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Cooling of Electrons in Semiconductors by a Static Electric Field

YU. G. GUREVICH AND S. I. SHEVCHENKO

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

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The possibility of cooling carriers in inhomogeneous semiconductors by applying a static electric field is investigated. The temperature distribution associated with the impurity distribution in a sample is obtained. It is shown that carrier cooling leads to descending segments of the average (for a sample) electron temperature as a function of the current. A descending segment can start at an arbitrarily low current.

ONE of the authors has shown[1] that under certain conditions charge carriers in inhomogeneous semiconductors can be cooled by a static electric field instead of being heated, and that the average energy of the carriers then drops below the lattice temperature T. This effect can be explained essentially as follows. Let the impurity concentration N in a semiconductor depend on a coordinate (x, for example). If the Debye radius is the smallest parameter having the dimension of length in the problem, the carrier concentration at each point will coincide with the impurity concentration[1)] (see [2,3], for example), thus establishing an electron concentration gradient (dN/dx)i (where i is the unit vector in the x direction). It is well known that an associated electrostatic field is then directed along the gradient. An external electric field applied to the sample induces a current in the same direction as this field.[4] If the current direction is counter to the direction of the concentration gradient and the external field is weaker than the field associated with dN/dx, the carriers will move in a (total) field that is oriented counter to their motion. The electric field then obviously removes energy from the carriers, which are thus "cooled." The energy removed from the carriers must be liberated outside the sample, in the current source, for example. The described effect is somewhat analogous to the Peltier effect, although the former is of spatial character.

In [1] carrier cooling was investigated without taking into account the dependence of carrier temperature on the coordinates. However, it is easily seen that the electric field E in a sample, and thus the carrier temperature, must be a function of x. The presence of a carrier temperature gradient can, generally speaking, bring about essential changes in the physical picture: thermal currents arise, the Thomson effect occurs, heat flux appears in the electronic subsystem etc.

In the present work carrier cooling in semiconductors is investigated with the aforementioned factors taken into account. We consider a nondegenerate (electronic) semiconductor in the shape of a parallepiped of length L in the x direction, while its dimensions in the transverse y and z directions considerably exceed the mean free electron path l_e associated with energy transfer.[3] The electron dispersion law is assumed to be isotropic and quadratic, and collisions between carriers and scattering centers are assumed to be quasielastic (with the momentum-transfer mean free carrier path $l_p \ll l_e$[5]).

The concentration of impurities, and therefore that of the carriers, depends on x and is given, for simplicity, by $N = N_0 e^{-x/l}$. The magnitude of N_0 is such that the frequency ν_{ee} of interelectronic collisions greatly exceeds the frequency ν_e of collisions between electrons and scattering centers accompanied by energy transfer. The electric current flows along the x axis with J ∥ i.

Under the foregoing assumptions our problem may be considered one-dimensional (i.e., all quantities will depend only on the x coordinate), and the symmetric part of the electron distribution is Maxwellian with the effective temperature Θ:[6] [2)]

$$f_0(\varepsilon) = N(2\pi m\Theta)^{-3/2}e^{-\varepsilon/\Theta} \tag{1}$$

where ε is the energy and m is the mass of the carriers.

The electron temperature as a function of the coordinates is determined from the equation of heat balance, which can be obtained from the kinetic equation (see [8,9], for example)

$$\frac{dQ_x}{dx} = \mathbf{JE} - NT\nu_e(u)(u-1). \tag{2}$$

Here

$$Q_x = \frac{N}{N_0}\varkappa u^{1+q}\left[\frac{eE_x}{T} - u\frac{1}{N}\frac{dN}{dx} - (2+q)\frac{du}{dx}\right] \tag{3}$$

is the x-component of heat flux Q, and E_x is the electric field strength in the sample (E has only an x-component);

$$J_x = \frac{N}{N_0}\sigma u^q\left[E_x - \frac{Tu}{e}\frac{1}{N}\frac{dN}{dx} - (1+q)\frac{T}{e}\frac{du}{dx}\right] \tag{4}$$

is the x-component of the current and u = Θ/T is the dimensionless electron temperature;

$$\varkappa = \frac{4\Gamma(7/2+q)N_0T^2}{3\sqrt{\pi}m\nu_0} \text{ and } \sigma = \frac{4\Gamma(5/2+q)}{3\sqrt{\pi}}\frac{N_0e^2}{m\nu_0}$$

are the electronic heat and the electric conductivity at u = 1, $\nu = \nu_0(T)(\epsilon/T)^{-q}$ is the frequency of collisions, with momentum transfer,[7] between electrons and scattering centers; $\nu_e(u) = \nu_{0e}(T)u^{r-1}$;[9] r and q are numerical coefficients of the order of unity (for which explicit

[1)]That is, of course, if we neglect processes such as impact ionization, variation of the recombination coefficient in the field, etc.

