Strong Electromagnetic Waves in Semiconductors Under Conditions of Inelastic Scattering of Current Carriers by Optical Phonons

O. N. CHAVCHANIDZE

Institute of Cybernetics, Georgian Academy of Sciences

Submitted July 26, 1971

Zh. Eksp. Teor. Fiz. 62, 797–805 (February, 1972)

The propagation of low frequency electromagnetic waves in semiconductors is investigated under conditions of inelastic scattering of the carriers by optical phonons. It is shown that the singularities in the components of the current, which occur under these conditions, cause the appearance of oscillating reflecting surfaces which are responsible for the existence of higher harmonics in the wave reflected into vacuum. The reflection coefficients of these harmonics are calculated.

It was shown in [1,2] that in semiconductors with strongly inelastic scattering of the carriers by optical phonons, the components of the current in crossed static electric and magnetic fields E and H possess singularities at a certain value of the ratio E/H. The influence of these singularities on the propagation of electromagnetic waves was investigated in [3]. However, only the propagation of circularly polarized waves was studied in [3]. In this case the electric field, which is damped upon its motion into the depths of the sample, at a certain point becomes equal to the critical value E_{cr} at which the singularities in the components of the current appear (it is assumed that the external magnetic field H, determining E_{cr}, is fixed and the electric field on the boundary of the sample is larger than E_{cr}). At this point, owing to the singularities, which consist mainly of interruptions of the dissipative current, the properties of the medium change abruptly, which leads to the appearance of an effective interface and consequently to the reflection of the wave.

The situation is considerably more complicated for the propagation of a wave of non-circular polarization. In this case, at each moment of time, the electric field already becomes equal to E_{cr} at many points of the space occupied by the semiconductor. Therefore a large number of surfaces appear, with $E > E_{cr}$ on one side of each of these surfaces and $E < E_{cr}$ on the other. In other words, the semiconductor breaks up into laminas, the properties of every other lamina are the same and those of neighboring laminas are markedly different.

The important distinctive feature of the propagation of a wave of non-circular polarization is that the coordinates of these interfaces change with time, that is, the reflection takes place from moving walls. It is obvious that this fact must lead to a change in the frequency of the reflected wave; harmonics with frequencies different from the frequency of the wave incident on the semiconductor should be present in the reflected field.

This article deals with the influence of the indicated peculiarities due to non-circular polarization of the wave on its propagation.

Let us consider the half-space $z > 0$, occupied by the semiconductor, with an external constant magnetic field H applied along the z axis. It is assumed that the magnetic field satisfies the inequalities

$$E^- \ll v_0 H / 2c \ll E^+. \qquad (1)$$

Here v_0 denotes the velocity of a carrier (in order to be definite, we shall talk about an electron) having an energy ϵ equal to the energy $\hbar\omega_0$ of an optical phonon, c is the velocity of light, and $E^\pm$ are the characteristic fields determined by the relations $eE^\pm\tau^\pm = mv_0$, where τ^+ and τ^- denote the electron relaxation times in the regions $\epsilon > \hbar\omega_0$ and $\epsilon < \hbar\omega_0$; m is the electron mass.

Inequality (1) means that the critical electric field, which is determined with the aid of the expression $E_{cr} = Hv_0/2c$,[1,2] has a value between the fields E^- and E^+; therefore at $E \approx E_{cr}$ very favorable conditions are created for extremely inelastic scattering of the carriers by optical phonons (for more details see [1,2]).

Now let a plane monochromatic wave of frequency ω be incident on the semiconductor in a direction normal to the interface. The polarization of the wave is assumed to be arbitrary, and the frequency is such that $\omega \ll \tau_E^{-1}$, where τ_E denotes the time required to accelerate the electron from $\epsilon = 0$ to $\epsilon = \hbar\omega_0$. If $E \approx E_{cr}$ then

$$\tau_E = mv_0 / |e|H \approx 2mc / |e|H = 2\omega_c^{-1}$$

(e denotes the electron charge and ω_0 is the cyclotron frequency). Then the low-frequency criterion takes the form

$$\omega \ll \omega_c. \qquad (2)$$

This condition means that during the time of revolution of the electron in the passive region ($\epsilon < \hbar\omega_0$), which coincides in order of magnitude with the time required to establish the needle-shaped distribution,[1] the electric field remains essentially constant. Therefore one can use the static conductivity tensor derived in [1,2].

