\omega t}\langle T a_p(t) a_{p'}^+(0)\rangle dt, \quad F_{pp'}(\omega) = \int e^{i\omega t}\langle T a_p(t) b_{p'}^+(0)\rangle dt,$$

$$F^+_{pp'}(\omega) = \int e^{i\omega t}\langle T b_p(t) a_{p'}^+(0)\rangle dt, \quad G^b_{pp'}(\omega) = -i\int e^{i\omega t}\langle T b_p(t) b_{p'}^+(0)\rangle dt. \tag{7}$$

From the equation of motion for the Heisenberg operators we obtain equations for the Fourier components of the Green's functions:

$$(\omega - \xi_p) G^a_{pp'}(\omega) = \delta_{pp'} - i\bar{\lambda}_p F^+_{pp'}(\omega) + \sum_{p''} U(p-p'') G^a_{p''p'}(\omega), \tag{8}$$

$$(\omega + \delta\mu) F^+_{pp'}(\omega) = i\sum_{p''} \bar{\lambda}^*_{p''}\Delta_{pp''} G_{p''p'}(\omega), \tag{9}$$

$$(\omega - \xi_p) F_{pp'}(\omega) = i\bar{\lambda}_p G^b_{pp'}(\omega) + \sum_{p''} U(p-p'') F_{p''p'}(\omega), \tag{10}$$

$$(\omega + \delta\mu) G^b_{pp'}(\omega) = \Delta_{pp'} - i\sum_{p''} \bar{\lambda}^*_{p''}\Delta_{pp''} F_{p''p'}(\omega). \tag{11}$$

2. GREEN'S FUNCTION OF DOPED SEMICONDUCTOR IN A STRONG ELECTROMAGNETIC FIELD WITH ALLOWANCE FOR IMPURITY SCATTERING

We shall be interested in Green's functions averaged over the coordinates of randomly distributed impurities. The averaging can be carried out with the aid of the diagram technique developed by Abrikosov and Gor'kov[8] and Vdovin and Galitskiĭ[9]. We begin the averaging with the function $G^a_{pp'}$, a closed equation for which was obtained by substituting F^+ from (9) in (8):

$$G^a_{pp'}(\omega) = \delta_{pp'} G_0^a(p) + G_0^a(p) \sum_{p''} \frac{\bar\lambda_p \bar\lambda^*_{p''}}{\omega+\delta\mu} \Delta_{pp''} G_{p''p'}(\omega) \qquad (12)$$

$$+ G_0^a(p) \sum_{p''} U(\mathbf{p}-\mathbf{p}'') G^a_{p''p'}(\omega),$$

with

$$G_0^a(p) = [\omega - \xi_p + i\delta s(\xi_p)]^{-1}, \quad s(x) \equiv \operatorname{sign}(x), \quad \delta \to 0^+.$$

With the exception of the last term, Eq. (12) coincides in form with the equation for the photon amplitude in a medium of resonant molecules[9], and it is therefore convenient to use the graphic notation of[9]. We let a dark circle correspond to the vertex

$$\frac{\bar\lambda_p \bar\lambda^*_{p'}}{\omega+\delta\mu} e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}_n}.$$

The impurity scattering $u_k(\mathbf{p}-\mathbf{p}')\exp[i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_k]$ will be denoted by a cross. A circle can be treated as a transition of an electron from a band to an impurity and then back to a band, with emission and subsequent absorption of a strong-field quantum. The circles and crosses pertaining to the same impurity atom are joined by a dashed line, which is taken to mean averaging over the coordinates of this impurity.

We carry out the calculation under the assumption that $(\mu_0\tau_a)^{-1} \ll 1$ and $|\bar\lambda|/\mu_0 \ll 1$. These conditions cause the "intersecting" diagrams to be small[8]. If the scattering is by charged impurities, then $(\mu_0\tau_a)^{-1} \ll 1$ is equivalent to the condition

$$2n^{\pm}e^2 / 3n_0 e v_a \ll 1, \qquad (13)$$

i.e. (at $n^{\pm} \sim n_0$), to the Born approximation for the impurity potential. Here $n^{\pm}$ is the concentration of all the charged impurities and $n_0 = p_0^3/3\pi^2$ is the maximum number of electrons that the electromagnetic wave can "pour into" the band.

The smallness of the Born parameter makes it possible to omit also diagrams in which a cross and a circle are joined by a dashed line. The order of relative smallness of such diagrams is $Le^2n^-/n\epsilon v_0 \ll 1$, n^-/n is the ratio of the number of charged impurities with level E_i to the total number of such impurities, and L is a logarithm of the order of unity. We note that simultaneous satisfaction of the conditions (2) and (13) is possible in a rather wide strong-field frequency interval, in view of the usually large difference between the masses of the electron and of the hole.