[2)]If the inequality $\nu_{ee} \gg \nu_e$ is not satisfied, the result obtained by the effective temperature method differs by a numerical coefficient of the order of unity from the rigorous result of the kinetic analysis.[7].

expressions are given in [3]), whose values depend on the scattering mechanism with respect to the carrier energy and momentum, respectively.

Because under stationary conditions the current J_x does not depend on the coordinate, it is convenient to express the field E_x in terms of J_x. Using (4), we obtain

$$E_x = \frac{J_x}{\sigma}\frac{N_0}{N}u^{-q} + \frac{Tu}{e}\frac{1}{N}\frac{dN}{dx} + (1+q)\frac{T}{e}\frac{du}{dx}. \tag{5}$$

Substituting E_x and Q_x into (2), we finally obtain the following equation for the electron temperature u:

$$\frac{d^2u}{dx^2} + (1+q)u^{-1}\left(\frac{du}{dx}\right)^2 - \left[\frac{1}{l} + \frac{3}{2}e^{x/l}\frac{TJ_x}{e\varkappa}u^{-(1+q)}\right]\frac{du}{dx}$$
$$= -\frac{TJ_x}{e\varkappa}e^{x/l}u^{-q}\left[-\frac{1}{l} + e^{x/l}\frac{TJ_x}{e\varkappa}\left(\frac{5}{2}+q\right)u^{-(1+q)}\right] + \frac{u^{r-q-2}(u-1)}{l^2}. \tag{6}$$

In deriving (6) it was assumed that $N = N_0e^{-x/l}$ and

$$\varkappa / N_0T\nu_e = l_e^2 \quad [9].$$

Equation (6) must be supplemented by the boundary conditions for temperature in the planes x = 0 and x = L, which describe the transfer of energy from electrons to scattering centers at the boundaries of the sample. As is shown in [3], these conditions are

$$\left.\frac{du}{dx}\right|_{x=0,L} = \pm\eta(u-1)|_{x=0,L}. \tag{7}$$

The parameter $\eta > 0$ is determined by the strength of the surface mechanisms.

Equation (6) shows that the two parameters l and l_e, having the dimension of length, are involved in the electron cooling problem. The case $l_e \gg l$ is of no special interest: First, for $\eta \neq 0$ and $L \lesssim l_e$, because of the boundary conditions the electrons are thermalized in the entire volume, i.e., u becomes of the order of unity; [8] secondly, in all semiconductors $l_e \sim 10^{-3}$–10^{-4} cm, and therefore the situation $l_e \gg l$ is unlikely to occur in experiments. Therefore we shall henceforth assume $l \gtrsim l_e$ and $L \sim l$.

The condition that the total electric field in the sample be directed counter to the current reduces, as is easily seen from [5], to the requirement

$$\frac{1}{l} > \frac{TJ_x}{e\varkappa}u^{-(1+q)}e^{L/l}. \tag{8}$$

It is obvious that (6) cannot be solved exactly; we shall therefore perform a linear analysis. It will be assumed that the electric current and the concentration gradient are consistent with

$$u = 1 - \vartheta, \quad \vartheta \ll 1. \tag{9}$$

Linearizing (6) with respect to ϑ and neglecting terms of the order ϑ/l_Θ^2 (where l_Θ is the characteristic length for electron temperature change), we obtain

$$d^2\vartheta/dx^2 - \left[\frac{1}{l} + \frac{3}{2}\frac{\delta}{l_e}e^{x/l}\right]\frac{d\vartheta}{dx} - \frac{\vartheta}{l_e^2}$$
$$= \frac{\delta}{l_e^2}e^{x/l}\left[-\frac{1}{l} + \frac{\delta}{l_e}\left(\frac{5}{2}+q\right)e^{x/l}\right]. \tag{10}$$

We have here introduced the notation $TJ_x/e\kappa = \delta/l_e$, with δ for the dimensionless current.

It will also be assumed henceforth that (8) becomes the extreme inequality

$$\delta \ll e^{-L/l}l_e/l. \tag{11}$$

In conjunction with (11) and the linearized boundary conditions (7) in the form

$$\left.\frac{d\vartheta}{dx}\right|_{x=0,L} = \pm\eta\vartheta|_{x=0,L}, \tag{12}$$

we easily solve (10) for all values of η. If, for simplicity, we assume $\eta = 0$ (in which case the effect should be maximal), we have

$$u = 1 - \delta\frac{l_e}{l}\left\{\frac{\lambda_2(e^{\lambda_2L} - e^{L/l})e^{\lambda_1x} - \lambda_1(e^{\lambda_1L} - e^{L/l})e^{\lambda_2x}}{l\lambda_1\lambda_2(e^{\lambda_1L} - e^{\lambda_2L})} + e^{x/l}\right\}, \tag{13}$$

where

$$\lambda_{1,2} = \frac{1}{2l} \pm \left(\frac{1}{4l^2} + \frac{1}{l_e^2}\right)^{1/2}.$$

Thus with $\delta \ll l_e/l \lesssim 1$ electron cooling occurs.