The basic equations of the problem are Maxwell's equations:

$$\frac{\partial^2 E_{x,y}}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 E_{x,y}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial j_{x,y}}{\partial t}. \qquad (3)$$

Here $n = \sqrt{\epsilon_L}$ denotes the index of refraction of the medium, ϵ_L is the dielectric constant of the lattice, and j is the conduction current, which after simple transformations can be written in the following form:

$$\mathbf{j} = \alpha\frac{n^2\omega}{4\pi}\left(\frac{j_\parallel}{j_0}\frac{E_{cr}}{E}\mathbf{E} + \frac{e}{|e|}\frac{j_\perp}{j_0 E}E_{cr}[\mathbf{E}\mathbf{e}_z]\right). \qquad (4)^*$$

*$[\mathbf{E}\mathbf{e}_z] \equiv \mathbf{E} \times \mathbf{e}_z$.

Here $\alpha = \omega_p^2/\omega\omega_c$; $\omega_p = \sqrt{4\pi e^2 N/m\epsilon_L}$ is the plasma frequency; N is the concentration; e_z is the unit vector along the z axis; $j_0 = \frac{1}{2}|e|Nv_0$; $j_{\parallel}$ is the dissipative current and $j_{\perp}$ is the Hall current. The values of $j_{\parallel}$ and $j_{\perp}$ as functions of E were obtained in [1] without taking interelectron collisions into account, and their values with allowance for these collisions were obtained in [2]. These functions depend weakly on the rate of interelectron collisions, and in both cases they can be well approximated by the expressions

$$\frac{j_{\parallel}}{j_0} = \Theta(E - E_{cr}), \quad \frac{j_{\perp}}{j_0}\frac{E_{cr}}{E} = 1 - \Theta(E - E_{cr}). \tag{5}$$

Here $\Theta(x)$ denotes the Heaviside function, which is equal to unity for $x > 0$ and equal to zero for $x < 0$.

Taking (4) and (5) into account, it is not difficult to see that the nonlinear equations (3) contain on the right-hand sides a sum of delta functions (that depend on the unknown electric field), which are equivalent to sources of electromagnetic waves. It is therefore convenient to change from differential to integral equations.

After applying the usual procedure (see, for example, [4]) for the construction of the solution of Eqs. (3) with the aid of the appropriate Green's function, one can easily obtain integral equations replacing the system (3):

$$E_{x,y}(z,t) = -\frac{1}{c^2}\int_{-\infty}^{t} dt_0 \int_0^{\infty} dz_0 \frac{\partial j_{x,y}(z_0,t_0)}{\partial t_0} G(z,t|z_0,t_0) + \frac{1}{4\pi}\int_{-\infty} dt_0 E_{x,y}(0,t_0) \left.\frac{\partial G(z,t|z_0,t_0)}{\partial z_0}\right|_{z_0=0}. \tag{6}$$

Here $G(z, t|z_0, t_0)$ is the Green's function of the scalar wave equation for semi-infinite ($z > 0$) space, satisfying the Dirichlet conditions. It has the form

$$G(z,t|z_0,t_0) = 2\frac{c}{n}\pi\left[\Theta\left(n\frac{|z+z_0|}{c} - (t-t_0)\right) - \Theta\left(n\frac{|z-z_0|}{c} - (t-t_0)\right)\right]; \tag{7}$$

$E_{x,y}(0, t_0)$ is the value of the electric field for $z = 0$.