Iterating and averaging in (12) we obtain, in accordance with the diagram of Fig. 1a,

$$\bar G_{pp'} = \delta_{pp'} G_p, \quad G_p^a = [(G_0^a)^{-1} - \bar G_\omega]^{-1}, \qquad (14)$$

$$\bar G_\omega = g_a N + \sum_k g_k,$$

where

$$g_a = \frac{|\lambda_p|^2}{\omega+\delta\mu}\left\{1 + \sum_{k=1}^{\infty}\left[\frac{\sum_{p'}|\lambda_{p'}|^2 G_0^a(\mathbf{p}',\omega)}{\omega+\delta\mu}\right]^k\right\}$$

$$= \frac{|\lambda_p|^2}{\omega+\delta\mu - \sum_{p'}|\lambda_{p'}|^2 G_0^a(\mathbf{p}',\omega)},$$

$$g_k = \sum_{p'} |u_k(\mathbf{p}-\mathbf{p}')|^2 G_0^a(\mathbf{p}',\omega).$$

Replacement of the intermediate G_0 in g_a and g_k by the more exact function leads to small corrections of the order of

$$1/\mu_0\tau_a, \quad |\bar\lambda|/\mu_0 \ll 1.$$

We shall be interested in what follows in the values $p \sim p_0$ and $\omega + \delta\mu \sim |\bar\lambda|$, i.e., the energy-spectrum region in which the gap is produced[7].

In calculation of g_a and g_k, integration regions that lie far from the surface $E^a_p = \mu_0$ can be excluded, since the integrals over all the regions do not depend on ω and yield real constants which, together with $u_k(0)$, renormalize E_i and μ_a. The integrals over the values of p′ close to p_0 lead to an imaginary part in G:

$$G_p^a(\omega) = \frac{1}{\omega - \xi_p - |\bar\lambda_p|^2 \mathscr{W}^{-1} + is(\omega)/2\tau_a},$$

$$\mathscr{W} = \omega + \delta\mu + iWs(\omega)/2, \qquad (15)$$

$$W = \frac{\rho_a(\mu_0)}{2}\int|\lambda(\theta)|^2 d\Omega, \quad \tau_a^{-1} = \frac{\rho_a(\mu_0)}{2}\sum_k\int|u_k(\theta)|^2 d\Omega,$$

$\rho_a(\epsilon) = \sum_p \delta(\epsilon - E^a_p)$—is the density of states in the band.

Averaging Eq. (9) in similar fashion, we obtain

$$F_p^+(\omega) = \frac{i\bar\lambda_p^*}{\mathscr{W}[\omega - \xi_p - |\bar\lambda_p|^2\mathscr{W}^{-1} + is(\omega)/2\tau_a]}. \qquad (16)$$

Let us find the averaged functions F and G^b. Substitution of G^b from (11) in (10) yields a close equation for F:

$$F_{pp'}(\omega) = \Delta_{pp'}F_0(p) + G_0^a(p)\sum_{p''}\frac{\bar\lambda_p\bar\lambda_{p''}}{\omega+\delta\mu}\Delta_{pp''}F_{p''p'}(\omega) \qquad (17)$$

$$+ G_0^a(p)\sum_{p''}U(\mathbf{p}-\mathbf{p}'')F_{p''p'}(\omega),$$

where

$$F_0(p) = i\bar\lambda_v G_0^a(p)/(\omega+\delta\mu).$$

As a result of the iterations we arrive at the diagrams of Fig. 1b, which the triangle corresponds to the quantity $N^{-1}\exp[i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_n]$, and the function F is a solution of Eq. (17) in which $\Delta_{pp'}$ in the first

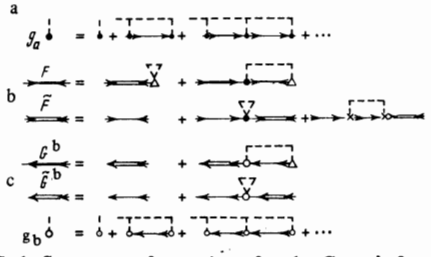

FIG. 1. Structure of equations for the Green's functions.

term is replaced by $\delta_{pp'}$. The diagrams in which the cross and the triangle are joined by a dashed line contain the Born parameter and are therefore discarded.

After averaging, in accordance with Fig. 1b, we obtain the following expression for F:

$$F_p(\omega)=\frac{i\bar{\lambda}_p}{\mathscr{V}[\omega-\xi_p-|\bar{\lambda}_p|^2\mathscr{V}^{-1}+is(\omega)/2\tau_a]}. \tag{18}$$

The function $G_p^b(\omega)$ can be determined by averaging Eq. (11) directly with the aid of the diagrams for F. However, bearing the subsequent calculations in mind, we obtain a closed equation for G^b, expressing F in terms of G^b. Taking $i\bar{\lambda}_p G_0^a(p)G^b$ in (10) to be the zeroth approximation for F and iterating, we obtain

$$F_{pp'}(\omega)=i\sum_{p''}G^{im}_{pp''}(\omega)\bar{\lambda}_{p''}G^b_{p''p'}(\omega). \tag{19}$$