When the inequality $l \gg l_e$ is satisfied, (11) becomes unnecessary. It is sufficient to require

$$\delta \lesssim l_e/l \ll 1. \tag{14}$$

Then in the regions $x \gg l_e$ and $l - x \gg l_e$ we have

$$u = 1 - \delta\frac{l_e}{l}e^{x/l}\left[1 - \delta\frac{l}{l_e}\left(\frac{5}{2}+q\right)e^{x/l}\right]. \tag{15}$$

It follows from (15) that for any fixed value of δ there exists a part of the sample,

$$l_e \ll x \ll l\ln\frac{l_e}{l}\frac{1}{\delta(5/2+q)},$$

where $u < 1$. This region becomes larger as $\delta l/l_e$ decreases.

Equations (14) and (15) show that the condition $\vartheta \ll 1$ is satisfied automatically. This indicates that even when (14) is fulfilled (6) has a solution with $u < 1$, although the cooling effect is small (in accordance with the smallness of $\delta l_e/l$). We note that the function u(x) in (15) corresponds to the solution of (10), where derivatives of u with respect to x, and thus the contribution of the temperature gradient to cooling, are neglected. Indeed, as has been shown in [9], if the foreign inhomogeneity greatly outweighs the energy-transfer mean free path l_e, the temperature and the inhomogeneity are related locally. Thus the result obtained in [1] is valid when (14) is satisfied, giving small cooling.[3)] If $l \sim l_e$, taking the specific value $l_e = 2l/\sqrt{3}$, which greatly simplifies the result, we have

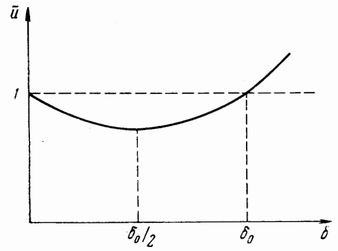
$$u = 1 - \frac{2}{\sqrt{3}}\delta[e^{x/l} - 1], \tag{16}$$

and, because of $\delta \ll l_e/l \sim p$, the condition $\vartheta \ll 1$ is satisfied.

From an analysis of (10) for $l \sim l_e$ it follows that appreciable cooling can occur if δ is sufficiently close to (but smaller than) the value $l_e/l \sim 1$. However, in this case all terms of (6) are of the same order of magnitude and an analytic form of u(x, δ) cannot be derived. Then in the absence of small parameters a solution $u \ll 1$ cannot exist. Therefore, if $l \sim l_e$ and the cooling condition (8) is fulfilled we have a temperature $u \lesssim 1$.

We shall consider how the electron temperature averaged over the length of the sample, $\bar{u}$, depends on the dimensionless electric current δ. This can be done

[3)]We note here that an incorrect volt-ampere characteristic was obtained in[1] under the conditions of strong electron cooling and neglect of ∇u.



more conveniently for the case $l_e \ll l$, because (15) is valid in a considerably broader range of δ than is the case for (13) and (16). It follows from (15) that

$$\bar{u} \equiv \frac{1}{L}\int_0^L u(x)\,dx$$

is represented by

$$\bar{u} = 1 - [e^{L/l} - 1]\frac{l_e}{L}\delta + [e^{2L/l} - 1]\left(\frac{5}{2} + q\right)\frac{l}{2L}\delta^2. \qquad (17)$$

We thus observe that $\bar{u} < 1$ for $0 < \delta < \delta_0$ and $\bar{u} > 1$ for $\delta > \delta_0$, where

$$\delta_0 = \frac{2l_e}{l}\left(\frac{5}{2} + q\right)\frac{e^{L/l} - 1}{e^{2L/l} - 1}.$$

At $\delta = \delta_0/2$ the temperature $\bar{u}$ reaches its minimum (see the figure). In the current range $0 < \delta < \delta_0/2$ the temperature $\bar{u}$ falls as the current increases; this must lead to instabilities of a different kind in the regime of the given field. It is of interest that the mechanism explained here is not the only mechanism that can produce a decline of temperature as the current increases. However, while for all other mechanisms, so far as we know, this decline begins when the current exceeds a certain critical value, in the present case we observe $d\bar{u}/d\delta < 0$ at an arbitrarily low current.

As the ratio l_e/l increases the minimum of $\bar{u}(\delta)$ is shifted to higher values of δ; $\bar{u}_{min}$ can be of the order of unity when $l_e/l \sim 1$.[4)] In this case the existence of a descending segment in the graph of $\bar{u}$ as a function of δ can lead to segments characterized by negative differential conductivity in the volt-ampere characteristic (VAC). If the electron temperature $\bar{u}$ depends on the current δ uniquely, it follows from (5) that the VAC with a descending segment can be only S-shaped. If $\bar{u}$ is not uniquely dependent on δ, the VAC can have both S-shaped and N-shaped segments. An explicit form of the volt-ampere characteristic can be obtained only by solving (6) numerically.

[4)]To avoid misunderstanding, we emphasize that the figure pertains only to cases of small currents δ. For large currents u can be a nonunique function of δ, but will necessarily possess a minimum.[1]

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