The following relations were used in the derivation of Eq. (6):

$$G(z,t|z_0,t) = \left.\frac{\partial G(z,t|z_0,t_0)}{\partial t_0}\right|_{t_0=t} = 0, \tag{8a}$$

$$G(z,t|z_0,t_0)|_{t_0\to-\infty} = \left.\frac{\partial G(z,t|z_0,t_0)}{\partial t_0}\right|_{t_0\to-\infty} = 0, \tag{8b}$$

$$\left.\frac{\partial G(z,t|z_0,t_0)}{\partial z_0}\right|_{z_0\to\infty} = 0. \tag{8c}$$

The validity of Eqs. (8) follows directly from the explicit form of the Green's function (7). One can also easily understand their physical meaning. Equation (8a) means that the effect of the source acting at the moment of observation, is equal to zero at any arbitrary point far away from the source. Equation (8b) means that the effect of the source acting at the moment $t_0 = -\infty$ dies down to zero at the present moment of time. Equation (8c) means that the effect of the source acting at an infinitely distant point dies down to zero for finite values of z.

Having integrated the first term of (6) by parts with respect to t_0, using the explicit form of the Green's function (7), and also using Eq. (8), we obtain instead of (6)

$$E_{x,y}(z,t) = E_{x,y}\left(0, t - \frac{n}{c}z\right) + \frac{2\pi}{nc}\int_0^{\infty} dz_0 \left[j_{x,y}\left(z_0, t - \frac{n}{c}|z+z_0|\right) - j_{x,y}\left(z_0, t - \frac{n}{c}|z-z_0|\right)\right]. \tag{9}$$

In what follows we shall assume that the electromagnetic wave is weakly attenuated. This occurs if the dissipative current due to the wave is considerably smaller than the displacement current. With Eqs. (4) and (5) taken into account, it is not difficult to find that the latter condition is equivalent to the inequality

$$\alpha \ll 1. \tag{10}$$

In accordance with (10) we represent the field in vacuum ($z < 0$) in the form

$$\begin{aligned} E_x(z,t) &= E_{x_0}[\cos(\omega t - kz) + R_x \cos(\omega t + kz)] + \alpha F_x(z,t), \\ E_y(z,t) &= E_{y0}[\sin(\omega t - kz) + R_y \sin(\omega t + kz)] + \alpha F_y(z,t). \end{aligned} \tag{11}$$

Here E_{x0} and E_{y0} are the amplitudes of the components of the electric field of the incident wave, R_x and R_y denote the reflection coefficients of the fundamental harmonic (of frequency ω) in the zero-order approximation with respect to α, $\alpha F_{x,y}(z, t)$ are the desired small corrections to the reflected wave, and $k = \omega/c$.

The usual continuity conditions of electrodynamics are satisfied across the vacuum-semiconductor interface ($z = 0$):

$$E_{x,y}(-0) = E_{x,y}(+0); \quad \partial E_{x,y}(-0)/\partial z = \partial E_{x,y}(+0)/\partial z. \tag{12}$$

Therefore, in the right hand side of Eq. (9), instead of the first term one can substitute expression (11) for $z \to 0$ and $t \to t - nz/c$. If in addition we utilize the second equation in (12) to the zero-order approximation in α, then we can determine R_x and R_y ($R_x = R_y = (1 - n)/(1 + n)$). Then (9) takes the form

$$\begin{aligned} E_x(z,t) &= E_{\text{кр}}(1+b)\cos s_+ + \alpha F_x\left(0, t - \frac{n}{c}z\right) + \frac{2\pi}{nc}\int_0^{\infty} dz_0 \left[j_x\left(z_0, t - \frac{n}{c}|z+z_0|\right) - j_x\left(z_0, t - \frac{n}{c}|z-z_0|\right)\right] \\ \times E_y(z,t) &= E_{\text{кр}}(1+d)\sin s_+ + \alpha F_y\left(0, t - \frac{n}{c}z\right) + \frac{2\pi}{nc}\int_0^{\infty} dz_0 \left[j_y\left(z_0, t - \frac{n}{c}|z+z_0|\right) - j_y\left(z_0, t - \frac{n}{c}|z-z_0|\right)\right]. \end{aligned} \tag{13}$$

The following notation has been used in obtaining expression (13):

$$E_{x0}(1+R_x) = E_{cr}(1+b), \quad E_{y0}(1+R_y) = E_{cr}(1+d), \quad s_+ = \omega t - knz. \tag{14}$$

The parameters b and d determine the degree of "supercriticality" of the electric field of the wave. In this connection three cases are possible: 1) b, d > 0; in this case the field at the surface z = 0 is always larger than E_{cr}; 2) b > 0 and d < 0 (or conversely); in this case the maximum value of the field at z = 0 is larger than E_{cr} but the minimum value is smaller than E_{cr}; 3) b, d < 0; in this case the electric field at z = 0, and consequently also for z > 0, is always smaller than E_{cr}. In the latter case the singularities in the current, which occur when $E = E_{cr}$, do not appear. Therefore we confine our attention to an investigation of the first two cases.