After substituting (19) in (11) and averaging, we can replace the Green's function of the electrons scattered by the impurities in the absence of an external electromagnetic field by its mean value[8]

$$\overline{G^{im}_{pp'}(\omega)}=\frac{\delta_{pp'}}{\omega-\xi_p+is(\omega)/2\tau_a}, \tag{20}$$

since the cross and the circle will be joined by a dashed line in all the discarded diagrams. We obtain in place of (11)

$$G^b_{pp'}(\omega)=\Delta_{pp'}G_0^b(p)+G_0^b(p)\sum_{p''}\frac{|\bar{\lambda}_{p''}|^2\Delta_{pp''}}{\omega-\xi_{p''}+is(\omega)/2\tau_a}G^b_{p''p'}(\omega), \tag{21}$$

$$G_0^b(p)=(\omega+\delta\mu)^{-1}.$$

Representing the quantity

$$|\lambda_{p'}|^2e^{i(p-p')r_n}/[\omega-\xi_{p'}+is(\omega)/2\tau_a]$$

by a light circle, we obtain in accordance with Fig. 1c

$$G_p^b(\omega)=\frac{\omega-\xi_p+is(\omega)/2\tau_a}{\mathscr{V}[\omega-\xi_p-|\bar{\lambda}_p|^2\mathscr{V}^{-1}+is(\omega)/2\tau_a]}. \tag{22}$$

Relations (15), (16), (18), and (22) determine the Green's functions of a doped semiconductor in the field of a strong electromagnetic wave with allowance for scattering by impurities.

3. ENERGY SPECTRUM, DAMPINGS, AND CHEMICAL POTENTIAL OF THE ELEMENTARY EXCITATIONS

The Green's functions (15) and (22) differ from those obtained in[7] in the presence of the dampings W and $1/\tau_a$. If W, $1/\tau_a \ll |\bar{\lambda}|$, then the Green's functions can be represented in the form

$$\begin{aligned}
G_p^a(\omega)&=\Theta(\omega)\left[\frac{u_p^2}{\omega-\varepsilon_p^\alpha+i\delta}+\frac{v_p^2}{\omega+\varepsilon_p^\beta+i\delta}\right]\\
&+\Theta(-\omega)\left[\frac{u_p^2}{\omega-\varepsilon_p^\alpha-i\delta}+\frac{v_p^2}{\omega+\varepsilon_p^\beta-i\delta}\right],\\
G_p^b(\omega)&=\Theta(\omega)\left[\frac{v_p^2}{\omega-\varepsilon_p^\alpha+i\delta}+\frac{u_p^2}{\omega+\varepsilon_p^\beta+i\delta}\right]\\
&+\Theta(-\omega)\left[\frac{v_p^2}{\omega-\varepsilon_p^\alpha-i\delta}+\frac{u_p^2}{\omega+\varepsilon_p^\beta-i\delta}\right],
\end{aligned} \tag{23}$$

$$u_p^2,v_p^2=\frac{1}{2}\left\{1\pm\frac{\xi_p+\delta\mu}{\sqrt{(\xi_p+\delta\mu)^2+4|\bar{\lambda}|^2}}\right\},\quad \Theta(x)=\frac{1}{2}(1+s(x)).$$

The poles of the Green's functions, which are the same for all four functions, determine the spectrum of the elementary excitations of the system:

$$\varepsilon_p^{\alpha,\beta}=\pm\tfrac{1}{2}(\xi_p-\delta\mu)+[\tfrac{1}{4}(\xi_p+\delta\mu)^2+|\bar{\lambda}_p|^2]^{1/2}. \tag{24}$$

The dependence of the poles of the function G_p^b, which describes the behavior of the impurity electrons as a function of the quasimomentum p denotes that the impurity level is transformed into an impurity band. The impurity band is produced as a result of impurity–band–other impurity transitions, as confirmed by the physical meaning of the damping W. According to the diagrams of Fig. 1, W results from transitions from the impurity to the band and back to the same impurity. Such transitions lead to a retention of the electron near the impurity and prevent delocalization, facts indeed manifest by the damping. Using the definition (15), we can show that W coincides with the probability of induced absorption of a strong-field quantum on the transition from the level of one impurity atom to the band.

The condition $W \ll |\bar{\lambda}|$, which is necessary for the production of the impurity band, can be represented (when $n_0 \sim n$) in the form

$$l_c=v_0/|\bar{\lambda}| \gg n^{-1/3},$$

which means that the coherence length l_c should exceed the mean distance between impurities. At an arbitrary ratio of the concentrations we have

$$\frac{W}{|\bar{\lambda}|}=\frac{3\pi}{2}\frac{n_0}{n}\frac{|\bar{\lambda}|}{\mu_0}. \tag{25}$$

Scattering by impurities, as seen from (15) and (32), leads to a damping $1/2\tau_a$, which is analogous to the damping in a normal metal[8].

The chemical potential μ_a, which depends on the frequency and intensity of the strong field, and also on the number of the electrons in the band (n_a^0) and at the impurity level (n_b^0) prior to turning on the electromagnetic field, i.e., in the equilibrium semiconductor, can be obtained from the electroneutrality condition. An analysis of the electroneutrality equation in the case of small η (usually $\eta \lesssim 10^{-4}$) shows that in the region of the strong-field frequencies

$$\begin{aligned}
&\Omega_1 \leqslant \Omega_0 \leqslant \Omega_2,\quad \Omega_1=E_i+\frac{[3\pi^2(n_a^0+n_b^0-n)]^{2/3}}{2m_a},\\
&\Omega_2=E_i+\frac{[3\pi^2(n_a^0+n_b^0)]^{2/3}}{2m_a}
\end{aligned} \tag{26}$$

the chemical potential is $\mu_a \approx \mu_0$ ($|\delta\mu| \ll |\bar{\lambda}|$). In this region, the excitations coincide with those obtained in[7]. At $\Omega_0 > \Omega_2$ all the impurities become free of electrons and $\mu_a < \mu_0$. The case $\Omega_0 < \Omega_1$ can be realized in a degenerate semiconductor, when the strong field couples the impurity level to a state in the band below the equilibrium Fermi level. In this case $\mu_a > \mu_0$.

4. ABSORPTION (AMPLIFICATION) OF A WEAK ELECTROMAGNETIC WAVE IN THE PRESENCE OF A STRONG ONE

Of considerable interest, particularly in the investigation of multimode lasing in a semiconductor laser, is the line shape of the impurity absorption of a weak electromagnetic field $\mathbf{E}\sin\Omega t$ of frequency Ω close to the frequency of the strong wave. Expressing in the usual manner the linear-in-E current of the transition

from the impurity level to the band in terms of the retarded two-particle Green's function G_R of pairwise coinciding arguments (see, for example,[12]), we obtain for the weak-field absorption coefficient

$$K(\Omega) = \mathrm{Re}\,\frac{8\pi\overline{\langle\langle \mathbf{jE}\sin\Omega t\rangle\rangle}}{\varkappa E^2}, \qquad (27)$$

$$K(\Omega) = -\frac{4\pi e^2}{3\varkappa\Omega V m^2}\sum_{\substack{nn'\\pp'}}\overline{\langle\Psi_p^*\hat{\mathbf{p}}\Psi_n\rangle\langle\Psi_{n'}\hat{\mathbf{p}}\Psi_{p'}\rangle\,\mathrm{Im}\,G_R^{nn'}(\mathbf{p},\mathbf{p}';\delta\Omega)},$$

where

$$G_R^{nn'}(\mathbf{p},\mathbf{p}';\delta\Omega) = -i\int e^{i\delta\Omega t}\Theta(t)\langle[b_{n'}^+(t)a_{p'}(t), a_p^+(0)b_n(0)]_-\rangle\,dt$$

is the Fourier component of the retarded Green's function at the frequency $\delta\Omega = \Omega - \Omega_0$. In deriving (27) we have carried out a canonical $\hat{U}(t)$-transformation and have used the fact that the transformed G_R in the stationary state depends only on the difference between the temporal arguments. The bar denotes averaging over the impurity positions, the brackets denote quantum-mechanical averaging, the double brackets denote additional averaging over the volume, time, and polarization of the wave, and κ is the refractive index.

According to[8], $G_R(\delta\Omega)$ is expressed in terms of the Fourier transform of the causal Green's function as follows:

$$G_R(\delta\Omega) = \mathrm{Re}\,G_c(\delta\Omega) + is(\delta\Omega)\,\mathrm{Im}\,G_c(\delta\Omega). \qquad (28)$$

Representing G_C in the form of a product of single-particle Green's functions and calculating the Fourier component, we rewrite (27) in the form

$$K(\Omega) = K_0(\Omega)I(\Omega),\ K_0(\Omega) = \frac{4\pi^2|V_{\alpha v}|^2M^2e^2}{3\Omega\varkappa V}\rho_a(\Omega - E_i) \qquad (29)$$

is the absorption coefficient in the absence of scattering, a strong field, and electrons in the band,

$$I(\Omega) = \frac{s(\delta\Omega)}{2\pi^2\rho_a(\Omega - E_i)}\,\mathrm{Re}\sum_{pp'}\overline{\int G_{pp'}^b(\omega - \delta\Omega)G_{p'p}^a(\omega)\,d\omega}, \qquad (30)$$

$I(\Omega)$ takes into account the influence of the strong field, the impurity scattering, and the populations on the absorption. We note that the term F^+F, which arises together with G^bG^a in the two-particle function

$$G_c = -i\langle T\{b_{p'}^+(t)a_{p'}(t),\ a_p^+(0)b_p(0)\}\rangle,$$

does not depend on the time and makes no contribution to the absorption.