By using the smallness of the parameter α, we shall solve the system of integral equations (13) by the method of successive approximations. The zero-order solutions in α correspond to the first terms in Eq. (13). In order to obtain expressions for the electric field with greater accuracy, it is necessary to substitute (4) (with Eq. (5) taken into account) with $E_{x,y}$ corresponding to the zero-order solutions in α, into the integrals appearing in (13). After the indicated substitution and simple calculations we obtain

$$E_x = E_{cr}(1+b)\cos s_+\left[1-\frac{\alpha knz}{2}\frac{\Theta(B(s_+))}{A(s_+)}\right] - \frac{e}{|e|}\frac{\alpha knz}{2}E_{cr}(1+d)\sin s_+[1-\Theta(B(s_+))] + \alpha F_x\left(0, t-\frac{n}{c}z\right) \quad (15)$$

$$E_y = E_{cr}(1+d)\sin s_+\left\{1-\frac{\alpha knz}{2}\frac{\Theta[B(s_+)]}{A(s_+)}\right\} + \frac{e}{|e|}\frac{\alpha knz}{2}E_{cr}(1+b)\cos s_+[1-\Theta(B(s_+))] + \alpha F_y\left(0, t-\frac{n}{c}z\right).$$

Here the following notation has been introduced:

$$A(\varphi) = [(1+b)^2\cos^2\varphi + (1+d)^2\sin^2\varphi]^{1/2},$$
$$B(\varphi)A(\varphi) = (2b+b^2)\cos^2\varphi + (2d+d^2)\sin^2\varphi. \quad (16)$$

Thr first terms in (15) describe the propagation of the fundamental harmonic with damping taken into account. As follows from (15) the damping mechanism is involved then when $B(s_+)$ becomes positive, which corresponds to the inequality $E > E_{cr}$, where E is to be understood as the solution of the zero-order approximation in α. The second terms in (15) describe the change of phase of the fundamental harmonic, and this change occurs only when the damping mechanism is switched off.

Formulas (15) give qualitatively correctly the behavior of the fundamental harmonic as it propagates into the interior of the semiconductor. However, in order to obtain qualitatively correct results with accuracy α (both for the fundamental as well as for the additional harmonics), it is necessary to make in Eqs. (13) one more step of the successive approximations. This is associated with the fact that the Θ function in the integrand of (13) (see Eqs. (4) and (5)) is very sensitive to a small change of its argument. Therefore, in order to obtain $E_{x,y}$ correct to order α, it is also necessary that the argument of the Θ-function be determined to the same accuracy. The necessity of the next step of the successive approximations can also be understood from a physical point of view. In fact, in order to obtain the wave which is reflected from the oscillating walls (consistent with the qualitative picture discussed during the formulation of the problem), it is necessary to use in the integrals in (13) the fundamental harmonic with its attenuation and change of phase taken into account, since it is precisely these factors which cause the appearance of the walls. The latter statement becomes especially clear for b, d > 0. In this case, if the attenuation of the fundamental harmonic is not taken into account in the integrals in (13) (which is equivalent to making only the first step of the successive approximations), the field will always be larger than E_{cr} and, consequently, singularities in the current will not have any influence on the propagation of the wave.

Let us substitute expressions (4) and (5) into (13), first having replaced $E_{x,y}$ in them with the aid of expressions (15). After integration and simple transformations we obtain

$$] E_x = E_{cr}(1+b)\cos s_+\left[1-\frac{L(z,t)}{2A(s_+)}\right] - \frac{e}{|e|}\frac{1}{2}(1+d)E_{cr}\sin s_+ \times[\alpha knz - L(z,t)] + \frac{\alpha}{4}E_{cr}\int_{-\infty}^{s_+} d\varphi\,\Phi(\varphi)\left[(1+b)\frac{\cos\varphi}{A(\varphi)} - \frac{e}{|e|}(1+d)\sin\varphi\right] + \alpha F_x\left(0, t-\frac{n}{c}z\right),$$

$$E_y = E_{cr}(1+d)\sin s_+\left[1-\frac{L(z,t)}{2A(s_+)}\right] + \frac{e}{|e|}\frac{1}{2}E_{cr}(1+b)\cos s_+[\alpha knz - L(z,t)] + \frac{\alpha}{4}E_{cr}\int_{-\infty}^{s_+} d\varphi\,\Phi(\varphi)\left[(1+d)\frac{\sin\varphi}{A(\varphi)} + \frac{e}{|e|}(1+b)\cos\varphi\right] + \alpha F_y\left(0, t-\frac{n}{c}z\right). \quad (17)$$

Here the following notation has been introduced:

$$L(z,t) = \Theta(s_+)[\alpha knz + B_1(z,t)],$$
$$B_1(z,t) = [B(s_+) - \alpha knz]\Theta[\alpha knz - B(s_+)],$$
$$\Phi(\varphi) = \{1-\Theta[B(\varphi) - y_2(\varphi)]\}\Theta[B(\varphi) - y_1(\varphi)], \quad (18)$$
$$y_1(\varphi) = -{}^1\!/_2\alpha(\varphi - s_+), \quad y_2(\varphi) = -{}^1\!/_2\alpha(\varphi - s_-),$$
$$s_- = \omega t + knz.$$

In deriving (17) we used the fact that $b \gg \alpha$ (for the sake of definiteness it is assumed that b > d). This inequality means that the damping is sufficiently weak so that the wave is attenuated to E_{cr} over a distance of the order of several wavelengths. Therefore the locations of the reflecting walls (in other words, the values of z at which the argument of the Θ-function in (5) vanishes) are determined by the behavior of the fundamental harmonic, but not by the behavior of the additional harmonics whose intensity is proportional to α.

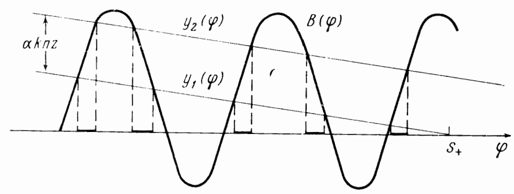
The first two terms in (17), just like the first two terms in (15), describe the propagation of the fundamental harmonic with damping and the change of its phase taken into account. However in (17) these processes are taken into consideration with greater precision (correct to order α) than in (15). The third terms appearing on the right-hand sides of expressions (17) correspond to waves due to the existence of oscillating surfaces, separating the region containing the dissipative current from the region without this current. Let us carry out an analysis of the last term in more detail.

It is not difficult to show that the integrand differs from zero only for values of φ satisfying the following inequalities:

$$y_1(\varphi) < B(\varphi) < y_2(\varphi). \quad (19)$$

The regions of φ satisfying the inequalities (19) are quite clear on the figure (they are shown by the heavy lines), on which all three functions appearing in (19) are plotted, namely the periodic function $B(\varphi)$ having a period equal to π, and the linear functions $y_1(\varphi)$ and $y_2(\varphi)$.

The boundary points of these regions, that is, the abscissas of the points of intersection of $B(\varphi)$ with $y_1(\varphi)$ or with $y_2(\varphi)$ are, respectively, the functions s_+ or s_-.



Graphical determination of the region of integration of the integrals in (17).