We consider first the absorption when the dampings are small and the Green's functions (23) can be used in the calculation. The average of the product in (30) is equal in this case to the product of the averages. Substituting (23) in (30), we obtain after integration

$$I(\Omega) = \frac{2s(\delta\Omega)}{\rho_a(\Omega - E_i)}\sum_p\Theta(\varepsilon_p^\alpha)\Theta(|\delta\Omega| - \varepsilon_p^\alpha)\,[u_p^4\delta(\varepsilon_p^\alpha + \varepsilon_p^\beta - \delta\Omega) + v_p^4\delta(\varepsilon_p^\alpha + \varepsilon_p^\beta + \delta\Omega)]. \qquad (31)$$

(31) coincides in form with the absorption coefficient for interband transitions[13], and in the frequency range $|\delta\Omega| < \min(\epsilon_p^\alpha + \epsilon_p^\beta) = 2|\bar{\lambda}|$ the absorption coefficient is equal to zero ($I = 0$). This agrees fully with the notions concerning the impurity band and the gap in the excitation spectrum[7].

Changing over in (31) from summation over p to integration with respect to ξ_p, we obtain

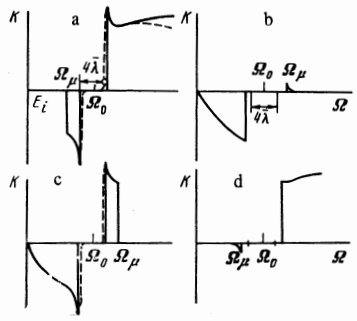

FIG. 2. Spectral dependence of the absorption (amplification) of a weak electromagnetic wave at different positions of the Fermi level: a) $0 < \delta\mu < |\bar{\lambda}|$, b) $\delta\mu < -|\bar{\lambda}|$, c) $-|\bar{\lambda}| < \delta\mu < 0$, d) $\delta\mu > |\bar{\lambda}|$. The dashed curve corresponds to absorption with allowance for the dampings.

$$I(\Omega) = \frac{\Theta(|\delta\Omega| - 2|\bar{\lambda}|)s(\delta\Omega)}{4\rho_a(\Omega - E_i)\,|\delta\Omega|\sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2}}\Big\{\Theta\Big(\frac{|\bar{\lambda}|^2 + (\delta\mu)^2}{|\delta\mu|} - |\delta\Omega|\Big) \times\rho_a(\mu_0 \pm \sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2})\,(\delta\Omega \pm \sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2})^2 + \rho_a(\mu_0 \mp \sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2})\,(\delta\Omega \mp \sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2})^2\Big\} \qquad (32)$$

for $|\delta\mu| \le |\bar{\lambda}|$, and

$$I(\Omega) = \frac{s(\delta\Omega)}{4\rho_a(\Omega - E_i)\,|\delta\Omega|\sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2}}\Theta\Big(|\delta\Omega| - \frac{|\delta\mu|^2 + |\bar{\lambda}|^2}{|\delta\mu|}\Big) \cdot\rho_a(\mu_0 \mp \sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2})\,(\delta\Omega \mp \sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2})^2 \qquad (33)$$

for $|\delta\mu| > |\bar{\lambda}|$.

In (32) and (33) it is necessary to take the upper sign for $\delta\mu < 0$ and the lower one for $\delta\mu > 0$.

We note that, as seen from (32) and (33), a contribution is made to absorption not only by the usual electronic transitions but also by transitions from the band to the impurity with emission of two strong-field quanta and absorption of one weak-field quantum, and from the impurity to the band with absorption of two strong-field quanta and emission of a weak-field quantum. It is precisely because of these transitions that we have absorption at the frequencies $\Omega > \Omega_\mu$ at $\delta\mu < -|\bar{\lambda}|$ when all the impurities are practically free of electrons, and amplification for the frequencies $\Omega < \Omega_\mu$ at $\delta\mu > |\bar{\lambda}|$, when the impurities are occupied by electrons.

In (32), these transitions correspond to the second term at $\delta\Omega > 0$ and to the first at $\delta\Omega < 0$, $\Omega_\mu = \Omega_0 - [|\bar{\lambda}|^2 + (\delta\mu)^2]/\delta\mu$.

The spectral dependence of $K(\Omega)$ is shown in Fig. 2. The gap singularities of the energy spectrum of the excitations are most generally pronounced in absorption at $|\delta\mu| < |\bar{\lambda}|$, which is satisfied in a wide strong-field frequency interval ($\Omega_1 \lesssim \Omega_0 \lesssim \Omega_2$).

Expression (32) becomes infinite as $|\delta\Omega| \to 2|\bar{\lambda}|$. This is the consequence of the singularity and the density of states in the strong field, the same as in interband transitions[13]. Obviously, it is impossible to neglect the damping in this frequency region.