Therefore the integrals appearing in (17) represent the sum of integrals whose limits of integration depend on s_+ or s_-. The limiting points, which are functions of s_-, cause the appearance in (17) of terms which depend on $\omega t + knz$ and consequently are responsible for the existence of waves traveling toward the negative direction of the z axis. It is not difficult to verify that with increasing values of z the intensity of such waves decreases. In fact, with increasing values of z the distance between the straight lines $y_1(\varphi)$ and $y_2(\varphi)$ increases (see the figure), which leads to a decrease in the number of boundary points depending on s_- and consequently leads to a decrease in the intensity of the reverse (with respect to the positive direction of the z axis) waves. It is natural to associate each point of intersection of the curves $B(\varphi)$ and $y_2(\varphi)$ with the existence of a reflecting surface which appears as a result of the cutoff of the dissipative current at $E = E_{cr}$. Therefore for a fixed value of z the number of boundary points, depending on s_-, determines the number of reflecting surfaces which appear to the right of the given value of z. Upon an increase of z, an instant sets in when $y_2(\varphi)$ starts to intersect $B(\varphi)$ past the point $\varphi = \omega t - knz$, which is the upper limit of integration in (17). For such values of z the field will not contain waves traveling in the negative direction of the z axis, because all boundary points will depend only on s_+. The disappearance of the reflected waves corresponds to a region inside the semiconductor, wherein the field is already smaller than E_{cr}, owing to the attenuation.

From an experimental point of view, the determination of the functions $F_{x,y}$ is of fundamental value; according to their definition (see Eqs. (11)) these functions correspond to the waves which are reflected into vacuum and which appear in the first-order approximation in α. It is convenient to represent these functions in the following form:

$$F_{x,y}(z,t) = E_{cr}\sum_{m=0}^{\infty}\tilde{R}_{x,ym}\cos m(\omega t + kz) + E_{cr}\sum_{m=1}^{\infty}R_{x,ym}\sin m(\omega t + kz). \quad (20)$$

Here $\tilde{R}_{xm}$, R_{xm}, $\tilde{R}_{ym}$, and R_{ym} denote the coefficients of reflection, subject to determination with the aid of the boundary conditions (12). We recall that the first conditions (12) were taken into account during the transition from differential equations to integral equations. It remains to utilize the equality of the derivatives with respect to z (for z = 0) of expressions (11) (with (21) taken into consideration) and of expressions (17).

Before utilizing the boundary conditions (12), let us show that for values of z close to z = 0 (such that $\alpha knz \ll b$) the term $B_1(z, t)$ in the first formula of (18) can be neglected in comparison with αknz. In fact, by taking the second formula of (18) into consideration it is not difficult to see that the ratio of the time interval, during the course of which the expression $\Theta[B(s_+)]B_1(z, t)$ does not vanish, to the time interval when $\Theta[B(s_+)]\,\alpha knz$ does not vanish will be of the order of $\alpha knz/b$ for small values of z. Therefore, "on the average" (with respect to time), the inequality

$$B_1(z,t) \approx \alpha knz \cdot \alpha knz/b \ll \alpha knz \quad (21)$$

holds, which permits us to neglect the correction $B_1(z, t)$ near the wall z = 0, where this correction appears in (17) after applying the second step of the successive approximations. This does not mean, however, that in (17) one can also neglect the second term, which is also a consequence of the second stage of successive approximations. The integrand of this term is of the order of unity (where it does not vanish). Therefore the integral term in (17) is of the order of $\alpha E_{cr}/4$, which is multiplied in the first place by the length of an individual segment of the region of integration (see the figure), which is of the order of $\alpha knz\pi/b$, and in the second place it is multiplied by the number of such segments, which is $\approx 2b/\alpha\pi$. Therefore this term $\approx \alpha E_{cr}knz$ and in order of magnitude it agrees with the other terms appearing in (17).