5. ABSORPTION OF WEAK ELECTROMAGNETIC WAVES WITH ALLOWANCE FOR IMPURITY SCATTERING

In this section we shall show that if $W/|\bar{\lambda}| \ll 1$ and $(\mu_0\tau_a)^{-1} \ll 1$ then the impurity scattering does not in-

fluence the absorption, so that the spectral singularities of the absorption coefficient remain also when such scattering is taken into account. We change over in (30) from summation over p to integration with respect to ξ_p. Since the main contribution to the integral is made by a narrow region near $\xi_p = \Omega - E_i - \mu_a$, the density of states $\rho_a(E_p)$, being a smooth function, can be taken outside the integral sign at $E_p = \Omega - E_i$. As a result we obtain

$$I(\Omega) = \frac{S(\delta\Omega)}{2\pi^2} \operatorname{Re} \int \Pi(\xi, \omega)\, d\xi\, d\omega, \quad \Pi(\xi, \omega) = \sum_{p'} \overline{G^b_{pp'}(\omega - \delta\Omega)\, G^a_{p'p}(\omega)}. \tag{34}$$

In the calculation of the double integral in (34) it is impossible to change the order of integration, since the integral converges only conditionally. We get around this known difficulty[8] by reducing the integration with respect to the frequency between infinite limits to integration between finite limits. To this purpose we note that

$$\int_{-\infty}^{\infty} \Pi d\omega = \int_0^{\delta\Omega} (\Pi - \Pi_R)\, d\omega + \int_{-\infty}^{\infty} \Pi_R d\omega + \int_{-\infty}^{0} (\Pi_A - \Pi_R)\, d\omega, \tag{35}$$

where

$$\Pi_R = \sum_{p'} \overline{G_R{}^b(\omega - \delta\Omega)\, G_R{}^a(\omega)}, \quad \Pi_A = \sum_{p'} \overline{G_A{}^b(\omega - \delta\Omega)\, G_A{}^a(\omega)}; \; G_R, G_A$$

are respectively the retarded and advanced Green's functions and are connected with the causal Green's function by the relation (28), and $G_A = G_R^*$. For concreteness we assume that $\delta\Omega > 0$.

Since Π_R is an analytic function in the upper half-plane and behaves like $1/\omega^2$ as $\omega \to \infty$, the second integral in the right-hand side of (35) vanishes. The last integral is pure imaginary, since $\Pi_A = \Pi_B^*$. Consequently

$$I(\Omega) = \frac{1}{2\pi^2} \int_0^{\delta\Omega} d\omega \int_{-\infty}^{\infty} d\xi \operatorname{Re}[\Pi(\xi, \omega) - \Pi_R(\xi, \omega)]. \tag{36}$$

The average of the product of two Green's functions is not equal to the product of the averages, and in averaging it is necessary to sum over a set of diagrams. In accordance with Figs. 3a and 3b, we obtain for $0 \le \omega \le \delta\Omega$:

$$\Pi(\xi, \omega) = G^b(\omega - \delta\Omega)\, G^a(\omega) \left[\frac{\omega + \delta\mu}{\tilde{\omega}} + \Lambda \right] - F^+(\omega - \delta\Omega)\, G^a(\omega)\, \Lambda_1, \tag{37}$$

$$\Pi_1(\xi, \omega) = F(\omega - \delta\Omega)\, G^a(\omega) \frac{\omega + \delta\mu}{\tilde{\omega}} + G^a(\omega - \delta\Omega)\, G^a(\omega)\, \Lambda_2, \tag{38}$$

where

$$\tilde{\omega} = \omega + \delta\mu + iW/2, \quad \tilde{\omega}_- = \omega - \delta\Omega + \delta\mu - iW/2,$$

$$\Lambda = \sum_{p'} \frac{|\lambda|^2 |\bar{\lambda}|^2}{\tilde{\omega}(\omega - \delta\Omega - \xi_{p'} - i/2\tau_a)} \Pi(\xi_{p'}, \omega),$$

$$\Lambda_1 = \sum_{p'k} |u_k(p - p')|^2 \Pi_1(\xi_{p'}, \omega),$$

$$\Lambda_2 = \sum_{p'} \left\{ \frac{|\lambda|^2 |\bar{\lambda}|^2}{\tilde{\omega}\tilde{\omega}_-} + \sum_k |u_k(p - p')|^2 \right\} \Pi_1(\xi_{p'}, \omega).$$

Multiplying (38) by

$$\sum_k |u_k(1 - p)|^2 + |\lambda|^2 |\bar{\lambda}|^2 / \tilde{\omega}\tilde{\omega}_-$$

and summing over p, we obtain Λ_2 and then analogously Λ_1, and from (37) we get Λ:

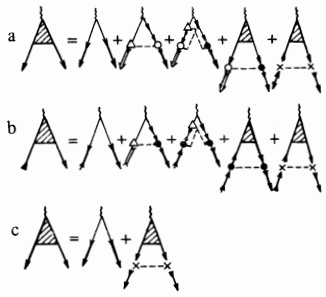

FIG. 3. Structure of the equations for the average product Green's functions.

$$\Lambda_2 = \frac{i(\omega + \delta\mu)\bar{\lambda}}{(\delta\Omega)\chi} \left[\frac{i|\bar{\lambda}|^2 W}{\tilde{\omega}\tilde{\omega}_-} + \frac{i}{\tau_a} \right], \tag{39}$$

$$\Lambda_1 = -\frac{\bar{\lambda}(\omega + \delta\mu)}{(\delta\Omega)\tau_a\chi}, \quad \Lambda = \frac{i|\bar{\lambda}|^2(\omega + \delta\mu)W}{(\delta\Omega)\tilde{\omega}\left[\chi + \frac{i\tilde{\omega}\tilde{\omega}_-}{\tau_a(\delta\Omega)}\right]} \left[1 + \frac{i|\bar{\lambda}|^2 \tilde{\omega}\chi^{-1}}{\tau_a(|\bar{\lambda}|^2/\tilde{\omega} - i/\tau_a - \delta\Omega)\delta\Omega} \right], \tag{40}$$

where $\chi = \tilde{\omega}\tilde{\omega} + |\bar{\lambda}|^2$.

Similar calculation for $\Pi_R(\xi, \omega)$ at $0 \le \omega \le \delta\Omega$ yield

$$\Pi_R(\xi, \omega) = G^{b*}(\omega - \delta\Omega)\, G^a(\omega) \frac{\omega + \delta\mu}{\tilde{\omega}}. \tag{41}$$

After substituting (37) and (41) in (36) and integrating with respect to ξ, we obtain

$$I(\Omega) = -\frac{1}{\pi} \int_0^{\delta\Omega} d\omega \operatorname{Re} \left\{ \frac{i(\omega + \delta\mu)^2}{\chi\delta\Omega} + \frac{i(\omega + \delta\mu)W|\bar{\lambda}|^2}{2(\delta\Omega)^2\tau_a\chi(\chi + i\tilde{\omega}\tilde{\omega}_-/\delta\Omega\tau_a)} \times \left(1 + \frac{|\bar{\lambda}|^2}{\tilde{\omega}_-(|\bar{\lambda}|^2/\tilde{\omega} - i/\tau_a - \delta\Omega)} \right) \right\}. \tag{42}$$

At $W \ll |\bar{\lambda}|$, the integral of the second term in (42) can be neglected, for even at $\bar{\lambda}\tau_a \ll 1$ its value is of the order of $W/\bar{\lambda} \ll 1$ for $\delta\Omega < 2|\bar{\lambda}|$ and $\sim 1/\mu_0\tau_a \ll 1$ for $\delta\Omega > 2|\bar{\lambda}|$. The first term in (42) does not depend on τ_a. Thus, impurity scattering at $(\mu_0\tau_a)^{-1} \ll 1$, $|\bar{\lambda}| \ll \mu_0 2n/3\pi n_0$ does not influence the absorption.

The integral of the first term (42) expressed in terms of elementary functions. We present the result at $\delta\mu = 0$ and $\delta\Omega - 2|\bar{\lambda}| \gg W$:

$$I(\Omega) = \frac{(\delta\Omega)^2 - 2|\bar{\lambda}|^2}{2\sqrt{(\delta\Omega)^2 - 4|\bar{\lambda}|^2}} \left[1 + \frac{2}{\pi} \operatorname{arctg} \frac{2|\bar{\lambda}|^2}{\delta\Omega W} \right]. \tag{43}$$

If $\delta\Omega < |\bar{\lambda}|^2/W$, then (43) coincides with (32). If $\delta\Omega \gg |\bar{\lambda}|^2/W$, then I is equal to 1/2 rather than unity as in (32). This difference is due to the fact that the formation of the impurity band ceases to influence the absorption at frequencies $\delta\Omega > |\bar{\lambda}|^2/W \gg |\bar{\lambda}|$. Indeed, the energy, in the impurity band, of an electron absorbing a weak field with such a frequency turns out to be smaller than the damping $W(\epsilon^\beta \approx |\bar{\lambda}|^2/\delta\Omega < W)$, and the energy of the final state in the conduction band does not depend on $|\bar{\lambda}|(\epsilon^\alpha \simeq \delta\Omega)$. Therefore the absorption is by independent impurities with electron populations equal to 1/2 owing to the transitions of the "impurity—band—the same impurity" type in the strong field.

At the frequency $\delta\Omega = 2|\bar{\lambda}|$ we have $I = \sqrt{|\bar{\lambda}|/8W}$,

i.e., W eliminates the divergence in (32). At $\delta\Omega < 2|\overline{\lambda}|$, the absorption decreases sharply practically to zero on the interval $\sim |\overline{\lambda}|(W/|\overline{\lambda}|)^{2/3}$

The spectral dependence of K, with allowance for the damping, is shown dashed in Figs. 2a and 2c.

We have considered transitions for an electron with one spin projection. If transitions of the electrons with opposite spin projections become equally probable, then it can be shown, taking the electrostatic repulsion of the impurities into consideration, that allowance for the spin leads only to a replacement of $\overline{\lambda}$ in all the excitations by $\overline{\lambda}/\sqrt{2}$.

It is useful to compare the results with absorption in the absence of a strong field. For concreteness, we consider an n-type semiconductor in which, owing to donor impurities, there are degenerate electrons in the conduction band and the acceptors are compensated. The electrons in the bands are described by the Green's function (20), and on the acceptor level by the function

$$G^{b}_{nn'}(\omega) = \delta_{nn'}/[\omega + \mu_a + E_i - i\delta]. \quad (44)$$

In calculating $I(\Omega)$, the average product of the Green's functions can be replaced by the product of the averages, since all the discarded diagrams contain the parameter $e^2/\epsilon v_a \ll 1$. As a result we obtain

$$I(\Omega) = \Theta(\Omega - \mu_a - E_i), \quad (45)$$

i.e., the impurity absorption near the edge has the form of a step and does not depend on the elastic scattering, just as in the presence of a strong wave. Allowance for the discarded diagrams decreases only the magnitude of the step, but does not smear it out.

Such a result becomes understandable if account is taken of the energy conservation law, which is satisfied in elastic collisions with impurities. Impurity absorption of a field of frequency lower than $(\mu a + E_i)$ is impossible, since the final state in the band for transitions with absorption of a field of such a frequency lies below the Fermi level and is occupied.

A new result is obtained in the case of intrinsic absorption. After averaging, in accordance with Fig. 3c, the product of the Green's functions of the valence-band and conduction-band electrons scattered by the impurities, and integrating, we obtain

$$K(\Omega) = \frac{4|V_{av}|^2 e^2}{3\Omega\varkappa}\left(2\frac{m_a m_v}{m_a + m_v}\right)^{3/2}\sqrt{\Omega - E_g}\,\frac{1}{\pi}\left[\frac{\pi}{2} + \operatorname{arctg}\frac{2(\Omega - \Delta)\tau_a m_a m_v}{(m_a + m_v)^2}\right], \quad (46)$$

where E_b is the width of the forbidden band and $\Delta = E_g + \mu_a(1 + m_a/m_v)$.

Expression (46) describes the smearing of the intrinsic absorption edge as a result of impurity scattering. When $\Omega = \Delta$, the absorption is smaller by a factor of two than in a pure semiconductor, and when $\Omega < \Delta$ there is an absorption "tail."

The smearing of the edge of the intrinsic absorption occurs as a result of indirect transitions (without momentum conservation) whose final states lie above the Fermi level. In interband absorption, unlike impurity absorption, such indirect transitions are present also for frequencies for which the direct transition is forbidden.

CONCLUSION

As explained in this paper, impurity–band transitions in the field of a strong electromagnetic wave have a number features that distinguish them from interband transitions. An important distinguishing feature is the fact that the impurity scattering does not disrupt the gap singularities of the absorption spectrum. Therefore the requirements imposed on the intensity of the strong field are much less stringent in the case of impurity–band transitions than for interband transitions.

The result obtained here, and also in[14] on the absorption and spectrum of recombination radiation can be directly used to analyze the conditions of multimode generation in a semiconductor laser, where, according to a number of experimental investigations, the radiative recombination occurs in a band–impurity level transition. Since the gain vanishes in an interval $2|\overline{\lambda}|$ near the frequency Ω_0 of the strong wave (first mode) and increases sharply on the boundary of this interval, the second mode occurs when the intensity of the first is such that the gap $2|\overline{\lambda}|$ reaches a value equal to the distance between the resonator modes[5].

We have disregarded in our analysis the broadening of the impurity level as a result of the possible scatter of the levels of the individual impurity atoms. Appropriate calculations show that when allowance is made for the broadening, the damping W is replaced in all the Green's functions by $\gamma = W + \delta$, if the distribution function of the impurity atoms over the levels is assumed to be[15]

$$f(E) = \frac{\delta/2\pi}{(E - E_i)^2 + \delta^2/4}.$$

Coherent interaction of the impurity electrons with a strong field and the associated singularities of the absorption of the weak wave should become manifest if $\gamma < |\overline{\lambda}|$. Numerical estimates for gallium arsenide at $E_0 = 10^4$ V/cm and at an acceptor concentration $n = 10^{18}$ cm^{-3} yield $|\overline{\lambda}| = 10^{-4}$ eV, and for InSb at the same strong-field intensity and $n = 10^{17}$ cm^{-3} we get $|\overline{\lambda}| \approx 2 \times 10^{-3}$ eV.

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91