By expanding all of the functions appearing in (17) near z = 0 in a Fourier series in the time, and by applying the boundary conditions (12) for the derivatives, we obtain equations for the reflection coefficients. The solutions of these equations become manageable only under certain additional assumptions. We cite the results in that particular case when the polarization of the wave is close to circular:

$$\gamma = \frac{2(b-d)}{\alpha}\left[1 + \frac{1}{(1+b)^2}\right] \ll 1. \quad (22)$$

Confining our attention to the first approximation in the parameter γ, we obtain the result that the coefficients of reflection of all the harmonics vanish except for the harmonics with frequencies ω and 3ω, for which we have

$$R_{y1} = \tilde{R}_{x1}, \quad \tilde{R}_{y1} = -R_{x1}, \quad R_{y3} = \tilde{R}_{x3}, \quad \tilde{R}_{y3} = -R_{x3}, \quad (23)$$

where

$$\begin{aligned}
\tilde{R}_{x1} &= \frac{n}{2(1+n)}\left[\sin s_0 + \frac{e}{|e|}(1+b)\cos s_0\right],\\
R_{x1} &= \frac{n}{2(1+n)}\left[-1 + \cos s_0 + \frac{e}{|e|}(1+b)\sin s_0\right],\\
\tilde{R}_{x3} &= -\frac{\gamma}{4}\frac{n}{2(1+n)}\left[\cos 3s_0 + \frac{e}{|e|}(1+b)\sin 3s_0\right],\\
R_{x3} &= \frac{\gamma}{4}\frac{n}{2(1+n)}\left[-\sin 3s_0 + \frac{e}{|e|}(1+b)\cos 3s_0\right].
\end{aligned} \quad (24)$$

Here

$$s_0 = \frac{2b}{\alpha}\frac{2+b}{1+b}. \quad (25)$$

From formulas (24) with (25) taken into account it is seen that the coefficients of reflection are rapidly oscillating functions of the parameter of supercriticality, the frequency of the incident wave, and the magnitude of the magnetic field (we recall that $\alpha = \omega_p^2/\omega\omega_H$). The smallness of the period of oscillation is a consequence of the condition $b \gg \alpha$.

Let us discuss the experimental conditions under which it is possible to observe the effects investigated in this article. As the investigated sample let us take p-Ge (where the inelasticity of the scattering is substantial[5]) with the parameters $m = 0.3m_0$ (m_0 denotes the mass of the free electron), $\hbar\omega_0 = 6 \times 10^{-14}$ erg, $\epsilon_L = 16$, and $\tau^+ = 10^{-12}$ sec. Then for $E_{cr} = 0.2E^+$ we obtain

$$\omega_c = \frac{eH}{mc} \approx \frac{2eE_{cr}}{mv_0} \approx \frac{2cE_{cr}}{eE^+\tau^+} = 4 \cdot 10^{11}\,\mathrm{sec}^{-1}.$$

Therefore condition (2) will be satisfied if the frequency of the incident wave is of the order of $\omega = 8 \times 10^{10}\,\mathrm{sec}^{-1}$, which corresponds to a length $\lambda = 6$ mm of the waves in the sample. The value of the magnetic field, at which the electric field $0.2E^+ = 650$ V/cm becomes critical, is of the order of 6.4 kOe. The condition that the parameter α be small will be satisfied if the hole concentration $N \ll 2.5 \times 10^{14}\,\mathrm{cm}^{-3}$. Finally, let us show that for $\gamma = 1/3$ the coefficient of reflection for the harmonic of frequency 3ω is of the order of $1/30$.

The author expresses his gratitude to F. G. Bass, I. M. Fuks, and Yu. G. Gurevich for helpful discussions.

[1] I. I. Vosilius and I. B. Levinson, Zh. Eksp. Teor. Fiz. **50**, 1660 (1966) [Sov. Phys. JETP **23**, 1104 (1966)].

[2] F. G. Bass, I. B. Levinson, and O. N. Chavchanidze, Zh. Eksp. Teor. Fiz. **52**, 1263 (1967) [Sov. Phys. JETP **25**, 839 (1967)].

[3] F. G. Bass, Yu. G. Gurevich, I. B. Levinson, and A. Yu. Matulis, Zh. Eksp. Teor. Fiz. **55**, 999 (1968) [Sov. Phys. JETP **28**, 518 (1969)].

[4] Philip M. Morse and Herman Feshbach, Methods of Theoretical Physics, Vol. 1, McGraw-Hill, 1953 [Russ. Transl., IIL, 1958].

[5] William E. Pinson and Ralph Bray, Phys. Rev. A (1964–1965) **136**, 1449 (1964).

Translated by H. H., Nickle
94